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ORIGINAL RESEARCH

Multiple-attribute decision making problems based on SVTNH methods

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Abstract
The neutrosophic set (NS) is a leading tool in modeling of situations involving incomplete, indeterminate and inconsistent information. The single-valued neutrosophic sets (SVNs) is more useful tool than neutrosophic sets in some applications of engineering and scientific problems. In this paper, we study Hamacher operations and operations between single-valued trapezoidal neutrosophic numbers. Then we propose the single-valued trapezoidal neutrosophic Hamacher weighted arithmetic averaging (SVTNHWA) operator, single-valued trapezoidal neutrosophic Hamacher ordered weighted arithmetic averaging (SVTNHOWA) operator, single-valued trapezoidal neutrosophic Hamacher hybrid weighted averaging (SVTNHHWA) operator, single-valued trapezoidal neutrosophic Hamacher weighted geometric averaging (SVTNHWGA) operator and single-valued trapezoidal neutrosophic Hamacher ordered weighted geometric averaging (SVTNHOWGA) operator and single-valued trapezoidal neutrosophic Hamacher hybrid weighted geometric averaging (SVTNHHWGA) operator, and obtain some of their properties. Furthermore, we developed a multiple-attribute decision-making method in single-valued trapezoidal neutrosophic (SVTN) environment based on these operators. Finally, we proposed an application of MADM problem in assessment of potential of software system commercialization.

Keywords Single-valued trapezoidal neutrosophic number · Hamacher operation · Arithmetic averaging operator · Geometric averaging operator · MADM method

1 Introduction

Multi-attribute decision-making (MADM) problem under different uncertain environments is an interesting research tool having received more and more attention by the researchers in the recent years (Gao et al. 2018; Lu et al. 2019; Wu et al. 2019; Tang and Wei 2019; Garg and Kumar 2018; Zhang et al. 2019; Jana and Pal 2019a, b; Jana et al. 2019b, c). The main aim of this technique is to choose the best alternative among the finite set of alternatives as claimed by the decision makers under the preference values of the alternatives. It has been extensively applied with quantitative or qualitative attribute values and has a board application in management model (Teixeira et al. 2018), economic analysis (Xu 1987), operation research (Xu 1988), analytic management (Levy et al. 2016), etc. As our modern society move forward with the decision-making process, so it always faces imprecise, vague and uncertain facts to take a decision in solving decision-making problems.

Neutrosophic set (NS) a tremendous branch of philosophy was proposed by Smarandache (1999, 2005). This proposed approach is characterized by three functions called (truth-, indeterminacy-, falsity)-membership functions, which is the extended form of the fuzzy sets (FS) defined by Zadeh (1965), and generalization of intuitionistic fuzzy (IFS) (Atanassov 1986). Even though FS and IFS are very powerful set to model decision problems containing uncertainties, in some cases these sets are not sufficient to overcome indeterminate and inconsistent information experience in real world problems. Therefore, NS has strong acceptance to develop models carrying indeterminate and inconsistent data. However, since codomain of membership functions of NS is real standard or nonstandard subsets of $]^{-}0, 1^{+}[$, in some applications areas engineering and real scientific

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fields they have some difficulties in modeling of problems. To overcome difficulties in these areas, Wang et al. (2010) defined the view of single-valued neutrosophic set (SVNs). The hypothesis of trapezoidal neutrosophic sets (TNs) and some of their operational rules such as score and accuracy functions were defined by Ye (2015). He introduced arithmetic and geometric weighted averaging operator and based on these operators, and also developed a methodology for a MADM problems. Deli and Subaş (2015) studied single-valued triangular neutrosophic numbers (SVTrNNs), which can be regarded as particular cases of (SVTNNs). Biswas et al. (2014) utilized expected interval of trapezoidal fuzzy numbers to study similarity measure in approach of decision making. To model a transportation problem Thamaraiselvi and Santhi (2016) made use of SVTrNNs. Liu and Wang (2014a) introduced a new methodology using normalized weighted bonferroni mean in neutrosophic environment for MCGDM problems. Liang et al. (2017) followed single-valued trapezoidal preference relations as a game plan for tackling MCDM problems, and proposed two operators named as SVTNWWAA operator and SVTNWWGA operator. They also gave a decision-making method based on SVTNPRs to discourse green supplier selection problems. The aggregation operators in information retrieval are important research areas. In 1988, the ordered OWA operator and studied some of their properties introduced by Yager (1988). Thereafter, the idea of OWA operator can be implemented to IFS and IVIFS environment, and developed MCDM, for more knowledge on other operators and terminology, the readers are referred to Beliakov et al. (2007), Hu and Wong (2013), He et al. (2013), Ji et al. (2018a), Li and Wang (2017), Liu (2013), Liu and Liu (2014), Liu and Wang (2014b), Liu et al. (2014a), Liu and Yu (2014), Wang and Liu (2011), Xia et al. (2012), Xu (2007), Xu and Yager (2006), Yu (2012, 2013a), Zhao et al. (2010), Gupta and Kohli (2016), and Garg and Kumar (2018).

Ye (2014c) proposed some novel weighted aggregation operators under simplified neutrosophic environment. Liu et al. (2014b) introduced some weighted Hamacher aggregation operators on generalized form neutrosophic numbers and investigated some properties of these operators. Peng et al. (2016) followed some aggregation functions based on the basis of new operational rules defined in Ye (2014c). Ye (2014d) focused to study on some arithmetic and geometric weighted aggregation operators on the basic of operational rules of interval neutrosophic linguistic numbers (INLNs) and investigated important properties of them. Broumi and Smarandache (2014) followed MADM methodology to make a decision by aggregating information related to neutrosophic trapezoidal linguistic arguments. Ji et al. (2018b) focused Frank operations of SVNNs, and constructed the SVN prioritized Bonferroni mean (SVNFNPBM) operator under Frank aggregation function. Zhang et al. (2016) introduced normal cloud method on neutrosophic set and other related conviction such as backward cloud generator, two aggregated operators, and an NNC distance measurement, and using these ideas to construct MADM approach under SVN environments. Nancy (2016) defined operations of SVNNs based on Frank norm operations, and they proposed a decision-making method after they define weighted aggregation operators. In Deschrijver et al. (2004); Deschrijver and Kerre (2002), proposed some aggregation operators based on algebraic operation of IFSs, which is a particular issue of t-norm (TN) and t-conorm (TCN). Wei et al. (2018) developed a MADM method based on bipolar fuzzy arithmetic and geometric weighted Hamacher aggregation operators and looked related properties of them. Gao et al. (2018) utilized Hamacher prioritized aggregation operators in the input arguments of dual hesitant bipolar fuzzy environment. Zhao and Wei (2013) applied hybrid operator using Einstein operations in multiple attribute decision-making method. Zhang (2017) introduced Frank aggregation operators for IVIFNs and develop a MAGDM problem. Yu (2013b) proposed Choquet aggregation operator on the basis of Einstein operational rules under IFNs. Jana et al. (2018) have utilized Dombi aggregation operator in bipolar fuzzy environment and then applying them to develop a MADM problems. Further, Jana et al. (2019a) utilized Dombi aggregation operator in MADM problems technique using picture fuzzy information. Liu (2016) applied some new operational rules for SVNNs based on Archimedean sum and product, and investigated some special properties of them. In Liu et al. (2016), constructed neutrosophic Bonferroni weighted geometric mean operator based on multi-valued functions. Ye (2016) take into account the expected values of neutrosophic linguistic numbers (NLN), and developed NLNWAA, NLNWGA operators using arithmetic and geometric average functions, and investigate their properties. Fan et al. (2017) constructed normalized weighted Bonferroni mean (LNNNWBM) operator and normalized weighted geometric Bonferroni mean (LNNNWGBM) operator under neutrosophic linguistic environment, and developed MAGDM problems using these operators. Lu and Ye (2017) proposed hybrid weighted arithmetic and geometric aggregation functions under SVN information and utilized these operators develop decision-making problems. Tan et al. (2017) introduced three generalized SVN linguistic operators which are followed as GSVNLWA, GSVNLOWA and GSVNLHA operator. Wu et al. (2018) defined the technique of SVN 2-tuple linguistic element and its operational rules. They also developed some SVN2TL weighted arithmetic and geometric Hamacher aggregation operators under SVN2TL environment. Furthermore, they developed an MAGDM method based on these new operations. SVTNNs have important role to model some real life problems including indeterminant and inconsistent data. In this paper, we propose some types of Hamacher arithmetic and geometric aggregating operators called (SVTNH) Hamacher weighted averaging (SVTNHWAA) operator, SVTNH ordered weighted arithmetic averaging (SVTNHOWAA) operator, SVTNH hybrid weighted arithmetic averaging (SVTNHHWAA) operator, SVTNH weighted geometric averaging (SVTNHWGA) operator, SVTNH ordered weighted geometric averaging (SVTNHOWGA) operator and SVTNH hybrid weighted geometric averaging (SVTNHHWGA) operator. We also investigate some of their properties and we give a multi attributive decision making method based on the new operators for SVTNNs. Finally, we present an approach of MADM technique for the selection of software systems of technology commercialization.

The rest of the article is organized as follows. In Sect. 2, some hypothesis and operations on the following environments IFNs, ITFNs and SVTNNs are depicted. In Sect. 3, Hamacher operations of SVTNNs are defined. In Sect. 4, some kinds of SVTNH arithmetic aggregating (SVTNHWAA) operators are introduced and some of their properties are discussed. In Sect. 5, some kinds of SVTNH geometric averaging (SVTNHWGA) operators are introduced and some of results are investigated. In Sect. 6, a MADM method are developed based on these aggregating operators defined in this paper. In Sect. 7, an application of developed MADM method is given. In Sect. 8, conclusions of the paper and studies that can be made in future are presented.

2 Preliminaries

In this section, we present briefly some concepts and operations related to intuitionistic fuzzy numbers (IFN), intuitionistic trapezoidal fuzzy numbers (ITFN) and single valued trapezoidal neutrosophic numbers.

2.1 Some concept of IFNs and ITFNs

Definition 1 (Wang and Zhang 2009) A intuitionistic trapezoidal fuzzy number $\hat{P}$ is an IF set on R (set of real numbers) which its membership functions is defined as follows:

$$\mu_{\hat{P}}(x) = \begin{cases} \hat{F}_P, & \text{if } \hat{e}_1 \le x < \hat{e}_2 \\ \mu_P, & \text{if } \hat{e}_2 \le x \le \hat{e}_3 \\ \hat{G}_P, & \text{if } \hat{e}_3 \le x < \hat{e}_4 \\ 0, & otherwise \end{cases} \tag{1}$$

and its non-membership function is defined as follows:

$$\nu_{\hat{P}}(x) = \begin{cases} \hat{H}_P, & \text{if } \hat{f}_1 \le x < \hat{f}_2 \\ \nu_P, & \text{if } \hat{f}_2 \le x \le \hat{f}_3 \\ \hat{K}_P, & \text{if } \hat{f}_3 \le x < \hat{f}_4 \\ 0, & otherwise \end{cases} \tag{2}$$

where $\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4, \hat{f}_1, \hat{f}_2, \hat{f}_3, \hat{f}_4 \in R$ and $\mu_{\hat{P}}, \nu_{\hat{P}} \subseteq [0,1]$ such that $0 \le \mu_{\hat{P}} + \nu_{\hat{P}} \le 1$.

The functions $\hat{F}_{\hat{P}}, \hat{G}_{\hat{P}}, \hat{H}_{\hat{P}}, \hat{K}_{\hat{P}} : R \to [0,1]$. Here $\hat{F}_{\hat{P}} : [\hat{e}_1, \hat{e}_2] \to [0,1]$, $\hat{K}_{\hat{P}} : [\hat{f}_3, \hat{f}_4] \to [0,1]$ are continuous increasing function and $\hat{G}_{\hat{P}} : [\hat{e}_3, \hat{e}_4] \to [0,1]$, $\hat{H}_{\hat{P}} : [\hat{f}_1, \hat{f}_2] \to [0,1]$ are continuous decreasing function. When continuous increasing and decreasing functions are linear, then ITFNs is preferred in practice.

Definition 2 (Wang and Zhang 2009) Let $\hat{P} = (\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4)$ be a ITFN. Then, membership value of $\hat{P}$ is defined by

$$\mu_{\tilde{P}} = \begin{cases} \frac{x-p_1}{\hat{e}_2-\hat{e}_1}\mu_{\hat{P}}, & \text{if } \hat{e}_1 \le x < \hat{e}_2 \\ \mu_{\hat{P}}, & \text{if } \hat{e}_2 \le x \le \hat{e}_3 \\ \frac{\hat{e}_4-x}{\hat{e}_3-\hat{e}_4}\mu_{\hat{P}}, & \text{if } \hat{e}_3 < x \le \hat{e}_4 \\ 0, & otherwise \end{cases} \tag{3}$$

and its non-membership value of $\hat{P}$ is defined as follows:

$$\nu_{\hat{P}} = \begin{cases} \frac{\hat{f}_2-x+\nu_{\hat{P}}(x-\hat{f}_1)}{\hat{f}_2-\hat{f}_1}, & \text{if } \hat{f}_1 \le x < \hat{f}_2 \\ \nu_{\hat{P}}, & \text{if } \hat{f}_2 \le x \le \hat{f}_3 \\ \frac{x-\hat{f}_3+\nu_{\hat{P}}(\hat{f}_4-x)}{\hat{f}_4-\hat{f}_3}\mu_{\hat{P}}, & \text{if } \hat{f}_3 < x \le \hat{f}_4 \\ 0, & otherwise. \end{cases} \tag{4}$$

where $\mu_{\hat{P}}, \nu_{\hat{P}} \subset [0,1]$, $0 \le \mu_{\hat{P}} + \nu_{\hat{P}} \le 1$ and $\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4, \hat{f}_1, \hat{f}_2, \hat{f}_3, \hat{f}_4 \in R$. If $[\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4] = [\hat{f}_1, \hat{f}_2, \hat{f}_3, \hat{f}_4]$ in an ITFNs $\hat{P}$, then ITFNs $\hat{P}$ is presented as $\hat{P} = \langle(\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4); \mu_{\hat{P}}, \nu_{\hat{P}}\rangle$.

Definition of NS is given in (Smarandache 1999) as follows:

Definition 3 (Smarandache 1999) Let X be finite, with a generic element in X denoted by x. A NS $\tilde{c}$ in X is defined by

$$\tilde{c} = \left\{\langle \hat{T}_c(x), \hat{I}_c(x), \hat{F}_c(x)\rangle | x \in X\right\},$$

where its truth-function $\hat{T}_c$ is presented by $\hat{T}_c : X \to]0^-, 1^+[$, indeterminacy-function $\hat{I}_c$ presented $\hat{I}_c : X \to]0^-, 1^+[$, and falsity- function $\hat{F}_c$ interpreted as $\hat{F}_c : X \to]0^-, 1^+[$. Also, $\hat{T}_c$, $\hat{I}_c$ and $\hat{F}_c$ are real standard or non-standard subsets of $]0^-, 1^+[$. There is no restriction on the sum of $\hat{T}_c$, $\hat{I}_c$ and $\hat{F}_c$, and so $0^- \le \hat{T}_c + \hat{I}_c + \hat{F}_c \le 3^+$.

For real applications of NS, Wang et al. (2010) introduced SVNs in the following definition.

Definition 4 (Wang et al. 2010) Let X be a finite set, with a generic element in X denoted by x. A SVNS is defined as:

$$\tilde{c} = \left\{ \langle \hat{T}_c(x), \hat{I}_c(x), \hat{F}_c(x) \rangle | x \in X \right\},$$

where $\hat{T}_c : X \to [0,1]$ indicated the truth, $\hat{I}_c : X \to [0,1]$ is the indeterminacy and $\hat{F}_c : X \to [0,1]$ is the falsity function of x to c with the condition $0 \le \hat{T}_c + \hat{I}_c + \hat{F}_c \le 3$.

The operational rules for SVNSs are given in Liu and Wang (2014a), Wang et al. (2010) and Ye (2014b).

Let $\tilde{c}_1 = (\hat{T}_{c_1}, \hat{I}_{c_1}, \hat{F}_{c_1})$ and $\tilde{c}_2 = (\hat{T}_{c_2}, \hat{I}_{c_2}, N_{c_2})$ be two SVNSs.

1. $\tilde{c}_1 \bigoplus \tilde{c}_2 = \left(\hat{T}_{c_1} + \hat{T}_{c_2} - \hat{T}_{c_1}\hat{T}_{c_2}, \hat{I}_{c_1}\hat{I}_{c_2}, \hat{F}_{c_1}\hat{F}_{c_2} \right)$
2. $\tilde{c}_1 \bigotimes c_2 = \left(\hat{T}_{c_1}\hat{T}_{c_2}(x), \hat{I}_{c_1} + \hat{I}_{c_2} - \hat{I}_{c_1}\hat{I}_{c_2}, \hat{F}_{c_1} + \hat{F}_{c_2} - \hat{F}_{c_1}\hat{F}_{c_2} \right)$
3. $\tilde{c}_1 \subseteq \tilde{c}_2$ if and only if following conditions are hold: $\hat{T}_{c_1} \le \hat{T}_{c_2}$; $\hat{I}_{c_1} \ge \hat{I}_{c_2}$; $\hat{F}_{c_1} \ge \hat{F}_{c_2}$
4. $\overline{c}_1$ is defined as follows: $\hat{T}_{\overline{c}_1} = \hat{F}_{c_1}$; $\hat{I}_{\overline{c}_1} = 1 - \hat{I}_{c_1}$; $\hat{F}_{\overline{c}_1} = \hat{T}_{c_1}$
5. $(\tilde{c}_1 \cap \tilde{c}_2) = \left(\min\{\hat{T}_{c_1}, \hat{T}_{c_2}\}; \max\{\hat{I}_{c_1}, \hat{I}_{c_2}\}; \max\{\hat{F}_{c_1}, \hat{F}_{c_2}\} \right)$
6. $(\tilde{c}_1 \cup \tilde{c}_2) = \left(\max\{\hat{T}_{c_1}, \hat{T}_{c_2}\}; \min\{\hat{I}_{c_1}, \hat{I}_{c_2}\}; \min\{\hat{F}_{c_1}, \hat{F}_{c_2}\} \right)$.

Definition 5 (Ye 2017) Let $\tilde{c} = (\hat{T}_c, \hat{I}_c, \hat{F}_c)$ be a SVNN in $\mathbb{R}$ (set of real numbers). Then, its (truth-, indeterminacy-, falsity)-membership functions are respectively defined as follows:

$$T_{\tilde{c}}(x) = \begin{cases} \hat{T}_c^L & \text{if } \hat{e} \le x < \hat{f} \\ \hat{T}_c & \text{if } \hat{f} \le x \le \hat{g} \\ \hat{T}_c^U & \text{if } \hat{g} < x \le \hat{h} \\ 0 & otherwise, \end{cases}$$

$$I_{\tilde{N}}(x) = \begin{cases} \hat{I}_c^L & \text{if } \hat{e} \le x < \hat{f} \\ \hat{I}_c & \text{if } \hat{f} \le x \le \hat{g} \\ \hat{I}_c^U & \text{if } \hat{g} < x \le \hat{h} \\ 0 & otherwise, \end{cases}$$

and

$$F_{\tilde{N}}(x) = \begin{cases} \hat{F}_c^L & \text{if } \hat{e} \le x < \hat{f} \\ \hat{F}_c & \text{if } \hat{f} \le x \le \hat{g} \\ \hat{F}_c^U & \text{if } \hat{g} < x \le \hat{h} \\ 0 & otherwise \end{cases},$$

respectively.

Definition 6 (Ye 2017) A single-valued trapezoidal neutrosophic number (SVTNN) is denoted by $\tilde{c} = \{(\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4), (\hat{f}_1, \hat{f}_2, \hat{f}_3, \hat{f}_4), (\hat{g}_1, \hat{g}_2, \hat{g}_3, \hat{g}_4)\}$ in a universe of discourse X, where parameters satisfy the relations $\hat{e}_1 \le \hat{e}_2 \le \hat{e}_3 \le \hat{e}_4$, $\hat{f}_1 \le \hat{f}_2 \le \hat{f}_3 \le \hat{f}_4$ and $\hat{g}_1 \le \hat{g}_2 \le \hat{g}_3 \le \hat{g}_4$. Then truth-$T_{\tilde{c}}$, indeterminacy $I_{\tilde{c}}$ and falsity memberships $F_{\tilde{c}}$ are respectively defined as follows:

$$T_{\tilde{c}}(x) = \begin{cases} \frac{x-\hat{e}_1}{\hat{e}_2-\hat{e}_1}\hat{T}_c & \text{if } \hat{e}_1 \le x < \hat{e}_2 \\ \hat{T}_c & \text{if } \hat{e}_2 \le x \le \hat{e}_3 \\ \frac{\hat{e}_4-x}{\hat{e}_4-\hat{e}_3}\hat{T}_c & \text{if } \hat{e}_3 < x \le \hat{e}_4 \\ 0 & otherwise, \end{cases} \tag{5}$$

$$I_{\tilde{c}}(x) = \begin{cases} \frac{\hat{f}_2-x+\hat{I}_c(x-\hat{f}_1)}{\hat{f}_2-\hat{f}_1} & \text{if } \hat{f}_1 \le x < \hat{f}_2 \\ \hat{I}_c & \text{if } \hat{f}_2 \le x \le \hat{f}_3 \\ \frac{x-\hat{f}_3+\hat{I}_c(\hat{f}_4-x)}{\hat{f}_4-\hat{f}_3} & \text{if } \hat{f}_3 \le x \le \hat{f}_4 \\ 0 & otherwise, \end{cases} \tag{6}$$

$$F_{\tilde{c}}(x) = \begin{cases} \frac{\hat{g}_2-x+\hat{F}_c(x-\hat{g}_1)}{\hat{g}_2-\hat{g}_1} & \text{if } \hat{g}_1 \le x < \hat{g}_2 \\ \hat{F}_c & \text{if } \hat{g}_2 \le x \le \hat{g}_3 \\ \frac{x-\hat{g}_3+\hat{F}_c(\hat{g}_4-x)}{\hat{g}_4-\hat{g}_3} & \text{if } \hat{g}_3 < x \le \hat{g}_4 \\ 0 & otherwise. \end{cases} \tag{7}$$

where $T_c, I_c, F_c \in [0,1]$ with $0 \le T_{\tilde{c}} + I_{\tilde{c}} + F_{\tilde{c}} \le 3$ and $\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4, \hat{f}_1, \hat{f}_2, \hat{f}_3, \hat{f}_4, \hat{g}_1, \hat{g}_2, \hat{g}_3, \hat{g}_4 \in R$. Then, $\tilde{N} = \langle ([\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4]; \hat{T}_c), ([\hat{f}_1, \hat{f}_2, \hat{f}_3, \hat{f}_4]; \hat{I}_c), ([\hat{g}_1, \hat{g}_2, \hat{g}_3, \hat{g}_4]; \hat{F}_c) \rangle$ is called SVTNNs. If $[\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4] = [\hat{f}_1, \hat{f}_2, \hat{f}_3, \hat{f}_4] = [\hat{g}_1, \hat{g}_2, \hat{g}_3, \hat{g}_4]$ in a SVTNNs $\tilde{N}$, then $\tilde{c}$ in SVTNNs can be denoted as $\tilde{c} = \langle ((\hat{e}_1, \hat{e}_2, \hat{e}_3, \hat{e}_4); \hat{T}_c, \hat{I}_c, \hat{F}_c) \rangle$.

If $\hat{e}_2 = \hat{e}_3$ in a SVTNN $\tilde{c}$, then SVTNN is reduces to SVTrNNs which is a special case of SVTNN $\tilde{c}$.

Definition 7 (Ye 2017) Let $\tilde{c}_1 = \langle (\hat{e}_1, \hat{f}_1, \hat{g}_1, \hat{h}_1); \hat{T}_{c_1}, \hat{T}_{c_1}, \hat{F}_{c_1} \rangle$ and $\tilde{c}_2 = \langle (\hat{e}_2, \hat{f}_2, \hat{g}_2, \hat{h}_2); \hat{T}_{c_2}, \hat{I}_{c_2}, \hat{F}_{c_2} \rangle$ be two SVTNNs and $\lambda > 0$. Then operational rules for $\tilde{c}_1$ and $\tilde{c}_2$ are defined as follows:

1. $\tilde{c}_1 \bigoplus \tilde{c}_2 = \langle (\hat{e}_1 + \hat{e}_2, \hat{f}_1 + \hat{f}_2, \hat{g}_1 + \hat{g}_2, \hat{h}_1 + \hat{h}_2); \hat{T}_{c_1} + \hat{T}_{c_2} - \hat{T}_{c_1}\hat{T}_{c_2}, \hat{I}_{c_1}\hat{I}_{c_2}, \hat{F}_{c_1}\hat{F}_{c_2} \rangle$

2. $\tilde{c}_1 \bigotimes \tilde{c}_2 = \langle(\hat{e}_1\hat{e}_2, \hat{f}_1\hat{f}_2, \hat{g}_1\hat{g}_2, \hat{h}_1\hat{h}_2); \hat{T}_{c_1}\hat{T}_{c_2}, \hat{I}_{c_1} + \hat{I}_{c_2} - \hat{I}_{c_1}\hat{I}_{c_2}, \hat{F}_{c_1} + \hat{F}_{c_2} - \hat{F}_{c_1}\hat{F}_{c_2}\rangle$
3. $\lambda\tilde{c}_1 = \langle(\lambda\hat{e}_1, \lambda\hat{f}_1, \lambda\hat{g}_1, \lambda\hat{h}_1); 1-(1-\hat{T}_{c_1})^\lambda, I^\lambda_{c_1}, F^\lambda_{c_1}\rangle$
4. $\tilde{c}_1^\lambda = \langle(\hat{e}_1^\lambda, \hat{f}_1^\lambda, \hat{g}_1^\lambda, \hat{h}_1^\lambda); T^\lambda_{c_1}, 1-(1-\hat{I}_{c_1})^\lambda, 1-(1-\hat{F}_{c_1})^\lambda\rangle$.

In Deli and Subaş (2014), introduced score and accuracy functions of SVTNNs.

Definition 8 (Deli and Subaş 2014) Let $\tilde{N} = \langle(\hat{e}, \hat{f}, \hat{g}, \hat{h}); \hat{T}_c, \hat{I}_c, \hat{F}_c\rangle$ be a SVTNN. Then score and accuracy function of $\tilde{c}$ are defined as follows:

$$\emptyset(\tilde{c}) = \frac{1}{16}\left[\hat{e} + \hat{f} + \hat{g} + \hat{h}\right] \times \left(2 + \hat{T}_c - \hat{I}_c - \hat{F}_c\right) \; \emptyset(\tilde{c}) \in [0,1] \tag{8}$$

$$\varphi(\tilde{N}) = \frac{1}{16}\left[\hat{e} + \hat{f} + \hat{g} + \hat{h}\right] \times \left(2 + \hat{T}_c - \hat{I}_c + \hat{F}_c\right), \varphi(\tilde{N}) \in [0,1] \tag{9}$$

respectively.

Based on the above functions, considering prioritized analysis between any two SVTNNs $\tilde{c}_1$ and $\tilde{c}_2$ is defined in Deli and Subaş (2014) as follows:

Let $\tilde{c}_1$ and $\tilde{c}_2$ be any two SVTNNs.

(i) If $\emptyset(\tilde{c}_1) < \emptyset(\tilde{c}_2)$, imply $\tilde{c}_1 \prec \tilde{c}_2$
(ii) If $\emptyset(\tilde{c}_1) > \emptyset(\tilde{c}_2)$, imply $\tilde{c}_1 \succ \tilde{c}_2$
(iii) If $\emptyset(\tilde{c}_1) = \emptyset(\tilde{c}_2)$, then

1. If $\varphi(\tilde{c}_1) < \varphi(\tilde{c}_2)$, imply $\tilde{c}_1 \prec \tilde{c}_2$.
2. If $\varphi(\tilde{c}_1) > \varphi(\tilde{c}_2)$, imply $\tilde{c}_1 \prec \tilde{c}_2$.
3. If $\varphi(\tilde{c}_1) = \varphi(\tilde{c}_2)$, imply $\tilde{c}_1 \sim \tilde{c}_2$.

3 Hamacher operations of single-valued trapezoidal neutrosophic sets

3.1 Hamacher operations

In FS theory, TN and TCN are the robust aid to present general union and intersection of FS (Deschrijver et al. 2004; Roychowdhury and Wang 1998). The generalized union and intersection of TN and TCN on IFS were provided by Deschrijver and Kerre (2002). Hamachar (1978) introduced Hamacher operations known as Hamacher (Ham) product ($\bigotimes$) and Hamacher (Ham) sum ($\bigoplus$), which are example of TN and TCN, respectively. Hamacher TN and TCN are provided in the following definition.

$$Ham(\zeta, \eta) = \zeta \bigotimes \eta = \frac{\zeta\eta}{\wp + (1-\wp)(\zeta + \eta - \zeta\eta)} \tag{10}$$

$$Ham*(\zeta, \eta) = \zeta \bigoplus \eta = \frac{\zeta + \eta - \zeta\eta - (1-\wp)\zeta\eta}{1-(1-\wp)\zeta\eta}. \tag{11}$$

Usually, when $\wp = 1$, then Hamacher TN and TCN reduce to the following forms:

$$Ham(\zeta, \eta) = \zeta \bigotimes \eta = \zeta\eta \tag{12}$$

$$Ham^*(\zeta, \eta) = \zeta \bigoplus \eta = \zeta + \eta - \zeta\eta. \tag{13}$$

are called algebraic TN and algebraic TCN, respectively.

When $\wp = 2$, then Hamacher TN and TCN reduces to the following forms:

$$Ham(\zeta, \eta) = \zeta \bigotimes \eta = \frac{\zeta\eta}{1 + (1-\zeta)(1-\eta)} \tag{14}$$

$$Ham^*(\zeta, \eta) = \zeta \bigoplus \eta = \frac{\zeta + \eta}{1 + \zeta\eta} \tag{15}$$

are known Einstein TN and Einstein TCN, respectively.

3.2 Hamacher operations of SVTNNs

To this part, we introduce the notion of Ham operations on SVTNNs and prove some properties of this operations. Let $\tilde{c}_1$ and $\tilde{c}_2$ be SVTNNs and $\lambda > 0$, then Ham product and Ham sum of $\tilde{c}_1 = \langle(\hat{e}_1, \hat{f}_1, \hat{g}_1, \hat{h}_1); \hat{T}_{c_1}, \hat{I}_{c_1}, \hat{F}_{c_1}\rangle$ and $\tilde{c}_2 = \langle(\hat{e}_2, \hat{f}_2, \hat{g}_2, \hat{h}_2); \hat{T}_{c_2}, \hat{I}_{c_2}, \hat{F}_{c_2}\rangle$ defined are as follows:

1. $$\tilde{c}_1 \bigoplus \tilde{c}_2 = \left\langle \left(\hat{e}_1 + \hat{e}_2, \hat{f}_1 + \hat{f}_2, \hat{g}_1 + \hat{g}_2, \hat{h}_1 + \hat{h}_2\right); \frac{T_{c_1} + T_{c_2} - T_{c_1}T_{c_2} - (1-\wp)T_{c_1}T_{c_2}}{1-(1-\wp)T_{c_1}T_{c_2}}, \frac{I_{c_1}I_{c_2}}{\wp + (1-\wp)(I_{c_1} + I_{c_2} - I_{c_1}I_{c_2})}, \frac{F_{c_1}F_{c_2}}{\wp + (1-\wp)(F_{c_1} + F_{c_2} - F_{c_1}F_{c_2})} \right\rangle$$

2. $$\tilde{c}_1 \bigotimes \tilde{c}_2 = \left\langle \left(\hat{e}_1\hat{e}_2, \hat{f}_1\hat{f}_2, \hat{g}_1\hat{g}_2, \hat{h}_1\hat{h}_2\right); \frac{T_{c_1}T_{c_2}}{\wp + (1-\wp)(T_{c_1} + T_{c_2} - T_{c_1}T_{c_2})}, \frac{I_{c_1} + I_{c_2} - I_{c_1}I_{c_2} - (1-\wp)I_{c_1}I_{c_2}}{1-(1-\wp)I_{c_1}I_{c_2}}, \frac{F_{c_1} + F_{c_2} - F_{c_1}F_{c_2} - (1-\wp)F_{c_1}F_{c_2}}{1-(1-\wp)F_{c_1}F_{c_2}} \right\rangle$$

3. $$\lambda\tilde{c}_1 = \Big\langle \big(\lambda\hat{e}_1, \lambda\hat{f}_1, \lambda\hat{g}_1, \lambda\hat{h}_1\big); \frac{(1+(\wp-1)T_{c_1})^\lambda - (1-T_{c_1})^\lambda}{(1+(\wp-1)T_{c_1})^\lambda + (\wp-1)(1-T_{c_1})^\lambda}, \frac{\wp(I_{c_1})^\lambda}{(1+(\wp-1)(1-I_{c_1}))^\lambda + (\wp-1)(I_{c_1})^\lambda}, \frac{\wp(F_{c_1})^\lambda}{(1+(\wp-1)(1-F_{c_1}))^\lambda + (\wp-1)(F_{c_1})^\lambda} \Big\rangle, \quad \lambda > 0$$

4. $$\tilde{c}_1^\lambda = \Big\langle \big(\hat{e}_1^\lambda, \hat{f}_1^\lambda, \hat{g}_1^\lambda, \hat{h}_1^\lambda\big); \frac{\wp(T_{c_1})^\lambda}{(1+(\wp-1)(1-T_{c_1}))^\lambda + (\wp-1)(T_{c_1})^\lambda}, \frac{(1+(\wp-1)I_{c_1})^\lambda - (1-I_{c_1})^\lambda}{(1+(\wp-1)I_{c_1})^\lambda + (\wp-1)(1-I_{c_1})^\lambda}, \frac{(1+(\wp-1)F_{c_1})^\lambda - (1-F_{c_1})^\lambda}{(1+(\wp-1)F_{c_1})^\lambda + (\wp-1)(1-F_{c_1})^\lambda} \Big\rangle, \quad \lambda > 0.$$

4 SVTN-Hamacher arithmetic aggregation operators

Based on the basis of Hamacher operation on SVTNNs, we propose single-valued trapezoidal neutrosophic Ham weighted arithmetic average (SVTNHWAA) operator, single-valued trapezoidal neutrosophic Ham ordered weighted arithmetic average (SVTNHOWAA) operator and single-valued trapezoidal neutrosophic Ham hybrid weighted arithmetic average (SVTNHHAA) operator.

Definition 9 Let $\tilde{c}_z = \Big\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \Big\rangle (z = 1, 2, \ldots, v)$ be a number of SVTNNs. Then, SVTNHWAA operator is a function $SVTNHWAA : \mathfrak{L}^v \to \mathfrak{L}$ defined as follows:

$$SVTNHWAA_\psi(\tilde{c}_1 \tilde{N}_2, \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v} (\psi_z \tilde{c}_z)$$

where $\psi = (\psi_1, \psi_2, \ldots, \psi_v)^T$ be the weight vector of $\tilde{c}_z (z = 1, 2, \ldots, v)$ with $\psi_z > 0$ and $\sum_{z=1}^{v} \psi_z = 1$.

By using Ham operations on SVTNNs, we get the following theorem.

Theorem 1 *Let* $\tilde{c}_z = \Big\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \Big\rangle (z = 1, 2, \ldots, v)$ *be a collection of SVTNNs. Then,*

$$SVTNHWAA_\psi(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v} (\psi_z \tilde{c}_z) = \Bigg\langle \Bigg(\sum_{z=1}^{v} \hat{e}_z\psi_z, \sum_{z=1}^{v} \hat{f}_z\psi_z, \sum_{z=1}^{v} \hat{g}_z\psi_z, \sum_{z=1}^{v} \hat{h}_z\psi_z \Bigg); \frac{\prod_{z=1}^{v}(1+(\wp-1)\hat{T}_z)^{\psi_z} - \prod_{z=1}^{v}(1-\hat{T}_z)^{\psi_z}}{\prod_{z=1}^{v}(1+(\wp-1)\hat{T}_z)^{\psi_z} + (\wp-1)\prod_{z=1}^{v}(1-\hat{T}_z)^{\psi_z}}, \frac{\wp\prod_{z=1}^{v}(\hat{I}_z)^{\psi_z}}{\prod_{z=1}^{v}(1+(\wp-1)(1-\hat{I}_z))^{\psi_z} + (\wp-1)\prod_{z=1}^{v}(\hat{I}_z)^{\psi_z}}, \frac{\wp\prod_{z=1}^{v}(\hat{F}_z)^{\psi_z}}{\prod_{z=1}^{v}(1+(\wp-1)(1-\hat{F}_z))^{\psi_z} + (\wp-1)\prod_{z=1}^{v}(\hat{F}_z)^{\psi_z}} \Bigg\rangle \tag{16}$$

where $\psi = (\psi_1, \psi_2, \ldots, \psi_v)$ *be the weight vector of* $\tilde{c}_z (z = 1, 2, \ldots, v)$ *such that* $\psi_z > 0$, *and* $\sum_{z=1}^{v} \psi_z = 1$.

By mathematical induction, We prove the Theorem 1 as follows:

Proof

(i) When $z = 1$, then $\psi_1 = 1$, therefore left side of the (16) becomes

$$SVTNHWAA_\psi(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_z) = \tilde{c}_1 = \langle (\hat{e}_1, \hat{f}_1, \hat{g}_1, \hat{h}_1); \hat{T}_1, \hat{I}_1, \hat{F}_1 \rangle$$

and for right side of (16), we have

$$\Bigg\langle (\hat{e}_1, \hat{f}_1, \hat{g}_1, \hat{h}_1); \frac{1+(\wp-1)\hat{T}_1 - (1-\hat{T}_1)}{(1+(\wp-1)\hat{T}_1) + (\wp-1)(1-\hat{T}_1)}, \frac{\wp\hat{I}_1}{1+(\wp-1)(1-\hat{I}_1) + (\wp-1)\hat{I}_1}, \frac{\wp\hat{F}_1}{1+(\wp-1)(1-\hat{F}_1) + (\wp-1)\hat{F}_1} \Bigg\rangle = \Big\langle (\hat{e}_1, \hat{f}_1, \hat{g}_1, \hat{h}_1); \hat{T}_1, \hat{I}_1, \hat{F}_1 \Big\rangle. \tag{17}$$

Hence, (16) holds for $z = 1$.

(ii) Assume that (16) holds for $z = t$, then

$$SVTNHWA_{\Psi}(\tilde{c}_1,\tilde{c}_2\dots,\tilde{c}_t)=\bigoplus_{z=1}^{t}(\psi_z\tilde{c}_z)$$
$$=\Bigg\langle\Bigg(\sum_{z=1}^{t}\hat{e}_z\psi_z,\sum_{z=1}^{t}\hat{f}_z\psi_z,\sum_{z=1}^{t}\hat{g}_z\psi_z,\sum_{z=1}^{t}\hat{h}_z\psi_z\Bigg);\frac{\prod_{z=1}^{t}(1+(\wp-1)\hat{T}_z)^{\psi_z}-\prod_{z=1}^{t}(1-\hat{T}_z)^{\psi_z}}{\prod_{z=1}^{t}(1+(\wp-1)\hat{T}_z)^{\psi_z}+(\wp-1)\prod_{z=1}^{t}(1-\hat{T}_z)^{\psi_z}},\frac{\wp\prod_{z=1}^{t}(\hat{I}_z)^{\psi_z}}{\prod_{z=1}^{t}(1+(\wp-1)(1-\hat{I}_z))^{\psi_z}+(\wp-1)\prod_{z=1}^{t}(\hat{I}_z)^{\psi_z}},\frac{\wp\prod_{z=1}^{t}(\hat{F}_z)^{\psi_z}}{\prod_{z=1}^{t}(1+(\wp-1)(1-\hat{F}_z))^{\psi_z}+(\wp-1)\prod_{z=1}^{t}(\hat{F}_z)^{\psi_z}}\Bigg\rangle. \tag{18}$$

Now for $z=t+1$, then

$$SVTNHWA_{\Psi}(\tilde{c}_1,\tilde{c}_2\dots,\tilde{c}_t,\tilde{c}_{t+1})=\bigoplus_{z=1}^{t}(\psi_z\tilde{c}_z)\bigoplus(\psi_{t+1}\tilde{c}_{t+1})$$
$$=\Bigg\langle\Bigg(\sum_{z=1}^{t}\hat{e}_z\psi_z,\sum_{z=1}^{t}\hat{f}_z\psi_z,\sum_{z=1}^{t}\hat{g}_z\psi_z,\sum_{z=1}^{t}\hat{h}_z\psi_z\Bigg);\frac{\prod_{z=1}^{t}(1+(\wp-1)\hat{T}_z)^{\psi_z}-\prod_{z=1}^{t}(1-\hat{T}_z)^{\psi_z}}{\prod_{z=1}^{t}(1+(\wp-1)\hat{T}_z)^{\psi_z}+(\wp-1)\prod_{z=1}^{t}(1-\hat{T}_z)^{\psi_z}},\frac{\wp\prod_{z=1}^{t}(\hat{I}_z)^{\psi_z}}{\prod_{z=1}^{t}(1+(\wp-1)(1-\hat{I}_z))^{\psi_z}+(\wp-1)\prod_{z=1}^{t}(\hat{I}_z)^{\psi_z}},\frac{\wp\prod_{z=1}^{t}(\hat{F}_z)^{\psi_z}}{(1+(\wp-1)\prod_{z=1}^{t}(1-\hat{F}_z))^{\psi_z}+(\wp-1)\prod_{z=1}^{t}(\hat{F}_z)^{\psi_z}}\Bigg\rangle$$
$$\bigoplus\Bigg\langle\Big(\hat{e}_{t+1}\psi_{t+1},\hat{f}_{t+1}\psi_{t+1},\hat{g}_{t+1}\psi_{t+1},\hat{h}_{t+1}\psi_{t+1}\Big);\frac{(1+(\wp-1)\hat{T}_{t+1})^{\psi_{t+1}}-(1-\hat{T}_{t+1})^{\psi_{t+1}}}{(1+(\wp-1)\hat{T}_{t+1})^{\psi_t+1}+(\wp-1)(1-\hat{T}_{t+1})^{\psi_{t+1}}},\frac{\wp(\hat{I}_{t+1})^{\psi_{t+1}}}{(1+(\wp-1)(1-\hat{I}_{t+1}))^{\psi_{t+1}}+(\wp-1)(\hat{I}_{t+1})^{\psi_{t+1}}},\frac{\wp(\hat{F}_{t+1})^{\psi_{t+1}}}{(1+(\wp-1)(1-\hat{F}_{t+1}))^{\psi_{t+1}}+(\wp-1)(\hat{F}_{t+1})^{\psi_{t+1}}}\Bigg\rangle$$
$$=\Bigg\langle\Bigg(\sum_{z=1}^{t+1}\hat{e}_z\psi_z,\sum_{z=1}^{t+1}\hat{f}_z\psi_z,\sum_{z=1}^{t+1}\hat{g}_z\psi_z,\sum_{z=1}^{t+1}\hat{h}_z\psi_z\Bigg);\frac{\prod_{v=1}^{k+1}(1+(\wp-1)\hat{T}_z)^{\psi_z}-\prod_{z=1}^{t+1}(1-\hat{T}_z)^{\psi_z}}{\prod_{z=1}^{t+1}(1+(\wp-1)\hat{T}_z)^{\psi_z}+(\wp-1)\prod_{z=1}^{t+1}(1-\hat{T}_z)^{\psi_z}},\frac{\wp\prod_{z=1}^{t+1}(\hat{I}_z)^{\psi_z}}{\prod_{z=1}^{t+1}(1+(\wp-1)(1-\hat{I}_z))^{\psi_z}+(\wp-1)\prod_{z=1}^{t+1}(\hat{I}_z)^{\psi_z}}, \tag{19}$$

$$\frac{\wp\prod_{z=1}^{t+1}(\hat{F}_z)^{\psi_z}}{\prod_{z=1}^{t+1}(1+(\wp-1)(1-\hat{F}_z))^{\psi_z}+(\wp-1)\prod_{z=1}^{t+1}(\hat{F}_z)^{\psi_z}}\Bigg\rangle. \tag{20}$$

Thus, $z=t+1$ holds for (16).

Hence, from steps (i) and (ii), we conclude that (16) holds for any $z\in N$. □

Theorem 2 (Idempotency) *Let* $\tilde{c}_z=\Big\langle(\hat{e}_z,\hat{f}_z,\hat{g}_z,\hat{h}_z);\hat{T}_z,\hat{I}_z,\hat{F}_z\Big\rangle$ $(z=1,2,\dots,v)$ *be a number of SVTNNs, where* $\tilde{c}_z=\tilde{c}$ *for all* z. *Then,*
$SVTNHWA_{\psi}(\tilde{c}_1,\tilde{c}_2\dots,\tilde{c}_v)=\tilde{c}$.

Theorem 3 (Boundedness) *Let* $\tilde{c}_z=\Big\langle(\hat{e}_z,\hat{f}_z,\hat{g}_z,\hat{h}_z);\hat{T}_z,\hat{I}_z,\hat{F}_z\Big\rangle$ $(z=1,2,\dots,v)$ *be a number of SVTNNs and let*

$$\tilde{c}^-=\min_z\tilde{c}_z=\Big\langle\Big(\min_z\hat{e}_z,\min_z\hat{f}_z,\min_z\hat{g}_z,\min_z\hat{h}_z\Big);\min_z(\hat{T}_z),\max_z(\hat{I}_z),\max_z(\hat{F}_z)\Big\rangle$$
$$\tilde{c}^+=\max_z\tilde{c}_z=\Big\langle\Big(\max_z\hat{e}_z,\max_z\hat{f}_z,\max_z\hat{g}_z,\max_z\hat{h}_z\Big);\max_z(\hat{T}_z),\min_z(\hat{I}_z),\min_z(\hat{F}_z)\Big\rangle.$$

Then,

$\tilde{c}^-\leq SVTNHWA_{\psi}(\tilde{c}_1,\tilde{c}_2\dots,\tilde{c}_v)\leq\tilde{c}^+$.

Theorem 4 (Monotonicity) *Let* $\tilde{c}_z=\Big\langle(\hat{e}_z,\hat{f}_z,\hat{g}_z,\hat{h}_z);\hat{T}_z,\hat{I}_z,\hat{F}_z\Big\rangle$ *and* $\tilde{c}'_v=\Big\langle(\hat{e}'_v,\hat{f}'_v,\hat{g}'_v,\hat{h}'_v);\hat{T}'_z,\hat{I}'_z,\hat{F}'_z\Big\rangle(z=1,2,\dots,v)$ *be two sets of SVTNNs. If* $\tilde{c}_z\leq\tilde{c}'_z$ *for all* z, *then*

$$SVTNHWA_{\psi}(\tilde{c}_1,\tilde{c}_2\dots,\tilde{c}_v)\leq SVTNHWA_{\psi}(\tilde{c}'_1,\tilde{c}'_2,\dots,\tilde{c}'_v). \tag{21}$$

Now, we considered two special cases subsequently for the SVTNHWA operator when the parameter $\wp$ *takes the values* 1 *or* 2.

Case 1 If $\wp=1$, then SVTNHWA is reduced to single-valued trapezoidal neutrosophic weighted arithmetic averaging (SVTNWA) operator

$$SVTNWA_{\psi}(\tilde{c}_1,\tilde{c}_2\dots,\tilde{c}_v)=\bigoplus_{z=1}^{v}(\psi_z\tilde{c}_z)$$
$$=\Bigg\langle\Bigg(\sum_{v}^{n}\hat{e}_z\psi_z,\sum_{j}^{n}\hat{f}_z\psi_z,\sum_{j}^{n}\hat{g}_z\psi_z,\sum_{v}^{n}\hat{h}_z\psi_z\Bigg);1-\prod_{z=1}^{v}(1-\hat{T}_z)^{\psi_z},\prod_{z=1}^{v}(\hat{I}_z)^{\psi_z},\prod_{z=1}^{v}(\hat{F}_z)^{\psi_z}\Bigg\rangle. \tag{22}$$

Case 2 If $\wp = 2$, then SVTNHWA is reduced to single-valued trapezoidal Einstein weighted arithmetic averaging (SVTNEWA) operator:

$$SVTNEWAA_{\Psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v}(\psi_z \tilde{c}_z)$$
$$= \Bigg\langle \Big(\sum_{v}^{n} \hat{e}_z \psi_z, \sum_{v}^{n} \hat{f}_z \psi_z, \sum_{v}^{n} \hat{g}_z \psi_z, \sum_{v}^{n} \hat{h}_z \psi_z \Big); \frac{\prod_{z=1}^{v}(1+\hat{T}_z)^{\psi_z} - \prod_{z=1}^{v}(1-\hat{T}_z)^{\Psi}}{\prod_{z=1}^{v}(1+\hat{T}_z)^{\psi_z} + \prod_{z=1}^{v}(1-\hat{T}_z)^{\Psi}}, \frac{2\prod_{z=1}^{v}(\hat{I}_z)^{\psi_z}}{\prod_{z=1}^{v}(2-\hat{I}_z)^{\psi_z} + \prod_{z=1}^{v}(\hat{I}_z)^{\psi_z}}, \frac{2\prod_{z=1}^{v}(\hat{F}_z)^{\psi_z}}{\prod_{z=1}^{v}(2-\hat{F}_z)^{\psi_z} + \prod_{z=1}^{v}(\hat{F}_z)^{\psi_z}} \Bigg\rangle. \tag{23}$$

Definition 10 Let $\tilde{c}_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle$ $(z = 1, 2, \ldots, v)$ be a number of SVTNNs. The SVTNHOWA operator of dimension v is a function $SVTNHOWA : \mathfrak{L}^v \to \mathfrak{L}$ with associated vector $\psi = (\psi_1, \psi_2, \ldots, \psi_v)^T$ such that $\psi_z > 0$, and $\sum_{z=1}^{v} \psi_z = 1$. Therefore,

$$SVTNHOWA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v}(\psi_z \tilde{c}_{\sigma(z)}) \tag{24}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ are the permutation of $\sigma(z)(z = 1, 2, \ldots, v)$, for which $\tilde{c}_{\sigma(z-1)} \geq \tilde{c}_{\sigma(z)}$ for all $z = 1, 2, \ldots, v$.

Based on Hamacher operation on SVTNNs, we can introduced the following.

Theorem 5 *Let* $c_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle (z = 1, 2, \ldots, v)$ *be a collection of SVTNNs. A SVTNHOWAA operator is a function* $SVTNHOWA : \mathfrak{L}^v \to \mathfrak{L}$ *with associated vector* $\psi = (\psi_1, \psi_2, \ldots, \psi_v)^T$ *such that* $\psi_z > 0$, *and* $\sum_{z=1}^{v} \psi_z = 1$. *Then,*

$$SVTNHOWA_{\Psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v}(\psi_z \tilde{c}_{\sigma(z)})$$
$$= \Bigg\langle \Big(\sum_{z=1}^{v} \hat{e}_{\sigma(v)} \psi_z, \sum_{z=1}^{v} \hat{f}_{\sigma(z)} \psi_z, \sum_{z=1}^{v} \hat{g}_{\sigma(z)} \psi_z, \sum_{z=1}^{v} \hat{h}_{\sigma(z)} \psi_z \Big); \frac{\prod_{z=1}^{v}(1+(\wp-1)\hat{T}_{\sigma(z)})^{\psi_z} - \prod_{z=1}^{v}(1-\hat{T}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v}(1+(\wp-1)\hat{T}_{\sigma(z)})^{\psi_z} + (\wp-1)\prod_{z=1}^{v}(1-\hat{T}_{\sigma(z)})^{\psi_z}}, \frac{\wp\prod_{z=1}^{v}(\hat{I}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v}(1+(\wp-1)(1-\hat{I}_{\sigma(z)}))^{\psi_z} + (\wp-1)\prod_{z=1}^{v}(\hat{I}_{\sigma(z)})^{\psi_z}}, \frac{\wp\prod_{z=1}^{v}(\hat{F}_{\sigma(z)})^{\psi_z}}{(1+(\wp-1)\prod_{z=1}^{v}(1-\hat{F}_{\sigma(z)})^{\psi_z} + (\wp-1)\prod_{z=1}^{v}(\hat{F}_{\sigma(z)})^{\psi_z}} \Bigg\rangle \tag{25}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ *are the permutation of* $\sigma(z)(z = 1, 2, \ldots, v)$, *for which* $\tilde{c}_{\sigma(z-1)} \geq \tilde{c}_{\sigma(z)}$ *for all* $z = 1, 2, \ldots, v$.

The SVTNHOWA operator follows these properties as:

Theorem 6 (Idempotency) *If* $\tilde{c}_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle$ $(z = 1, 2, \ldots, v)$ *be a number of SVTNNs such that* $\tilde{c}_z = \tilde{c}$ *for all z. Then,*

$$SVTNHOWA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \tilde{c}. \tag{26}$$

Theorem 7 (Boundedness) *Let* $\tilde{c}_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle$ $(z = 1, 2, \ldots, v)$ *be a number of SVTNNs and let*

$$\tilde{c}^- = \min_z \tilde{c}_z, \quad \tilde{c}^+ = \max_z \tilde{c}_z.$$

Then,

$$\tilde{c}^- \leq SVTNHOWA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) \leq \tilde{c}^+.$$

Theorem 8 (Monotonicity property) *Let* $\tilde{c}_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle$ *and* $\tilde{c}'_z = \left\langle (\hat{e}'_z, \hat{f}'_z, \hat{g}'_z, \hat{h}'_z); \hat{T}'_z, \hat{I}'_z, \hat{F}'_z \right\rangle (z = 1, 2, \ldots, v)$ *be two sets of SVTNNs, if* $\tilde{c}_z \leq \tilde{c}'_z$ *for all z, then*

$$SVTNHOWA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) \leq SVTNHOWA_{\psi}(\tilde{c}'_1, \tilde{c}'_2, \ldots, \tilde{c}'_v).$$

Theorem 9 (Commutativity) *Let* $\tilde{c}_z(z = 1, 2, \ldots, v)$ *and* $\tilde{c}'_z$ $(z = 1, 2, \ldots, v)$ *be two sets of SVTNNs. Then,*

$$SVTNHOWA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = SVTHOWA_{\psi}(\tilde{c}'_1, \tilde{c}'_2, \ldots, \tilde{c}'_v)$$

where $\tilde{c}'_z(z = 1, 2, \ldots, v)$ *is any permutation of* $\tilde{c}_z(z = 1, 2, \ldots, v)$.

There are two cases arises when the parameter $\wp$ takes 1 or 2.

Case 1 If $\wp = 1$, then SVTNHOWA is reduced to single-valued trapezoidal neutrosophic ordered weighted arithmetic averaging (SVTNOWAA) operator

$$SVTNHOWA_{\Psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v}(\psi_z \tilde{c}_z)$$
$$= \left\langle \Big(\sum_{z=1}^{v} \hat{e}_{\sigma(v)}\psi_z, \sum_{z=1}^{v} \hat{f}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{g}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{h}_{\sigma(z)}\psi_z \Big); \right.$$
$$\left. 1 - \prod_{z=1}^{v}(1 - T_{\sigma}(v))^{\psi_z}, \prod_{z=1}^{v}(I_{\sigma}(v))^{\psi_z}, \prod_{z=1}^{v}(F_{\sigma}(v))^{\psi_z} \right\rangle.$$

Case 2 If $\wp = 2$, then SVTNHOWA is transformed to the SVTNEOWA operator:

$$SVTNEOWA_{\Psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v}(\psi_z \tilde{c}_z)$$
$$= \left\langle \Big(\sum_{z=1}^{v} \hat{e}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{f}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{g}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{h}_{\sigma(z)}\psi_z \Big); \right.$$
$$\frac{\prod_{z=1}^{v}(1 + \hat{T}_{\sigma}(z))^{\psi_z} - \prod_{z=1}^{v}(1 - \hat{T}_{\sigma}(z))^{\Psi}}{\prod_{z=1}^{v}(1 + \hat{T}_{\sigma}(z))^{\psi_z} + \prod_{z=1}^{v}(1 - \hat{T}_{\sigma}(z))^{\Psi}},$$
$$\frac{2\prod_{z=1}^{v}(\hat{I}_{\sigma}(z))^{\psi_z}}{\prod_{z=1}^{v}(2 - \hat{I}_{\sigma}(z))^{\psi_z} + \prod_{z=1}^{v}(\hat{I}_{\sigma}(z))^{\psi_z}},$$
$$\left. \frac{2\prod_{z=1}^{v}(\hat{F}_{\sigma}(z))^{\psi_z}}{\prod_{z=1}^{v}(2 - \hat{F}_{\sigma}(z))^{\psi_z} + \prod_{z=1}^{v}(\hat{F}_{\sigma}(z))^{\psi_z}} \right\rangle.$$

Above Definitions 12 and 13, we see that SVTNHWA operator considered the weights of SVTN values, other hand SVTNHOWAA imply weights of the given ordered positions of SVTN values instead of weights of the SVTN values. Therefore, weights represent both the operators SVTNHWA and SVTNHOWA are in different ways. But, they are examined only one of them.

To overcome this difficulties, we introduce SVTN-Hamacher hybrid arithmetic averaging (SVTNHHA) operator.

Definition 11 A SVTN-Ham hybrid arithmetic averaging (SVTNHHA) operator of dimension v is a function $SVTNHHA : \mathfrak{L}^v \rightarrow \mathfrak{L}$, with associated weight vector $\psi = (\psi_1, \psi_2, \ldots, \psi_v)$ such that $\psi_z > 0$, and $\sum_{z=1}^{v} \psi_z = 1$. Further,

$$SVTNHHA_{\psi,\Psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v}(\psi_z \dot{\tilde{c}}_{\sigma(z)})$$
$$= \left\langle \Big(\sum_{z=1}^{v} \hat{e}_{\sigma(v)}\psi_z, \sum_{z=1}^{v} \hat{f}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{g}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{h}_{\sigma(z)}\psi_z \Big); \right.$$
$$\frac{\prod_{z=1}^{v}(1 + (\wp - 1)\dot{\hat{T}}_{\sigma(z)})^{\psi_z} - \prod_{z=1}^{v}(1 - \dot{\hat{T}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v}(1 + (\wp - 1)\dot{\hat{T}}_{\sigma(z)})^{\psi_z} + (\wp - 1)\prod_{z=1}^{v}(1 - \dot{\hat{T}}_{\sigma(z)})^{\psi_z}},$$
$$\frac{\wp \prod_{z=1}^{v}(\dot{\hat{I}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v}(1 + (\wp - 1)(1 - \dot{\hat{I}}_{\sigma(z)}))^{\psi_z} + (\wp - 1)\prod_{z=1}^{v}(\dot{\hat{I}}_{\sigma(z)})^{\psi_z}},$$
$$\left. \frac{\wp \prod_{z=1}^{v}(\dot{\hat{F}}_{\sigma(z)})^{\psi_z}}{(1 + (\wp - 1)\prod_{z=1}^{v}(1 - \dot{\hat{F}}_{\sigma(z)})^{\psi_z} + (\wp - 1)\prod_{z=1}^{v}(\dot{\hat{F}}_{\sigma(z)})^{\psi_z}} \right\rangle$$

where $\dot{\tilde{c}}_{\sigma(z)}$ is the zth largest weighted SVTN values $\dot{\tilde{c}}_z(\dot{\tilde{c}}_z = vw_z\tilde{c}_z, z = 1, 2, \ldots, v)$, and $w = (w_1, w_2, \ldots, w_v)^T$ be the v weight vector of $\dot{\tilde{c}}_z$ with $w_z > 0$ and $\sum_{z=1}^{v} w_z = 1$, where v is follows as balancing coefficient. When $w = (1/v, 1/v, \ldots, 1/v)$, then SVTNHWAA operator is a particular issue of SVTNHHAA operator.

Let $\psi = (1/v, 1/v, \ldots, 1/v)$, then SVTNHOWAA is a particular issue of the operator SVTNHHWAA. Thus, SVTNHHWAA operator is a generalization of SVTNHWAA and SVTNHOWAA, which review the degrees of the given class and their ordered positions.

Now we describe two cases of the SVTNHHWA operator for the values of $\wp$:

Case 1 If $\wp = 1$, then SVTNHHWA is reduced to the SVTN-hybrid weighted arithmetic averaging (SVTrNHWA) operator given as follows:

$$SVTNHWA_{\Psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v}(\psi_z \dot{\tilde{N}}_v)$$
$$= \left\langle \Big(\sum_{z=1}^{v} \hat{e}_{\sigma(v)}\psi_z, \sum_{z=1}^{v} \hat{f}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{g}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{h}_{\sigma(z)}\psi_z \Big); \right.$$
$$\left. 1 - \prod_{z=1}^{v}(1 - \dot{\hat{T}}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v}(\dot{\hat{I}}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v}(\dot{F}_{\sigma(v)})^{\psi_z} \right\rangle.$$

Case 2 If $\wp = 2$, then SVTNHHWA operator is reduced to the SVTN-Einstein hybrid weighted arithmetic averaging (SVTNEHWA) operator given as follows:

$$SVTNEHWAA_{\Psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigoplus_{z=1}^{v}(\psi_z \dot{\hat{N}}_v)$$
$$= \Bigg\langle \Bigg(\sum_{z=1}^{v} \hat{e}_{\sigma(v)}\psi_z, \sum_{z=1}^{v} \hat{f}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{g}_{\sigma(z)}\psi_z, \sum_{z=1}^{v} \hat{h}_{\sigma(z)}\psi_z \Bigg);$$
$$\frac{\prod_{z=1}^{v}(1+\dot{\hat{T}}_{\sigma(z)})^{\psi_z} - \prod_{z=1}^{v}(1-\dot{\hat{T}}_{\sigma(z)})^{\Psi}}{\prod_{z=1}^{v}(1+\dot{\hat{T}}_{\sigma(z)})^{\psi_z} + \prod_{z=1}^{v}(1-\dot{\hat{T}}_{\sigma(z)})^{\Psi}},$$
$$\frac{2\prod_{z=1}^{v}(\dot{\hat{I}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v}(2-\dot{\hat{I}}_{\sigma(z)})^{\psi_z} + \prod_{z=1}^{v}(\dot{\hat{I}}_{\sigma(z)})^{\psi_z}}, \frac{2\prod_{z=1}^{v}(\dot{\hat{F}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v}(2-\dot{\hat{F}}_{\sigma(z)})^{\psi_z} + \prod_{z=1}^{v}(\dot{\hat{F}}_{\sigma(z)})^{\psi_z}} \Bigg\rangle. \tag{27}$$

5 SVTN-Hamacher geometric aggregation operators

To this part, we introduce Hamacher geometric aggregation operators under SVTN information such as (SVTNHWGA)operator, (SVTNHOWGA) operator and (SVTNHHWGA) operator.

Definition 12 Let $\tilde{c}_z = \Big\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \Big\rangle (z = 1, 2, \ldots, v)$ be a number of SVTNNs. Then, SVTNHWGA operator is a function $\mathfrak{L}^v \to \mathfrak{L}$ such that

$$SVTNHWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigotimes_{z=1}^{v}(\tilde{c}_z)^{\psi_z}$$

where $\psi = (\psi_1, \psi_2, \ldots, \psi_v)^T$ be the weight vector of $\tilde{c}_z (z = 1, 2, \ldots, v)$ with $\psi_z > 0$ and $\sum_{z=1}^{v} \psi_z = 1$.

We have drawn the following theorem using Hamacher operations on SVTNNs.

Theorem 10 *Let* $\tilde{c}_z = \Big\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \Big\rangle$ $(z = 1, 2, \ldots, v)$ *be a number of SVTNNs. Then, aggregated value of them using the SVTNHWGA operation is also a SVTNN, and*

$$SVTNHWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigotimes_{v=1}^{n}(\tilde{c}_z)^{\psi_z}$$
$$= \Bigg\langle \Bigg(\prod_{z=1}^{v}(\hat{e}_z)^{\psi_z}, \prod_{z=1}^{v}(\hat{f}_z)^{\psi_z}, \prod_{z=1}^{v}(\hat{g}_z)^{\psi_z}, \prod_{z=1}^{v}(\hat{h}_z)^{\psi_z} \Bigg);$$
$$\frac{\wp \prod_{z=1}^{v}(\hat{T}_z)^{\psi_z}}{\prod_{z=1}^{v}(1+(\wp-1)(1-\hat{T}_z))^{\psi_z} + (\wp-1)\prod_{z=1}^{v}(\hat{T}_z)^{\psi_z}}$$
$$\frac{\prod_{z=1}^{v}(1+(\wp-1)\hat{T}_z)^{\psi_z} - \prod_{z=1}^{v}(1-\hat{I}_z)^{\psi_z}}{\prod_{z=1}^{v}(1+(\wp-1)\hat{I}_z)^{\psi_z} + (\wp-1)\prod_{z=1}^{v}(1-\hat{I}_z)^{\psi_z}},$$
$$\frac{\prod_{z=1}^{v}(1+(\wp-1)\hat{F}_z)^{\psi_z} - \prod_{z=1}^{v}(1-\hat{F}_z)^{\psi_z}}{\prod_{z=1}^{v}(1+(\wp-1)\hat{F}_z)^{\psi_z} + (\wp-1)\prod_{z=1}^{v}(1-\hat{F}_z)^{\psi_z}} \Bigg\rangle \tag{28}$$

where $\psi = (\psi_1, \psi_2, \ldots, \psi_v)$ *be the weight vector of* $\tilde{c}_z (z = 1, 2, \ldots, v)$ *such that* $\psi_z > 0$, *and* $\sum_{z=1}^{v} \psi_z = 1$.

Proof Proved by mathematical induction follows from Theorem 1. □

Theorem 11 (Idempotency) *If* $\tilde{c}_z = \Big\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \Big\rangle (z = 1, 2, \ldots, v)$ *are all equal, i.e.* $\tilde{c}_z = \tilde{c}$ *for all z, then*

$$SVTNHWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \tilde{c}.$$

Theorem 12 (Boundedness) *Let* $\tilde{c}_z = \Big\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \Big\rangle (z = 1, 2, \ldots, v)$ *be a number of SVTNNs and let*

$$\tilde{c}^- = \min_z \tilde{c}_z = \Big\langle \Big(\min_z \hat{e}_z, \min_z \hat{f}_z, \min_z \hat{g}_z, \min_z \hat{h}_z \Big);$$
$$\min_z(\hat{T}_z), \max_z(\hat{I}_z), \max_z(\hat{F}_z) \Big\rangle$$
$$\tilde{c}^+ = \max_z \tilde{c}_z = \Big\langle \Big(\max_z \hat{e}_z, \max_z \hat{f}_z, \max_z \hat{g}_z, \max_z \hat{h}_z \Big);$$
$$\max_z(\hat{T}_z), \min_z(\hat{I}_z), \min_z(\hat{F}_z) \Big\rangle.$$

Then,

$$\tilde{c}^- \le SVTNHWGA_{\Psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) \le \tilde{c}^+.$$

Theorem 13 (Monotonicity) *Let* $\tilde{c}_z = \Big\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \Big\rangle (z = 1, 2, \ldots, v)$ *and* $\tilde{c}'_z (z = 1, 2, \ldots, v)$ *be two sets of SVTNNs. If* $\tilde{c}_z \le \tilde{c}'_z$ *for all z, then*

$$SVTNHWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) \le SVTNHWGA_{\psi}(\tilde{c}'_1, \tilde{c}'_2, \ldots, \tilde{c}'_v).$$

Now, we considered two special cases subsequently for the SVTNHWGA operator when the parameter $\wp$ *takes the values* 1 *or* 2.

Case 1 If $\wp = 1$, then SVTNHWGA operator will reduce to (SVTNWGA) operator:

$$SVTNWGA_{\Psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigotimes_{v=1}^{n} (\tilde{c}_z)^{\psi_z}$$
$$= \left\langle \Big(\prod_{z=1}^{v} (\hat{e}_z)^{\psi}_z, \prod_{v}^{n} (\hat{f}_z)^{\psi}_z, \prod_{z=1}^{v} (\hat{g}_z)^{\psi}_z, \prod_{z=1}^{v} (\hat{h}_z)^{\psi}_z \Big); \prod_{z=1}^{v} (\hat{T}_z)^{\psi_z}, 1 - \prod_{z=1}^{v} (1 - \hat{I}_z)^{\psi_z}, 1 - \prod_{z=1}^{v} (1 - \hat{F}_z)^{\psi_z} \right\rangle.$$

Case 2 If $\wp = 2$, then SVTNHWGA operator is reduces to SVTNEWGA operator:

$$SVTNEWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigotimes_{z=1}^{v} (\tilde{c}_z)^{\psi_z}$$
$$= \left\langle \Big(\prod_{z=1}^{v} (\hat{e}_z)^{\psi_z}, \prod_{z=1}^{v} (\hat{f}_z)^{\psi_z}, \prod_{z=1}^{v} (\hat{g}_z)^{\psi_z}, \prod_{z=1}^{v} (\hat{h}_z)^{\psi_z} \Big); \frac{2 \prod_{z=1}^{v} (\hat{T}_z)^{\psi_z}}{\prod_{z=1}^{v} (2 - \hat{T}_z)^{\psi_z} + \prod_{z=1}^{v} (\hat{T}_z)^{\psi_z}}, \frac{\prod_{z=1}^{v} (1 + \hat{I}_z)^{\psi_z} - \prod_{z=1}^{v} (1 - \hat{I}_z)^{\psi_z}}{\prod_{z=1}^{v} (1 + \hat{I}_z)^{\psi_z} + \prod_{z=1}^{v} (1 - \hat{I}_z)^{\psi}}, \frac{\prod_{z=1}^{v} (1 + \hat{F}_z)^{\psi_z} - \prod_{z=1}^{v} (1 - \hat{F}_z)^{\psi_z}}{\prod_{z=1}^{v} (1 + \hat{F}_z)^{\psi_z} + \prod_{z=1}^{v} (1 - \hat{F}_z)^{\psi_z}} \right\rangle. \tag{29}$$

Definition 13 Let $\tilde{c}_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle$ $(z = 1, 2, \ldots, v)$ be a number of SVTNNs. A SVTNHOWGA operator is a function $SVTNHOWGA : \mathfrak{L}^v \to \mathfrak{L}$ with associated vector $\psi = (\psi_1, \psi_2, \ldots, \psi_v)^T$ such that $\psi_z > 0$, and $\sum_{z=1}^{v} \psi_z = 1$. Therefore,

$$SVTNHOWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigotimes_{z=1}^{v} (\tilde{c}_{\sigma(z)})^{\psi_z} \tag{30}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(v))$ are the permutation of $\sigma(z)(z = 1, 2, \ldots, v)$, for which $\tilde{c}_{\sigma(z-1)} \geq \tilde{c}_{\sigma(z)}$ for all $z = 1, 2, \ldots, v$.

The following theorem is develop based on Ham-operation on SVTNNs.

Theorem 14 *Let* $\tilde{c}_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle (z = 1, 2, \ldots, v)$ *be a number of SVTNNs. A SVTNHOWGA operator of dimension v is a function* $SVTNHOWGA : \mathfrak{L}^v \to \mathfrak{L}$ *with associated vector* $\psi = (\psi_1, \psi_2, \ldots, \psi_v)^T$ *such that* $\psi_z > 0$, *and* $\sum_{z=1}^{v} \psi_z = 1$. *Furthermore,*

$$SVTNHOWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \bigotimes_{z=1}^{v} (\tilde{c}_{\sigma(z)})^{\psi_z}$$
$$= \left\langle \Big(\prod_{z=1}^{v} (\hat{e}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{f}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{g}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{h}_{\sigma(z)})^{\psi_z} \Big); \frac{\wp \prod_{z=1}^{v} (\hat{T}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)(1 - \hat{T}_{\sigma(z)}))^{\psi_z} + (\wp - 1) \prod_{z=1}^{v} (\hat{T}_{\sigma(z)})^{\psi_z}}, \frac{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{I}_{\sigma(z)})^{\psi_z} - \prod_{z=1}^{v} (1 - \hat{I}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{I}_{\sigma(z)})^{\psi_z} + (\wp - 1) \prod_{z=1}^{v} (1 - \hat{I}_{\sigma(z)})^{\psi_z}}, \frac{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{F}_{\sigma(z)})^{\psi_z} - \prod_{z=1}^{v} (1 - \hat{F}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{F}_{\sigma(z)})^{\psi_z} + (\wp - 1) \prod_{z=1}^{v} (1 - \hat{F}_{\sigma(z)})^{\psi_z}} \right\rangle \tag{31}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(v))$ *are the permutation of* $\sigma(z)(z = 1, 2, \ldots, v)$, *for which* $\tilde{c}_{\sigma(z-1)} \geq \tilde{c}_{\sigma(z)}$ *for all* $z = 1, 2, \ldots, v$.

The following properties can be easily proved for SVTNHOWGA operator.

Theorem 15 (Idempotency) *If* $\tilde{c}_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle (z = 1, 2, \ldots, v)$ *such that* $\tilde{c}_z = \tilde{c}$ *for all z. Then,*

$$SVTNHOWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) = \tilde{c}.$$

Theorem 16 (Boundedness) *Let* $\tilde{c}_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle (z = 1, 2, \ldots, v)$ *be a number of SVTNNs and let* $\tilde{c}^- = \min_z \tilde{c}_z, \quad \tilde{c}^+ = \max_z \tilde{c}_z$. *Then,*

$$\tilde{c}^- \leq SVTNHOWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) \leq \tilde{c}^+.$$

Theorem 17 (Monotonicity) *Let* $\tilde{c}_z = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle (z = 1, 2, \ldots, v)$ *and* $\tilde{c}'_z (z = 1, 2, \ldots, v)$ *be two sets of SVTNN. If* $\tilde{c}_z \leq \tilde{c}'_z$ *for all z. Then,*

$$SVTNHOWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \ldots, \tilde{c}_v) \leq SVTNHOWGA_{\psi}(\tilde{c}'_1, \tilde{c}'_2, \ldots, \tilde{c}'_v).$$

Theorem 18 (Commutativity) *Let* $\tilde{c}_z (z = 1, 2, \dots, v)$ *and* $\tilde{c}'_z$ $(z = 1, 2, \dots, v)$ *be two sets of SVTNNs. Then,*

$$SVTNHOWGA_\psi(\tilde{c}_1, \tilde{c}_2 \dots, \tilde{c}_v) = SVTNHOWGA_\psi(\tilde{c}'_1, \tilde{c}'_2, \dots, \tilde{c}'_v)$$

where $\tilde{c}'_z (z = 1, 2, \dots, v)$ *is any permutation of* $\tilde{c}_z (z = 1, 2, \dots, v)$.

If it is taken the 1 and 2, then there are two cases for the parameter $\wp$.

Case 1 If $\wp = 1$, then SVTNHOWGA operator reduces to SVTNOWGA operator

$$SVTNOWGA_\psi(\tilde{c}_1, \tilde{c}_2 \dots, \tilde{c}_v) = \bigotimes_{z=1}^{v} (\tilde{c}_z)^{\psi_z} = \left\langle \left(\prod_{z=1}^{v} (\hat{e}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{f}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{g}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{h}_{\sigma(z)})^{\psi_z} \right); \prod_{z=1}^{v} (\hat{T}_\sigma(z))^{\psi}_z, 1 - \prod_{z=1}^{v} (1 - \hat{I}_\sigma(z))^{\psi_z}, 1 - \prod_{z=1}^{v} (1 - \hat{F}_\sigma(z))^{\psi_z} \right\rangle.$$

Case 2 If $\wp = 2$, then SVTNHOWGA operator is reduced to the SVTNEOWGA operator:

$$SVTNEOWGA_\psi(\tilde{c}_1, \tilde{c}_2 \dots, \tilde{c}_v) = \bigotimes_{z=1}^{v} (\tilde{c}_z)^{\psi_z} = \left\langle \left(\prod_{z=1}^{v} (\hat{e}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{f}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{g}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{h}_{\sigma(z)})^{\psi_z} \right); \frac{2 \prod_{z=1}^{v} (\hat{T}_\sigma(z))^{\psi_z}}{\prod_{z=1}^{v} (2 - \hat{T}_\sigma(z))^{\psi_z} + \prod_{z=1}^{v} (\hat{T}_\sigma(v))^{\psi_z}}, \frac{\prod_{z=1}^{v} (1 + \hat{I}_\sigma(z))^{\psi_z} - \prod_{z=1}^{v} (1 - \hat{I}_\sigma(z))^{\psi_z}}{\prod_{z=1}^{v} (1 + \hat{I}_\sigma(z))^{\psi_z} + \prod_{z=1}^{v} (1 - \hat{I}_\sigma(z))^{\psi_z}}, \frac{\prod_{z=1}^{v} (1 + \hat{F}_\sigma(z))^{\psi_z} - \prod_{z=1}^{v} (1 - \hat{F}_\sigma(z))^{\psi_z}}{\prod_{z=1}^{v} (1 + \hat{F}_\sigma(z))^{\psi_z} + \prod_{z=1}^{v} (1 - \hat{F}_\sigma(z))^{\psi_z}} \right\rangle. \tag{32}$$

In Definitions 12 and 13, we see that SVTNHWGA operator considered weights only the SVTN values, other hand the SVTNHOWGA operator weights imply the given ordered positions of the weights of SVTN values themselves. Therefore, weights interpreted in SVTNHWGAA and SVTNHOWGA are in different view. But, they are examine only one of them. To overcome this problems, we introduced SVTN-Hamacher hybrid geometric averaging (SVTNHHGA) operator.

Definition 14 Let $N_j = \left\langle (\hat{e}_z, \hat{f}_z, \hat{g}_z, \hat{h}_z); \hat{T}_z, \hat{I}_z, \hat{F}_z \right\rangle$ be a number of SVTNNs. Then, SVTNHHGA operator of dimension v is a function $SVTNHHGA : \mathfrak{L}^v \to \mathfrak{L}$, with associated weight vector $\psi = (\psi_1, \psi_2, \dots, \psi_v)$ such that $\psi_z > 0$, and $\sum_{z=1}^{v} \psi_z = 1$. Therefore, SVTNHHWGA operator can be evaluated as

$$SVTNHHWGA_\psi(\tilde{c}_1, \tilde{c}_2 \dots, \tilde{c}_v) = \bigotimes_{v=1}^{n} (\dot{\tilde{c}}_{\sigma(z)})^{\psi_z} = \left\langle \left(\prod_{z=1}^{v} \hat{e}_{\sigma(z)} \psi_z, \prod_{z=1}^{v} \hat{f}_{\sigma(z)} \psi_z, \prod_{z=1}^{v} \hat{g}_{\sigma(z)} \psi_z, \prod_{z=1}^{v} \hat{h}_{\sigma(z)} \psi_z \right); \frac{\wp \prod_{z=1}^{v} (\dot{\hat{T}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)(1 - \dot{\hat{T}}_{\sigma(z)}))^{\psi_z} + (\wp - 1) \prod_{z=1}^{v} (\dot{\hat{T}}_{\sigma(z)})^{\psi_z}}, \frac{\prod_{z=1}^{v} (1 + (\wp - 1)\dot{\hat{I}}_{\sigma(z)})^{\psi_z} - \prod_{z=1}^{v} (1 - \dot{\hat{I}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)\dot{\hat{I}}_{\sigma(z)})^{\psi_z} + (\wp - 1) \prod_{z=1}^{v} (1 - \dot{\hat{I}}_{\sigma(z)})^{\psi_z}}, \frac{\prod_{z=1}^{v} (1 + (\wp - 1)\dot{\hat{F}}_{\sigma(z)})^{\psi_z} - \prod_{z=1}^{v} (1 - \dot{\hat{F}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)\dot{\hat{F}}_{\sigma(z)})^{\psi_z} + (\wp - 1) \prod_{z=1}^{v} (1 - \dot{\hat{F}}_{\sigma(z)})^{\psi_z}} \right\rangle \tag{33}$$

where $\dot{\tilde{c}}_{\sigma(z)}$ is the zth largest weighted trapezoidal neutrosophic values $\dot{\tilde{c}}_z$ $(\dot{\tilde{c}}_z = v w_z \tilde{c}_z, z = 1, 2, \dots, v)$, and $w = (w_1, w_2, \dots, w_v)^T$ be the weight vector of $\dot{\tilde{c}}_z$ with $w_z > 0$ and $\sum_{z=1}^{v} w_z = 1$, where v is the balancing coefficient. When $w = (1/v, 1/v, \dots, 1/v)$, then SVTNHWGA operator is a particular issue of SVTNHHGA operator. Let $\psi = (1/v, 1/v, \dots, 1/v)$, then SVTNHOWGA is a usual issue of the SVTNHHGA operator. Thus, SVTNHHGA is a extension of both the SVTNHWGA and SVTNHOWGA operators, which reflects the degrees of the given arguments and their ordered positions.

Now we describe two cases of the SVTNHHGA operator for the values of $\wp$:

Case 1 If $\wp = 1$, then SVTNHHWGA operator is reduced to the SVTNHGA operator:

$$SVTNHWGA_\psi(\tilde{c}_1, \tilde{c}_2 \dots, \tilde{c}_v) = \bigotimes_{z=1}^{v} (\dot{\tilde{c}}_z)^{\psi_z} = \left\langle \left(\prod_{z=1}^{v} (\hat{e}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{f}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{g}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{h}_{\sigma(z)})^{\psi_z} \right); \prod_{z=1}^{v} (\dot{\hat{T}}_{\sigma(z)})^{\psi_z}, 1 - \prod_{z=1}^{v} (1 - (\dot{\hat{I}}_{\sigma(z)})^{\psi_z}, 1 - \prod_{z=1}^{v} (1 - \dot{\hat{F}}_{\sigma(z)})^{\psi_z} \right\rangle.$$

Case 2 If $\wp = 2$, then SVTNHHWGA operator is reduced to the SVTNEHWGA operator as follows:

$$SVTNEHbWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \dots, \tilde{c}_v) = \bigotimes_{z=1}^{v} (\dot{\tilde{c}}_z)^{\psi_z}$$

$$= \Bigg\langle \Bigg(\prod_{z=1}^{v} (\hat{e}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{f}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{g}_{\sigma(z)})^{\psi_z}, \prod_{z=1}^{v} (\hat{h}_{\sigma(z)})^{\psi_z} \Bigg);$$
$$\frac{2\prod_{z=1}^{v} (\dot{\hat{T}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v} (2 - \dot{\hat{T}}_{\sigma(z)})^{\psi_z} + \prod_{z=1}^{v} (\dot{\hat{T}}_{\sigma(z)})^{\psi_z}},$$
$$\frac{\prod_{z=1}^{v} (1 + \dot{\hat{I}}_{\sigma(z)})^{\psi_z} - \prod_{z=1}^{v} (1 - \dot{\hat{I}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v} (1 + \dot{\hat{I}}_{\sigma(z)})^{\psi_z} + \prod_{z=1}^{v} (1 - \dot{\hat{I}}_{\sigma(z)})^{\psi_z}},$$
$$\frac{\prod_{z=1}^{v} (1 + \dot{\hat{F}}_{\sigma(z)})^{\psi_z} - \prod_{z=1}^{v} (1 - \dot{\hat{F}}_{\sigma(z)})^{\psi_z}}{\prod_{z=1}^{v} (1 + \dot{\hat{F}}_{\sigma(z)})^{\psi_z} + \prod_{z=1}^{v} (1 - \dot{\hat{F}}_{\sigma(z)})^{\psi_z}} \Bigg\rangle.$$

6 Model for MADM in trapezoidal neutrosophic environment

In this study, we propose MADM method using SVTNH aggregation operators in which weights of the attributes values real numbers under SVTN environment. Here MADM method is used to developed usefulness of evaluation emerging software systems commercialization under SVTN information. Let $A = \{A_1, A_2, \dots, A_m\}$ be the set of alternatives, $T = \{T_1, T_2, \dots, T_n\}$ be the set of attributes. Let $\psi = (\psi_1, \psi_2, \dots, \psi_v)$ be the weight vector of the attribute A_z $(z = 1, 2, \dots, v)$ are completely known such that $\psi_z > 0$ and $\sum_{z=1}^{v} \psi_z = 1$. Suppose that $\tilde{D} = (\hat{T}_{hz}, \hat{I}_{hz}, \hat{F}_{hz})_{u\times v}$ is the trapezoidal neutrosophic decision matrix, where $\hat{T}_{hz}$ is the truth-membership degree for which alternative A_h satisfies the attribute T_z given by the decision makers, $\hat{I}_{hz}$ denote the degree of indeterminacy-membership such that alternative A_h does not satisfies the attribute T_z, and $\hat{F}_{hz}$ falsity-membership degree that the alternative A_h does not satisfy the attribute T_z given by the decision maker, where $\hat{T}_{hz} \subset [0, 1]$, $\hat{I}_{hz} \subset [0, 1]$ and $\hat{F}_{hz} \subset [0, 1]$ for which $0 \le \hat{T}_{hz} + \hat{I}_{hz} + \hat{F}_{hz} \le 1$, $(h = 1, 2, \dots, u)$ and $(z = 1, 2, \dots, v)$.

The algorithm follows a method to interpret MADM problem under SVTN information using SVTNHWA and SVTNHWGA operators.

Algorithm

Input: SVTN information.

Output: To get desired alternative.

Step 1. We introduce the decision matrix $\tilde{D}$, and use the operator SVTNHWA

$$\delta_h = SVTNHWA_{\psi}(\tilde{c}_1, \tilde{c}_2 \dots, \tilde{c}_v) = \bigoplus_{z=1}^{v} (\psi_z \tilde{c}_z)$$
$$= \Bigg\langle \Bigg(\sum_{z=1}^{v} \hat{e}_z \psi_z, \sum_{z=1}^{v} \hat{f}_z \psi_z, \sum_{z=1}^{v} \hat{g}_z \psi_z, \sum_{z=1}^{v} \hat{h}_z \psi_z \Bigg);$$
$$\frac{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{T}_z)^{\psi_z} - \prod_{z=1}^{v} (1 - \hat{T}_z)^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{T}_z)^{\psi_z} + (\wp - 1)\prod_{z=1}^{v} (1 - \hat{T}_z)^{\psi_z}},$$
$$\frac{\wp \prod_{z=1}^{v} (\hat{I}_z)^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)(1 - \hat{I}_z))^{\psi_z} + (\wp - 1)\prod_{z=1}^{v} (\hat{I}_z)^{\psi_z}},$$
$$\frac{\wp \prod_{z=1}^{v} (\hat{F}_z)^{\psi_z}}{(1 + (\wp - 1)\prod_{z=1}^{v} (1 - \hat{F}_z))^{\psi_z} + (\wp - 1)\prod_{z=1}^{v} (\hat{F}_z)^{\psi_z}} \Bigg\rangle \tag{34}$$

or $$\delta_h = SVTNHWGA_{\psi}(\tilde{c}_1, \tilde{c}_2 \dots, \tilde{c}_v) = \bigotimes_{z=1}^{v} (\tilde{c}_z)^{\psi_z}$$
$$= \Bigg\langle \Bigg(\prod_{z=1}^{v} (\hat{e}_z)^{\psi_z}, \prod_{z=1}^{v} (\hat{f}_z)^{\psi_z}, \prod_{z=1}^{v} (\hat{g}_z)^{\psi_z}, \prod_{z=1}^{v} (\hat{h}_z)^{\psi_z} \Bigg);$$
$$\frac{\wp \prod_{z=1}^{v} (\hat{T}_z)^{\psi_z}}{(1 + (\wp - 1)\prod_{z=1}^{v} (1 - \hat{T}_z))^{\psi_z} + (\wp - 1)\prod_{z=1}^{v} (\hat{T}_z)^{\psi_z}}$$
$$\frac{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{T}_z)^{\psi_z} - \prod_{z=1}^{v} (1 - \hat{I}_z)^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{I}_z)^{\psi_z} + (\wp - 1)\prod_{z=1}^{v} (1 - \hat{I}_z)^{\psi_z}},$$
$$\frac{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{F}_z)^{\psi_z} - \prod_{z=1}^{v} (1 - \hat{F}_z)^{\psi_z}}{\prod_{z=1}^{v} (1 + (\wp - 1)\hat{F}_z)^{\psi_z} + (\wp - 1)\prod_{z=1}^{v} (1 - \hat{F}_z)^{\psi_z}} \Bigg\rangle \tag{35}$$

to obtained the over all preference values δ_h $(h = 1, 2, \dots, u)$ of the alternative A_h.

Step 2. Evaluation of the score $\emptyset(\delta_h)$ $(h = 1, 2, \dots, u)$ based on over all SVTN information δ_h $(h = 1, 2, \dots, u)$ to determine the ranking of all the alternatives $A_h (h = 1, 2, \dots, u)$ to select desirable choice A_h. If the value of $\emptyset(\delta_h)$ and $\emptyset(\delta)$ are same, then we next proceed to evaluate degrees of accuracy $\varphi(\delta_h)$ and $\varphi(\delta_z)$ based on over all SVTN information of β_h and δ_z, and rank the alternative A_h depending with the accuracy degrees of $\varphi(\delta_h)$ and $\varphi(\delta_h)$.

Step 3. Rank all the alternative A_h $(h = 1, 2, \dots, u)$ in order to choice the best one(s) in accordance with $\emptyset(\delta_h)(h = 1, 2, \dots, u)$.

Step 4. End.

Table 1 Evaluations of decision makers

	A_1	A_2	A_3	A_4
T_1	$\langle(0.3, 0.4, 0.5, 0.6);0.6, 0.2, 0.2\rangle$	$\langle(0.5, 0.6, 0.7, 0.8);0.8, 0.2, 0.3\rangle$	$\langle(0.2, 0.4, 0.4, 0, 5);0.5, 0.3, 0.4\rangle$	$\langle(0.6, 0.7, 0.8, 0.9);0.7, 0.3, 0.3\rangle$
T_2	$\langle(0.4, 0.5, 0.6, 0.7);0.7, 0.1, 0.1\rangle$	$\langle(0.4, 0.5, 0.6, 0.7);0.7, 0.4, 0.4\rangle$	$\langle(0.5, 0.6, 0.7, 0.8);0.7, 0.2, 0.2\rangle$	$\langle(0.4, 0.5, 0.6, 0.7);0.8, 0.4, 0.4\rangle$
T_3	$\langle(0.2, 0.3, 0.4, 0.5);0.5, 0.2, 0.2\rangle$	$\langle(0.4, 0.5, 0.6, 0.7);0.6, 0.2, 0.2\rangle$	$\langle(0.2, 0.3, 0.4, 0.5);0.8, 0.1, 0.1\rangle$	$\langle(0.3, 0.4, 0.5, 0.6);0.7, 0.3, 0.3\rangle$
T_4	$\langle(0.5, 0.6, 0.7, 0.8);0.4, 0.2, 0.4\rangle$	$\langle(0.2, 0.4, 0.6, 0.7);0.4, 0.2, 0.5\rangle$	$\langle(0.4, 0.5, 0.5, 0.6);0.3, 0.3;0.4\rangle$	$\langle(0.5, 0.5, 0.5, 0.5);1, 0, 0\rangle$

7 Numerical example and comparative analysis

7.1 Numerical example

With the rapid progress and huge application of information technology, the selection of emerging software systems becomes more and more important. The aim of the project is to predict the best software systems based on their performances, that provide alternatives of four candidates. Therefore, to this section, we shall present a numerical result to establish the potential assessment of software technology systems depicted in Ye (2014a) under SVTN environment in order to investigate our proposed method. There is a committee which selects four possible software systems $\tilde{A}_h(h = 1, 2, \ldots, 4)$. They choose four attributes to assess four possible software as follows:

T_1: Contribution about organization performance.
T_2: Effort to transform from current system.
T_3: Costs of hardware and software investment.
T_4: Outsourcing software developer reliability.

According to above attributes of which weight vector is $\psi = (0.25, 0.22, 0.35, 0.18)^T$, alternatives A_1, A_2, A_3 and A_4 are evaluated with SVTNNs by decision makers which have same dominance degree. Evaluation of decision makers is as in Table 1.

In order to select most desirable software A_h $(h = 1, 2, \ldots, m)$, we use the SVTNHWA and SVTNHWGA operators. SVTN values in Table 1 are evaluated as follows:

– Step 1: Let $\wp = 3$. Then, by using the SVTNHWAA operator to aggregate preferences values δ_h of software systems A_h for $(h = 1, 2, 3, 4)$ are as follows:

$$\widetilde{\delta}_1 = \Big\langle (0.3230,\ 0.4230,\ 0.5230,\ 0.6230);\ 0.5582,\ 0.1725,\ 0.1976 \Big\rangle$$
$$\widetilde{\delta}_2 = \Big\langle (0.3890,\ 0.5070,\ 0.6250,\ 0.7250);\ 0.6528,\ 0.2351,\ 0.3091 \Big\rangle$$
$$\widetilde{\delta}_3 = \Big\langle (0.3020,\ 0.4270,\ 0.4840,\ 0.5840);\ 0.6413,\ 0.1899,\ 0.2182 \Big\rangle$$
$$\widetilde{\delta}_4 = \Big\langle (0.4330,\ 0.5150,\ 0.5970,\ 0.6790);\ 0.7636,\ 0.2671,\ 0.2671 \Big\rangle.$$

– Step 2: By using the equation given in Definition 8, for each δ_h $(h = 1, 2, 3, 4)$ score $S(\tilde{\delta}_h)$ is obtained as follows:

$$S(\widetilde{\delta}_1) = 0.2587, \quad S(\widetilde{\delta}_2) = 0.2960,$$
$$S(\widetilde{\delta}_3) = 0.2508, \quad S(\widetilde{\delta}_4) = 0.3099.$$

– Step 3: Based on the scores of the software systems A_h for $(h = 1, 2, 3, 4)$ ranking order of the emerging software systems A_h is obtained as $A_4 \succ A_2 \succ A_1 \succ A_3$
– Step 4: According to ranking order of the alternatives A_4 is selected as the best choice software system.

If SVTNHWGA operator is used for the same problem, then the problem can be solved in similar way as follows:

– Step 1: Let $\wp = 3$. Then, by using the SVTNHWGA operator to aggregate δ_h of emerging software systems A_h for $(h = 1, 2, 3, 4)$ are calculated as follows:

$$\tilde{\delta}_1 = \Big\langle (0.3040,\ 0.4086,\ 0.5114,\ 0.6133);\ 0.5469,\ 0.1777,\ 0.2139 \Big\rangle$$
$$\tilde{\delta}_2 = \Big\langle (0.3733,\ 0.5027,\ 0.6236,\ 0.7238);\ 0.6310,\ 0.2444,\ 0.3250 \Big\rangle$$
$$\tilde{\delta}_3 = \Big\langle (0.2772,\ 0.4116,\ 0.4709,\ 0.5730);\ 0.6000,\ 0.2074,\ 0.2511 \Big\rangle$$
$$\tilde{\delta}_4 = \Big\langle (0.4167,\ 0.5030,\ 0.5854,\ 0.6647);\ 0.7777,\ 0.2861,\ 0.2861 \Big\rangle.$$

– Step 2: By using the equation given in Definition 8, for each δ_h $(h = 1, 2, 3, 4)$ score $S(\tilde{\delta}_h)$ is obtained as follows:

$$S(\tilde{\delta}_1) = 0.2457,$$
$$S(\tilde{\delta}_2) = 0.2865, S(\tilde{\delta}_3) = 0.2319,$$
$$S(\tilde{\delta}_4) = 0.29901.$$

– Step 3: Based on the scores of software systems A_h for $(h = 1, 2, 3, 4)$ ranking order of the software systems A_h is obtained as $A_4 \succ A_2 \succ A_1 \succ A_3$.
– Step 4: According to ranking order of the alternatives A_4 is selected as the best software system.

Note that, although scores of alternatives are different for obtained $\tilde{\beta}_h(h = 1, 2, 3, 4)$ by using SVTNWA and SVTNWGA, ranking order of the alternatives is same. A_4 is the most desirable alternative in either events. To compare with the existing work (Wang and Zhang 2009) which develop decision making approach using ITFN information whereas in this proposed decision making problems using SVTNs information. It is noted that SVTN is a generalization ITFN. The results of the decision making method in this paper is more classic and general in applications. Also, compared with the existing works (Biswas et al. 2014; Ye 2013, 2014c; Zhang et al. 2014) in which evaluated decision making results are in the domain of discrete sets of literatures but not existing continuous sets of literatures, whereas this paper proposed decision making approach can be suitable to solve decision making problems with triangular and trapezoidal neutrosophic information. Therefore, propose method in this paper is a generation of the existing methods and have a advantages to solve decision making problems.

8 Conclusion

In this article, we study about the method to solve a MADM problem under SVTN information. We introduce arithmetic and geometric averaging operations to utilize some SVTN Hamacher aggregation operators from the motivation of Hamacher operations as: SVTNHWAA operator, SVTNHOWAA operator, SVTNHHWAA operator, SVTrNHWGA operator, SVTNHOWGA operator and SVTNHHWGA operator. The different characteristic of these proposed operators are studied. Then, we have used these operators to develop some approaches to solve MADM problems. Lastly, a practical example for emerging software system selection is given to verify our proposed method and to illustrate the application and effectiveness of the proposed method. In next study, the proposed model can be applied in decision support systems, risk analysis and other domains containing uncertainties.

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