Journal of Ambient Intelligence and Humanized Computing (2020) 11:2187–2201
https://doi.org/10.1007/s12652-019-01251-z

ORIGINAL RESEARCH

Multi-criteria PROMETHEE method based on possibility degree with Z-numbers under uncertain linguistic environment

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Received: 13 October 2018 / Accepted: 12 February 2019 / Published online: 26 February 2019

Abstract
People make decisions based on their cognitive information about the objective world. Zadeh's Z-number allows people to better express their cognition of the real world by considering the fuzzy restriction and reliability restriction of information. However, the Z-number is a complex construct, and some important issues must be discussed in its study. Here, a computationally simple method of ranking Z-numbers for multi-criteria decision-making (MCDM) problems is proposed, and a comprehensive possibility degree of Z-numbers is defined, as inspired by the possibility degree concept of interval numbers. The outranking relations of Z-numbers are also discussed on the basis of the proposed method. Then, a weight acquisition algorithm relative to the possibility degree of Z-numbers is presented. Finally, an extended Preference Ranking Organization Method for Enrichment Evaluation II (PROMETHEE II) based on the possibility degree of Z-numbers is developed for the MCDM problem under Z-evaluation, and a numerical example about the selection of travel plans is used to illustrate the validity of the proposed method. The applicability and superiority of the proposed method is demonstrated through sensitivity and comparative analyses along with other existing methods.

Keywords Z-numbers · Multi-criteria decision-making · PROMETHEE II · Possibility degree

1 Introduction

Humans participate in relevant decision-making activities based on the information they perceive (Ji et al. 2018; Li and Wang 2017; Wang et al. 2017). However, the perceived information is usually fuzzy and partially reliable, which deeply affects humans' decision-making activities (Hu et al. 2017; Tian et al. 2017; Wang et al. 2015, 2017, 2018, 2019; Zadeh 1965). Zadeh (2011) proposed the Z-number in consideration of the fuzzy restriction and information reliability. A Z-number, Z, is composed of an ordered pair, (A, B), which is used to describe a real-valued uncertain variable, X. The component A is a fuzzy restriction on the values of X, and B reflects the reliability restriction of the first component. A Z-number can be denoted as 'X is $Z = (A, B)$' or (X, A, B). According to Zadeh (2011), A and B in the natural language can be converted into trapezoidal fuzzy numbers (Bakar and Gegov 2015) or triangular fuzzy numbers (TFNs) (Aliev et al. 2016a, b) for computation purposes. Therefore, studying the Z-number-based decision-making problem is meaningful under uncertain linguistic environments.

Decision makers (DMs) are often more willing to express their decision-making perspectives by using natural language terms rather than specific numerical scores due to the ambiguity of the decision-making process and its uncertainties. Recently, many scholars have conducted in-depth research on uncertain linguistic decision-making problems (Hu et al. 2018; Huang et al. 2018; Li et al. 2018; Mardani et al. 2015; Xue et al. 2016). Ding and Liu (2018) used the decision-making trial and evaluation laboratory method to identify critical success factors in emergency management, in which evaluation values were represented by two-dimensional uncertain linguistic variables. Liu et al. (2019) built a robot selection model by combining quality function development theory and the qualitative flexible multiple criteria method, by which the DMs expressed their views through interval-valued Pythagorean uncertain linguistic sets. Peng and Wang (2018) studied the applications of Z-numbers and cloud model to address multi-criteria group decision-making

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problems under uncertain linguistic environments. Peng et al. (2019) combined linguistic variables and Z-numbers to establish a multi-criteria game model.

Zadeh (2011) stressed that stating the problem described by Z-numbers is relatively easy, but solving for the Z-numbers is difficult in terms of computation. Many scholars have studied the generation and operation of Z-numbers (Kang et al. 2018a, b, c, d). For example, Yager (2012) studied specific underlying probability distributions to solve certain decision problems involving Z-numbers. Aliev et al. (2015) developed the basic arithmetic of discrete Z-numbers according to discrete fuzzy number theory (Casasnovas and Riera 2006; Chou 2003; Voxman 2001) and the general principle of Z-number calculation (Zadeh 2011). Aliev et al. (2016a, b) implemented a comprehensive study on continuous Z-numbers. Shen and Wang (2018) constructed a comprehensively weighted Z-distance measure that only considers the reliability restriction and the underlying probability distribution of Z-numbers. The above studies focused on the direct calculation of Z-numbers according to the basic properties of the Z-number. However, these methods are complicated because of the need to satisfy the requirements of goal programming and convolution operations (Peng and Wang 2017).

Some scholars have attempted to simplify the operations of Z-numbers by converting them into simpler forms. Kang et al. (2012a, b) developed a typical method for converting the Z-number into a trapezoidal fuzzy number or a triangular fuzzy number. Kang et al. (2012a, b) and Kang et al. (2018a, b, c, d) constructed a formula for converting the Z-number into a crisp value. Aliyev (2016) proposed a single-distance measure by considering the Z-number as a pair of two classical fuzzy numbers. Different from the explanations of Aliev et al. (2015); Aliev et al. (2016a, b); Yang and Wang (2018), Kang et al. (2018a, b, c, d) regarded the Z-number as an ordered pair of two fuzzy numbers, and they proposed an improved fuzzy measure for calculating the uncertainty of this Z-number. The above conversion methods effectively reduced the complexity involved in Z-information fusion and hence can be applied to many practical decision problems (Kang et al. 2018a, b, c, d; Tavakkoli-Moghaddam et al. 2015; Wu et al. 2018; Yaakob and Gegov 2016). However, the above studies ignored the different effects of both fuzzy and reliability restrictions of the Z-number on the decision-making process.

The Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) method developed by Brans et al. (1986) is one of the most applicable outranking methods for solving decision-making problems. Many studies on fuzzy multi-criteria decision-making (MCDM) have used the PROMETHEE method. Liu et al. (2017) performed a failure mode and effect analysis of a risk identification problem by combining the cloud model and PROMETHEE. Liang et al. (2018) proposed a projection-based PROMETHEE method with hesitant fuzzy linguistic term sets. Li and Wang (2017) extended the PROMETHEE method to hesitant probabilistic fuzzy environments. Peng et al. (2016) presented a novel MCDM method based on hesitant fuzzy sets and prospect theory. Tavakkoli-Moghaddam et al. (2015) investigated the problem of facility location selection under Z-evaluation by developing the Z-PROMETHEE method. Although the fuzzy PROMETHEE decision-making method has been thoroughly studied, the research on extending PROMETHEE by combining it with the Z-number and under Z-environments is still rare; hence, such research is necessary.

Here, a novel ranking method for Z-numbers is developed to deal with the MCDM problem under Z-evaluation. Firstly, the concept of possibility degree of TFNs is defined, as inspired by the possibility degree concept of interval numbers (Xu and Da 2003, 2002). Secondly, a joint possibility degree of Z-numbers is developed on the basis of the possibility degree of TFNs. In addition, a pairwise comparative matrix that reflects the additive preference relation is constructed to acquire the criteria weights (Xu 2001). Thirdly, an extended PROMETHEE method based on the proposed possibility degree of Z-numbers is presented for the MCDM problem, in which the evaluation values are described by using Z-numbers. Finally, as the most important research aspect, a numerical example about the selection of travel plans is used to illustrate the validity and feasibility of the proposed Z-PROMETHEE approach. Compared with those in the existing literature, some pivotal innovations can be derived from the present work as follows:

1. The possibility degree of two TFNs is developed, as inspired by the possibility degree concept of interval fuzzy numbers. Then, a comprehensively weighted possibility degree of two Z-numbers is constructed. This possibility degree is used to compare the fuzzy restriction and the reliability restriction of two Z-numbers without converting the Z-number into a fuzzy number or a crisp value.
2. For the MCDM problem with the Z-number as the evaluation value, if the criteria weights are expressed by using Z-numbers, then a method of converting the Z-weight vector into a real-weight vector is necessary. In light of the possibility degree of Z-numbers, a weight acquisition algorithm is introduced on the basis of the fuzzy complementary judgment matrix to obtain the real weight vector.
3. The possibility degree between two Z-numbers can reveal their partial order relationship. The PROMETHEE II method, combined with the possibility degree of Z-numbers, is used to present a Z-valued multi-criteria PROMETHEE method under uncertain

linguistic environments. The extended Z-PROMETHEE method can provide the full order of all the alternatives. Moreover, a travel example is used to illustrate its effectiveness

The remainder of this paper is constructed as follows. In Sect. 2, some basic concepts are reviewed for subsequent discussions. In Sect. 3, a possibility degree definition of Z-numbers is developed to construct their outranking relations. Furthermore, a Z-PROMETHEE approach based on the possibility degree concept of Z-numbers is presented for MCDM in Sect. 4. In Sect. 5, a numerical example is used to illustrate the feasibility of the proposed method. Sensitivity and comparative analyses are implemented to verify the validity and reasonability of the proposed approach. The conclusion is presented in Sect. 6.

2 Preliminaries

For convenience of subsequent discussions, some basic concepts are presented for background information such as triangular fuzzy number, continuous Z-number, and interval number.

2.1 Continuous fuzzy number and triangular fuzzy number

Definition 1 (Aliev et al. 2016a, b): Let X be a universe of discourse. The fuzzy set A on X, whose membership function is the mapping of $\mu_A : R \to [0,1]$, is a continuous fuzzy number if it fulfils the following conditions:

1. A is a normal fuzzy set;
2. A is a convex fuzzy set;
3. α-cut A^α of A is a closed interval for any $\alpha \in [0,1]$;
4. The support *supp* (A) of A is bounded.

Definition 2 (Abbasbandy and Hajjari 2009): Let X be a universe of discourse. The membership function of the TFN $A=(a, b, c)$ is

$$\mu_A(x)\begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x < b \\ 1 & x = b \\ \frac{c-x}{c-b} & b \le x < c \\ 0 & x > c \end{cases}. \tag{1}$$

Evidently, the TFN is a type of continuous fuzzy number.

2.2 Z-number and continuous Z-number

Definition 3 (Zadeh 2011): A Z-number is an ordered pair of fuzzy numbers, (A, B), which is used to characterise a real-valued uncertain variable X. The first component A, which is allowed to be taken by X, plays a role in the fuzzy restriction of values. The second component B is a reliability restriction of the first component. A Z-number is usually expressed as 'X is (A, B)', 'X is $Z = (A, B)$' or '(X, A, B)'.

Definition 4 (Aliev et al. 2016a, b): A continuous Z-number is an ordered pair $Z=(A, B)$, in which some conditions need to be satisfied as follows:

1. A is a continuous fuzzy number whose membership function is the mapping of $\mu_A : R \to [0,1]$, where R_X is the support *supp* (A) of A;
2. B is a continuous fuzzy number whose membership function is the mapping of $\mu_B : [0,1] \to [0,1]$.

2.3 Possibility degree of interval numbers

Definition 5 (Xu and Da 2003): Let X be a universe of discourse. An interval number a on X can be defined as

$$a = [a^-, a^+] = \left\{x | a^- \le x \le a^+\right\}. \tag{2}$$

If $a^- = a^+$, then the interval number $a = [a^-, a^+]$ will degenerate to a real number. Moreover, for any two interval numbers $a = \left[a^-, a^+\right]$ and $b = \left[b^-, b^+\right]$, a is strictly equivalent to b marked as $a = b$ if $a^- = b^-$ and $a^+ = b^+$.

Definition 6 (Gao 2013; Xu 2001; Xu and Da 2003): Let $a = [a^-, a^+]$ and $b = [b^-, b^+]$ be any two interval numbers. The possibility degree of $a \ge b$ is defined as

$$p(a \ge b) = \max\left\{1 - \max\left(\frac{b^+ - a^-}{(b^+ - b^-) + (a^+ - a^-)}, 0\right), 0\right\}. \tag{3}$$

For any real number a and any interval number $b = \left[b^-, b^+\right]$, the possibility degree formula is also applicable to the calculation of the possibility degree of $a \ge b$ or $b \ge a$, provided that the real number a is viewed as $a = \left[a^-, a^+\right]$.

According to Xu and Da (2003), the possibility degree between any two real numbers can be defined. However, some discordant points in the properties of the possibility degree may exist. For example, obtaining $p(a \ge b) = 0$ is inappropriate when a and b are real numbers and equal, as the definition will not satisfy the reflexivity condition (i.e. $p(a \ge b) \ne \frac{1}{2}$). Therefore, the possibility degree of any two real numbers has been redefined according to Gao (2013).

Definition 7 (Gao 2013): Let a and b be any two real numbers. The possibility degree of $a \geq b$ is defined as

$$P(a \geq b) = \begin{cases} 0 & a < b \\ 0.5 & a = b \\ 1 & a > b \end{cases}. \tag{4}$$

Theorem 1 (Gao 2013): Let $a = [a^-, a^+](a^- \leq a^+)$, $b = [b^-, b^+](b^- \leq b^+)$ and $c = [c^-, c^+](c^- \leq c^+)$ be any three interval numbers. Some properties of the possibility degree are satisfied as follows:

1. Normative: $0 \leq p(a \geq b) \leq 1$;
2. Complementary: $p(a \geq b) + p(b \geq a) = 1$;
3. Reflexivity: $p(a \geq b) = p(b \geq a) = 0.5$ if $a = b$;
4. Transitivity: if $p(a \geq b) \leq 0.5$ and $p(b \geq c) \leq 0.5$, then $p(a \geq c) \leq 0.5$.

3 Outranking relations of Z-numbers

A novel concept named possibility degree of Z-numbers is proposed on the basis of two closely connected subsections. Furthermore, the outranking relations between the Z-numbers are defined on the basis of the possibility degree of Z-numbers.

3.1 Possibility degree of TFNs

Definition 8 Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be any two TFNs. Some comparative relations can be defined as follows:

1. If $a_1 = b_1$, $a_2 = b_2$ and $a_3 = b_3$, then $\tilde{a}$ is strictly equivalent to $\tilde{b}$, and they are marked as $\tilde{a} = \tilde{b}$.
2. If $a_1 \geq b_3$, then $\tilde{a}$ is strictly larger than $\tilde{b}$, and it is marked as b^-, b^+.

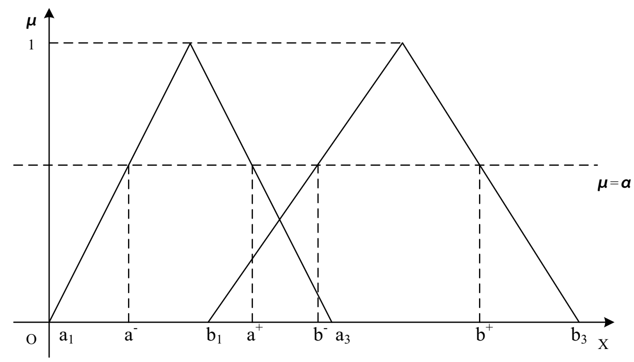

Fig. 1 Possibility degree of two TFNs

where $p^\alpha(\tilde{a} \geq \tilde{b})$ is the possibility degree of the cut set $[a^-, a^+]$ of $\tilde{a}$ and the cut set $[b^-, b^+]$ of $\tilde{b}$ in α level (Yao and Chiang 2003; Yao et al. 2003), as shown in Fig. 1. The analytic solution of the possibility degree of TFNs is shown in the Appendix A.

Note 1 The TFNs in Definition 9 are regular fuzzy numbers, which indicates that the range of the integration interval is from 0 to 1. If the uncertain information is expressed by irregular fuzzy numbers (e.g. the maximum membership is less than one), then the cut set of the irregular fuzzy number is assumed to be a crisp value when the level of cut set is greater than its maximum membership. For example, for an irregular fuzzy number (1, 2, 3;0.5) with the maximum membership of 0.5, the interval of the cut set will always be [2, 2] or when the level α of the cut set belongs to (0.5, 1]. Therefore, Eq. (5) in Definition 9 can be used to rank two irregular fuzzy numbers.

Example 1 Let $\tilde{a} = (1, 2, 3)$ and $\tilde{b} = (2, 3, 4)$ be two TFNs. According to Definition 9,

$$\tilde{a}^\alpha = [1 + \alpha, 3 - \alpha]$$

$$\tilde{b}^\alpha = [2 + \alpha, 4 - \alpha];$$

$$p^\alpha(\tilde{a} \geq \tilde{b}) = \max\left\{1 - \max\left\{\frac{(4-\alpha) - (1+\alpha)}{[(4-\alpha) - (2+\alpha)] + [(3-\alpha) - (1+\alpha)]}\right\}, 0\right\};$$

Definition 9 Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be any two TFNs. The possibility degree of $\tilde{a} \geq \tilde{b}$ is defined as follows:

$$p(\tilde{a} \geq \tilde{b}) = \int_0^1 p^\alpha(\tilde{a} \geq \tilde{b}) d\alpha, \tag{5}$$

$$p(\tilde{a} \geq \tilde{b}) = \int_0^1 p^\alpha(\tilde{a} \geq \tilde{b}) d\alpha = 0.0767.$$

The possibility degree between TFNs can be directly computed by adopting the formula in Appendix A.

$$p(\tilde{a} \geq \tilde{b}) = \frac{3\text{-}2}{(4-2)+(3-1)} + \frac{3\text{-}2}{(4-2)+(3-1)} \ln \frac{3-2}{(3-2)+(3-2)} = \frac{1}{4} + \frac{1}{4} \ln \frac{1}{2} = 0.0767$$

Definition 10 Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be any two TFNs. Some comparative relations can be defined as follows:

1. If $p(\tilde{a} \geq \tilde{b}) = p(\tilde{b} \geq \tilde{a}) = 0.5$, then $\tilde{a}$ is indifferent to $\tilde{b}$, and it is marked as $\tilde{a} \sim \tilde{b}$.
2. If $p(\tilde{a} \geq \tilde{b}) > 0.5$, then $\tilde{a}$ is weakly larger than $\tilde{b}$, and it is marked as $\tilde{a} \succ \tilde{b}$.

Theorem 2 Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be any two TFNs. Some properties of the possibility degree of TFNs are satisfied as follows:

1. Normative: $0 \leq p(\tilde{a} \geq \tilde{b}) \leq 1$;
2. Complementary: $p(\tilde{a} \geq \tilde{b}) + p(\tilde{b} \geq \tilde{a}) = 1$;
3. Reflexivity: $p(\tilde{a} \geq \tilde{b}) = p(\tilde{b} \geq \tilde{a}) = 0.5$ if $\tilde{a} = \tilde{b}$.

Proof

1. On the basis of Definition 9 and the first property (normative property) of Theorem 1,

$\forall \alpha \in [0,1],\ 0 \leq p^{\alpha}(\tilde{a} \geq \tilde{b}) \leq 1$. Consequently, $\int_0^1 0 d\alpha \leq p(\tilde{a} \geq \tilde{b}) = \int_0^1 p^{\alpha}(\tilde{a} \geq \tilde{b}) d\alpha \leq \int_0^1 1 d\alpha$.

Therefore, $0 \leq p(\tilde{a} \geq \tilde{b}) \leq 1$.

Thus, the first property (normative property) is proven.

2. In accordance with Definition 9 and the second property (complementary property) of Theorem 1,

$\forall \alpha \in [0,1],\ p^{\alpha}(\tilde{a} \geq \tilde{b}) + p^{\alpha}(\tilde{b} \geq \tilde{a}) = 1$. Consequently,

$$
\begin{aligned}
p(\tilde{a} \geq \tilde{b}) + p(\tilde{b} \geq \tilde{a}) &= \int_0^1 p^{\alpha}(\tilde{a} \geq \tilde{b}) d\alpha + \int_0^1 p^{\alpha}(\tilde{b} \geq \tilde{a}) d\alpha \\
&= \int_0^1 \left[p^{\alpha}(\tilde{a} \geq \tilde{b}) + p^{\alpha}(\tilde{b} \geq \tilde{a}) \right] d\alpha \\
&= \int_0^1 1 d\alpha
\end{aligned}
.
$$

Therefore, $p(\tilde{a} \geq \tilde{b}) + p(\tilde{b} \geq \tilde{a}) = 1$.

Thus, the second property (complementary property) is proven.

3. According to Definition 9 and the third property (reflexivity) of Theorem 1,

if $\tilde{a} = \tilde{b}$, then $\forall \alpha \in [0,1], p^{\alpha}(\tilde{a} \geq \tilde{b}) = 0.5$ and $p^{\alpha}(\tilde{b} \geq \tilde{a}) = 0.5$ Consequently, $p(\tilde{a} \geq \tilde{b}) = \int_0^1 p^{\alpha}(\tilde{a} \geq \tilde{b}) d\alpha = \int_0^1 0.5 d\alpha$ and $p(\tilde{b} \geq \tilde{a}) = \int_0^1 p^{\alpha}(\tilde{b} \geq \tilde{a}) d\alpha = \int_0^1 0.5 d\alpha$.

Therefore, $P(\tilde{a} \geq \tilde{b}) = P(\tilde{b} \geq \tilde{a}) = 0.5$ if $\tilde{a} = \tilde{b}$.

Hence, the reflexivity property of Theorem 1 is proven.

Remark 1 The normative, complementary and reflexivity properties of the defined possibility degree of TFNs have been discussed in Theorem 2. Here, the transitivity property is demonstrated below.

A classic example is shown in Fig. 2. In many studies of fuzzy MCDM, a sequence of TFNs is often used as an uncertain information expression to evaluate alternatives. A TFN is a special interval fuzzy number. The TFNs in such a sequence are usually partially intersecting or non-intersecting.

The sequence of TFNs shown in Fig. 2 is represented by $A_i, i = 1, 2, 3, 4$. For any three different TFNs such as A_i, A_j and A_k, finding $p(A_i \geq A_j) < 0.5$ is easy if and only if $i < j$

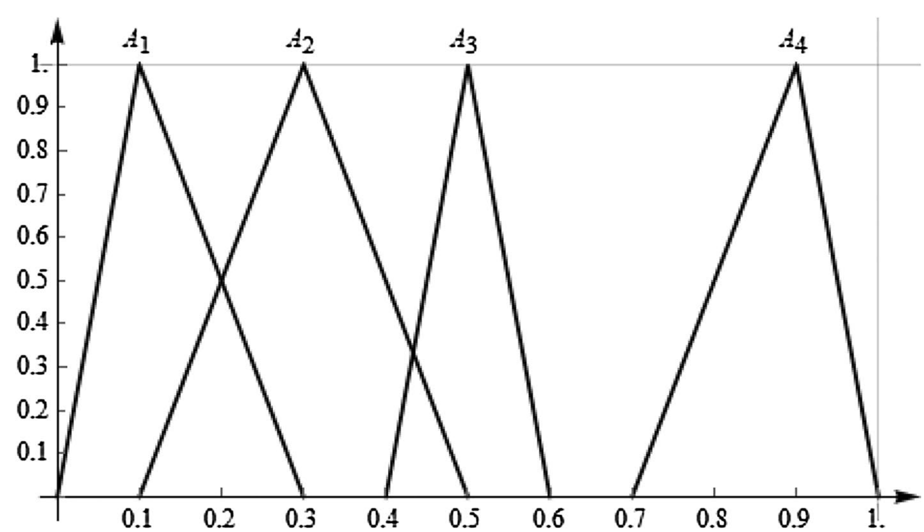

Fig. 2 Sequence of TFNs

(see brief proof in Appendix B). Thus, if $p\left(A_i \geq A_j\right) < 0.5$ and $p\left(A_j \geq A_k\right) < 0.5$, then $i < j$ and $j < k$. Consequently, $i < k$ is satisfied. Therefore, $p\left(A_i \geq A_k\right) < 0.5$.

3.2 Possibility degree of Z-numbers

Definition 11 Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be any two Z-numbers where all the elements in $\{A_1, B_1, A_2, B_2\}$ are TFNs. Some comparative relations can be defined as follows:

1. If $A_1 = A_2$ and $B_1 = B_2$, then Z_1 is absolutely equivalent to Z_2, and they are marked as $Z_1 \equiv Z_2$.
2. If $A_1 \sim A_2$ and $B_1 \sim B_2$, then Z_1 is strictly equivalent to Z_2, and they are marked as $Z_1 = Z_2$.
3. If $A_1 > A_2$ and $B_1 > B_2$, then Z_1 is absolutely larger than Z_2, and they are marked as $Z_1 \gg Z_2$.
4. If $A_1 \succ A_2$ and $B_1 \succ B_2$, then Z_1 is strictly larger than Z_2, and they are marked as $Z_1 > Z_2$.

Definition 12 Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be any two Z-numbers where all the elements in $\{A_1, B_1 A_2, B_2\}$ are TFNs. The possibility degree of $Z_1 \geq Z_2$ can be defined as

$$p(z_1 \geq z_2) = \omega p(A_1 \geq A_2) + (1 - \omega) p(B_1 \geq B_2), \tag{6}$$

where the value of ω lies in the interval of [0,1], and it represents the concern degree of a DM towards the first component A of the Z-number. The equivalent formula is given by

$$p(Z_1 \geq Z_2) = \omega \int_0^1 p^{\alpha}(A_1 \geq A_2) d\alpha + (1 - \omega) \int_0^1 p^{\beta}(B_1 \geq B_2) d\beta. \tag{7}$$

The parameter ω in Eq. (6) can reflect the varying preferences of different DMs. When $0 < \omega < 0.5$, a DM perceives the reliability of an information as more important than the other properties. When $\omega = 0.5$, the ambiguity and reliability of information are equally significant for the DM. When $0.5 < \omega < 1$, the DM is more concerned on the ambiguity of the information. In particular, $\omega = 0$ indicates that the DM only considers the reliability restriction of the information, whereas $\omega = 1$ indicates that the DM is only concerned with the fuzzy restriction of the information.

A Z-number simultaneously considers the ambiguity and reliability properties of an information. According to Zadeh (2011), the first component of Z-number reflects the fuzzy restriction, whereas the second component plays a role in reliability restriction. These two characteristics must be considered when ranking Z-numbers. Furthermore, different DMs have varying risk preferences towards fuzziness and reliability. Therefore, the associated possibility degree formula that has been developed is rational.

Example 2 Let $Z_1 = ((0.1, 0.2, 0.3), (0.2, 0.3, 0.4))$ and $Z_2 = ((0.2, 0.3, 0.4), (0.1, 0.2, 0.3))$ be two Z-numbers. Then, $p(Z_1 \geq Z_2)|_{\omega=0.5} = \omega p(A_1 \geq A_2) + (1 - \omega) p(B_1 \geq B_2) = 0.5 \times 0.0767 + (1 - 0.5) \times 0.9233 = 0.5$, $p(Z_1 \geq Z_2)|_{\omega=0.4} = 0.5847$ and $p(Z_1 \geq Z_2)|_{\omega=0.6} = 0.4153$.

Definition 13 Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be any two Z-numbers where all the elements in $\{A_1, B_1, A_2, B_2\}$ are TFNs. Some comparative relations can be defined as follows:

1. If $p(Z_1 \geq Z_2) = p(Z_2 \geq Z_1) = 0.5$, then Z_1 is indifferent to Z_2, and it is marked as $Z_1 \sim Z_2$.
2. If $p(Z_1 \geq Z_2) > 0.5$, then Z_1 is weakly larger than Z_2, and it is marked as $Z_1 \succ Z_2$.

Theorem 3 Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be any two Z-numbers where the components A and B are TFNs. Some properties about the possibility degree of Z-numbers are satisfied as follows:

1. Normative: $0 \leq p(Z_1 \geq Z_2) \leq 1$;
2. Complementary: $p(Z_1 \geq Z_2) + p(Z_2 \geq Z_1) = 1$;
3. Reflexivity: $p(Z_1 \geq Z_2) = p(Z_2 \geq Z_1) = 0.5$ if $Z_1 = Z_2$.

Proof

1. In accordance with Definition 12 and the first property (normative property) of Theorem 2,

$0 \leq p(A_1 \geq A_2) \leq 1$ and $0 \leq p(B_1 \geq B_2) \leq 1$ Consequently, $\omega \times 0 + (1 - \omega) \times 0 \leq p(Z_1 \geq Z_2) = \omega p(A_1 \geq A_2) + (1 - \omega) p(B_1 \geq B_2) \leq \omega \times 1 + (1 - \omega) \times 1$.

Therefore, the first property (normative property) is proven.

2. According to Definition 12 and the second property (complementary property) of Theorem 2,

$p(A_1 \geq A_2) + p(A_2 \geq A_1) = 1$ and $p(B_1 \geq B_2) + p(B_2 \geq B_1) = 1$. Consequently,

$$\begin{aligned}
&p(Z_1 \geq Z_2) + p(Z_2 \geq Z_1)\\
&= \omega p(A_1 \geq A_2) + (1-\omega)p(B_1 \geq B_2) + \omega p(A_2 \geq A_1) + (1-\omega)p(B_2 \geq B_1)\\
&= \omega\left[p(A_1 \geq A_2) + p(A_2 \geq A_1)\right] + (1-\omega)[p(B_1 \geq B_2) + p(B_2 \geq B_1)]\\
&= \omega + (1+\omega)\\
&= 1
\end{aligned}$$

Hence, the second property (complementary property) is proven.

(i) On the basis of Definition 12 and the third property (reflexivity property) of Theorem 2,

if $Z_1 = Z_2$, then $p(A_1 \geq A_2) = 0.5$, $p(A_2 \geq A_1) = 0.5$, $p(B_1 \geq B_2) = 0.5$ and $p(B_2 \geq B_1) = 0.5$. Consequently, $p(Z_1 \geq Z_2) = \omega p(A_1 \geq A_2) + (1-\omega)p(B_1 \geq B_2) = \omega \times 0.5 +(1-\omega)\times 0.5$. and $p(Z_2 \geq Z_1) = \omega p(A_2 \geq A_1) + (1-\omega)p(B_2 \geq B_1) = \omega \times 0.5 + (1-\omega)\times 0.5$.

Therefore, the third property (reflexivity property) is proven.

Remark 2 Let $Z_1 = \left(A_1, B_1\right)$, $Z_2 = \left(A_2, B_2\right)$ and $Z_3 = \left(A_3, B_3\right)$ be any three Z-numbers, in which $A_1 = (a_1^i, a_2^i, a_3^i)(i = 1, 2, 3)$ and $B_1 = (b_1^i, b_2^i, b_3^i)(i = 1, 2, 3)$ are TFNs. Moreover, $p\left(A_1 \geq A_2\right) < 0.5$, $p\left(A_2 \geq A_3\right) < 0.5$, $p(B_1 \geq B_2) < 0.5$ and $p(B_2 \geq B_3) < 0.5$ are satisfied. Consequently, $p(Z_1 \geq Z_2) \leq 0.5$, $p(Z_2 \geq Z_3) \leq 0.5$ and $p(Z_1 \geq Z_3) \leq 0.5$.

Example 3 Let $Z_1 = ((1, 2, 3), (0.2, 0.3, 0.4))$, $Z_2 = ((2, 3, 4), (0.4, 0.6, 0.7))$ and $Z_3 = ((3, 4, 5), (0.7, 0.8, 1))$ be three given continuous Z-numbers.

According to Definition 12, $p(Z_1 \geq Z_2) = 0.0384 < 0.5$, $p(Z_2 \geq Z_3) = 0.0442 < 0.5$ and $p(Z_2 \geq Z_3) = 0 < 0.5$.

4 Z-PROMETHEE approach for solving MCDM problems

The possibility degree of Z-numbers can meaningfully solve the MCDM problem based on Z-numbers. The possibility degree of Z-numbers can also reflect the difference between two Z-numbers. An extended Z-PROMETHEE approach is therefore developed as follows.

For a MCDM problem using Z-numbers, let $A = \{a_i | i = 1, 2, \dots, m\}$ be a set that includes all the provided alternatives; $c = \{c_j | j = 1, 2, \dots, n\}$ be the collection of criteria; $D = [z_{ij}]_{m\times n} = \left[(A_{ij}, B_{ij})\right]_{m\times n}$ be the decision-making matrix, in which $z_{ij} = (A_{ij}, B_{ij})$ denotes the evaluation of alternative a_i under the criteria c_j; and $W = [Z_j]_{1\times n} = \left[(A_j, B_j)\right]_{1\times n}$ be the weight matrix, in which $z_{ij} = (A_j, B_j)$ reflects the importance of criteria c_j.

Step 1. Normalise the decision-making matrix.

Different criteria require different scales. Moreover, two different sets of criteria exist (i.e. benefit criteria and cost criteria). For discussion purposes, the linear transformation (Kang et al. 2018a, b, c, d; Yaakob and Gegov 2016) is used to eliminate the effects of the differentiations.

$$A_{ij}^N = \begin{cases} \left(\frac{a_{ij}^1}{c_j^+}, \frac{a_{ij}^2}{c_j^+}, \frac{a_{ij}^3}{c_j^+}\right) & j \in B \\ \left(1 - \frac{a_{ij}^3}{c_j^+}, 1 - \frac{a_{ij}^2}{c_j^+}, 1 - \frac{a_{ij}^1}{c_j^+}\right) & j \in C \end{cases}, \tag{8}$$

where B and C are the collections of benefit criteria and cost criteria, respectively, and $C_j^+ = \max_i \left\{a_{ij}^3\right\}$. Thus, the normalised matrix can be denoted as $D^N = \left[z_{ij}^N\right]_{m\times n} = \left[\left(A_{ij}^N, B_{ij}\right)\right]_{m\times n}$.

Step 2. Compute the criteria weights.

When the importance of the criteria is evaluated by using Z-numbers, an appropriate method for acquiring the criteria weight vector must be developed under Z-environment. Xu (2001) proposed an algorithm for the priority of fuzzy complementary judgment matrix. The discussion in Sect. 3 suggests that the possibility degree of Z-numbers is suitable in Xu's algorithm. Hence, the criteria weights can be obtained as follows:

$$w_i = \frac{\sum_{j=1}^n p(c_i, c_j) + \frac{n}{2} - 1}{n(n-1)}, \ i = 1, 2, \dots n, \tag{9}$$

where $p(c_i, c_j) = p(z_i \geq z_j) = \omega p(A_i \geq A_j) + (1-\omega)p(B_i \geq B_j)$; w_i reflects the importance of criteria c_i; and $\sum_{i=1}^n w_i = 1$.

Step 3. Calculate the priority index $\psi\left(a_i, a_k\right)$ of the alternative a_i over the alternative a_k.

$$\psi(a_i, a_k) = \frac{\sum_{j=1}^n w_j p_j(a_i, a_k)}{\sum_{j=1}^n w_j}, \tag{10}$$

where $p_j(a_i, a_k) = p(z_{ij} \geq z_{kj}) - 0.5$. If $p_j\left(a_i, a_k\right) > 0$, which indicates that $p(z_{ij} \geq z_{kj}) > 0.5$, then a_i is better than relative to c_j. Thus, a_i is undifferentiated from a_k for criterion c_j if $p_j(a_i, a_k) = 0$ (i.e. $p_j(z_{ij}, z_{kj}) = 0.5$). In addition, a_i is inferior

Table 1 Decision matrix with linguistic values

	c_1 (VS,C)	c_2 (S,VC)	c_3 (S,C)	c_4 (S,C)	c_5 (S,VC)
a_1	((115,120,125),VU)	(F,C)	(SD,C)	(F,U)	((200,225,250),C)
a_2	((85,90,95),C)	(S,VC)	(D,VU)	(SD,C)	((100,150,200),VC)
a_3	((105,110,115),U)	(VS,C)	(F,C)	(SD,U)	((50,100,150),C)
a_4	((75,80,85),VC)	(F,C)	(S,C)	(S,C)	((100,150,200),N)
a_5	((95,100,105),U)	(VS,N)	(SD,U)	(F,N)	((225,250,300),U)
a_6	((75,80,85),C)	(S,U)	(VS,C)	(F,C)	((50,75,100),C)
a_7	((105,110,115),VU)	(F,C)	(VS,U)	(S,VC)	((150,200,225),U)
a_8	((95,100,105),U)	(SS,U)	(F,VU)	(VS,U)	((50,75,100),VC)

to a_k under c_j if the condition is $p_j\left(a_i, a_k\right) < 0$, which indicates that $p_j(Z_{ij} \geq Z_{kj}) < 0.5$.

Step 4. Compute the outgoing flow, the incoming flow and the net flow of alternative a_i.

(i) Outgoing flow:

$$\phi^+\left(a_i\right) = \sum_{k=1}^{m} \psi\left(a_i, a_k\right) \tag{11}$$

(ii) Incoming flow:

$$\phi^-\left(a_i\right) = \sum_{k=1}^{m} \psi\left(a_k, a_i\right) \tag{12}$$

(iii) Net flow:

$$\phi\left(a_i\right) = \phi^+\left(a_i\right) - \phi^-\left(a_i\right) \tag{13}$$

Step 5. Rank all of the provided alternatives.

The ranking of alternatives can be acquired in light of each alternative's net flow. The larger $\phi\left(a_i\right)$ is, the better a_i will be.

5 Illustrative example

The living standards of people in China have further improved with the prosperity of the Chinese economy. In accordance with Maslow's hierarchy of needs (Maslow 1972), human beings pursue higher spiritual enjoyment to enhance their happiness. The Chinese government has attempted to increase statutory holidays to encourage consumption, indicating a solution akin to 'two birds hit with one stone'. On the one hand, the economy of China has become increasingly dynamic. On the other hand, the life quality of the people has also been greatly improved.

The celebration of the National Day of China is the longest festival in the country. During National Day, many people prefer to go on a short trip with family or friends. To enhance the planned tour, the most suitable travel option should be considered. The selection of travel plans often

Table 2 Codebook of linguistic terms for fuzzy restriction

Linguistic term in S	TFN
VD: very dissatisfied	(0,1,2)
D: dissatisfied	(1,2,3)
SD: slightly dissatisfied	(2,3,4)
F: fare	(3,4,5)
SS: slightly satisfied	(4,5,6)
S: satisfied	(5,6,7)
VS: very satisfied	(6,7,8)

Table 3 Codebook of linguistic terms for reliability restriction

Linguistic term in S'	TFN
VU: very uncertain	(0,0.1,0.3)
U: uncertain	(0.1,0.3,0.5)
N: neutral	(0.3,0.5,0.7)
C: certain	(0.5,0.7,0.9)
VC: very certain	(0.7,0.9,1)

Table 4 Criteria weight distribution matrix

Criterion	c_1	c_2	c_3	c_4	c_5
Weight value	0.2217	0.2203	0.1688	0.1688	0.2203

involves some major factors (Gardashova 2014; Kang et al. 2018a, b, c, d), such as price, service and destination. When evaluating the different travel plans, uncertainties are important aspects of the decision information. On the one hand, the evaluation values of travel planning by using the above criteria are usually fuzzy and imprecise. On the other hand, the reliability of decision information must be considered due to the subjectivity of the evaluation. The issue of travel plan selection can be described appropriately by using Z-numbers combined with a natural language (Zadeh 2011).

A particular example on travel plan selection is used to explain the proposed method. Moreover, to evaluate the different travel plans, the five most critical criteria are determined: c_1 (basic budget), c_2 (location preference), c_3 (scenic

Table 5 Priority index matrix

$\psi(a_i, a_k)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
a_1	0	−0.2404	−0.2113	−0.2459	−0.0095	−0.2503	−0.1331	−0.2289
a_2	0.2404	0	0.0662	−0.0721	0.1590	−0.1301	0.1579	0.1153
a_3	0.2113	−0.0662	0	−0.0891	0.1079	−0.1678	0.1266	−0.0292
a_4	0.2459	0.0721	0.0891	0	0.3665	−0.0495	0.1746	0.1000
a_5	0.0095	−0.1590	−0.1079	−0.3665	0	−0.2479	−0.0691	−0.0159
a_6	0.2503	0.1301	0.1678	0.0495	0.2479	0	0.1862	0.1965
a_7	0.1331	−0.1579	−0.1266	−0.1746	0.0691	−0.1862	0	−0.1194
a_8	0.2289	−0.1153	0.0292	−0.1000	0.0159	−0.1965	0.1194	0

Table 6 Flow matrix

	Outgoing flow	Incoming flow	Net flow
a_1	−1.3195	1.3195	−2.6389
a_2	0.5365	−0.5365	1.0730
a_3	0.0934	−0.0934	0.1867
a_4	0.9987	−0.9987	1.9973
a_5	−0.9566	0.9566	−1.9132
a_6	1.2284	−1.2284	2.4568
a_7	−0.5625	0.5625	−1.1249
a_8	−0.0184	0.0184	−0.0368

Table 7 Ranking results when $0 \le \omega < 0.5$

Ranking	0	0.1	0.2	0.3	0.4
1	a_2	a_2	a_2	a_4	a_6
2	a_4	a_4	a_4	a_2	a_4
3	a_6	a_6	a_6	a_6	a_2
4	a_3	a_3	a_3	a_3	a_3
5	a_1	a_1	a_1	a_8	a_8
6	a_7	a_7	a_8	a_7	a_7
7	a_8	a_8	a_7	a_1	a_5
8	a_5	a_5	a_5	a_5	a_1

Table 8 Ranking results when $0 < \omega \le 0.5$

Ranking	0.6	0.7	0.8	0.9	1
1	a_6	a_6	a_6	a_6	a_6
2	a_4	a_4	a_4	a_4	a_4
3	a_2	a_8	a_8	a_8	a_8
4	a_8	a_2	a_2	a_3	a_3
5	a_3	a_3	a_3	a_2	a_7
6	a_7	a_7	a_7	a_7	a_2
7	a_5	a_5	a_5	a_5	a_5
8	a_1	a_1	a_1	a_1	a_1

security), c_4 (scenic service) and c_5 (invisible consumption). The decision matrix with linguistic values shown in Table 1 is based on these five criteria. The codebooks of linguistic terms are shown in Tables 2 and 3.

5.1 Application of the proposed approach

A numerical example is used to illustrate the feasibility of the proposed approach. The particular procedure is as follows.

Note 2. The DM's risk preference parameter ω is set to be equal to 0.5.

Step 1. Normalise the decision-making matrix.

For simplicity, criteria c_1 and c_5 denote the cost criteria, whilst criteria c_2, c_3 and c_4 represent the benefit criteria. Therefore, the decision matrix can be normalised by using Eq. (8).

Step 2. Compute the criteria weights.

On the basis of Eq. (9) in Sect. 3, the weight values of the criteria are calculated (Table 4).

Step 3. Construct the possibility degree matrix under each criterion and calculate the priority index $\psi(a_i, a_k)$ of alternative a_i over alternative a_k. The obtained results are shown in Table 5.

Step 4. Compute the outgoing flow, incoming flow and net flow of each alternative (Table 6).

Step 5. Rank all of the provided alternatives.

On the basis of Table 6, the order of all the provided alternatives is obtained.

$$a_6 \succ a_4 \succ a_2 \succ a_3 \succ a_8 \succ a_7 \succ a_5 \succ a_1 \quad (14)$$

5.2 Sensitivity analysis

A sensitivity analysis of ω for the possibility degree between Z-numbers is implemented to determine the influence of this parameter to the ranking result. As stated in Definition 12, the value of ω reflects the concern degree towards the fuzzy restriction A of a Z-number. For simplicity, the values of ω are acquired in the collection of $\{\omega | \omega = 0.1k, 0 \le 10, k \in N\}$. The results are shown in Tables 7 and 8.

As shown in Tables 7 and 8, the ranking results of all travel plans change as ω changes. When $0 \le \omega \le 0.2$, the optimal travel plan is a_2; when $0.5 < \omega \le 1$, the optimal

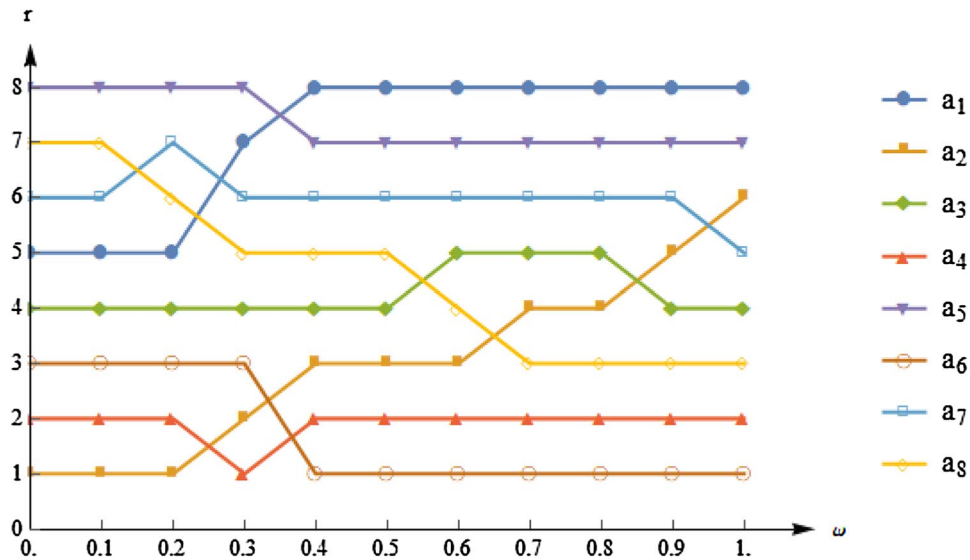

Fig. 3 Ranking results of all travel plans under different ω values

Table 9 Rankings acquired from different methods

Method	Ranking
Aliyev's method in Aliyev (2016)	$a_6 \succ a_4 \succ a_2 \succ a_3 \succ a_8 \succ a_7 \succ a_1 \succ a_5$
Yaakob's method in Yaakob and Gegov (2016)	$a_7 \succ a_6 \succ a_3 \succ a_4 \succ a_8 \succ a_5 \succ a_1 \succ a_2$
Kang's method in Kang et al. (2012a, b)	$a_4 \succ a_6 \succ a_2 \succ a_3 \succ a_8 \succ a_7 \succ a_5 \succ a_1$
Kang's method in Kang et al. (2018a, b, c, d)	$a_4 \succ a_6 \succ a_2 \succ a_3 \succ a_8 \succ a_7 \succ a_5 \succ a_1$
The proposed approach ($\omega = 0.5$)	$a_6 \succ a_4 \succ a_2 \succ a_3 \succ a_8 \succ a_7 \succ a_5 \succ a_1$

Table 10 Comparison of evaluation information among different PROMETHEE methods

	Method I: Fuzzy PROMETHEE	Method II: Z-PROMETHEE	Proposed method
Criteria	TFNs	Z-numbers	Z-numbers
Alternatives	TFNs	TFNs	Z-numbers

travel plan is a_6. Therefore, ω affects the ranking of travel plans, which preliminarily illustrates the rationality of setting the ω parameter. The ranking diagram of all travel plans under different ω values is shown in Fig. 3.

As shown in Fig. 3, the ranking results of all travel plans vary when ω takes different values. In the diagram, the optimal travel plan is a_2 when $0 \le \omega \le 0.2$. The DM at this time pays more attention to the reliability restriction rather than the fuzzy restriction of information. Consequently, the criteria, including c_2 and c_5, become even more important. Criteria c_2 and c_5 considerably affect the travel plan rankings compared with the c_1, c_3 and c_5. Moreover, on the basis of Table 1, the reliability restriction of the evaluation values of travel plan a_2 under criteria c_2 and c_5 is more positive than those of the other travel plans. Thus, the optimal travel plan is a_2 when $0 \le \omega \le 0.2$.

As shown in Fig. 3, the optimal travel plan becomes a_6 when ω is larger than 0.3. The increase in ω indicates that the DM has focused on the fuzzy restriction of information whilst reducing the concern for information reliability, thereby providing a two-pay impact on the ranking of travel plans. On the one hand, the weight of criterion c_1 continues to increase. Moreover, as shown in Table 1, the evaluation value of travel plan a_6 under criterion c_1 is better than any other travel plans. Therefore, the increase in ω is a positive contribution to a_6, which becomes the optimal option. On the other hand, the fuzzy restriction of the evaluation values

Table 11 Rankings acquired from different methods

Methods	Rankings
Fuzzy PROMETHEE in Chen et al. (2011)	$a_6 \succ a_8 \succ a_4 \succ a_7 \succ a_3 \succ a_2 \succ a_5 \succ a_1$
Z–PROMETHEE in Tavakkoli-Moghaddam et al. (2015)	$a_6 \succ a_8 \succ a_4 \succ a_3 \succ a_7 \succ a_2 \succ a_5 \succ a_1$
The proposed PROMETHEE approach ($\omega = 0.5$)	$a_6 \succ a_4 \succ a_2 \succ a_3 \succ a_8 \succ a_7 \succ a_5 \succ a_1$

of travel plan a_6 under the other criteria is generally better than those of the other travel plans. Thus, travel plan a_6 is the optimal solution. Consequently, travel plan a_6 is more likely to be the most satisfactory plan when DMs focus on information reliability.

The proposed Z-PROMETHEE approach shows good stability and feasibility when ω is used as a parameter in sensitivity analysis. On the one hand, the ranking results are relatively consistent when ω lies in some intervals. On the other hand, the ranking results change when ω changes. This finding is generally consistent with our expectation. Therefore, setting ω as a parameter is necessary to rationally reflect a DM's preference. Consequently, the decision-making method based on the possibility degree of Z-numbers can be effectively applied to actual decision scenarios.

6 Comparative analysis

Other methods are for comparison with the proposed method. Here, the comparative analysis has two components. The first component compares the proposed method with the existing method, whilst the second component compares the proposed method with the previously developed PROMETHEE method.

Part I: Comparison of the proposed method with existing methods of Z-information fusion.

Method I. Yaakob and Gegov (2016) presented a TOPSIS method in Z-environments by converting the Z-number into a classical fuzzy number (Kang et al. 2018a, b, c, d). The following formula is used to convert the reliability restriction B of the Z-number into a real number:

$$\alpha = \frac{\int x\mu_B dx}{\int \mu_B dx}. \tag{15}$$

Method II. The method proposed by Kang et al. (2012a, b) is based on another conversion concept, and it is expressed as

$$\begin{aligned} &P(\tilde{A} \otimes \tilde{B}) \\ &= P(\tilde{A}) \times (\tilde{B}) \\ &= \frac{1}{6}(a_1 + 4 \times a_2 + a_3) \times \frac{1}{6}(b_1 + 4 \times b_2 + b_3) \end{aligned} \tag{16}$$

Method III. Kang et al. (2018a, b, c, d) argued that a Z-number can be evaluated by using a real value based on utility theory. The utility formula of a Z-number is

$$\begin{aligned} &TU(Z) \\ &= TU\left(\tilde{A}, \tilde{R}\right) \\ &= \int_0^1 \int_0^1 \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} \left\{ \begin{array}{l} \left[\frac{\tilde{A}^-(\alpha)+\tilde{A}^+(\alpha)}{2} + x\left(\tilde{A}^+(\alpha) - \tilde{A}^-(\alpha)\right)\right] e^{-\left[\tilde{A}^+(\alpha)-\tilde{A}^-(\alpha)\right]^2} \\ \times\left[\frac{\tilde{R}^-(\beta)+\tilde{R}^+(\beta)}{2} + x\left(\tilde{R}^+(\beta) - \tilde{R}^-(\beta)\right)\right] e^{-\left[\tilde{R}^+(\beta)-\tilde{R}^-(\beta)\right]^2} \end{array} \right\} dxdyd\alpha d\beta \end{aligned} \tag{17}$$

Method IV. The decision-making method developed by Aliyev (2016) is based on their proposed distance measure for Z-numbers, and it is expressed as

$$D(Z_1, Z_2) = \frac{1}{n+1} \sum_{k=1}^{n} \left\{ \left| a_{1a_k}^L - a_{2a_k}^L \right| + \left| a_{1a_k}^R - a_{2a_k}^R \right| \right\} + \frac{1}{m+1} \sum_{k=1}^{m} \left\{ \left| b_{1a_k}^L - b_{2a_k}^L \right| + \left| b_{1a_k}^R - b_{2a_k}^R \right| \right\}. \tag{18}$$

The rankings acquired from the abovementioned four methods are shown in Table 9.

The four other methods, which result in the different rankings, can be further explained as follows.

As shown in Table 9, the ranking result obtained by Aliyev (2016) is somewhat identical to that of the proposed method. In addition, the optimal travel plans of the two methods are the same. The ranking method developed by Aliyev (2016) and the proposed ranking method are used to compare the fuzzy restriction and the reliability restriction of the Z-numbers based on the cut-set theory of trapezoidal/triangular fuzzy numbers. However, Aliyev considered a Z-number as a pair of equally important trapezoidal/triangular fuzzy numbers. This approach is inconsistent with that of actual decision-making. In general, DMs have different risk preferences and different degrees of emphasis towards information reliability. Therefore, the ranking result generated by Aliyev (2016) may deviate from actual decision-making.

The ranking result generated in Yaakob's method is somewhat inconsistent with the proposed method. Yaakob and Gegov (2016) presented an extended Z-TOPSIS based on the conversion method of Kang et al. (2012a, b). The conversion method shows two characteristics. Firstly, it fully retains the information of fuzzy restriction A in the Z-number.

Secondly, reliability restriction B is converted into a real number. Although this conversion method has attempted to reduce the complexity of comparing Z-numbers, it results in the information loss of Z-numbers to some extent in relation to the reliability restriction. Therefore, the ranking result produced by the above decision-making method is unconvincing.

The optimal travel plans acquired by the methods of Kang et al. (2018a, b, c, d); Kang et al. (2012a, b) are inconsistent with that of the proposed method. The extant two methods developed two formulas to convert the Z-number into a real value. Consequently, some unreasonable decision results may arise if these two ranking methods are followed. For example, no difference exists between $Z_1 = ((0.1, 0.2, 0.3), (0.4, 0.5, 0.6))$ and $Z_2 = ((0.4, 0.5, 0.6), (0.1, 0.2, 0.3))$ when the methods by Kang et al. (2018a, b, c, d); Kang et al. (2012a, b) are adopted. Obviously, the decision results do not accord with actual decision-making. In fact, the fuzzy restriction and reliability restriction of Z-numbers represent completely different meanings. Therefore, their ranking results may have some deviation from the actual decision-making.

The proposed method based on the possibility degree of Z-numbers does not convert the Z-number into a classical fuzzy number or crisp value. Furthermore, it can cater to the different risk preferences of DMs by adjusting the decision preference parameter. Therefore, the proposed method of ranking Z-numbers based on the possibility degree is more applicable when considering the risk preferences of DMs.

Part II: Comparison of the proposed method with the existing PROMETHEE methods.

Method I. Chen et al. (2011) developed an extended multi-criteria PROMETHEE approach based on Zadeh's fuzzy logic. In their approach, TFNs were used as the uncertain information for criteria and alternative evaluation. The maximum set and minimum set method proposed by Chen (1985) was used to rank TFNs when the net flow of each alternative was calculated.

Method II. Tavakkoli-Moghaddam et al. (2015) proposed an extended multi-criteria Z-PROMETHEE group decision-making method. Criteria evaluation was conducted by using the Z-number, and the alternative evaluation under each criterion was expressed by TFNs. The conversion method in Kang et al. (2012a, b) was initially used to convert the Z-information of the evaluation criteria into TFNs, and the subsequent steps of their Z-PROMETHEE method were the same as those performed by Chen et al. (2011).

A comparison of different sets of evaluation information from the various PROMETHEE methods is shown in Table 10.

The rankings acquired from the different methods are shown in Table 11.

Z-numbers consider the fuzzy restriction and the reliability restriction of the decision information, and this approach differs from those that use traditional fuzzy sets. To effectively compare and analyse the extended PROMETHEE approaches under different information situations, the information for alternative evaluation is obtained and adjusted. Firstly, the reliability restrictions of the evaluation information of the alternatives and the criteria were ignored, and only the fuzzy restriction was considered when all the provided alternatives were sorted by the fuzzy PROMETHEE method of Chen et al. (2011). For example, for the evaluation ((115, 120, 125), *VU*) of a_1 under c_1, the reliability '*VU*' was removed, and only the fuzzy restriction (115, 120, 125) was retained. Secondly, in the Z-PROMETHEE method of Tavakkoli-Moghaddam et al. (2015), the evaluation information of the criteria was adjusted to render it fully reliable, and only the fuzzy restriction was retained. For instance, after removing the reliability restriction during criteria evaluation, all of the criteria except c_1 were regarded equally important because their fuzzy restriction was 'S'.

As shown in Table 11, the best and worst alternatives (i.e. a_6 and a_1, respectively) derived from the two existing PROMETHEE methods are the same as those obtained by the proposed method. Thus, the proposed method and the existing methods are consistent to some extent.

However, some inconsistencies exist between the proposed method and the existing PROMETHEE methods. Firstly, a_3 and a_7 have different priorities when the two existing PROMETHEE methods are used to rank the alternatives. Although the fuzzy restriction of the evaluation information of all the criteria is the same except for c_1, the reliability restrictions of the evaluation information of c_2 and c_5 are higher. Consequently, c_2 and c_5 are more important. Moreover, Table 1 shows that the evaluations of a_3 under c_2 and c_5 are better than those of a_7. Therefore, the existing Z-PROMETHEE methods yielded a_3 with a higher priority compared that of the fuzzy PROMETHEE method. Secondly, the priority of a_8 obtained by the proposed method is lower than the priority of a_8 generated by the two existing methods, particularly because the two existing methods did not consider the reliability of their alternatives' evaluation information. Evidently, as shown in Table 1, the reliability restriction of the evaluation information of a_8 under most criteria is very low. Therefore, a_8 in the proposed approach has a lower priority than the previously developed PROMETHEE methods.

The fuzzy PROMETHEE method proposed by Chen et al. (2011) did not consider the influence of information reliability on MCDM problems. In addition, in the Z-PROMETHEE method of Tavakkoli-Moghaddam et al. (2015), only the criteria were evaluated by the Z-number, whereas the alternative evaluation was expressed by TFNs. The comparative analysis shows that the proposed PROMETHEE method is better than the existing method when the information of the alternatives and the criteria are evaluated by using a Z-number. Furthermore, the proposed PROMETHEE approach is based on the comprehensively weighted possibility degree of Z-numbers, which considers the different effects of fuzzy and reliability restrictions of information during actual decision-making. The previously developed Z-PROETHEE method converted the Z-number into a TFN, which caused information loss to some extent. Overall, the proposed PROMETHEE method is better than the existing PROMETHEE methods.

On the basis of the comparative analysis, some conclusions about the proposed method can be drawn.

1. Adopting the different preferences of DMs indicates good applicability. The numerical example shows that although the decision matrix does not change all the time, the ranking results can vary when ω changes. In other words, even if two DMs use the same Z-evaluation, their expressions can still differ because of their varying preferences. The proposed ranking method is valid and therefore applicable in this case.
2. The information loss of Z-numbers can be reduced to some extent. In particular, the proposed approach does not convert the Z-number into a classical fuzzy number and/or a real number but instead considers the relation between two components of the Z-number. Thus, the proposed method is more faithful to the original concept of the Z-number, and it reduces information distortion.
3. Each extended PROMETHEE method has its own application scope. When the reliability restriction of an information is difficult to obtain under certain decision-making environments, using the traditional fuzzy PROMETHEE decision-making method may be more appropriate. The existing Z-PROMETHEE approach only considered the reliability restriction during criteria evaluation but not during alternative evaluation, which indicates research deficiency. By contrast, as an innovative work, the proposed PROMETHEE method simultaneously considers the reliability restriction during both alternative evaluation and criteria evaluation. Therefore, the proposed PROMETHEE is superior to the existing Z-PROMETHEE methods.

7 Conclusions

Z-number simultaneously considers the ambiguity and the reliability of an information. To fully use Z-numbers, efficient methods for Z-information fusion must be developed to support decision-making activities. A novel concept called the possibility degree of Z-numbers is proposed on the basis of the possibility degree concept of interval numbers to discuss the outranking relations of Z-numbers. The possibility degree formula for the Z-numbers is constructed in two steps. Firstly, the possibility degree of TFNs based on cut-set theory and the possibility degree of interval numbers is defined to serve as the basis of the proposed method. Secondly, the possibility degree formula of the Z-numbers is constructed by combining the possibility degrees of two restriction components of the Z-number by using a single adjustable risk preference parameter. In addition, the numerical example also illustrates the effectiveness of the proposed method to enhance the practical application of cognitive information during decision making under Z-evaluation.

The topics about the possibility degree of Z-numbers are worth studying in future research. Firstly, parameter determination, as a manner of reflecting the varying preferences of different DMs, continues to be important, especially in group decision making. The possible future direction is to determine the value of the risk preference parameter by using intelligent optimisation algorithms. Secondly, considering that Z-numbers can much better describe the objective world when combined with natural language, the proposed ranking method for Z-numbers may play a significant role in the fields of computing with words, artificial intelligence and cognitive computing.

Acknowledgements The authors would like to thank the editors and anonymous reviewers for their great help on this study. This work was supported by the National Natural Science Foundation of China (No. 71871228).

Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interest regarding the publication of this paper.

Appendix A. Special computation of the possibility degree of triangular fuzzy numbers

Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be any two TFNs. The possibility degree of $\tilde{a} \geq \tilde{b}$ can be computed as follows:

If $a_1 = a_2$, $a_2 = a_3$, $b_1 = b_2$ and $b_2 = b_3$, then

$$p(\tilde{a} \geq \tilde{b}) = \begin{cases} 0 & a_2 < b_2 \\ 0.5 & a_2 = b_2 \\ 1 & a_2 > b_2 \end{cases}$$

Otherwise,

$$p(\tilde{a} \geq \tilde{b}) = \begin{cases} 0 & a_2 < b_2 \text{ and } a_3 \leq b_1 \\ \frac{a_3-b_1}{(b_3-b_1)+(a_3-a_1)} + \frac{b_2-a_2}{(b_3-b_1)+(a_3-a_1)} \ln\left(\frac{b_2-a_2}{(b_2-b_1)+(a_3-a_1)}\right) & a_2 < b_2 \text{ and } a_3 > b_1 \\ \frac{a_3-b_1}{(b_3-b_1)+(a_3-a_1)} & a_2 = b_2 \\ \frac{a_3-b_1}{(b_3-b_1)+(a_3-a_1)} + \frac{a_2-b_2}{(b_3-b_1)+(a_3-a_1)} \ln\left(\frac{a_2-b_2}{(a_2-a_1)+(b_3-b_2)}\right) & a_2 > b_2 \text{ and } b_3 > a_1 \\ 1 & a_2 < b_2 \text{ and } b_3 > a_1 \end{cases}$$

Appendix B. Proof for the conclusion in Remark 1.

As shown in Fig. 2, two TFNs, denoted by A_i and $(i < j)$, exist.

If A_i and A_j are non-intersecting (e.g. A_3 and A_4), then $p^\alpha(A_i \geq A_j) = 0$, $\forall \alpha \in [0,1]$ Consequently, $p(A_i \geq A_j) < 0.5$ is satisfied according to Definition 9.

If A_i and A_j are partially intersecting (e.g. A_1 and A_2), whose cut sets under level α are $\left[A_{i\alpha}^-, A_{i\alpha}^+\right]$ and $\left[A_{j\alpha}^-, A_{j\alpha}^+\right]$, then $p^\alpha(A_i \geq A_j) = \begin{cases} 0 & A_{i\alpha}^+ - A_{j\alpha}^- \leq 0 \\ \frac{A_{i\alpha}^+ - A_{j\alpha}^-}{(A_{j\alpha}^+ - A_{j\alpha}^-)+(A_{i\alpha}^+ - A_{i\alpha}^-)} & A_{i\alpha}^+ - A_{j\alpha}^- > 0 \end{cases}$. Furthermore, the following is obtained:

$$\frac{A_{i\alpha}^+ - A_{i\alpha}^-}{(A_{j\alpha}^+ - A_{j\alpha}^-)+(A_{i\alpha}^+ + A_{i\alpha}^-)} \leq \frac{A_{i\alpha}^+ - A_{j\alpha}^- + \left[\frac{A_{j\alpha}^+ + A_{j\alpha}^-}{2} - \frac{A_{i\alpha}^+ + A_{i\alpha}^-}{2}\right]}{(A_{j\alpha}^+ - A_{j\alpha}^+)+(A_{i\alpha}^+ - A_{i\alpha}^-)} = \frac{\frac{1}{2}\left[(A_{j\alpha}^+ - A_{j\alpha}^-)+(A_{i\alpha}^+ - A_{i\alpha}^-)\right]}{(A_{j\alpha}^+ - A_{j\alpha}^-)+(A_{i\alpha}^+ - A_{i\alpha}^-)} = 0.5.$$

Consequently, $p^\alpha(A_i \geq A_j) \leq 0.5$, $\forall \alpha \in [0,1]$ is always true. Therefore, $p(A_i \geq A_j) < 0.5$ is satisfied according to Definition 9.

On the basis of the above definitions, the relevant conclusion can be obtained.

References

Abbasbandy S, Hajjari T (2009) A new approach for ranking of trapezoidal fuzzy numbers. Comput Math Appl 57(3):413–419

Aliev RA, Alizadeh AV, Huseynov OH (2015) The arithmetic of discrete Z-numbers. Inf Sci 290(C):134–155

Aliev RA, Huseynov OH, Serdaroglu R (2016a) Ranking of Z-numbers and its application in decision making. Int J Inf Technol Decis Mak 15(06):1503–1519

Aliev RA, Huseynov OH, Zeinalova LM (2016b) The arithmetic of continuous Z-numbers. Inf Sci 373(C):441–460

Aliyev RR (2016) Multi-attribute decision making based on Z-valuation. Procedia Comput Sci 102(C):218–222

Bakar ASA, Gegov A (2015) Multi-Layer decision methodology for ranking Z-numbers. Int J Comput Intell Syst 8(2):395–406

Brans JP, Vincke P, Mareschal B (1986) How to select and how to rank projects: the promethee method. Eur J Oper Res 24(2):228–238

Casasnovas J, Riera JV, On the addition of discrete fuzzy numbers, In: Proceedings of the 5th WSEAS international conference on Telecommunications and informatics, World Scientific and Engineering Academy and Society (WSEAS), Istanbul, Turkey, 2006, pp. 432–437

Chen S-H (1985) Ranking fuzzy numbers with maximizing set and minimizing set. Fuzzy Sets Syst 17(2):113–129

Chen Y-H, Wang T-C, Wu C-Y (2011) Strategic decisions using the fuzzy PROMETHEE for IS outsourcing. Expert Syst Appl 38(10):13216–13222

Chou C-C (2003) The canonical representation of multiplication operation on triangular fuzzy numbers. Comput Math Appl 45(10–11):1601–1610

Ding X-F, Liu H-C (2018) A 2-dimension uncertain linguistic DEMATEL method for identifying critical success factors in emergency management. Appl Soft Comput 71:386–395

Gao F-j (2013) Possibility degree and comprehensive priority of interval numbers. Syst Eng Theory Pract 33(8):2033–2040

Gardashova LA (2014) Application of operational approaches to solving decision making problem using Z-numbers. Appl Math 05(09):1323–1334

Hu J-h, Yang Y, Zhang X-l, Chen X-h (2018) Similarity and entropy measures for hesitant fuzzy sets. Int Trans Oper Res 25(3):857–886

Hu Y-P, You X-Y, Wang L, Liu H-C (2018) An integrated approach for failure mode and effect analysis based on uncertain linguistic GRA–TOPSIS method. Soft Comput. https://doi.org/10.1007/s00500-018-3480-7

Huang J, You X-Y, Liu H-C, Si S-L (2018) New approach for quality function deployment based on proportional hesitant fuzzy linguistic term sets and prospect theory. Int J Production Res. https://doi.org/10.1080/00207543.2018.1470343

Ji P, Zhang H, Wang J (2018) A fuzzy decision support model with sentiment analysis for items comparison in e-commerce: the case study of PConline.com. IEEE Trans Syst Man Cybernetics. https://doi.org/10.1109/TSMC.2018.2875163

Kang B, Wei D, Li Y, Deng Y (2012a) Decision making using Z-numbers under uncertain environment. J Comput Inf Syst 8(7):2807–2814

Kang B, Wei D, Li Y, Deng Y (2012b) A method of converting Z-number to classical fuzzy number. J Inf Comput Sci 9(3):703–709

Kang B, Chhipi-Shrestha G, Deng Y, Hewage K, Sadiq R (2018a) Stable strategies analysis based on the utility of Z-number in the evolutionary games. Appl Math Comput 324:202–217

Kang B, Deng Y, Hewage K, Sadiq R (2018b) Generating Z-number based on OWA weights using maximum entropy. Int J Intell Syst 33(8):1745–1755

Kang B, Deng Y, Hewage K, Sadiq R (2018c) A method of measuring uncertainty for Z-number. IEEE Trans Fuzzy Syst. https://doi.org/10.1109/TFUZZ.2018.2868496

Kang B, Deng Y, Sadiq R (2018d) Total utility of Z-number. Appl Intell 48(3):703–729

Li J, Wang J (2017) Multi-criteria outranking methods with hesitant probabilistic fuzzy sets. Cognit Comput 9(5):611–625

Li J, Wang J, Hu J (2018) Multi-criteria decision-making method based on dominance degree and BWM with probabilistic hesitant fuzzy information. Int J Mach Learn Cybernetics. https://doi.org/10.1007/s13042-018-0845-2

Liang R, Wang J, Zhang H. Projection-Based PROMETHEE (2018) Methods based on hesitant fuzzy linguistic term sets. Int J Fuzzy Syst 20(7):2161–2174

Liu H, Li Z, Song W, Su Q (2017) Failure mode and effect analysis using cloud model theory and PROMETHEE method. IEEE Trans Reliab 66(4):1058–1072

Liu H-C, Quan M-Y, Shi H, Guo C (2019) An integrated MCDM method for robot selection under interval-valued pythagorean uncertain linguistic environment. Int J Intell Syst 34(2):188–214

Mardani A, Jusoh A, Zavadskas EK (2015) Fuzzy multiple criteria decision-making techniques and applications—two decades review from 1994 to 2014. Expert Syst Appl 42(8):4126–4148

Maslow AH (1972) A theory of human motivation. Psychol Rev 50(1):370–396

Peng H-g, Wang J-q (2017) Hesitant uncertain linguistic Z-numbers and their application in multi-criteria group decision-making problems. Int J Fuzzy Syst 19(5):1300–1316

Peng H-g, Wang J-q (2018) A multicriteria group decision-making method based on the normal cloud model With Zadeh's Z-numbers. IEEE Trans Fuzzy Syst 26(6):3246–3260

Peng J-J, Wang J-Q, Wu X-H (2016) Novel Multi-criteria decision-making approaches based on hesitant fuzzy sets and prospect theory. Int J Inform Technol Decis Mak 15(03):621–643

Peng H-g, Wang X-k, Wang T-l, Wang J-q (2019) Multi-criteria game model based on the pairwise comparisons of strategies with Z-numbers. Appl Soft Comput 74:451–465

Shen K-w, Wang J-q (2018) Z-VIKOR method based on a new weighted comprehensive distance measure of Z-number and its application. IEEE Trans Fuzzy Syst 26(6):3232–3245

Tavakkoli-Moghaddam R, Sotoudeh-Anvari A, Siadat A (2015) A multi-criteria group decision-making approach for facility location selection using PROMETHEE under a fuzzy environment, In: B. Kamiński, G.E. Kersten, T. Szapiro (Eds.) Outlooks and insights on group decision and negotiation: 15th international conference, GDN 2015, Warsaw, Poland, June 22–26, Proceedings, Springer International Publishing, Cham, 2015, pp. 145–156

Tian Z, Wang J, Wang J, Zhang H (2017) Simplified neutrosophic linguistic multi-criteria group decision-making approach to green product development. Group Decis Negot 26(3):597–627

Voxman W (2001) Canonical representations of discrete fuzzy numbers. Fuzzy sets Syst 118(3):457–466

Wang J-q, Peng J-j, Zhang H-y, Liu T, Chen X-h (2015) An uncertain linguistic multi-criteria group decision-making method based on a cloud model. Group Decis Negot 24(1):171–192

Wang J-q, Cao Y-x, Zhang H-y (2017) Multi-criteria decision-making method based on distance measure and choquet integral for linguistic Z-numbers. Cognitive Comput 9(6):827–842

Wang J, Wang J-q, Tian Z-p, Zhao D-y (2018) A multi-hesitant fuzzy linguistic multicriteria decision-making approach for logistics outsourcing with incomplete weight information. Int Trans Oper Res 25(3):831–856

Wang X, Wang J, Zhang H (2019) Distance-based multicriteria group decision-making approach with probabilistic linguistic term sets. Expert Syst 36:e12352

Wu D, Liu X, Xue F, Zheng H, Shou Y, Jiang W (2018) A new medical diagnosis method based on Z-numbers. Appl Intell 48(4):854–867

Xu Z-s (2001) Algorithm for priority of fuzzy complementary judgement matrix. J Syst Eng 16(4):311–314

Xu Z, Da Q (2002) Multi-attribute decision making based on fuzzy linguistic assessments. J Southeast Univ (Nat Sci Ed) 32(4):656–658

Xu Z-s, Da Q-l (2003) Possibility degree method for ranking interval numbers and its application. J Syst Eng 18(1):67–70

Xue Y-x, You J-x, Zhao X-f Liu H-c (2016) An integrated linguistic MCDM approach for robot evaluation and selection with incomplete weight information. Int J Prod Res 54(18):5452–5467

Yaakob AM, Gegov A (2016) Interactive TOPSIS based group decision making methodology using Z-numbers. Int J Comput Intell Syst 9(2):311–324

Yager RR (2012) On Z-valuations using Zadeh's Z-numbers. Int J Intell Syst 27(3):259–278

Yang Y, Wang J-q (2018) SMAA-based model for decision aiding using regret theory in discrete Z-number context. Appl Soft Comput 65:590–602

Yao J-S, Chiang J (2003) Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. Eur J Oper Res 148(2):401–409

Yao J-S, Ouyang L-Y, Chang H-C (2003) Models for a fuzzy inventory of two replaceable merchandises without backorder based on the signed distance of fuzzy sets. Eur J Oper Res 150(3):601–616

Zadeh LA (1965) Fuzzy sets. Inform Control 8(3):338–353

Zadeh LA (2011) A note on Z-numbers. Inf Sci 181(14):2923–2932