



Determination of optimal reserve contribution of thermal units to afford the wind power uncertainty

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Abstract

Due to unforeseen variations in wind speed profiles, wind farm integrations are recognized as intermittent and uncertain energy contributors. More specifically, integration of such renewable energy resources aligned with the conventional thermal units although reduces the emissions and brings about a clean environment, it introduces serious problems in assigning optimal and reliable level of these units in load supplying and spinning reserve provision. This situation is more intensified considering the uncertainties arisen by the power system loading demand. To facilitate such operational hurdles, the ongoing study puts forward an efficient model for assigning the optimal spinning reserve which accommodates the uncertainties in both the wind speed and load profiles. Stochastic behavior of these parameters is simulated by generating a proper number of scenarios through the Monte Carlo simulation (MCS) approach. Then, each of these scenarios is evaluated based on the established linear mixed integer approach in a deterministic fashion. Accordingly, a computationally efficient approach is obtained paving the way for real-world implementations and assuring the global optimum results. The proposed approach is applied to a 12-unit test system including 10 thermal units and 2 wind farms. Results are reflected in terms of the commitment status, energy dispatches, and reserve contributions of each committed unit. A comprehensive discussion is conducted to disclose the possible improvements.

Keywords Wind farm integrations · Wind speed and load uncertainties · Stochastic analysis · Mixed-integer linear programming · Increased wind energy deployment · Emission reduction.

List of symbols

Sets and indices

g	Index of generating units.
t, T	Index and set of time intervals
s	Index of scenarios
d	Index of load points
w	Index of wind farms

Constants and parameters

$P_d(t)$	Active power demand at load point d at time t
$v_i(t)$	Wind speed at wind farm i at time t
$A(g)$	Coefficient of the piecewise linear production cost function of unit g
a_g, b_g, c_g	Coefficients of the quadratic production cost function of unit g
$\alpha_g, \beta_g, \gamma_g$	Coefficients of the quadratic emission function of unit g
$f(l, g)$	Slope of block l of the piecewise linear production cost function of unit g
NL	Number of segments in piecewise linearization approach
$OFC(g)$	Operation and maintenance fixed cost of thermal unit g
$OFC(w)$	Operation and maintenance fixed cost of wind farm w
$OVC(g)$	Operation and maintenance variable cost of thermal unit g
$OVC(w)$	Operation and maintenance variable cost of wind farm w

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$P_G^{\max}(g)$	Maximum power generating capacity of thermal unit g
$P_G^{\min}(g)$	Minimum power generating limit of thermal unit g
P_r^S	Probability of scenario s
RW	A fraction of total wind power considered as the reserve requirement due to wind power prediction errors
$T_g(l)$	Upper limit in each segment of the linearized cost function of thermal unit g
$n(t)$	Number of hours at time interval t
Variables	
$P_{GD}^S(g, t)$	Load contribution of thermal unit g at time t in scenario s
$P_{GR}^S(g, t)$	Reserve contribution of thermal unit g at time t in scenario s
$P_R^S(t)$	Fraction of total system load as the reserve requirement at time t in scenario s
$P_W^S(w, t)$	Generation of wind farm w at time t in scenario s
ρ^S	Probability of scenario s
$u_{i,t}^S$	Decision variables of unit i at time t in scenario s . (on = 1, off = 0)
$x_{i,t}^S$	State variables of unit i at time t in scenario s
ξ^S	Vector of scenario s

1 Introduction

In power system engineering, unit commitment (UC) problem is an important optimization process which is carried out to determine on/off state of generating units over a specific time horizon and also to indicate the contribution of thermal units in spinning reserve provision. The main purpose is to minimize the power system operation costs. However, the generation-consumption balance along with other technical constraints should be satisfied, too (Handschin and Slomski 1990). Moreover, in recent years, the looming energy crisis and the environmental concerns have expedited a swift integration of renewable energy resources in power systems. Among the existing renewable resources, wind energy stands as one of the prosperous resources in affording a remarkable volume of clean and costless energy for the societies. Although, the wind energy contributes to an economically and environmentally friendly solution, its intermittent and uncertain nature poses serious technical concerns in power system studies (Liu et al. 2018). Moreover, the power system loading demand has a varying nature which could instigate the generation-consumption imbalances (Reddy et al. 2015). Although development of efficient forecasting mechanisms (Ghadimi et al. 2017) depresses such impediments, there is a substantial need for establishing well-defined operational

platforms. Such a task is more highlighted in simultaneous commitment of wind farm integrations with the conventional thermal units. In these situations and considering the real-time operations, several technical constraints such as the spinning reserve and the ramping capabilities could restrict these units from efficiently responding to the variations in wind power generation (Zou et al. 2015). Therefore, UC problem should be solved such that the optimal level of spinning reserve would be determined to respond to the wind power uncertainties.

UC problem demonstrates a nonlinear, large-scale, and mixed integer feature which could be tackled based on different approaches. Devising efficient UC approaches grants extra monetary savings for power system operators and generating companies. To assure such economic improvements, this problem has been explored by several academicians and industry researchers in the past decades. Heuristics methods (Kjeldsen and Chiarandini 2012), dynamic programming (Kumar and Palanisamy 2007), intelligent algorithms such as particle swarm optimization (Zhang et al. 2016), harmony search (Afkousi et al. 2010), neural networks (Dieu and Ongsakul 2007), mixed integer non-linear programming (MINLP) (Yang et al. 2012), mixed integer linear programming (MILP) (Li et al. 2014), and Lagrangian relaxation (Yu and Zhang 2014) are some of the implemented approaches. Lagrangian relaxation approach is more consistent for large-scale and sizeable problems. Thus, it is one of the mostly deployed approaches for solving UC problems. What should be mentioned is that this method requires the application of heuristics algorithms to achieve a feasible solution. Consequently, the obtained results might be steered to suboptimal solutions which could be intensified due to nonconvex nature of the UC problem. Likewise, some authors have proposed the application of commercial solvers to obtain the numeric solution of the proposed UC model. In this regard, Vieira et al. have developed a UC formulation taking into account the presence of wind farms and pump storage units (Vieira et al. 2016). The developed model points out a deterministic nature with MINLP fashion. Similar to Lagrangian relaxation approach, the MINLP problems are faced with optimality concerns and do not grant the global optimum solutions. Contrarily, the MILP formulation is shown to be explicitly handled by the available solvers and guarantees global optimal solutions. As well, the convergence process is terminated at a more reduced computational time (Li and Shahidehpour 2005).

In the literatures, different strategies have been devised to accommodate the uncertain nature of wind speed and load profiles. The two most applicable approaches are reported as fuzzy modeling and stochastic evaluation mechanism (Aien et al. 2016). Stochastic programming considers the existing uncertain parameters which could be integrated in the developed UC mechanisms. In this approach, the uncertain

behaviour of different parameters is modeled based on probability density functions (PDFs). Then, adequate scenarios are generated based on these PDFs and included in the probabilistic analysis of the problem. Several authors have endeavoured to include the existing uncertainties in assessing the UC problem. Although the authors in Siahkali and Vakilian (2010) have considered the wind power and load uncertainties in UC problem, the established models lie within the MINLP manners. Thus, the optimality concerns of the obtained results are still present and the computational burden of the problem could depreciate the practical worth of the proposed approaches. In an attempt to avert the nonlinearity feature of UC problem, authors in Carrión and Arroyo (2006) have presents a new MILP formulation for the unit commitment problem of thermal units where the cost function of thermal unit is accurately approximated by a set of piecewise blocks. However, all of the parameter are considered deterministic. A two-stage linear model is explored in Huang et al. (2014) which addresses the possible uncertainties in load demand and storage units although no any attention is paid on renewable and intermittent energy resources. Moreover, a linear formulation is adopted for stochastic UC problem which includes the uncertainties in demand response participation by end users (Liu and Tomovic 2015). The proposed model although is a well-established approach, it does not address the substantial trend in renewable and intermittent energy resources. Besides, the authors in Kazemi et al. (2016) have developed a stochastic UC problem which includes the uncertainties contributed by renewable energy resources. The established approach however deploys the priority list of generating units in assigning the on/off state of generating units. This practice although lessens the computational burden of the problem, could depreciate the optimality of the results.

With respect to the outlined context, this paper intends to develop a linear stochastic approach for UC problem in thermal power plants along with the wind farm integrations so that the optimal spinning reserves are assigned to thermal units. To preserve the mixed integer linear characteristics of the proposed model, a set of piecewise and linear representations are replaced with the nonlinear expressions. The proposed approach not only considers the typical constraints in thermal power plants but also accommodates the relevant constraints of wind power generation. At each time interval, wind speed uncertainty is modeled based on several scenarios extracted from the corresponding PDFs based on Monte Carlo simulation (MCS) approach. Each of these scenarios is then evaluated based on the established approach in a deterministic fashion. Considering the stochastic nature of the wind power generation and also the variations in power system loading demand, the proposed approach determines the

commitment status, energy dispatches, and reserve contributions of each committed unit. Thus, an optimal commitment strategy is obtained which minimizes the overall costs of energy and reserve provisions. In brief, the main contributions of this approach could be listed as follows:

- Considering the uncertainties of load profile and wind speed in UC problem with MILP formulation;
- The application of MCS approach affords a proper handling of uncertainties in wind speed and load profiles;
- The linear feature of the proposed approach assures the global optimum result for the commitment solution of the thermal units and wind farm integrations;
- The linear feature of the proposed model is consistent with several commercial solvers paving the way for real-world implementations of the proposed approach.

This papers proceeds as follow. Section 2 addresses the uncertainty modeling and scenario generation approaches. Section 3 develops the proposed stochastic UC approach adopted for the thermal and wind farm integrations. The fundamental mathematical formulation, running constraints, and the linearization schemes are discussed in depth. Section 4 presents extensive numerical results to validate the anticipated performance of the proposed model. Precise discussions are provided to highlight the outperformance of the proposed approach. Eventually, the concluding remarks are provided in the last section.

2 Uncertainty considerations

A brief glossary of this section could be found as follows. As clarified earlier, the inevitable partial imperfectness of forecasting mechanisms introduces several uncertainties in UC problem. In wind-thermal generating combinations, as the case of the ongoing study, the wind speed and load uncertainties are recognized as the most effective parameters in UC solutions. Generally, the stochastic behavior of these parameters is studied based on properly devised PDFs. Deploying the MCS approach, a set of scenarios are generated based on the founded PDFs. To assure the global optimal solutions and also to decrease the computational burden of the problem, the proposed UC approach is made linearized. Then, each of the scenarios is analyzed to compute the total operation cost based on the proposed model on mid-term intervals. Eventually, the expected values are determined according to the obtained solutions and considering the corresponding probability of each scenario. A detailed representation of these procedures is provided in the following subsections.

2.1 Load uncertainty modeling

Typically, the normal distribution function is the most widely deployed PDF for representing the uncertainties in load forecasting errors. Let assume that $P_d(t)$ is the forecasted weekly peak load at time interval t . Then, its forecasting error is modeled with a normal distribution function in which $\mu_d(t)$ and $\sigma_d(t)$ are the mean and standard deviation of the forecasted load patterns, respectively. Equation (1) is the mathematical representation of these statements (Billinton and Allan 1992). At each time interval of t , these quantities could be determined based on the historic statistical analyses.

$$f(P_d(t)) = \frac{1}{\sqrt{2\pi \times \sigma_d(t)^2}} \exp\left(\frac{-(P_d(t) - \mu_d(t))^2}{2\sigma_d(t)^2}\right) \quad (1)$$

2.2 Uncertainty modeling in wind farm integrations

In a similar manner, the wind speed forecasting error is represented through a suitable PDF. Generally, the Gaussian distribution is recognized as the most suitable one for representing the differences among the predicted and the measured wind speeds (Lange 2005). A simple mathematical modification of the Gaussian PDF ends in the Rayleigh distribution represented based on the average wind speed $\bar{v}_i(t)$. Equation (2) is the mathematical representation of the Rayleigh distribution at time interval of t adopted for i th wind farm (Liang 2014).

$$f(v_i(t)) = \left(\frac{\pi \times v_i(t)}{2 \times \bar{v}_i(t)^2}\right) \exp\left(-\frac{\pi}{4} \left(\frac{v_i(t)}{\bar{v}_i(t)}\right)^2\right) \quad (2)$$

Considering a specific wind speed value, the output power of each wind turbine is determined based on the corresponding power curve depicted in Fig. 1. As can be seen, a wind turbine is designed such that it starts the power generation at a specific wind speed called as cut-in speed v_{ci} . Moreover, the maximum generating capacity of a wind turbine is occurred at the rated wind speed denoted by v_r . For the wind speeds extended in the span of v_{ci} to v_r , this figure reveals a nonlinear relationship for the output power generation versus the wind speed value. This notice is mathematically represented based on Eq. (3) (Siahkali and Vakilian 2010). At each time interval of t and considering one specific wind turbine, the maximum durable wind speed is limited to v_{co} known as the cut-out speed. At this point, the output power generation of wind turbine is stopped due to safety reasons. It should be mentioned that for the wind speeds located between the rated and cut-out speeds, the wind turbine produces its rated output power.

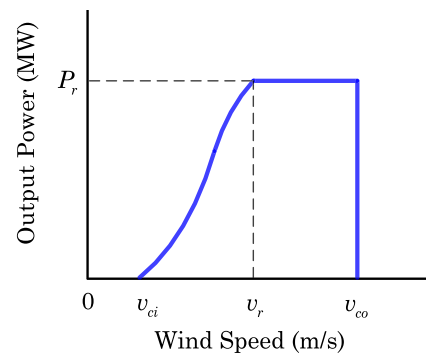


Fig. 1 Power curve of a single wind turbine unit

$$P_{wi}(t) = \begin{cases} 0 & 0 \leq v_i(t) \leq v_{ci} \\ P_r \times (A + B \times v_i(t) + C \times v_i(t)^2) & v_{ci} \leq v_i(t) \leq v_r \\ P_r & v_r \leq v_i(t) \leq v_{co} \\ 0 & v_i(t) \geq v_{co} \end{cases} \quad (3)$$

In this study, the cut-in, rated, and cut-out wind speeds are respectively equal to 3, 11.5, and 25 m/s. Also, the rated output power of each wind turbine equals to 2 MW. Moreover, each wind farm is assumed to contain 40 identical wind turbines.

2.3 Scenario generation approach

To attain the most proper arrangement of scenarios, the inverse transform approach is deployed in scenario generation task (Siahkali and Vakilian 2010). In this process, it is assumed that at each time interval of t , the load profile and wind speed PDFs are known. Consequently, the cumulative distribution functions (CDFs) are easily attained for each of the embedded PDFs. Then, a random decimal number located between zero and one is selected and mapped to the CDF curves. By this way, it is possible to capture the corresponding value of each uncertain parameter. In this study, there are 3 uncertain parameters. One refers back to the loading demand of the system and the next two parameters allude to the wind speed values at 2 different farms. As well, in mid-term commitment horizon, 12 time intervals are assumed in a weekly basis. Accordingly, each scenario (ξ^S) contains 36 components and is described by $\xi^S = \{\xi_d^S, \xi_{W1}^S, \xi_{W2}^S\}$ constructed over the whole time period. Each component is a random variable generated based on the clarified approach and dealt within the established UC framework.

3 Stochastic UC approach: mathematical formulation and linearization process

To determine the optimal spinning reserve of thermal power plants, solving UC problem is required since in reserve provision only those thermal units could contribute which are committed. In this section, the stochastic model of UC problem is introduced.

3.1 Objective function and the running constraints

This section develops the fundamental mathematical basis adopted for the stochastic mid-term UC problem. The time horizon is assumed as one season in a weekly basis. With respect to the mid-term scheduling time intervals, the ramping constraints and the minimum up/down concerns are overlooked. The proposed UC problem seeks for the minimum cost operational commitment of generating units. Equation (4) formulates the contemplated objective function. As can be seen, at each scenario of s , the total costs includes the following:

- Fuel cost of thermal units;
- Variable operation cost for generating power and spinning reserve in thermal units;
- Fixed operation cost of thermal power plants;
- Variable and fixed operation cost of wind turbines.

$$\begin{aligned}
 & \text{Min } J(x_{i,t}^s, u_{i,t}^s, \xi^s) \\
 & = \sum_{t=1}^T \sum_{g=1}^{N_G} \{FC(P_{GD}^S(g, t)) \times n(t)\} \times u_{g,t}^s \\
 & + \sum_{t=1}^T \sum_{g=1}^{N_G} \{(P_{GD}^S(g, t) + P_{GR}^S(g, t)) \times OVC(g) \times n(t)\} \times u_{g,t}^s \\
 & + \sum_{t=1}^T \sum_{g=1}^{N_G} \left\{ P_G^{\max}(g) \times OFC(g) \times \frac{n(t)}{8760} \right\} \\
 & + \sum_{t=1}^T \sum_{w=1}^{N_W} \{P_W^S(w, t) \times OVC(w) \times n(t)\} \times u_{w,t}^s \\
 & + \sum_{t=1}^T \sum_{w=1}^{N_W} \left\{ P_W^{\max}(w) \times OFC(w) \times \frac{n(t)}{8760} \right\}
 \end{aligned} \tag{4}$$

where decision variables $u_{i,t}^s$ indicate the units' status (on/off) and the state variables $x_{i,t}^s$ determine the units' generation including both thermal units and wind farms. To assure a technically satisfied solution, the established approach

should satisfy the generation-consumption balance. Also, it should preserve the constraints due to the required reserve quantities and the constraints in wind power generation and also the thermal generating units. These constraints can be formulated as follows.

$$\sum_{g=1}^{N_G} P_{GD}^S(g, t) + \sum_{w=1}^{N_W} P_W^S(w, t) = P_d^S(t) \quad t = 1, 2, \dots, T \tag{5}$$

This constraint describes the balance of generated power by thermal units and wind turbines versus the load demand within each scenario.

$$\sum_{g=1}^{N_G} P_{GR}^S(g, t) \geq RW \times \sum_{w=1}^{N_W} P_W^S(w, t) + P_R^S(t) \tag{6}$$

To compensate the fluctuations in wind power generation, a suitable percentage of its total generation is included as reserve capacity. This point is modeled by RW in Eq. (6). In this study, 10% fluctuation is considered in wind power generation. Also, the second term in Eq. (6) covers the uncertainties arisen by the forecasting mechanisms errors. It is assumed to be a specified percentage of total load demand e.g. 5%.

$$P_{GD}^S(g, t) + P_{GR}^S(g, t) \leq P_G^{\max}(g) \times u_{g,t}^s \tag{7}$$

$$P_G^{\min}(g) \times u_{g,t}^s \leq P_{GD}^S(g, t) \tag{8}$$

The generated power of thermal units should be greater than a lower limit when they are committed due to technical issues. In addition, the summation of generated power and spinning reserve should be lower than thermal unit capacity. These two constraints determine the lower and upper limits for thermal power plant described in above equations. It should be mentioned that some UC constraints such as unit ramp up/ramp down and also the unit starting cost are not considered in this study as the investigated horizon lies in a mid-time basis. These assumptions are in accordance to Siahkali and Vakilian (2010). Specifically speaking, within a weekly time interval in mid-term UC, these constraints do not affect the results.

3.2 Linearization process

In UC problem, the fuel cost function of thermal units is typically represented based on a quadratic function. This feature is represented by the following equation.

$$FC(P_{GD}(g, t)) = a_g + b_g \times P_{GD}(g, t) + c_g \times (P_{GD}(g, t))^2 \tag{9}$$

This expression demonstrates a nonlinear feature which threatens the optimality conditions and increases the computational burden of the problem. To avert such impediments, a linear representation is devised for this equation. The main notion is based on the piecewise linear segmentation of the quadratic curve represented in Fig. 2.

Equations (10)–(16) address the mathematical statements of the proposed approach. It should be mentioned that adequate number of pieces ends in negligible approximation errors without sacrificing the modeling precision (Carrión and Arroyo 2006).

$$FC(P_{GD}(g,t)) = A(g) \times u_{g,t} + \sum_{l=1}^{NL} f(l,g) \times Pl(l,g,t) \quad (10)$$

$$A(g) = a_g + b_g \times P_G^{\min}(g) + c_g \times (P_G^{\min}(g))^2 \quad (11)$$

$$P_G(g,t) = \sum_{l=1}^{NL} Pl(l,g,t) + P_G^{\min}(g) \times u_{g,t} \quad (12)$$

$$Pl(1,g,t) \leq T_g(1) - P_G^{\min}(g) \quad (13)$$

$$Pl(l,g,t) \leq T_g(l) - T_g(l-1), l = 2, \dots, NL - 1 \quad (14)$$

$$Pl(NL,g,t) \leq P_G^{\max}(g) - T_g(NL - 1) \quad (15)$$

$$Pl(l,g,t) \geq 0 \quad (16)$$

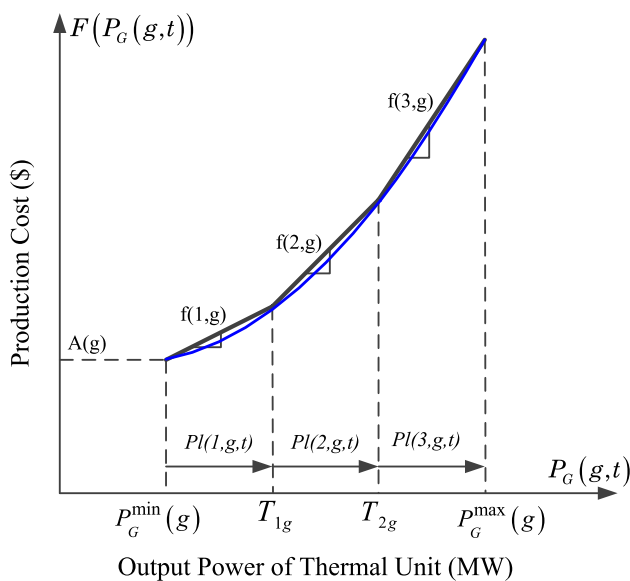


Fig. 2 Piecewise linearization approach adopted for the quadratic cost function representation

3.3 Thermal units emission

Thermal power plants are one of the main sources of emissions. Therefore, development of renewable energy resources could reduce the emission and help to realize a clean environment. The amount of emission due to a thermal power plant could be expressed as a polynomial function whose order depends on the preferred accuracy (Saber and Venayagamoorthy 2012). In this study, a quadratic function is considered for emission estimation which is represented as follows (Saber and Venayagamoorthy 2012).

$$EW(P_{GD}(g,t)) = \alpha_g + \beta_g \times P_{GD}(g,t) + \gamma_g \times (P_{GD}(g,t))^2 \quad (17)$$

By increasing the share of wind power in demand supply, the generated power from thermal units and hence the emission would be diminished.

3.4 Stochastic UC approach

The stochastic UC problem analyzes a different set of scenarios, stochastically. Mathematically speaking, each of the investigated scenarios is assessed in a deterministic manner based on the developed UC formulation abbreviated in Eq. (18).

$$\begin{aligned} &\text{Min } J(x_{i,t}^S, u_{i,t}^S, \xi_i^S) \\ &\text{s.t. all constraints (5) to (8)} \end{aligned} \quad (18)$$

For each of the scenarios, an optimal solution is obtained which is denoted by $u_{i,t}^s = u_{i,1}^s, u_{i,2}^s, \dots, u_{i,T}^s$ and $x_{i,t}^s = x_{i,1}^s, x_{i,2}^s, \dots, x_{i,T}^s$. However, considering the overall scenarios, a combination of the obtained results with the corresponding probabilities (ρ^s) determines the stochastic solution. Thus, the objective function of stochastic UC will be the weighted average of the obtained solution in Eq. (18) for each scenario. In the same manner, the expected output for each variable is the weighted average of those solutions. In brief, the simulation process of the proposed approach contains the following steps:

- Scenario generation process considering the uncertainties in load profiles and wind speed values at different time intervals;
- Computing a deterministic solution based on the proposed UC formulation for each of the generated scenarios;
- Computing the expected values for the output variables based on the obtained solutions at each scenario and its corresponding probability.

As clarified, in stochastic UC problem each output variable is determined based on weighted mean of those values at each scenario. Decision variable of a unit ($u_{i,t}^S$) is considered as one of the investigated output variables. This variable determines the commitment state of each unit (on = 1, off = 0). It should be mentioned that this variable is a binary one. This is while; the obtained mean value may not contain a binary feature. Accordingly, same decision variables are deployed for all scenarios while the state variables could adopt different continuous values. Hence, the established objective function is renewed as follows to consider the overall scenarios. As can be seen, the expected cost is used and the decision variable is the same for all scenarios.

$$\text{Min } \sum_{s=1}^{N_s} \rho^S \times J(x_{i,t}^S, u_{i,t}^S, \xi_t^S) \tag{19}$$

s.t. all constraints (5) to (8)

4 Numerical studies: performance validations and general discussions

To attest the expected functionalities of the proposed approach, a typical 12-unit test system is numerically analyzed in depth. This system contains 10 thermal units and 2 wind farm integrations. The input data for the thermal units and their emission coefficients are given in Kazarlis et al. (1996) and Saber and Venayagamoorthy (2010), respectively. Moreover, the technical information of two wind farm integrations is added to this system. As mentioned earlier, the fuel cost function of thermal units is linearized based on the presented piecewise linear representation. The mean values and the standard deviations of weekly peak load and estimated wind speeds at each of the wind farms are extracted based on the results in Siahkali and Vakilian (2010).

To provide a fruitful numerical analysis, at first the developed piecewise linear approach is studied against the nonlinear UC model in a deterministic manner. The obtained

results are discussed in terms of the reduced computational time and attaining the global optimal solutions. Afterwards, the linear and nonlinear stochastic UC problem is tailored and the obtained results are compared in detail.

4.1 Optimization without renewable resources

In this case, there is no renewable energy resource and the demand is supplied by thermal units, totally. A MINLP solver in GAMS software is used to calculate the schedule and dispatch the power and the corresponding emission. Total operation cost and emission are obtained as M\$57.81 and 2,562,753 tons, respectively. In this case, the spinning reserve which us required to compensate the uncertainty of wind power is not of significance.

4.2 Deterministic solution

In deterministic analyses, the intermittency of the wind power generation and the uncertainties in loading demands are overlooked. Thus, the mean values of these parameters are considered at each time interval and the UC problem is analyzed based on the proposed deterministic approach. The standard branch and bound (SBB) method is deployed to tackle the nonlinear feature of the developed UC problem. Table 1 reports the commitment states of the generating units over the planning horizon. This problem is then analyzed based on the proposed linear model. Thus, linear programming methods are deployed efficiently to handle the optimization processes. The commitment state of each generating unit is represented in Table 2. A slight difference is occurred at weeks 8 and 9 in which the 6th unit is replaced with the 5th unit in nonlinear programming results. These differences are highlighted in Tables 1 and 2 by shady region.

Moreover, the generated power of each thermal unit based on the proposed nonlinear and linear models are demonstrated in Figs. 3 and 4. As can be seen, the obtained results

Table 1 Commitment states of units based on deterministic nonlinear model, bold italic region indicates the difference of linear and nonlinear model (1 = on, 0 = off)

Units	Time interval											
	1	2	3	4	5	6	7	8	9	10	11	12
Th1	1	1	1	1	1	1	1	1	1	1	1	1
Th2	1	1	1	1	1	1	1	1	1	1	1	1
Th3	1	1	0	1	1	1	1	1	1	1	1	1
Th4	1	1	1	1	1	1	1	1	1	1	1	1
Th5	1	1	1	1	1	1	1	1	1	1	1	1
Th6	1	1	0	0	1	1	1	0	0	1	1	1
Th7	0	0	0	0	0	0	0	0	0	0	1	1
Th8	0	0	0	0	0	0	0	0	0	0	0	1
Th9	0	0	0	0	0	0	0	0	0	0	0	0
Th10	0	0	0	0	0	0	0	0	0	0	0	0

Table 2 Commitment states of units based on deterministic linear model, bold italic region indicates the difference of linear and nonlinear model (1 = on, 0 = off)

Units	Time intervals											
	1	2	3	4	5	6	7	8	9	10	11	12
Th1	1	1	1	1	1	1	1	1	1	1	1	1
Th2	1	1	1	1	1	1	1	1	1	1	1	1
Th3	1	1	0	1	1	1	1	1	1	1	1	1
Th4	1	1	1	1	1	1	1	1	1	1	1	1
Th5	1	1	1	1	1	1	1	<i>0</i>	<i>0</i>	1	1	1
Th6	1	1	0	0	1	1	1	<i>1</i>	<i>1</i>	1	1	1
Th7	0	0	0	0	0	0	0	0	0	0	1	1
Th8	0	0	0	0	0	0	0	0	0	0	0	1
Th9	0	0	0	0	0	0	0	0	0	0	0	0
Th10	0	0	0	0	0	0	0	0	0	0	0	0

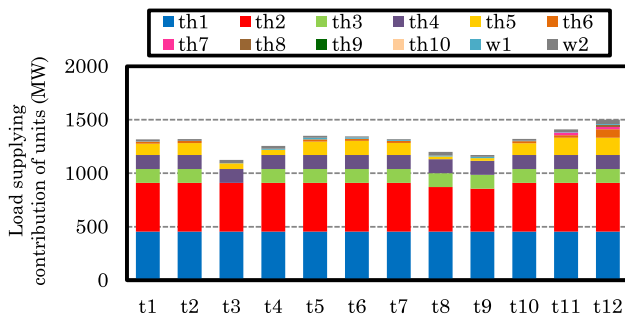


Fig. 3 Mid-term load supplying contributions of units based on deterministic nonlinear model

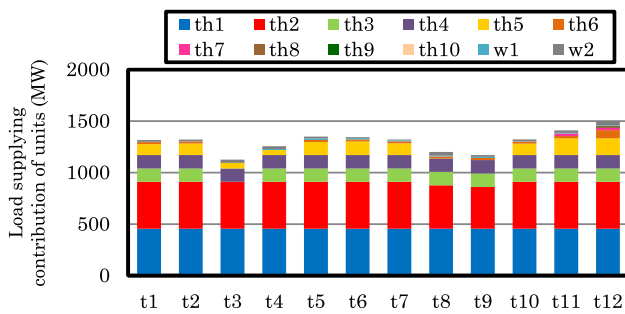


Fig. 4 Mid-term load supplying contributions of units based on deterministic linear model

are in close agreement with each other assuring the accurate linearization of the quadratic expressions.

4.2.1 Cost and emission

It should be mentioned that the total operation cost is obtained as M\$56.67 based on the executed nonlinear UC approach. This is while; the linear approach yields lower cost equal to M\$56.66. In other words, the global optimal solution is granted based on the linear approach. In addition,

Table 3 Computational burden of the deterministic UC problem

UC modeling approach	Computation time (s)
Nonlinear	7
Piecewise linear	1

the total amount of emission in the presence of renewable resources is reduced to 2,508,814. It means that if 10% of power sources are of renewable resources type, e.g., wind farms, then the emission volume is reduced by more than 2%. Accordingly, wind farm integration can significantly decrease the emission and contribute to a clean environment.

4.2.2 Computational burden

Regarding the computational burden of the problem, Table 3 reports the calculation times achieved based on the proposed approaches. As can be seen, the linearization trick diminishes the computational burden of the problem, drastically. Thus, an efficient approach is developed more suitable for recently evolved real-time mechanisms.

4.2.3 Spinning reserve

The reserve contributions of each thermal unit based on the proposed nonlinear and linear models are demonstrated in Figs. 5 and 6 which are in close agreement with each other. As can be seen, 5th and 6th units contribute for supplying the spinning reserve in most of the intervals. However, some units never contribute for spinning reserve and some of them e.g. the 2nd and 7th contribute only in two intervals.

As described in Eq. (6), the *RW* factor is taken to determine the reserve capacity for compensating the fluctuations in wind power. The typical value of *RW* is assumed to be 0.1. By increasing the *RW*, additional spinning reserve is

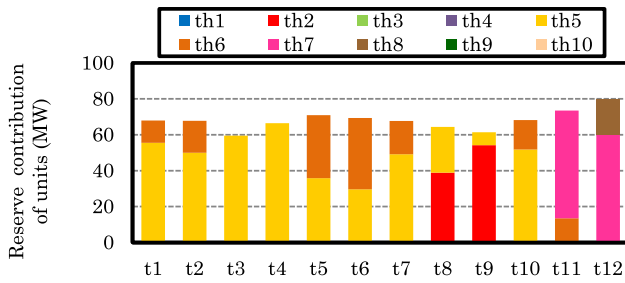


Fig. 5 Mid-term reserve contributions of units based on deterministic nonlinear model

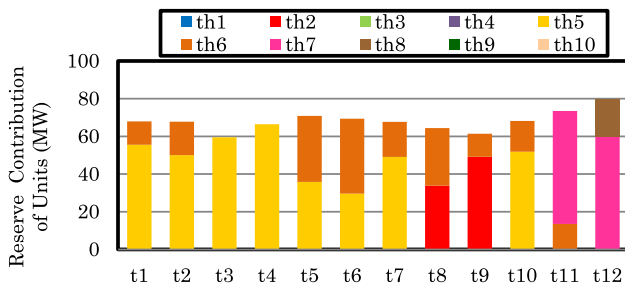


Fig. 6 Mid-term reserve contribution of units based on deterministic linear model

Fig. 7 Total operation cost for different values of compensating factor for wind uncertainty

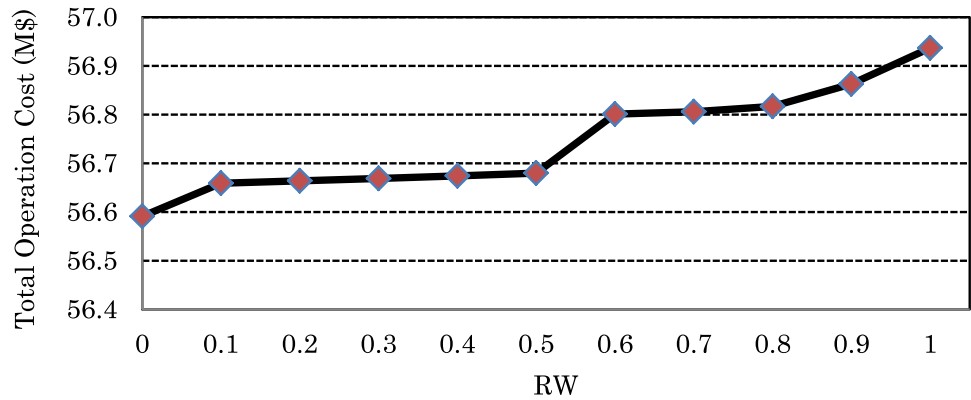
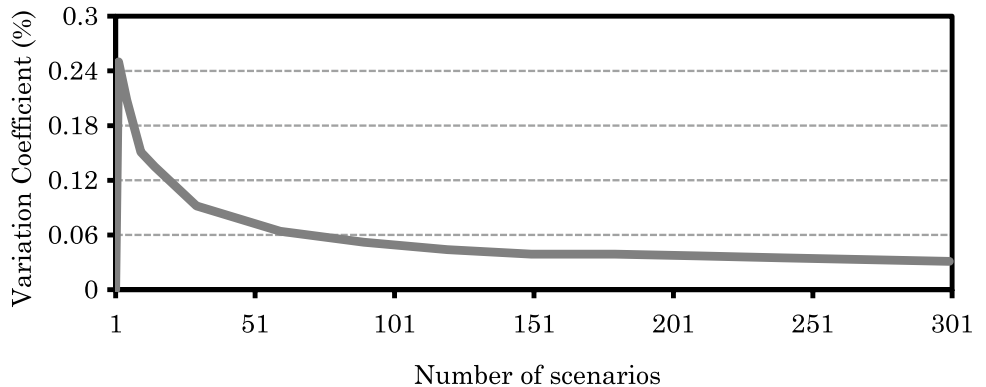


Fig. 8 The variation coefficient curve to determine the suitable number of scenarios



required and hence, the operational cost is increased. The procedure of these variations are described in Fig. 7.

4.3 Stochastic solution

As mentioned earlier, the wind speed and load uncertainties are modeled based on a number of dissimilar scenarios. The MCS-based inverse transform approach assigns equal occurrence probabilities for these scenarios. The number of scenarios is selected such that the variation coefficient approaches to its minimum value and stabilizes around it. Considering an uncertain parameter X , the variation coefficient is defined as follows (Billinton and Allan 1992):

$$cv_X = \frac{\sigma_X}{\mu_X \cdot \sqrt{N_s}} \tag{20}$$

Figure 8 depicts the variation coefficient versus the number of scenarios. As can be seen, the number of 180 scenarios is adequate to assess the stochastic UC problem.

Each of these scenarios is assessed based on the proposed deterministic linear approach. The stochastic solutions are then computed based on the expected values considering the overall scenarios and their occurrence probabilities. Figure 9 demonstrates the probability distribution function of the operation cost based on the linear

Table 4 Stochastic UC results at different scenarios based on linear model

Number of scenarios	Total cost (M\$)		Variation coefficient (%)
	Mean (μ)	Standard deviation (σ)	
30	57.414	1.762	0.560
60	56.962	1.699	0.385
90	56.952	1.562	0.289
120	56.827	1.612	0.259
150	56.731	1.619	0.233
180	56.688	1.638	0.215

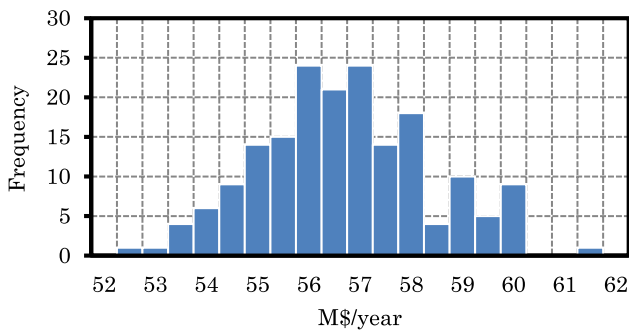


Fig. 9 Distribution of total operation cost based on the proposed linear model

Table 5 Committing states of units based on stochastic UC approach, bold italic regions indicate the differences of stochastic and deterministic results (1 = on, 0 = off)

Units	Time interval											
	1	2	3	4	5	6	7	8	9	10	11	12
Th1	1	1	1	1	1	1	1	1	1	1	1	1
Th2	1	1	1	1	1	1	1	1	1	1	1	1
Th3	1	1	<i>1</i>	1	1	1	1	1	1	1	1	1
Th4	1	1	1	1	1	1	1	1	1	1	1	1
Th5	1	1	1	1	1	1	1	<i>1</i>	<i>1</i>	1	1	1
Th6	1	1	0	<i>1</i>	1	1	1	<i>0</i>	<i>0</i>	1	1	1
Th7	<i>1</i>	<i>1</i>	0	0	<i>1</i>	<i>1</i>	<i>1</i>	0	0	<i>1</i>	1	1
Th8	0	0	0	0	0	0	0	0	0	0	<i>1</i>	1
Th9	0	0	0	0	0	0	0	0	0	0	0	<i>1</i>
Th10	0	0	0	0	0	0	0	0	0	0	0	<i>1</i>

Table 6 Total capacity of committed units in deterministic and stochastic UC approach

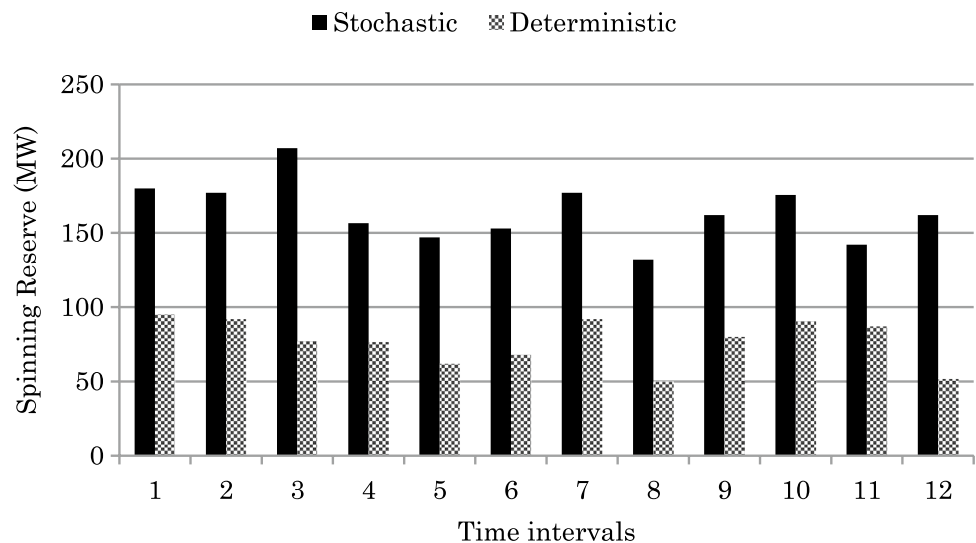
	Time intervals											
	1	2	3	4	5	6	7	8	9	10	11	12
Expected demand	1317	1320	1125	1255.5	1350	1344	1320	1200	1170	1321.5	1410	1500
Total cap. in deterministic	1412	1412	1202	1332	1412	1412	1412	1250	1250	1412	1497	1552
Total cap. in stochastic	1497	1497	1332	1412	1497	1497	1497	1332	1332	1497	1552	1662

UC approach. The indices related to total cost analysis are presented in Table 4. The investigated indices include mean value, standard deviation, and variation coefficient at different number of scenarios. As can be seen, the total operation cost decreases as the number of scenarios increases. This notice is emanated by the increased precision of the proposed model as the higher number of scenarios gives a proper sense of uncertainties.

The commitment states of all generating units in linear stochastic UC approach is obtained based on Table 5. As can be seen, there are some record of dissimilar commitment patterns against the proposed linear deterministic UC approach in Table 2. These differences, highlighted by shady region are noticed due to uncertainty handlings.

By comparing Tables 5 and 2, it can be noticed that the committed thermal units and hence the capability of power generating is increased in stochastic approach to compensate the uncertainty of wind power. The total capacity of committed units in both approaches and the expected values of load demand are presented in Table 6. The difference between the total capacity and the expected demand can be defined as the spinning reserve. The values of spinning reserve in each interval is presented in Fig. 10.

Fig. 10 Optimal values of spinning reserve in stochastic and deterministic approaches



5 Concluding remarks

Optimal spinning reserve was assigned to thermal units using a linear optimization approach proposed for stochastic UC problem. The uncertainties in wind power generation and power system's loading demand are considered through some scenarios. In this context, the quadratic cost functions were represented based on suitable number of piecewise linearized segmentations. At first, the technical superiority of the linear approach was deduced against the nonlinear method in a deterministic manner. Results demonstrated a decreased computational burden along with an improved economical metric. As well, the linear approach granted a global optimal solution and averted the divergence issue in nonlinear problems. The initial model was extended to include the possible uncertainties in wind speed and load profiles. MCS-based inverse transform was applied to generate different scenarios whose number was assigned based on the proposed variation coefficient index. Each of the scenarios was investigated based on the established model and the stochastic solutions were attained in the form of expected values considering the overall scenarios and their specific occurrence probabilities. The spinning reserve in the stochastic approach was increased to compensate the uncertainties of wind power. The obtained results revealed the practical merits of the proposed approach in real-world stochastic UC problems.

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