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ORIGINAL RESEARCH

Fuzzy neural network approach to optimizing process performance by using multiple responses

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Abstract This research proposes a method for optimizing process performance; the method involves the use of multiple quality characteristics, fuzzy logic, and radial basis function neural networks (RBFNNs). In the method, each quality characteristic is transformed into a signal-to-noise ratio, and all the ratios are then provided as inputs to a fuzzy model to obtain a single comprehensive output measure (COM). The RBFNNs are used to generate a full factorial design. Finally, the average COM values are calculated for different factor levels, where for each factor, the level that maximizes the COM value is identified as the optimal level. Three case studies are presented for illustrating the method, and for all of them, the proposed method affords the largest total anticipated improvements in multiple quality responses compared with previously used methods, including the fuzzy, grey-Taguchi, Taguchi, and principal component analysis methods. The main advantages of the proposed method are its effectiveness in obtaining global optimal factor levels, its applicability and the requirement of less computational effort, and its efficiency in improving performance. In conclusion, the proposed method may enable practitioners optimize process performance by using multiple quality characteristics.

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1 Introduction

Over the past few years, the optimization of multiple responses of a product or process has received considerable attention as a potential technique that can help survive today's intense competition (Çakıroğlu and Acır 2013; Otebolaku and Andrade 2015). The Taguchi method, which has been traditionally widely used, is applicable only for the optimization of a single quality characteristic of a product or process (Li et al. 2008; Dasgupta et al. 2014). Consequently, several methods have been proposed for optimizing process performance by using multiple quality responses; the methods include data envelopment analysis (Al-Refaie et al. 2009), fuzzy regression (Al-Refaie 2013), artificial neural networks (Zăvoianu et al. 2013; Chen 2015; Wu and Chen 2015), fuzzy methods (Bose et al. 2013), the utility method (Sivasakthivel et al. 2014), and goal programming (Al-Refaie and Li 2011; Al-Refaie et al. 2014). Nevertheless, such methods have several limitations, such as relying on nonparametric evaluation, dealing with a limited number of experiments, achieving local optimality, and requiring good mathematical skills. These limitations and weaknesses of existing methods continue to motivate researchers to develop more efficient methods for solving multiresponse problem in robust design.

In the Taguchi method, finding the combination of optimal factor settings that optimizes the process performance is critical. Although several methods have been proposed in the literature, they have limitations (Al-Refaie 2014a, b, c, d, 2015; Al-Refaie et al. 2012), which include (1) providing local optima; for example, fuzzy logic and regression methods, (2) the methods being nonparametric methods such as the utility concept and grey relational analysis, (3) the relationship between process settings and responses is ignored, (4) insufficiency of concurrent improvements in multiple responses, and (5) requirement of mathematical skills such as knowledge of regression techniques. Accordingly, the present research makes an extension to ongoing research by proposing an approach for optimizing process performance by using multiple quality responses, fuzzy logic, and artificial neural networks (ANNs) techniques. The remainder of this paper is presented in the following sequence. "Fuzzy logic and ANN techniques in the optimization procedure" introduces fuzzy logic, ANN techniques, and the optimization procedure. "Illustrations" provides illustrative case studies. "Research results" summarizes the research results. Finally, the conclusions are presented in "Conclusions".

2 Fuzzy logic and ANN techniques in the optimization procedure

2.1 Fuzzy logic

The fuzzy logic principle is widely used to handle vague and uncertain information. Two common types of fuzzy systems are used: Takagi–Sugeno (T–S) and Mamdani fuzzy systems. Mamdani fuzzy systems are special cases of T–S fuzzy systems, which involve mathematical expressions that contain a linear function (Lilly 2010). The architecture of a fuzzy system is shown in Fig. 1 (Lilly 2010). The functions of a fuzzy system include fuzzification, which depends on the membership function (MF), rule evaluation, defining the MF for the output, setting the fuzzy rules, and defuzzification to transform the fuzzy value into a comprehensible output measure (Sun and Hsueh 2011; de Pontes et al. 2012).

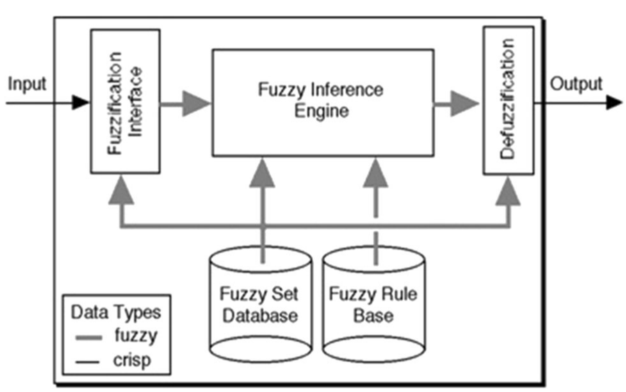


Fig. 1 Fuzzy logic system. **a** MFs for the two inputs. **b** fuzzification of the two inputs

In the proposed method, the center of gravity (COG) defuzzification method is used. Suppose two input variables are to be converted into a COM value by using Mamdani's fuzzy inference method, the method sets two MFs, high and low, for each input. Let the range of input 1 be [A, D] and the range of input 2 be [B, C]. Then, the fuzzification of the input variables is as shown in Fig. 2. Generally, the rules that relate the two inputs with the output are set as follows:

If input 1 is Low and input 2 is Low then the output is Low.

If input 1 is Low and input 2 is High then the output is Normal.

If input 1 is High and input 2 is Low then the output is Normal.

If input 1 is High and input 2 is High then the output is High.

Applying these rules to fuzzy values yields the following results:

$$\mu_{\text{low}}(\text{input 1})^{\wedge}\mu_{\text{low}}(\text{input 2}) = \mu_{\text{low}}(\text{output})$$
$$\mu_{\text{low}}(\text{input 1})^{\wedge}\mu_{\text{high}}(\text{input 2}) = \mu_{\text{normal}}(\text{output})$$
$$\mu_{\text{high}}(\text{input 1})^{\wedge}\mu_{\text{low}}(\text{input 2}) = \mu_{\text{normal}}(\text{output})$$
$$\mu_{\text{high}}(\text{input 1})^{\wedge}\mu_{\text{high}}(\text{input 2}) = \mu_{\text{high}}(\text{output})$$

For two values A and B, as the representation A ^ B denotes min(A, B). For example,

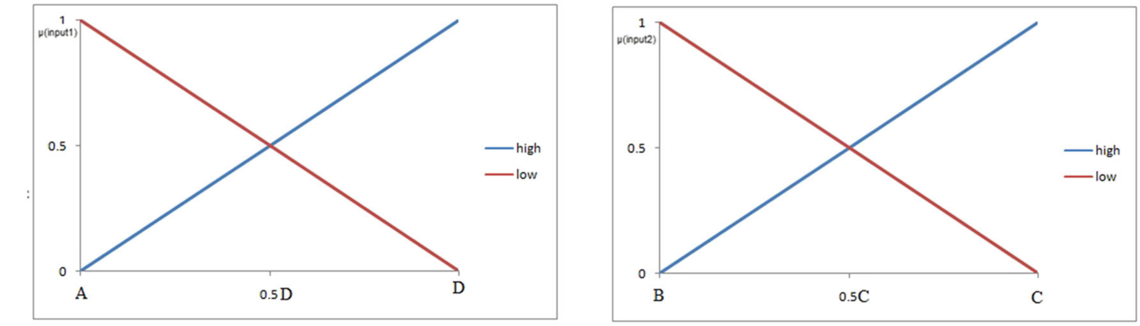
$\mu_{\text{low}}(\text{input 1}) \wedge \mu_{\text{low}}(\text{input 2}) = \min(\mu_{\text{low}}(\text{input 1}), \mu_{\text{low}}(\text{input 2}))$.

The MFs for the output are then set as shown in Fig. 3, where S and Q are in the range (0, 1). Finally, output defuzzification is performed by using the COG method to compute the COM value, as shown in Fig. 4.

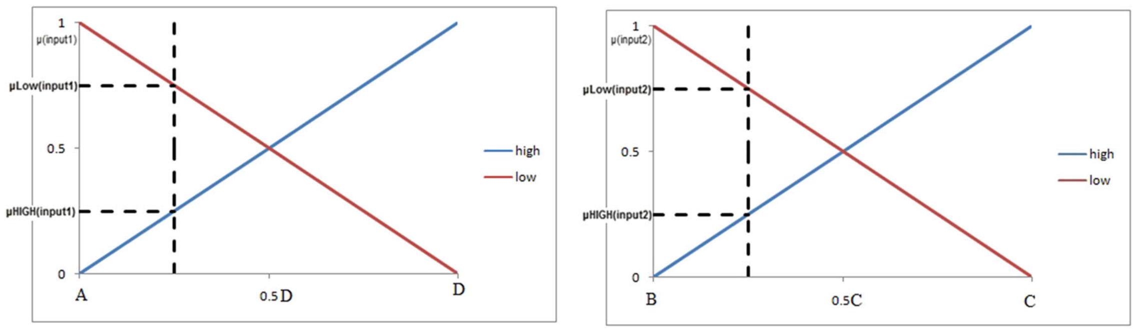
Several studies have used the fuzzy logic approach for optimizing performance by considering multiple quality characteristics (Azadeh et al. 2011; Sun et al. 2012; Mandic et al. 2014).

2.2 Artificial neural networks

The ANNs are soft computing techniques based on certain aspects of human behavior; they involve the use of a finite number of layers (which serve as the computing elements) with different neurons. The capabilities of the ANNs are stored in inter-unit connections, strengths, or weights, which are all handled and tuned in the learning process (Asiltürk and Çunkaş 2011; Moosavi and Soltani 2013). The type of ANN that is the most widespread consists of input, hidden, and output layers. The input and output layers represent the variables, and the hidden layer represents the relationship between the input and the output variables. The weights of the nodes are random for a given input pattern and are updated to obtain predicted responses



(a) MFs for the two inputs.



(b) Fuzzification of the two inputs.

Fig. 2 MFs for and fuzzification of the two inputs

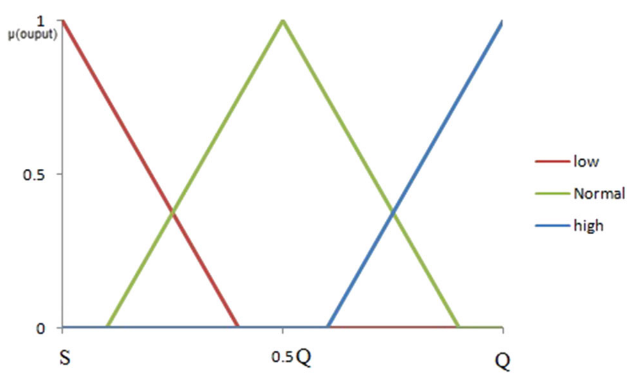


Fig. 3 MFs for the output

that are less susceptible to errors (Almonacid et al. 2011). In many types of ANNs, such as the backward propagation neural network (BPNN), the gradient descent method, the Levenberg–Marquardt algorithm, the Broyden–Fletcher–Goldfarb–Shanno algorithm, or the resilient back propagation algorithm can be applied to adjust the weights used in the approximation. A multilayer perceptron model is mainly based on the BPNN. The drawbacks of the BPNN are overfitting, and immature decisions that result from the use of local minima rather than global minima. Moreover, a BPNN model shows slow convergence. In the BPNN, a fixed number of neurons should be set before data training, and a wide range of inputs can be covered since sigmoid neurons are used in the hidden layer (Xia et al. 2010). By contrast, the radial basis function neural network (RBFNN) can approximate the desired outputs predicted without requiring a mathematical expression of the relationship between the outputs and the inputs; hence, radial basis functions are called model-free estimators. A high convergence speed is also achievable by using a radial basis function where only one hidden layer is present. The RBFNN architecture is shown in Fig. 5 (Peng et al. 2014). The parameters Ud and yi are the inputs and outputs of the RBFNN for the training data and G_l represents the Gaussian function in hidden layer l at the center where the number of controllable factors ranges from 1 to D, n is the number of experiments, the number of hidden layers is L, and w_{lz} is the weight. The problem of local minima is not encountered in the RBFNN, unlike the BPNN. Because of the advantages and powerful features of the RBFNN, it is widely used for nonlinear functional approximation and pattern classification. The RBFNN has other advantages: local and optimal approximations can be obtained; it has a high convergence rate, high precision, and an adaptive structure; and the output is independent of the weight value set initially (Javan et al. 2013). In the RBFNN, the network output vector is a linear combination of the outputs of the basis function, and the neurons in the hidden layer use the

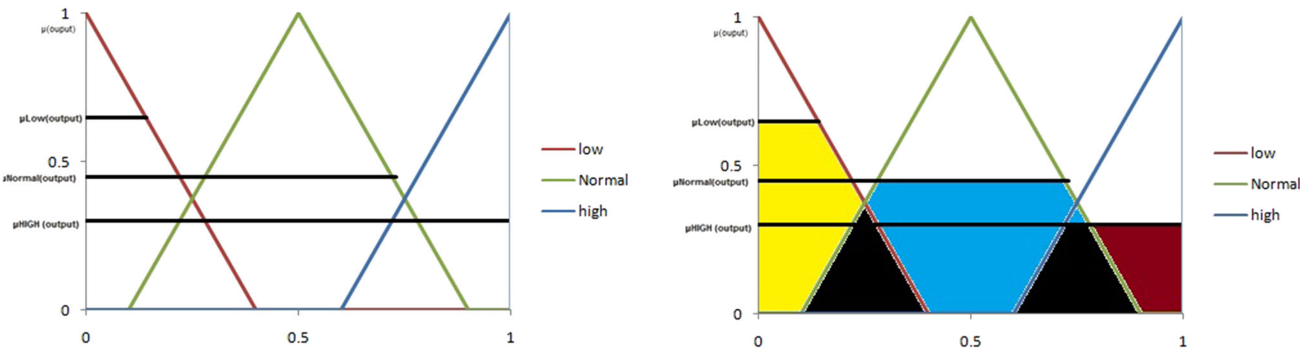


Fig. 4 COG method

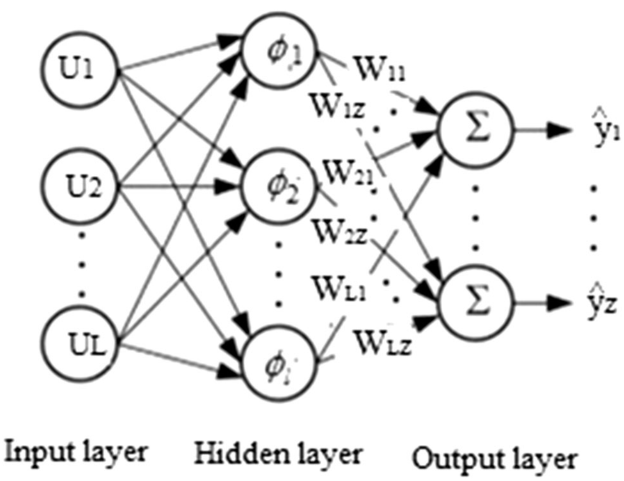


Fig. 5 RBFNN architecture

radial basis function as an activation function. If a Gaussian function is chosen as the radial basis function, the outputs can have the following form:

$$\hat{y}_z(s+1) = \sum_{l=1}^{L} \varphi_l w_{lz} = \sum_{l=1}^{L} w_{lz} \exp\left(-\frac{\left\|\hat{U} - m_l\right\|^2}{2\sigma_l^2}\right) \quad \text{for } z = 1, 2, \ldots, Z \tag{1}$$

where $\hat{U}(s) = \left[\underset{1}{\hat{U}}(s)\ldots\ldots\underset{m}{\hat{U}}(s)\right]^T$ is the input vector, $\hat{y}_z(s+1)$ is the zth output, l represents the hidden neuron, and the zth output neuron is linked by the weight w_{lz}. Furthermore, L denotes the number of Gaussian functions, which is equal to the number of hidden layer nodes, φ_l represents the Gaussian function in hidden layer l at the center, m_l is the width of the Gaussian function, and σ_l represents the standard deviation of u_l. In the training algorithm applied to the initial RBFNN, many iterations are performed to find the optimal values of w_{lz} and m_l. Therefore, it is imperative to know the initial structure of the network. The input vectors are used for determining the basic functions, and then both input and output data are used to set the weights connected to the output layer. This two-stage training methodology prevents the problem of local minima, and the RBFNN has a high divergence rate (Tsai 2014).

RBFNNs have been widely used in many business applications (Tatar et al. 2013; Chen et al. 2013). Furthermore, several studies have employed the ANN approach for optimizing performance by using multiple quality characteristics. Furtuna et al. (2011) used the ANN method along with a genetic algorithm to optimize the polysiloxane synthesis process. Lin et al. (2012) conducted parameter optimization of a solar energy selective absorption film continuous sputtering process by using Taguchi methods, neural networks, a desirability function, and genetic algorithms. Marvuglia et al. (2014) proposed a new approach involving the combination of a fuzzy logic controller and ANNs for the dynamic and automatic regulation of the indoor temperature.

2.3 The optimization procedure

In the Taguchi method, the columns of the orthogonal array (OA) represent the controllable factors to be studied, and the rows represent combinations of factor levels used in experiments. Typically, there are three types of quality characteristics: smaller-the-better (STB), larger-the-better (LTB), and nominal-the-best (NTB) responses. The proposed approach for optimizing process performance by using multiple quality characteristics is illustrated in Fig. 6 and is outlined as follows:

Step 1: Assume that in an OA, L controllable factors are studied by conducting n experiments to improve J quality characteristics, as shown in Table 1; y_{ijk} is the kth replicate of the jth response in the ith experiment, where $i = 1, \ldots, n$; $j = 1, \ldots, J$; and $k = 1, \ldots, K$. Compute the signal-to-noise ratio (SNR) (η_{ij}) for the jth response in experiment i by using the appropriate formula as follows.

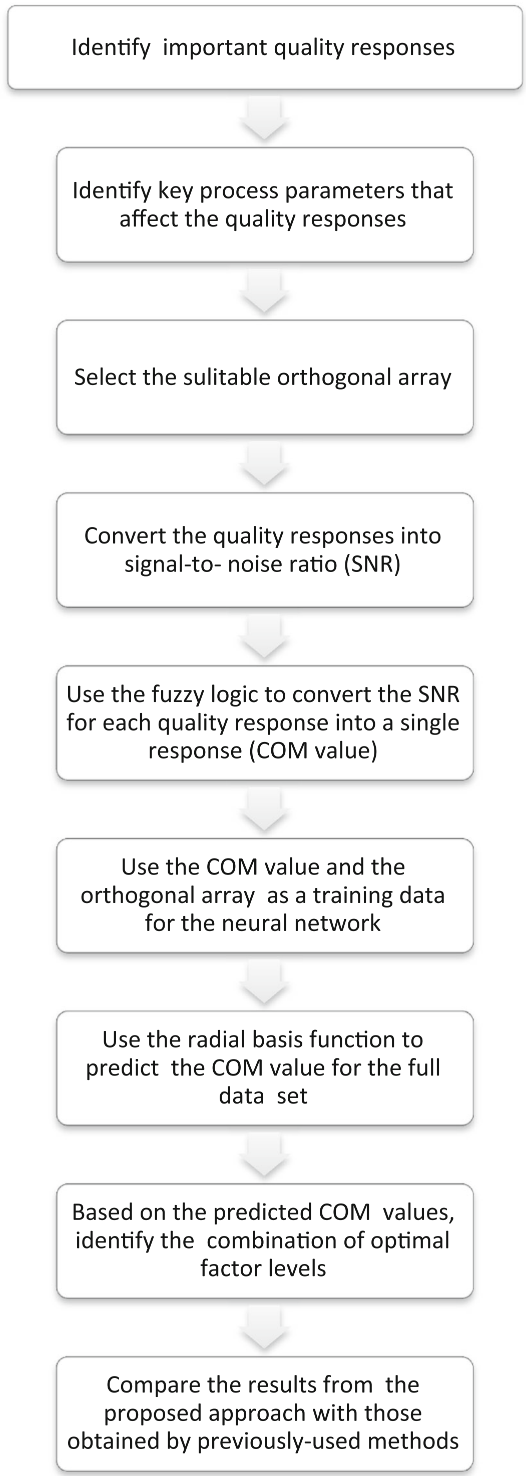


Fig. 6 The proposed methodology

For STB-type quality characteristic, η_i is calculated as (Al-Refaie 2010, 2011):

$$\eta_i = -10\log\left[1/K\left(\sum_{k=1}^{K} y_{ik}^2\right)\right] \quad \forall i \tag{2}$$

For NTB-type quality characteristic, η_i is calculated as follows:

$$\eta_i = -10\log\left[\overline{y}_i^2/s_i^2\right] \quad \forall i \tag{3}$$

where $\overline{y_i}$ and s_i are the estimated average and standard deviation of response j in experiment i, respectively. For LTB-type quality characteristic, η_i is calculated as

$$\eta_i = -10\log\left[(1/K)\sum_{k=1}^{K} 1/y_{ik}^2\right] \quad \forall i \tag{4}$$

The parameter η_i is computed for each response in experiment i, as shown in Table 2.

Step 2: Convert the multiple quality characteristics into a single response by using fuzzy logic (or Mamdani-style fuzzy inference) in which the inputs and output MFs are linear. The inputs are the η_{ij} values, whereas the outputs are the COM_i values. The Mamdani-style fuzzy inference process is performed in four stages:

(1) Fuzzification of the inputs
Define the MF for each quality characteristic using the corresponding η_{ij} values. Let the values G_{i1}, G_{i2}, …, G_{iJ} represent the fuzzy subsets defined by the MFs $\mu_{G_{i1}}$, $\mu_{G_{i1}}$, …, and $\mu_{G_{ij}}$. Use the minimum and maximum values of η_{ij} to generate the MF for each quality characteristic, as shown in Fig. 7.

(2) Rule evaluation
Generate the fuzzy rules that relate the inputs to the output. The fuzzy rule base consists of a set of J inputs, one output measure F, and T rules; for example, for the ith experiment, the rules may be formulated as follows:
Rule 1: If η_{i1} is G_{11} and η_{i2} is G_{12}… and η_{iJ} is G_{1J} then F_1 is M_1 else
Rule 2: If η_{i1} is G_{21} and η_{i2} is G_{22}…. and η_{iJ} is G_{2J} then F_2 is M_2 else

⋮ ⋮ ⋮ ⋮ ⋮ ⋮ ⋮ ⋮

Rule T: If η_{i1} is G_{T1} and η_{i2} is G_{T2} …and η_{iJ} is G_{TJ} then F_T is M_T.

(3) Aggregation of the rule outputs
From the fuzzy rules, the MFs of the output are identified. For example, the value of M_t that is obtained from the fuzzy rules represents the fuzzy subsets defined by MFs, μ_{M_t}. To compute M_t for each rule t, the η_{ij} value for each quality characteristic is used as an input variable of the rules. The fuzzy reasoning of the rules yields the output by using the max–min composition operation. The MFs of the output of fuzzy reasoning can be expressed as

Table 1 Experiment layout

Exp i	Response j									
	y_{i1}			y_{i2}			$\cdots$	y_{iJ}		
1	y_{111}	$y_{112}\cdots$	y_{11k}	y_{121}	$y_{122}\cdots$	y_{12k}	$\cdots$	y_{1J1}	$y_{1J2}\cdots$	y_{1JK}
2	y_{211}	$y_{212}\cdots.$	y_{21k}	y_{221}	$y_{222}\cdots.$	y_{22k}	$\cdots$	y_{2J1}	$y_{2J2}\cdots.$	y_{2JK}
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.		.	.	.
n	y_{n11}	$y_{n12}\cdots$	y_{n1k}	y_{n21}	$y_{n22}\cdots$	y_{n2k}	$\cdots$	y_{nJ1}	$y_{nJ2}\cdots$	y_{nJK}

Table 2 Signal-to-noise ratios

Exp i	Response j			
	η_{i1}	η_{i2}	$\cdots$	η_{iJ}
1	η_{11}	η_{12}	$\cdots$	η_{1J}
2	η_{21}	η_{22}	$\cdots$	η_{2J}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
n	η_{n1}	η_{n2}	$\cdots$	η_{nJ}
Min	min η_{i1}	min η_{i2}	$\cdots$	min η_{iJ}
Max	max η_{i1}	max η_{i2}	$\cdots$	max η_{iJ}

$$\mu_{C_0}(F) = \big(\mu_{G_{11}}(\eta_{i1}) \wedge \mu_{G_{12}}(\eta_{i2}) \wedge \ldots\ldots \wedge \mu_{G_{1J}}(\eta_{iJ})\mu_{M_1}(F_1)\big) \vee \ldots\ldots \big(\mu_{G_{T1}}(\eta_{i1}) \wedge \mu_{G_{T2}}(\eta_{i2}) \wedge \ldots\ldots \wedge \mu_{G_{TJ}}(\eta_{iJ})\mu_{M_T}(F_T)\big) \tag{5}$$

where $\wedge$ is the minimum operation used in the AND fuzzy operation and $\vee$ is the maximum operation used in the OR fuzzy operation. Figure 8 shows the fuzzy value for each quality characteristic in experiment i. Suppose two quality characteristics are studied, with the first rule being low for both quality characteristics. This results in low for the output, which means min (low G_{i1} and low G_{i2}).

(4) Defuzzification

A defuzzification method is used to convert the fuzzy inference output μ_{C_0} into a nonfuzzy value COM_i. The conversion is performed using the COG method. The larger the COM value the better the performance. For each experiment i, the COM_i value is calculated using the following equation, and it is displayed in Fig. 9.

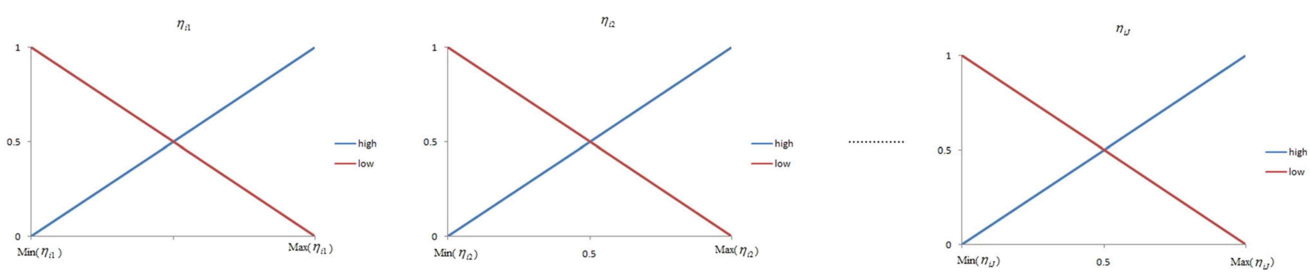


Fig. 7 MFs for the input variables

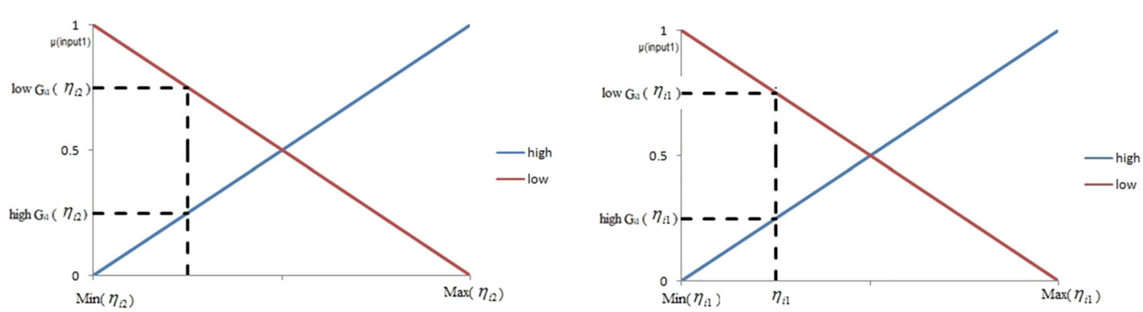


Fig. 8 Fuzzy value for each quality characteristic in experiment i

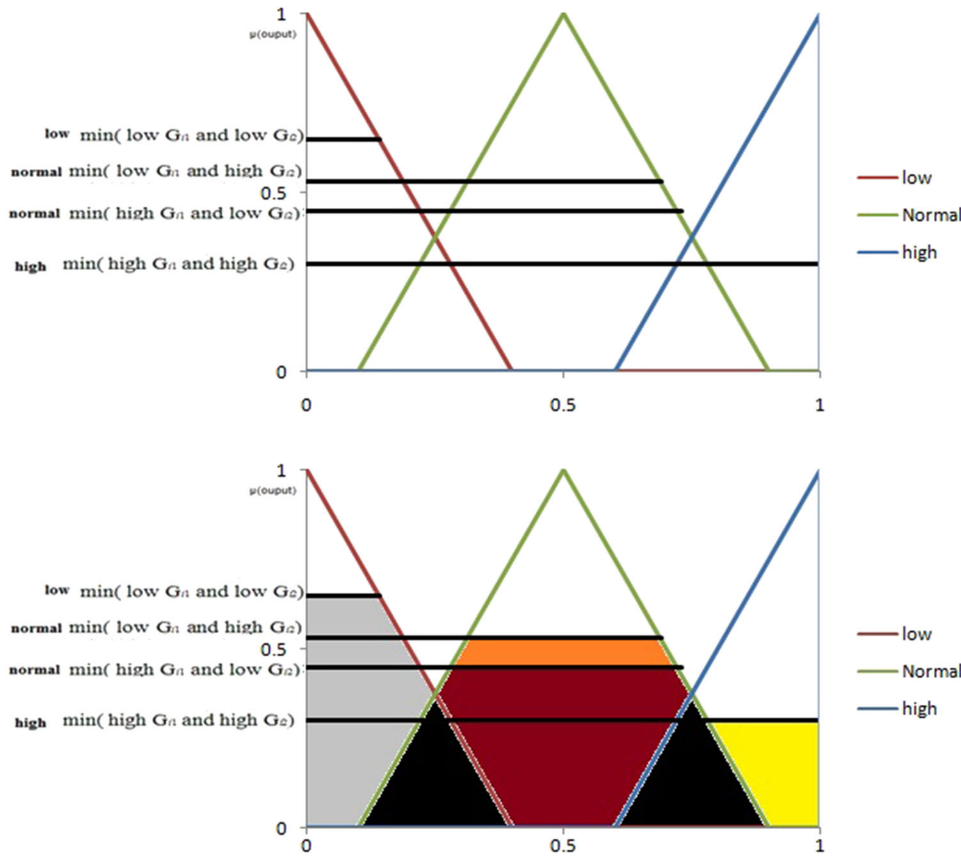


Fig. 9 Defuzzification using the COG method. **a** MFs for the MMR, **b** MFs for the SR and **c** MFs for the EWR

Table 3 Training and predicted data

Exp_i	The orthogonal array for fractional factorial with COM value					
1	X_{11}	X_{12}	…	X_{1L}	COM_1	Original
2	X_{21}	X_{22}	…	X_{2L}	COM_2	
.	.	.	…	.	.	
.	.	.	…	.	.	
n	X_{n1}	X_{n2}	…	X_{nL}	COM_n	
.	.	.	…	.	.	Predicted
.	.	.	…	.	.	
H	X_{H1}	X_{H2}	…	X_{HL}	COM_H	

$$COM = \frac{\sum F\mu_{C_0}(F)}{\sum \mu_{C_0}(F)} \tag{6}$$

Step 3: To obtain the global optimal solution, the full data set of the *COM* is generated by the RBFNN technique on the basis of the input and output matrices; the OA is used as the input matrix and the *COM* values are employed as the output matrix. The full data set of the *COM* value is predicted with less error for the full factorial design. Table 3 shows the layout of the two matrices used as the training data and the predicted data for the full factorial design; n and H are the number of experiments in the OA and the number of full factorial runs, respectively. In this approach, the function type of the RBFNN is newrbe, in which the number of hidden layers is two and the spread constant with a smoothing factor of one. The number of neurons is equal to the number of input vectors or the number of experiments. The first layer represents the nonlinear function (a Gaussian function is used as an activation function) and the second layer represents the linear function.

Step 4: Calculate the average *COM* values for each factor level, as shown in Table 4. The level that has the largest average *COM* value is identified as the optimal level for the factor.

Step 5: Compare the total anticipated improvement in each quality response obtained using the proposed approach with those obtained using previously used approaches. The anticipated improvement in each quality response is calculated by subtracting the sum of average SNRs for the combination of optimal factor levels from that for the combination of initial factor levels. Furthermore, calculate the total anticipated improvement in multiple responses and then compare the results.

Table 4 COM averages for each factor level

Controllable factors	Level			
	Level_{l1}	Level_{l2}	…	Level_{lD}
X_1	$\text{Avg}(COM_{11})$	$\text{Avg}(COM_{12})$	…	$\text{Avg}(COM_{1D})$
X_2	$\text{Avg}(COM_{21})$	$\text{Avg}(COM_{22})$	…	$\text{Avg}(COM_{2D})$
.	.	.	…	.
.	.	.	…	.
X_L	$\text{Avg}(COM_{L1})$	$\text{Avg}(COM_{L2})$	…	$\text{Avg}(COM_{LD})$

Table 5 Experimental parameters of an electro-erosion process

Factors	Level 1	Level 2	Level 3
Voltage (V)	40.000	60.000	70.000
Current (C)	9.000	12.000	15.000
Duty factor (DF)	0.400	0.600	0.800
Tool (T)	Br	Cu	WC

3 Illustrations

Three real case studies that were investigated in previous studies are adopted for illustrating the proposed method.

3.1 Optimization of process parameters in electro-erosion

Muthuramalingam and Mohan (2014) optimized the parameters of an electro-erosion process by using three quality characteristics: material removal rate (MRR, mm^3/min), electrode wear rate (EWR, mm^3/min), and surface roughness (SR, μm). The parameters are shown in Table 5. The experimental results of the L_{27} array are shown in Table 6. In this case study, the quality characteristics are classified into two categories: the EWR and SR are STB and the MRR is LTB. The proposed method is then employed as follows:

Step 1: The η_{ij} values are calculated for each of the three quality responses for the 27 experiments by using the appropriate formula from Eqs. (2)–(4). Table 6 lists the obtained results.

Step 2: The Mamdani-style fuzzy inference is adopted to convert the three quality characteristics into a single response. First, the η_{ij} values of the MRR, EWR, and SR were fuzzified. The two fuzzy subsets low and high are assigned to the η_{ij} values of the MRR, EWR, and SR. Table 7 displays the high and low representations for η_{i2} of the SR response as an example. Figure 10 displays the MFs for the three quality responses. The rules that communicate between the MFs of the responses and the output are shown in Table 8. Next, the rule outputs are aggregated, and the four fuzzy subsets are assigned to the output COM value, as shown in Fig. 11. Finally, the fuzzy value of the output is defuzzified to convert it to a crisp COM value for each experiment using the COG method. Table 9 shows the calculated COM_i value for each experiment.

Step 3: The RBFNN technique shown in Fig. 12 is used to generate the full data set for the COM

Table 6 Estimated η_{ij} values for Case Sudy 3.1

Exp. *i*	MRR (mm^3/min)	SR (μm)	EWR (mm^3/min)	η_{i1} (MRR)	η_{i2} (SR)	η_{i3} (EWR)
1	0.783	0.326	0.016	−2.125	9.736	36.082
2	4.896	3.724	1.322	13.797	−11.420	−2.424
3	8.097	5.286	0.972	18.166	−14.463	0.250
4	4.673	5.523	1.262	13.392	−14.844	−2.019
5	8.811	5.604	1.057	18.901	−14.970	−0.484
6	0.971	0.725	0.019	−0.256	2.793	34.244
7	7.142	5.124	0.857	17.076	−14.192	1.340
8	0.982	0.731	0.020	−0.158	2.722	34.155
⋮	⋮	⋮	⋮	⋮	⋮	⋮
25	7.865	7.653	2.124	17.914	−17.677	−6.541
26	13.803	8.364	1.656	22.799	−18.448	−4.383
27	1.568	0.905	0.031	3.907	0.867	30.061

Table 7 High and low representations of η_{i2} for the SR response (μm)

η_{i2}	High	Low	η_{i2}	High	Low
−2.125	**0.0**	**100.0**	18.951	79.4	20.6
13.797	60.0	40.0	−0.238	7.1	92.9
18.166	76.5	23.5	17.807	75.1	24.9
13.392	58.5	41.5	23.798	97.7	2.3
18.901	79.2	20.8	19.428	81.2	18.8
−0.256	7.0	93.0	0.914	11.5	88.5
17.076	72.4	27.6	20.545	85.4	14.6
−0.158	7.4	92.6	1.100	12.2	87.8
16.341	69.6	30.4	18.540	77.9	22.1
12.726	56.0	44.0	**24.413**	**100.0**	**0.0**
18.406	77.4	22.6	17.914	75.5	24.5
1.267	12.8	87.2	22.799	93.9	6.1
17.577	74.2	25.8	3.907	22.7	77.3
1.046	11.9	88.1			

values. Table 10 displays the full data set obtained.

Step 4: The average COM values are calculated for each factor level, as shown in Table 11. It is found that the combination of optimal factor levels is $V_1C_1DF_1T_3$, whereas it is identified as $V_3C_3DF_2T_2$ using the grey-Taguchi approach.

Step 5: For each quality characteristics, the anticipated improvements obtained using the neural–fuzzy approach are compared with those determined using the grey-Taguchi concept.

3.2 Optimization of binary mixture removal

Zolgharnein et al. (2014) employed the Taguchi method and principal component analysis for the optimization of the removal of a binary mixture of alizarin red and alizarin yellow by using multiple LTB quality characteristics: the sorbent capacity of alizarin red (Qr), sorbent capacity of alizarin yellow (Qy), removal percentage of alizarin red (Rr), and removal percentage of alizarin yellow (Ry). The L_{27} array was used to investigate the process parameters shown in Table 12. The η_{ij} values are calculated and displayed in Table 13.

The Mamdani-style fuzzy inference process is performed, in which the two fuzzy subsets low and high are assigned to the four η_{ij} values for each quality

Table 8 Generated fuzzy rules for case study 3.1

η_{i1}	η_{i2}	η_{i3}	*COM*
Low	Low	Low	Lowest
		High	Low
	High	Low	Low
		High	High
High	Low	Low	Low
		High	High
	High	Low	High
		High	Highest

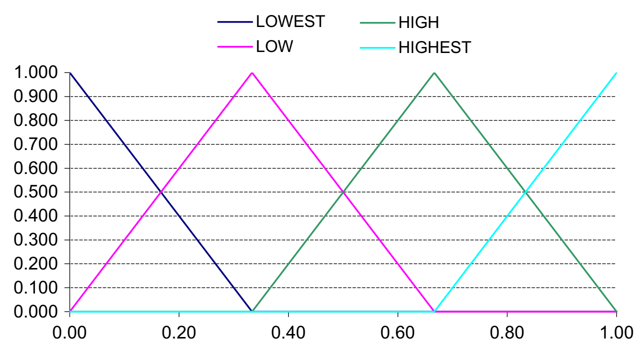
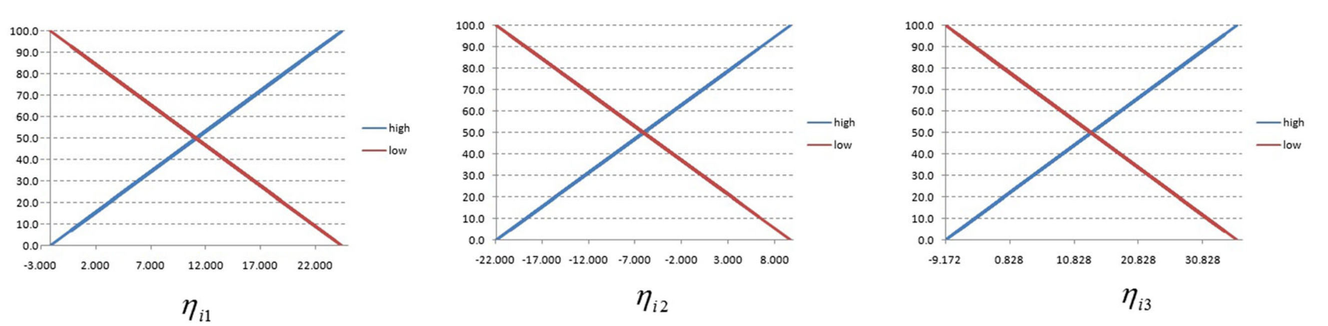


Fig. 11 MFs for the *COM* (Case study 3.1)



(a) MFs for the MMR **(b)** MFs for the SR **(c)** MFs for the EWR

Fig. 10 MFs for each quality response of Case study 3.1

Table 9 Calculated *COM* values for Case study 3.1

Exp_i	COM_i	Exp_i	COM_i	Exp_i	COM_i
1	0.667	10	0.435	19	0.468
2	0.436	11	0.440	20	0.599
3	0.426	12	0.596	21	0.370
4	0.408	13	0.456	22	0.620
5	0.422	14	0.492	23	0.378
6	0.580	15	0.352	24	0.386
7	0.429	16	0.599	25	0.383
8	0.580	17	0.353	26	0.384
9	0.362	18	0.372	27	0.566

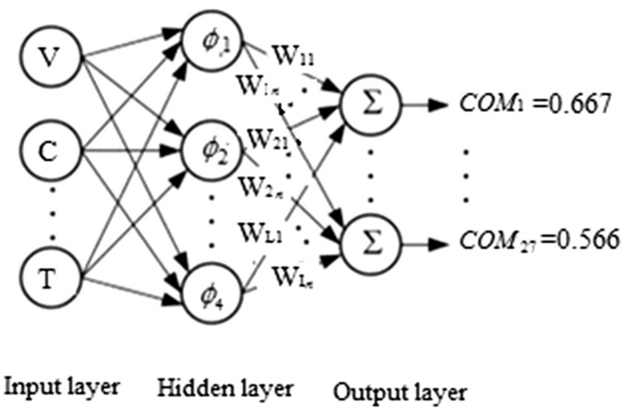


Fig. 12 RBFNN architecture for Case study 3.1

Table 10 Full data set for the COM value for Case study 3.1

Run	V	C	DF	T	COM_i
1	1	1	1	1	0.667
2	1	1	1	2	0.4746
3	1	1	1	3	0.4328
4	1	1	2	1	0.5855
5	1	1	2	2	0.436
6	1	1	2	3	0.4292
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
79	3	3	3	1	0.566
80	3	3	3	2	0.4468
81	3	3	3	3	0.4123

characteristic (Qr, Qy, Rr, and Ry). The MFs for the COM values are displayed in Fig. 13.

To convert the fuzzy value of the output to a nonfuzzy value (the COM value), the COG defuzzification method is used. Finally, the RBFNN shown in Fig. 14 is used to complete the full data set for the COM value, shown in Table 14.

Table 11 Average COM values for the full factorial design for Case study 3.1

COM value			
Factors	Level 1	Level 2	Level 3
V	**0.473**	0.453	0.460
C	**0.485**	0.454	0.447
DF	**0.488**	0.453	0.445
T	0.409	0.414	**0.562**

Table 12 Controllable factors and their levels for Case study 3.2

Factors	Level 1	Level 2	Level 3
pH	5	8	12
Sorbent dose (M)	1	8	15
Initial alizarin red conc (Cr)	0.1606	0.68	1.3
Initial alizarin yellow conc (Cy)	0.06	0.68	1.3
Temperature (T)	25	40	55
Shaker rat (S)	20	110	200
Time (Ti)	20	40	90

Table 13 Values of η_{ij} for the quality characteristics of Case study 3.2

Exp*i*	η_{i1}(Ry)	η_{i2}(Rr)	η_{i3}(Qy)	η_{i4}(Qr)
1	37.8419	39.2758	−26.5951	−25.1612
2	38.4856	39.7354	−25.9514	−24.7015
3	38.0618	39.3697	−26.3752	−25.0673
4	18.0618	26.0206	−25.288	−17.3292
5	29.5424	33.0643	−13.8074	−10.2856
6	23.5218	29.5424	−19.828	−13.8074
7	6.0206	7.9588	−31.7005	−29.7623
⋮	⋮	⋮	⋮	⋮
21	35.8478	23.5218	−25.3951	−64.437
22	26.0206	33.0643	−61.9382	−33.8074
23	13.9794	31.5957	−73.9794	−35.276
24	13.9794	28.9432	−73.9794	−37.9285
25	36.902	37.5012	−29.9697	−23.7417
26	36.6272	38.0618	−30.2445	−23.1812
27	36.537	39.8387	−30.3346	−21.4043

The average COM value for each factor level is calculated and displayed in Table 15. The combination of optimal factor levels is identified as $M_2Cr_2Cy_2Ph_1S_2Ti_2T_2$ by selecting the levels having the maximum COM value for each factor.

The anticipated improvement in each of the quality characteristics (Qr, Qy, Rr, and Ry) is calculated using the neural–fuzzy approach and compared with that obtained using the Taguchi method and principal component analysis.

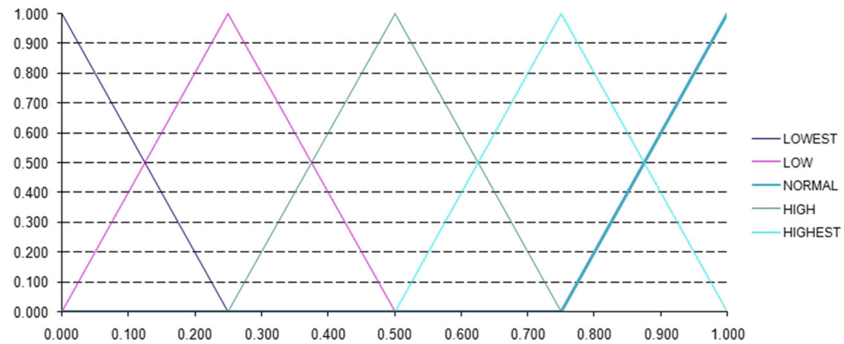


Fig. 13 MFs for the COM for Case study 3.2

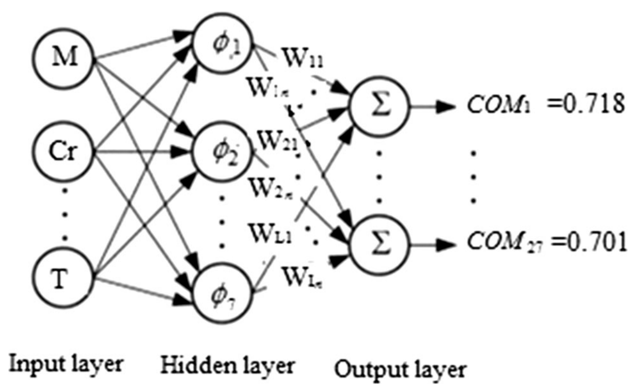


Fig. 14 RBFNN architecture for Case study 3.2. **a** SR, **b** TL, **c** CF and **d** PC

Table 14 Full data set for the COM value for Case study 3.2

Exp. i	M	Cr	Cy	Ph	S	Ti	T	COM_i
1	1	1	1	1	1	1	1	0.718
2	1	1	1	1	1	1	2	0.662
3	1	1	1	1	1	1	3	0.5802
4	1	1	1	1	1	2	1	0.6616
5	1	1	1	1	1	2	2	0.6629
6	1	1	1	1	1	2	3	0.5989
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
2185	3	3	3	3	3	3	1	0.5512
2186	3	3	3	3	3	3	2	0.5485
2187	3	3	3	3	3	3	3	0.5468

3.3 Optimization of high-speed CNC turning parameters

Gupta et al. (2011) performed the optimization of high-speed CNC machining of AISI P-20 tool steel by using four LTB-type quality characteristics—SR (µm), tool life

Table 15 Average COM values for the full factorial design for Case study 3.2

Average value of COM			
Factors	1	2	3
M	0.577964	**0.580266**	0.5698728
Cr	0.574258	**0.582319**	0.5715251
Cy	0.572857	**0.582416**	0.5728299
PH	**0.598846**	0.583079	0.5461774
S	0.576153	**0.580715**	0.5712344
Ti	0.571862	**0.581743**	0.5744975
T	0.568287	**0.582114**	0.5777007

Table 16 Machining parameters and their levels for Case study 3.3

Factor/process parameter	Level 1	Level 2	Level 3
S Cutting speed (m/min)	120	160	200
F Feed rate (mm/rev)	0.10	0.12	0.14
D Depth of cut (mm)	0.20	0.35	0.50
N Nose radius (mm)	0.40	0.80	1.20
E Environment	Dry	Wet	Cryo

(TL, min), cutting force (CF, N), and power consumption (PC, W)—and the Taguchi-fuzzy approach. The selected process parameters presented in Table 16 are studied using the L_{27} array. The η_{ij} values are calculated for each quality characteristic, and they are shown in Table 17. The two fuzzy subsets low and high are assigned to the four inputs by using the η_{ij} values of SR, TL, CF, and PC, as shown in Fig. 15. The two fuzzy subsets low and high are assigned to the four inputs by using the η_{ij} values of SR, TL, CF, and PC, as shown in Fig. 15. The five fuzzy subsets are then assigned to the output, COM value, as shown in Fig. 16. The COG method is used to convert the fuzzy value to a nonfuzzy value that is called the COM value.

Table 17 Experimental results for Case study 3.3

Exp_i	SR		TL		CF		PC	
	μm	η_{i1}	Min	η_{i2}	N	η_{i3}	W	η_{i4}
1	1.410	−2.985	29.000	38.489	171.300	−40.731	1066.000	−58.759
2	0.710	2.894	34.000	40.175	147.500	−43.379	1560.000	−63.862
3	0.600	4.485	54.670	44.297	111.740	−44.677	866.000	−60.562
4	0.470	6.554	34.670	40.341	120.300	−41.611	1493.000	−63.484
5	0.190	14.256	51.660	43.809	180.600	−45.135	987.000	−59.885
6	1.180	−1.414	27.000	38.174	236.200	−47.468	1187.000	−61.488
7	0.670	3.522	50.000	43.523	157.700	−43.959	960.000	−59.650
8	1.160	−1.264	24.660	37.391	214.400	−46.627	1134.000	−61.088
9	0.920	0.724	28.330	38.590	286.900	−49.157	1813.000	−65.170
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
24	0.180	14.886	37.660	41.062	168.700	−44.546	1613.000	−64.155
25	0.640	3.831	18.000	34.657	162.000	−44.196	1573.000	−63.937
26	0.310	10.170	34.330	40.258	162.500	−44.217	1453.000	−63.248
27	0.480	6.374	16.660	30.111	276.160	−48.827	1667.000	−64.438

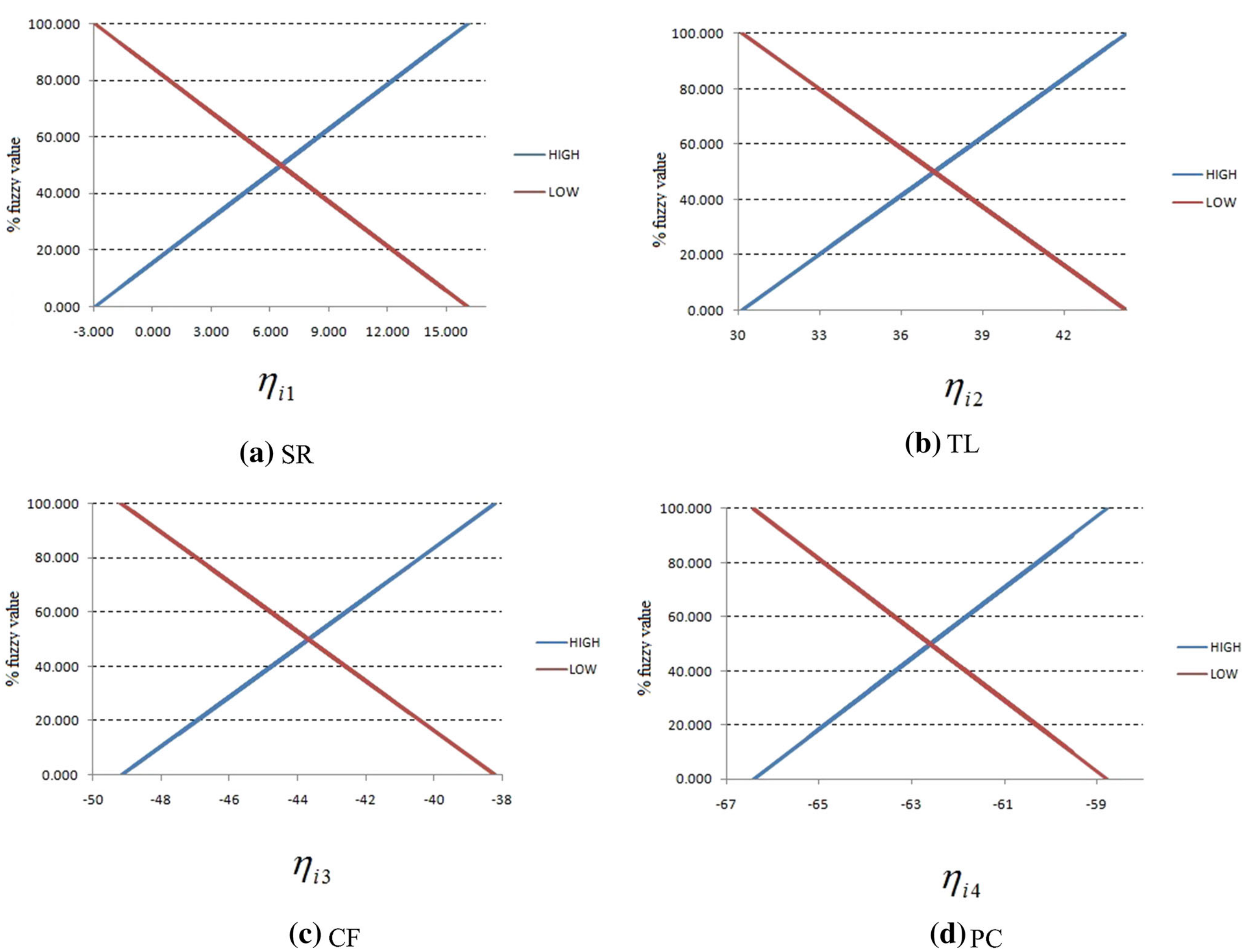


Fig. 15 MFs for the inputs of Case study 3.3

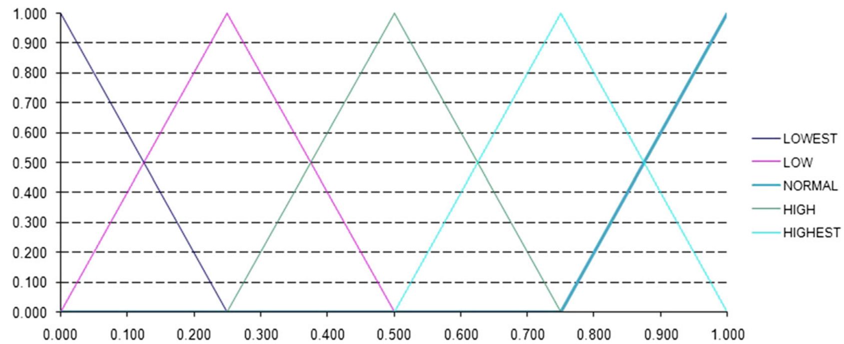


Fig. 16 MFs of COM for Case study 3.3

Table 18 COM averages for the full factorial design for Case study 3.3

Factor	Levels		
	1	2	3
S	0.5097	**0.5151**	0.4981
F	**0.5317**	0.5142	0.4771
D	**0.5434**	0.5096	0.4699
N	0.5022	**0.5121**	0.5086
E	0.471	0.4935	**0.5584**

The RBFNN technique is used to generate the full factorial data. The COM averages are calculated for each factor level, as shown in Table 18, where the combination of optimal factor levels is identified as $S_2F_1D_1N_2E_3$. The anticipated improvement for the optimal combination is obtained using the neural–fuzzy approach, and it is compared with that obtained using the Taguchi-fuzzy approach.

4 Research results

4.1 Optimization results for electro-erosion

The anticipated improvement for electro-erosion is shown in Table 19, where it is clear that for the initial factor settings ($V_1C_1DF_1T_1$), the values of the sum of η_{ij} averages for the MMR (LTB), EWR (STB), and SR (STB) are 49.458, −39.041, and 25.680, respectively. For the grey-Taguchi concept ($V_3C_3DF_2T_2$), they are 38.452, −24.495, and 19.809 dB, respectively. Finally, the fuzzy-RBFNN approach ($V_1C_1DF_1T_3$) yields the values of the sum for the MMR, SR, and EWR are 33.396, −17.464, and 62.124 dB, respectively. The anticipated improvements in the MMR, SR, and EWR determined using the grey-Taguchi concept (fuzzy-RBFNN) are −11.006 (−16.062), 14.546 (21.577), and −5.870 (36.444) dB, respectively. It is found that the grey-Taguchi concept provides larger anticipated improvement in the MMR (LTB type) compared with the fuzzy-RBFNN approach. By contrast, the fuzzy-RBFNN approach outperforms the grey-Taguchi concept in improving SR and EWR. Finally, the total anticipated improvements for the fuzzy-RBFNN approach (=13.987 dB) are considerably larger than those obtained (=−7.988 dB) using the grey-Taguchi concept.

4.2 Results for optimization of binary mixture removal

The anticipated improvements in Ry, Rr, Qy, and Qr using the Taguchi method and principal component analysis (fuzzy-RBFNN) are 7.4 (1.6), 6.0 (6.4), −10.7 (4.6), and −12.1 (9.5) dB, respectively, as shown in Table 20. It is clear that the Taguchi method and principal component analysis ($M_2Cr_1Cy_1PH1S_1Ti_1T_1$) provide a larger

Table 19 Comparison between the anticipated improvements for Case study 3.1

Method	Average η_{ij}	Total improvement	Response	The sum of η_{ij}	Individual improvement
Initial setting $V_1C_1DF_1T_1$	12		MMR	49.458	
			SR	−39.041	
			EWR	25.680	
A grey-Taguchi concept $V_3C_3DF_2T_2$	4.044	−7.988	MMR	38.452	−11.006
			SR	−24.495	14.546
			EWR	19.809	−5.870
Proposed neural-fuzzy $V_1C_1DF_1T_3$	26.019	13.987	MMR	33.396	−16.062
			SR	−17.464	21.577
			EWR	62.124	36.444

Table 20 Comparison of the improvements for Case Study 3.2

Method	Average η_{ij}	Overall improvement	Response	Total η_{ij}	Individual improvement
Initial setting	−5.1		Ry	201.7	
			Rr	226.8	
			Qy	−236.9	
			Qr	−211.9	
Taguchi and principle component $M_2Cr_1Cy_1PH1S_1Ti_1T_1$	−7.4	−2.4	Ry	209.1	7.4
			Rr	232.7	6.0
			Qy	−247.6	−10.7
			Qr	−224.0	−12.1
Neural-fuzzy $M_2Cr_2Cy_2Ph_1S_2Ti_2T_2$	0.4	5.5	Ry	203.3	1.6
			Rr	233.2	6.4
			Qy	−232.3	4.6
			Qr	−202.4	9.5

Table 21 Comparison of the improvements for Case study 3.3

Approach	η_{ij} averages	Overall improvement	Quality characteristics	The sum of η_{ij} averages	Individual improvement
Initial setting	−78.47		SR	23.07	
			TL	192.37	
			CF	−220.44	
			PC	−308.87	
Taguchi fuzzy $S_1F_1D_1N_2E_3$	−73.60	4.86	SR	32.58	9.51
			TL	199.64	7.26
			CF	−217.69	2.75
			PC	−308.95	−0.08
Neural-fuzzy $S_2F_1D_1N_2E_3$	−73.22	5.25	SR	35.90	12.83
			TL	197.76	5.39
			CF	−216.99	3.45
			PC	−309.55	−0.68

anticipated improvement in Ry compared with the fuzzy-RBFNN approach ($M_2Cr_2Cy_2Ph_1S_2Ti_2T_2$). By contrast, the fuzzy-RBFNN outperforms the Taguchi method and principal component analysis in improving Rr, Qy, and Qr. Finally, the total anticipated improvement for the fuzzy-RBFNN approach (=5.5 dB) is considerably larger than that (=−2.4 dB) for the Taguchi method and principal component analysis.

4.3 Results for optimization of high-speed CNC turning

The anticipated improvements in the SR, TL, CF, and PC using the Taguchi-fuzzy approach (fuzzy-RBFNN) are 9.51 (12.83), 7.26 (5.39), 2.75 (3.45), and −0.08 (−0.68) dB, respectively, as shown in Table 21. It is clear that the Taguchi-fuzzy approach ($S_1F_1D_1N_2E_3$) provides larger anticipated improvements in SF and PC compared with the fuzzy-RBFNN approach ($S_2F_1D_1N_2E_3$). By contrast, the fuzzy-RBFNN approach outperforms the Taguchi-fuzzy approach in improving the SR and CF. Finally, the total anticipated improvement obtained using the fuzzy-RBFNN approach (=5.5 dB) is considerably larger than that (=4.86 dB) obtained using the Taguchi-fuzzy approach.

5 Conclusions

This study successfully integrated fuzzy logic and the RBFNN and used multiple quality responses for the optimization of process performance. For each quality characteristic, η_{ij} values are calculated for each quality response, and these values are set as the input to the fuzzy model for obtaining a single response COM. The ANN is then used to predict the full data for the fractional design, for which the full factorial array is used as input to predict the COM value. The average COM values are calculated for each factor level and then adopted to identify the combination of optimal factor levels. Illustrative case studies from past studies are provided. The proposed approach shows appreciable improvements compared with previously used techniques, such as the grey-Taguchi concept, Taguchi method and principal component analysis, and Taguchi-fuzzy approach. Moreover, the proposed technique has the following advantages: (1) it provides a global optimal solution, (2) provides the largest overall improvement, and (3) shows flexibility in dealing with different data sets, increasing number of factors, levels, and factor weights. Future work should consider using the loss function instead of the SNR and/or using genetic algorithms instead of the RBFNN.

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