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ORIGINAL PAPER

Multiple solitons solutions, lump solutions and rogue wave solutions of the complex cubic Ginzburg–Landau equation with the Hirota bilinear method

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Abstract: In this paper, we obtain the exact solutions of the (1+1) dimensional complex cubic Ginzburg–Landau equation (CCGLE) by using the Hirota bilinear method, this equation is a universal model for the evolution of the envelope of slowly varying waves packets in nonlinear dissipative media. Different types of solutions including solitons, lump and rogue wave solutions are gotten by taking different transformations. Besides, the physical significance of these solutions of CCGLE can be understood better through drawing 3D, 2D and contour plots.

Keywords: Ginzburg–Landau equation; Hirota bilinear method; Solitons solutions; Lump solutions; Rogue wave solutions

1. Introduction

In many nonlinear physical systems, the diffusion of local pulses or solitons are controlled by different types of complex Ginzburg–Landau equations (CGLEs) including the dispersion and nonlinear effects of the conservative and dissipative forms[1, 2]. As universal models of mode formation in nonlinear dissipative media [3], complex Ginzburg–Landau equations (CGLEs) describe a variety of phenomena at the qualitative and even quantitative levels, from nonlinear wave to second–order transitions, from superconducting supercurrent, Bose-Einstein condensates (BEC) to strings and plasmas in liquid crystal field theory [4–8]. Therefore, it is very important to solve the exact solutions of partial differential equations, especially CGLEs [9–11]. So far, different types of solutions to one–dimensional complex Ginzburg–Landau equations have been analyzed such as impulse, quasi–impact, source, sink, periodic and quasi–periodic solutions [1].

As a kind of general amplitude equations in physics [12], complex cubic Ginzburg–Landau equations (CCGLEs) are also used to research many practical problems. For example, chemical turbulence, Rayleigh–Beynard convection [13], Taylor–Coutte flow, Poiseuille flow, reaction–diffusion systems, nonlinear optical and hydromechanical stability problems [14]. Specially, Its dark solitary waves were also discovered by Nazaki and Bekki, although they are unstable [16].

In this paper, we investigate the (1+1) dimensional complex cubic Ginzburg–Landau equation (CCGLE) which is used to describe dissipative systems above the bifurcation point [17], examples of applications include pulse generation by passive mode–locked soliton lasers [18], signal transmission in optical communications lines, traveling waves in binary fluid mixtures [19] and pattern formation [20] in many other physical systems [12]. The (1+1) dimensional complex cubic Ginzburg–Landau equation has the following form [14, 21]

$$\chi_t = r\chi - (1+ib)|\chi|^2\chi + (1+ia)\chi_{xx}, \tag{1}$$

where $\chi(x,t)$ is a complex function from R^2 to C representing the modulation of the oscillating field, b and r are two real control parameters [14]. $r\chi$ and $(1+ib)|\chi|^2\chi$ describe the local dynamics of the oscillators, $r\chi$ leads to the linear instability mechanism of the oscillation and $(1+ib)|\chi|^2\chi$ produces nonlinear amplitude saturation and frequency renormalization. $(1+ia)\chi_{xx}$ is the spatial coupling which explains diffusion and dispersion of oscillating motion [21].

As far as we know, many different types of solutions of the (1+1) dimensional CCGLE have been given. Such as,

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the traveling wave solutions were obtained by the first integral and the (G′/G)–expansion methods [12], the implicit and explicit solutions were obtained by using the methods of variable separation and plane wave solutions [14], the singular bright solitons, the hybrid periodic solitons, the dark solitons and the periodic M-shaped solitons solutions were acquired via the Sardar sub-equation method [15], the periodic solitons solutions were also gotten through the energy balance method [15], the existence of time quasi–periodic solutions under periodic boundary conditions has been discussed [22]. In addition, by constructing the KAM theorem for dissipative systems with unbounded pertubations and multi–normal frequencies, a Contorian branches of the two dimensional invariant torus for CCGLE also have been obtained [22]. Finally, the kink solitons and periodic solitons solutions were attained based on the foundation of the notions of the planar dynamical theory for CCGLE [23].

However, there are still some solutions that are not used by the (1+1) dimensional CCGLE, such as the Hirota bilinear method has remarkable characteristics, which make it suitable for determining different kinds of solutions [24–32]. In this paper, we will attain the multiple solitons, lump and rogue wave solutions of Eq. (1) by using the Hirota bilinear method and setting different forms of auxiliary equations.

In order to gain the exact solutions of Eq. (1), we choose following equations to convert it

$$\chi = u + iv, \qquad |\chi| = \sqrt{u^2 + v^2}, \tag{2}$$

where u, v are undetermined functions of independent variables x and t.

Inserting Eq. (2) into Eq. (1) and seperating its real and imaginary parts, we can gain

$$\mathrm{Re}\ : u_t - u_{xx} + av_{xx} - ru + u^3 + v^2u - bu^2v - bv^3 = 0. \tag{3}$$

$$\mathrm{Im}\ : v_t - v_{xx} - au_{xx} - rv + v^3 + u^2v + bv^2u + bu^3 = 0. \tag{4}$$

The structure of this article is shown below. In Sect. 2, we can obtain the multiple solitons solutions of the (1+1) dimensional CCGLE through the Hirota bilinear method. The lump solutions can be given in Sect. 3 and the lump–one stripe soliton interaction solutions also can be obtained in Sect. 4. In Sect. 5, the rogue wave solutions can be acquired. In Sect. 6, we discuss the physical meanings of images that we have obtained. Finally, the conclusion is stated in Sect. 7.

2. Multiple solitons solutions

In this section, we will apply the Hirota bilinear method to attain the multiple solitons solutions of the (1+1) dimensional CCGLE.

Firstly, substituting

$$u = e^{\theta_i}, \quad \theta_i = k_i x + m_i t + c_i, \quad i = 1, 2, \ldots, N, \tag{5}$$

where k_i, m_i and c_i are undetermined constants, into the linear terms of Eq. (3) and Eq. (4) and solving the corresponding equations, we can gain the dispersion relations as follows

$$m_i = k_i^2 + r, \quad \theta_i = k_i x + (k_i^2 + r)t + c_i, \quad i = 1, 2, \ldots, N, \tag{6}$$

where k_i and c_i are undetermined constants.

From Eq. (6), we can notice that the dispersion relations m_i only depend on the coefficients k_i of x.

In order to get the multiple solitons solutions of the (1+1) dimensional CCGLE, we substitute

$$\begin{aligned} u &= R_1(\ \mathrm{In}\, f)_x, \\ v &= R_2(\ \mathrm{In}\, f)_x, \end{aligned} \tag{7}$$

where R_1 and R_2 are undetermined constants. f as the auxiliary functions will be determined later.

2.1. Single soliton solutions

To gain the single soliton solutions of the (1+1) dimensional CCGLE, we take account the form of f like following

$$f = 1 + e^{\theta_1}, \tag{8}$$

where $\theta_1 = k_1 x + (k_1^2 + r)t + c_1$ and k_1, c_1 are undetermined constants.

Inserting Eq. (8) with Eq. (7) into Eq. (3) and Eq. (4), collecting all power coefficients of e^{θ_1} and making their results to zero, we can get

$$\begin{aligned} &a = 0, \quad b = 0, \quad R_1 = \sqrt{2 - R_2^2}, \\ &k_1 = \sqrt{\frac{r}{2}}, \quad r = r, \quad R_2 = R_2. \end{aligned} \tag{9}$$

Substituting Eq. (9) with Eq. (8) into Eq. (7), we know

$$f = 1 + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1},$$

$$u = \frac{\sqrt{\frac{r(2-R_2^2)}{2}}\ e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1}}{1 + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1}}, v = \frac{R_2\sqrt{\frac{r}{2}}e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1}}{1 + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1}}, \tag{10}$$

$$\chi = \frac{(\sqrt{2 - R_2^2} + R_2 i)\sqrt{\frac{r}{2}}e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1}}{1 + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1}},$$

where c_1, r and R_2 are arbitrary constants.

2.2. Double solitons solutions

The auxiliary function f for the double solitons solutions is given by

$$f = 1 + e^{\theta_1} + e^{\theta_2} + a_{12}e^{\theta_1+\theta_2}, \tag{11}$$

where $\theta_1 = k_1x + (k_1^2 + r)t + c_1, \theta_2 = k_2x + (k_2^2 + r)t + c_2$ and k_1, k_2, c_1, c_2 are undetermined constants.

Interposing Eq. (11) with Eq. (7) into Eq. (3) and Eq. (4), collecting all coefficients of the same power and making them to zero, we can gain

Set-1:

$$\begin{gathered} a = 0, \quad b = 0, \quad k_1 = -\sqrt{\frac{r}{2}}, \quad k_2 = \sqrt{\frac{r}{2}}, \\ R_1 = \sqrt{2 - R_2^2}, \quad a_{12} = 0, \quad r = r, \quad R_2 = R_2. \end{gathered} \tag{12}$$

Set-2:

$$\begin{gathered} a = 0, \quad b = 0, \quad k_1 = \sqrt{\frac{r}{2}}, \quad k_2 = -\sqrt{\frac{r}{2}}, \\ R_1 = \sqrt{\frac{1 - 2R_2^2}{2}}, \quad a_{12} = 1, \quad r = r, \quad R_2 = R_2. \end{gathered} \tag{13}$$

Plugging above equations with Eq. (11) into Eq. (7), different double solitons solutions can be given.

Case 1: $a_{12} = 0$.

$$\begin{aligned} f &= 1 + e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2}, \\ v &= \frac{R_2\sqrt{\frac{r}{2}}(e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2} - e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1})}{1 + e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2}}, \\ u &= \frac{\sqrt{\frac{r(2-R_2^2)}{2}}(e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2} - e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1})}{1 + e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2}}, \\ \chi &= \frac{(\sqrt{2 - R_2^2} + R_2 i)\sqrt{\frac{r}{2}}(e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2} - e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1})}{1 + e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2}}, \end{aligned} \tag{14}$$

where R_2, r, c_1 and c_2 are arbitrary constants.

Case 2: $a_{12} \neq 0$.

$$\begin{aligned} f &= 1 + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} + e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2} + e^{3rt + c_1 + c_2}, \\ v &= \frac{R_2\sqrt{\frac{r}{2}}(e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} - e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2})}{1 + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} + e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2} + e^{3rt + c_1 + c_2}}, \\ u &= \frac{\frac{\sqrt{r(1-2R_2^2)}}{2}(e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} - e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2})}{1 + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} + e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2} + e^{3rt + c_1 + c_2}}, \\ \chi &= \frac{(\sqrt{\frac{1-2R_2^2}{2}} + R_2 i)\sqrt{\frac{r}{2}}(e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} - e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2})}{1 + e^{\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_1} + e^{-\sqrt{\frac{r}{2}}x + \frac{3}{2}rt + c_2} + e^{3rt + c_1 + c_2}}. \end{aligned} \tag{15}$$

where R_2, r, c_1 and c_2 are arbitrary constants.

2.3. Three solitons solutions

The auxiliary function f for the three solitons solutions is given in the following form

$$\begin{aligned} f = {} & 1 + e^{\theta_1} + e^{\theta_2} + e^{\theta_3} + a_{12}e^{\theta_1+\theta_2} \\ & + a_{13}e^{\theta_1+\theta_3} + a_{23}e^{\theta_2+\theta_3} \end{aligned} \tag{16}$$

where $\theta_1 = k_1x + (k_1^2 + r)t + c_1, \theta_2 = k_2x + (k_2^2 + r)t + c_2, \theta_3 = k_3x + (k_3^2 + r)t + c_3$ and $k_1, k_2, k_3, c_1, c_2, c_3$ are undetermined constants.

Inserting Eq. (16) with Eq. (7) into Eq. (3) and Eq. (4), combining similar terms and making their coefficients to zero, we can know

$$\begin{gathered} a = 0, b = 0, k_1 = -\sqrt{\frac{r}{2}}, k_2 = \sqrt{\frac{r}{2}}, k_3 = \sqrt{\frac{r}{2}}, \\ a_{12} = 1, a_{23} = 1, a_{13} = 1, b_{123} = 1, \ r = r, R_1 = \frac{\sqrt{2 - 9R_2^2}}{3}. \end{gathered} \tag{17}$$

Substituting Eq. (17) with Eq. (16) into Eq. (7), three solitons solutions can be obtained as follows

$$
\begin{aligned}
f &= 1 + e^{-\sqrt{\frac{r}{2}}x+\frac{3}{2}rt+c_1} + e^{\sqrt{\frac{r}{2}}x+\frac{3}{2}rt}W_1 + e^{3rt}W_2 \\
&\quad + e^{2\sqrt{\frac{r}{2}}x+3rt+A_{23}} + e^{\sqrt{\frac{r}{2}}x+\frac{9}{2}rt+B_{123}}, \\
v &= \frac{R_2\sqrt{\frac{r}{2}}(-e^{-\sqrt{\frac{r}{2}}x+\frac{3}{2}rt+c_1} + e^{\sqrt{\frac{r}{2}}x+\frac{3}{2}rt}W_1 + 2e^{2\sqrt{\frac{r}{2}}x+3rt+A_{23}} + e^{\sqrt{\frac{r}{2}}x+\frac{9}{2}rt+B_{123}})}{1 + e^{-\sqrt{\frac{r}{2}}x+\frac{3}{2}rt+c_1} + e^{\sqrt{\frac{r}{2}}x+\frac{3}{2}rt}W_1 + e^{3rt}W_2 + e^{2\sqrt{\frac{r}{2}}x+3rt+A_{23}} + e^{\sqrt{\frac{r}{2}}x+\frac{9}{2}rt+B_{123}}}, \\
u &= \frac{\frac{\sqrt{r(2-9R_2^2)}}{3\sqrt{2}}(-e^{-\sqrt{\frac{r}{2}}x+\frac{3}{2}rt+c_1} + e^{\sqrt{\frac{r}{2}}x+\frac{3}{2}rt}W_1 + 2e^{2\sqrt{\frac{r}{2}}x+3rt+A_{23}} + e^{\sqrt{\frac{r}{2}}x+\frac{9}{2}rt+B_{123}})}{1 + e^{-\sqrt{\frac{r}{2}}x+\frac{3}{2}rt+c_1} + e^{\sqrt{\frac{r}{2}}x+\frac{3}{2}rt}W_1 + e^{3rt}W_2 + e^{2\sqrt{\frac{r}{2}}x+3rt+A_{23}} + e^{\sqrt{\frac{r}{2}}x+\frac{9}{2}rt+B_{123}}}, \\
\chi &= \frac{(\frac{\sqrt{r(2-9R_2^2)}}{3\sqrt{2}} + R_2 i)(-e^{-\sqrt{\frac{r}{2}}x+\frac{3}{2}rt+c_1} + e^{\sqrt{\frac{r}{2}}x+\frac{3}{2}rt}W_1 + 2e^{2\sqrt{\frac{r}{2}}x+3rt+A_{23}} + e^{\sqrt{\frac{r}{2}}x+\frac{9}{2}rt+B_{123}})}{1 + e^{-\sqrt{\frac{r}{2}}x+\frac{3}{2}rt+c_1} + e^{\sqrt{\frac{r}{2}}x+\frac{3}{2}rt}W_1 + e^{3rt}W_2 + e^{2\sqrt{\frac{r}{2}}x+3rt+A_{23}} + e^{\sqrt{\frac{r}{2}}x+\frac{9}{2}rt+B_{123}}},
\end{aligned}
\tag{18}
$$

where $W_1 = e^{c_2} + e^{c_3}, W_2 = e^{A_{12}} + e^{A_{13}}, A_{12} = c_1 + c_2, A_{13} = c_1 + c_3, A_{23} = c_2 + c_3, B_{123} = c_1 + c_2 + c_3$ and R_2, r, c_1, c_2, c_3 are arbitrary constants.

3. Lump solutions

To gain the lump solutions of the (1+1) dimensional CCGLE, we use the following transformations

$$u = (In\ f)_x, v = (In\ g)_x, \tag{19}$$

where f, g as the auxiliary functions will be decided later.

Equation(19) transforms Eq. (3) and Eq. (4) into the following bilinear forms

$$
\begin{aligned}
& f^2g^3(f_{xt} - f_{xxx}) + fg^3(-f_xf_t + 3f_xf_{xx} \\
&\quad - 2g^3f_x^3 + af^3g^2g_{xxx} - 3af^3gg_xg_{xx} + 2af^3g_x^3 \\
&- rf^2g^3f_x + g^3f_x^3 + f^2gg_x^2f_x - bfg^2f_x^2g_x - bf^3g_x^3 = 0.
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
& g^2f^3(g_{xt} - g_{xxx}) + gf^3(-g_xg_t + 3g_xg_{xx} \\
&\quad - 2f^3g_x^3 - ag^3f^2f_{xxx} + 3ag^3ff_xf_{xx} + 2ag^3f_x^3 \\
&- rg^2f^3g_x + f^3g_x^3 + g^2ff_x^2g_x + bgf^2g_x^2f_x + bg^3f_x^3.
\end{aligned}
\tag{21}
$$

For lump solutions, we assume f and g have the following bilinear forms

$$f = \xi_1^2 + \xi_2^2 + \alpha,\ g = \xi_1^2 + \xi_2^2 + \beta, \tag{22}$$

where $\xi = p_1x + l_1t, \xi_2 = p_2x + l_2t$ and $p_1, p_2, l_1, l_2, \alpha, \beta$ are undecided constants.

Inserting Eq. (22) into Eq. (20) and Eq. (21) and solving corresponding equations, we can attain

$$p_1 = p_1,\quad l_1 = l_1,\quad \alpha = \alpha,\quad \beta = \beta,\quad p_2 = -p_1i,\quad l_2 = l_1i. \tag{23}$$

Interposing Eq. (23) into Eq. (22) and Eq. (19), the form of lump solution can be given

$$
\begin{aligned}
& f = 4p_1l_1xt + \alpha,\ g = 4p_1l_1xt + \beta, \\
& u = \frac{4p_1l_1t}{4p_1l_1xt + \alpha},\ v = \frac{4p_1l_1t}{4p_1l_1xt + \beta}, \\
& \chi = \frac{4p_1l_1t}{4p_1l_1xt + \alpha} + i\frac{4p_1l_1t}{4p_1l_1xt + \beta},
\end{aligned}
\tag{24}
$$

where p_1, l_1, α, β are arbitrary constants.

4. Lump-one stripe soliton interaction solutions

The auxiliary functions f and g for the lump-one stripe soliton interaction solutions can be given as

$$f = \xi_3^2 + \xi_4^2 + e^{\xi_5} + \delta,\ g = \xi_3^2 + \xi_4^2 + e^{\xi_5} + \epsilon, \tag{25}$$

where $\xi_3 = p_3x + l_3t, \xi_4 = p_4x + l_4t, \xi_5 = p_5x + l_5t$ and $p_3, p_4, p_5, l_3, l_4, l_5, \delta, \epsilon$ are undecided constants.

Plugging Eq. (25) into Eq. (20) and Eq. (21), the values of undetermined constants can be gotten.

Set-1:

$$
\begin{aligned}
& a = 1, b = \frac{1}{2}, l_3 = i, l_4 = 1, r = r, \\
& \delta = 0, \epsilon = 0, p_3 = p_3, p_5 = \sqrt{r}, l_5 = r, p_4 = p_3i.
\end{aligned}
\tag{26}
$$

Set-2:

$$
\begin{aligned}
& a = 1, b = 1, r = 0, p_5 = 0, \delta = 0, \epsilon = 0, \\
& l_3 = l_3, p_3 = p_3, p_4 = p_3i, l_4 = -l_3i, l_5 = l_5.
\end{aligned}
\tag{27}
$$

Inserting these results into Eq. (25) and Eq. (19), we can get following different lump-one stripe soliton interaction solutions of the (1+1) dimensional CCGLE.

Case 1: $r \neq 0, p_5 \neq 0$.

$$
\begin{aligned}
&f = 4p_3ixt + e^{\sqrt{r}x+rt}, g = 4p_3ixt + e^{\sqrt{r}x+rt},\\
&u = \frac{4p_3it + \sqrt{r}e^{\sqrt{r}x+rt}}{4p_3ixt + e^{\sqrt{r}x+rt}}, v = \frac{4p_3it + \sqrt{r}e^{\sqrt{r}x+rt}}{4p_3ixt + e^{\sqrt{r}x+rt}},\\
&\chi = \frac{(1+i)(4p_3it + \sqrt{r}e^{\sqrt{r}x+rt})}{4p_3ixt + e^{\sqrt{r}x+rt}},
\end{aligned} \tag{28}
$$

where p_3, r are arbitrary constants.

Case 2: $r = 0, p_5 = 0$.

$$
\begin{aligned}
&f = 4p_3l_3xt + e^{l_5t}, g = 4p_3l_3xt + e^{l_5t},\\
&u = \frac{4p_3l_3t}{4p_3l_3xt + e^{l_5t}}, v = \frac{4p_3l_3t}{4p_3l_3xt + e^{l_5t}},\\
&\chi = \frac{(1+i)4p_3l_3t}{4p_3l_3xt + e^{l_5t}},
\end{aligned} \tag{29}
$$

where p_3, l_3, l_5 are arbitrary constants.

5. Rogue wave solutions

To gain rogue wave solutions, the bilinear forms of auxiliary functions f and g can be assumed as follows

$$
f = \xi_6^2 + \xi_7^2 + \cosh(\xi_8) + \sigma, g = \xi_6^2 + \xi_7^2 + \cosh(\xi_8) + \zeta, \tag{30}
$$

where $\xi_6 = p_6x + l_6t, \xi_7 = p_7x + l_7t, \xi_8 = p_8x + l_8t$ and $p_6, p_7, p_8, l_6, l_7, l_8, \sigma, \zeta$ are undecided constants.

Inserting Eq. (30) into Eq. (20) and Eq. (21), we can understand

Set-1:

$$
\begin{aligned}
&b = a, p_7 = p_6i, l_7 = -l_6i, p_6 = p_6, l_6 = l_6,\\
&\sigma = 0, \quad \zeta = 0, r = r, l_8 = 0, p_8 = \frac{\sqrt{r}}{\sqrt{2(1-a)}}.
\end{aligned} \tag{31}
$$

Set-2:

$$
\begin{aligned}
&b = 0, a = 0, l_8 = 0, p_8 = \sqrt{\frac{r}{2}}, r = r,\\
&\sigma = 0, \zeta = 0, \quad p_7 = p_6i, l_7 = l_6i, p_6 = p_6, l_6 = l_6,
\end{aligned} \tag{32}
$$

where p_6, l_6, r are arbitrary constants.

Plugging above results into Eq. (30) and Eq. (19), we can acquire different forms of rogue wave solutions of the (1+1) dimensional CCGLE.

Case 1: $a \neq 0$.

$$
\begin{aligned}
&f = 4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2(1-a)}}x\right), \ g = 4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2(1-a)}}x\right),\\
&u = \frac{4p_6l_6t + \sqrt{\frac{r}{2(1-a)}}\cosh\left(\sqrt{\frac{r}{2(1-a)}}x\right)}{4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2(1-a)}}x\right)}, \ v = \frac{4p_6l_6t + \sqrt{\frac{r}{2(1-a)}}\cosh\left(\sqrt{\frac{r}{2(1-a)}}x\right)}{4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2(1-a)}}x\right)},\\
&\chi = \frac{(1+i)(4p_6l_6t + \sqrt{\frac{r}{2(1-a)}}\cosh\left(\sqrt{\frac{r}{2(1-a)}}x\right))}{4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2(1-a)}}x\right)},
\end{aligned} \tag{33}
$$

where r, p_6, l_6 are arbitrary constants.

Case 2: $a = 0$.

$$
\begin{aligned}
&f = 4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2}}x\right), g = 4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2}}x\right),\\
&u = \frac{4p_6l_6t + \sqrt{\frac{r}{2}}\cosh\left(\sqrt{\frac{r}{2}}x\right)}{4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2}}x\right)}, \ v = \frac{4p_6l_6t + \sqrt{\frac{r}{2}}\cosh\left(\sqrt{\frac{r}{2}}x\right)}{4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2}}x\right)},\\
&\chi = \frac{(1+i)(4p_6l_6t + \sqrt{\frac{r}{2}}\cosh\left(\sqrt{\frac{r}{2}}x\right))}{4p_6l_6xt + \cosh\left(\sqrt{\frac{r}{2}}x\right)},
\end{aligned} \tag{34}
$$

where r, p_6, l_6 are arbitrary constants.

6. Images analyses

By taking special values to r, R_2 and c_1, we can gain the single soliton of Eq. (1) as shown in Fig. 1. We can know that a list of single solitons that are symmetric about the plane $x = 0$ appear in the $x - t$ plane and the solitons only have one peak, which can be seen in Fig. 1b. When the values of t are fixed, it is clear that the solitons with the same shapes and amplitudes appear periodically and the same situation doesn't occur when the values of x are fixed, which indicates that the single solitons propagate along the x-axis. Besides, we can see that the single solitons here are optical solitons from the density plot.

Similarly, as Figs. 2 and 3 showing, the double and three solitons of Eq. (1) can be given by assigning special values to r, R_2 and c_j. We can observe that the double and three solitons also propagate periodically along the x-axis and the shapes and amplitudes keep unchange when the values of t are fixed. However, unlike the single solitons, the double solitons have two peaks and the three solitons have

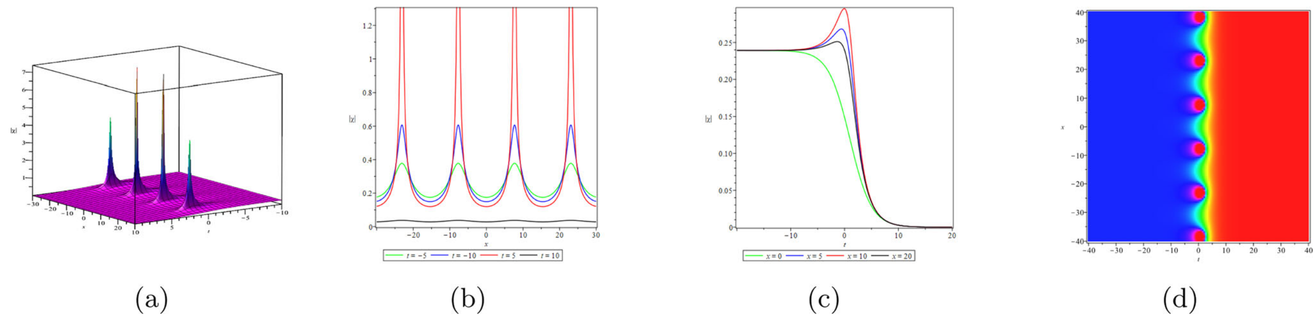

Fig. 1 Single soliton solution $\chi(x,t)$ with $r=-\frac{1}{3},\ c_1=\frac{1}{2},\ R_2=-2$. (**a**) 3 D-plot . (**b**) 2 D-plot .(**c**) 2 D-plot . (**d**) density plot

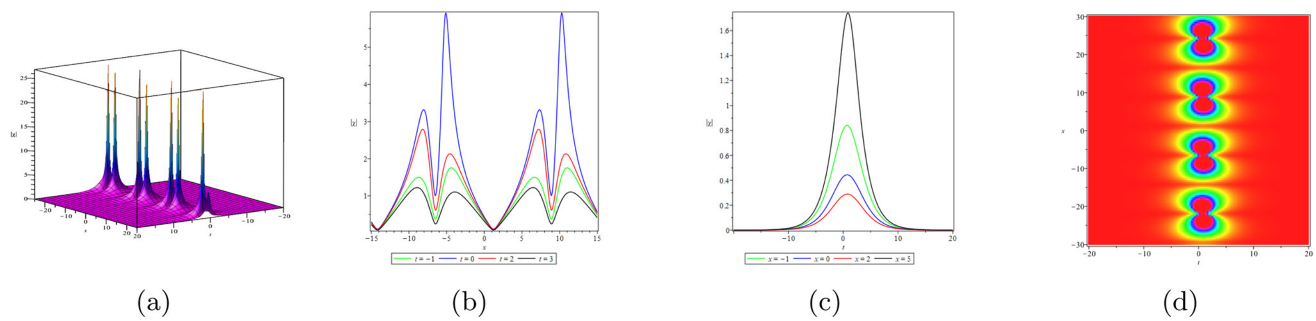

Fig. 2 Double solitons solution $\chi(x,t)$ with $r=-\frac{1}{3},\ c_1=\frac{1}{2},\ c_2=\dfrac{1}{4}+i,\ R_2=2$. (**a**) 3 D-plot .(**b**) 2 D-plot . (**c**) 2 D-plot . (**d**) density plot

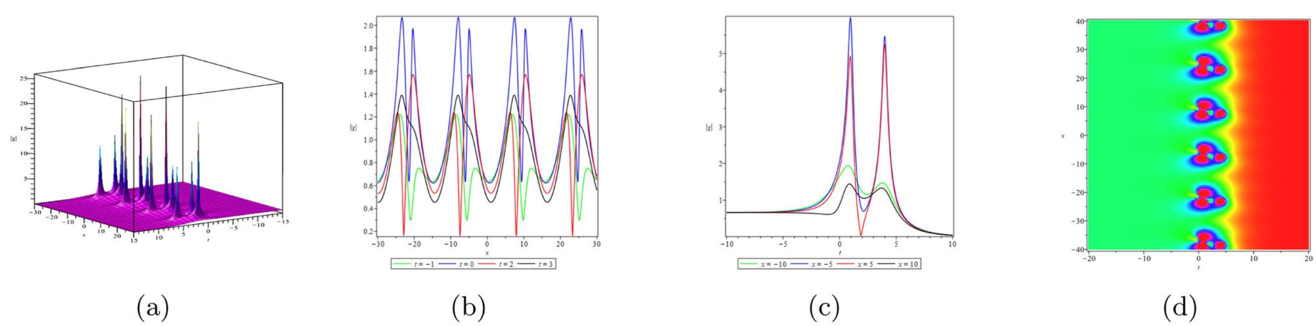

Fig. 3 Three solitons solution $\chi(x,t)$ with $r=-\frac{1}{3},\ c_1=\frac{1}{4}+i,\ c_2=2,\ c_3=\frac{1}{2}, R_2=\frac{1}{4}$. (**a**)3 D-plot . (**b**) 2 D-plot . (**c**) 2 D-plot . (**d**) density plot

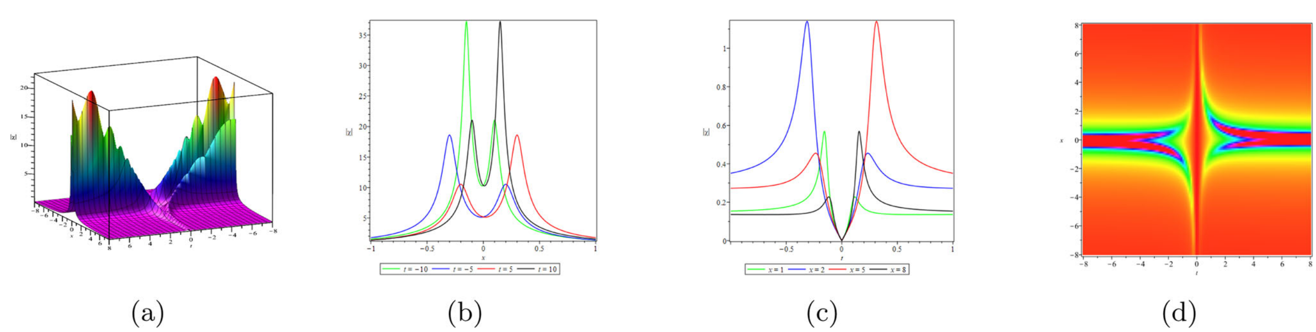

Fig. 4 Lump solution $\chi(x,t)$ with $\alpha=1-0.4i,\ l_1=0.2+0.1i,\ p_1=1-0.5i,\ \beta=-1.5+0.3i$. (**a**) 3 D-plot . (**b**) 2 D-plot . (**c**) 2 D-plot . (**d**) density plot

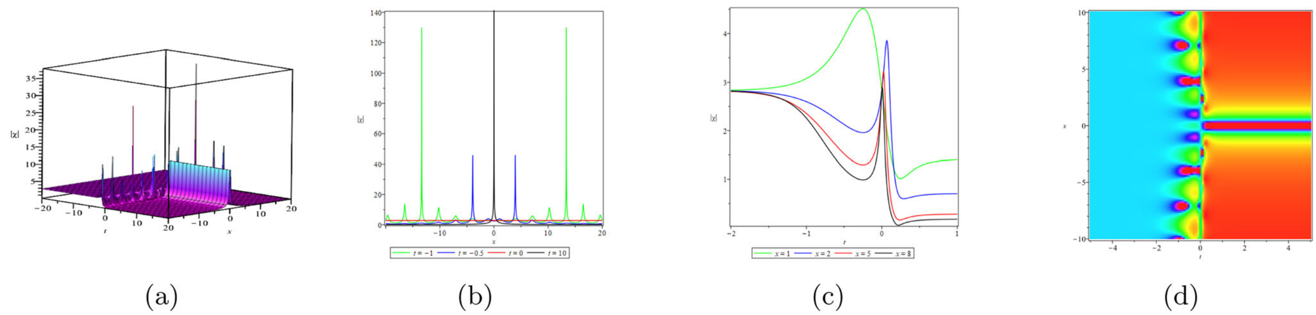

Fig. 5 Lump-one stripe soliton interaction solution $\chi(x,t)$ with $p_3 = 1,\ r = -4$. (**a**) 3 D-plot .(**b**) 2 D-plot . (**c**) 2 D-plot . (**d**) density plot

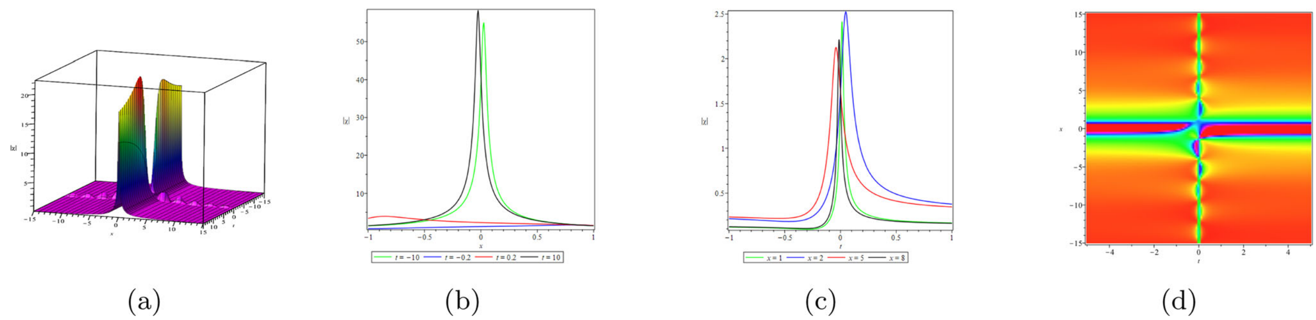

Fig. 6 Rogue solution $\chi(x,t)$ with $a = \frac{1}{4},\ l_6 = \frac{1}{2},\ p_6 = 1 + i,\ r = -2$. (**a**) 3 D-plot . (**b**) 2 D-plot .(**c**) 2 D-plot . (**d**) density plot

three peaks in the propagation process, which can be seen in Figs. 2a and 3a. In general, each line in Fig. 2b should contain two peaks and in Fig. 3b should contain three peaks. However, we can see that the lines only have two peaks in Fig. 3b and this is due to the arrangement of the three solitons in Fig. 3a. To prove this point, we find that each line in Fig. 3c also contains two peaks.

We can acquire the image of the lump solution as shown in Fig. 4 by giving α, β, l_1 and p_1 special values. It seems that the image of the lump solution is simliar to the single soliton solution from Fig. 4b, but we also can notice that the image appears in chunk at the base of the soliton. When the values of x are fixed, it is obvious that the two lump solitons appear symmetrically on either side of the line $t = 0$ and the function values gradually approache 0 as t approache 0. Finally, we can know that there exists a lump region between the two lump solitons.

The image of the lump-one stripe soliton interaction solution can be given as shown in Fig. 5 by taking special values to p_3 and r. As t gradually approache $t = 0$ from the negative half-axis of t, solitons that propagate periodically along the x-axis appear. While $x = 0$, soliton is periodic along the t-axis only if $t > 0$. This result also can be drawn from the density plot.

As Fig. 6 showing, the diagram of the rogue wave solution can be obtained by giving a, l_6, p_6 and r. It is obvious that there is a list of waves with the same amplitudes in the middle of the plane and the amplitudes of the waves increase for a short period of time, which is exactly the characteristic of the rogue wave. In addition, we also can find that at the amplitudes of the waves $t = -0.2$ and 0.2 are significantly lower than those at $t = -10$ and 10 from Fig. 6b, which also show that the amplitudes of the waves increase sharply as they approache the line $t = 0$.

7. Conclusion

In this article, we gain solitons solutions of the (1+1) dimensional CCGLE via the transformations of $u = (\ \mathrm{In}\, f)_x$ and $v = (\ \mathrm{In}\, f)_x$. On this basis, by taking different forms of f, the single, double and three solitons solutions can be obtained. Besides, we also attain the lump and rogue wave solutions of the (1+1) dimensional CCGLE by applying the transformations $u = (\ \mathrm{In}\, f)_x$ and $v = (\ \mathrm{In}\, g)_x$ and taking different functions forms of f and g. Finally, through 3D , 2D, density images and proper interpretation, we can know the features and the contours differences of different solutions. This article is different from previous literatures that need to transform partial differential equations into ordinary differential equations. At the same time, this article also provides ideas and ways for those equations that can't be completely reduced to bilinear forms but whose solitons solutions are desired.

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Declarations

Conflict of interest The authors declare that they have no Conflict of interest.

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