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ORIGINAL PAPER

Influence of f$(\mathcal{R}, \mathcal{T}, \mathcal{Q})$ gravity on cylindrical collapse

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Abstract: This article examines the dynamics of gravitational collapse in $f(\mathcal{R}, \mathcal{T}, \mathcal{Q})$ gravity, where $\mathcal{Q} = \mathcal{R}_{ab}\mathcal{T}^{ab}$. We consider self-gravitating anisotropic cylindrical geometry whose interior is filled with dissipative matter configuration and match it with exterior cylindrically symmetric spacetime at the hypersurface through junction conditions. We employ the Misner–Sharp and Müler–Israel–Stewart formalisms to derive the dynamical as well as transport equations corresponding to the model $\mathcal{R} + \Phi\sqrt{\mathcal{T}} + \Psi\mathcal{Q}$, where Φ and Ψ are arbitrary coupling constants. We then establish some relations between these equations through which the impact of effective matter variables, heat dissipation and the bulk viscosity on the collapse rate is studied. Further, we express the Weyl scalar in terms of the effective matter sector. We also obtain the conformal flatness by applying some restrictions on the considered model and taking dust configuration into the account. Finally, we investigate various cases to check whether the modified corrections increase or decrease the collapse rate.

Keywords: $f(\mathcal{R}, \mathcal{T}, \mathcal{R}_{ab}\mathcal{T}^{ab}$; Gravitational collapse; Self-gravitating structures

1. Introduction

Cosmological observations reveal that our universe is originated by the expansion of superheated matter and energy. Cosmologists explored that a considerable portion of this unfathomable universe is made up of stars, planets and galaxies. The most appealing and promising phenomenon in the structural formation of these celestial objects is the gravitational collapse. The pioneer work of Chandrasekhar [1] on this phenomenon helps scientists to understand its importance in the field of relativistic astrophysics. He found that a star remains stable until its external pressure and internal force of attraction (due to its mass) are counterbalanced by each other. The dynamics of dust collapse has been discussed by Oppenheimer and Snyder [2], from which they found that such collapse eventually results in a black hole. Misner and Sharp [3] studied spherical geometry coupled with anisotropic fluid and checked how the collapse rate is affected by pressure anisotropy. The Misner–Sharp technique has been employed by Herrera and Santos [4] to investigate the collapsing rate of a sphere and found that the energy dissipates in the form of heat/radiations. Herrera et al. [5] analyzed the impact of anisotropy on the collapse of cylindrically symmetric matter source. Sharif and his collaborators [6] studied the dynamics of uncharged and charged spherical/cylindrical systems and deduced that the collapse rate is reduced in the presence of electric charge.

As the process of gravitational collapse is highly dissipative, the effects of heat dissipation in this phenomenon cannot be ignored [7, 8]. Chan [9] explored the collapsing phenomenon for a radiating compact object and revealed that the shear viscosity increases anisotropy of the fluid distribution. Di Prisco et al. [10] studied anisotropic matter configuration and disclosed that the explosion in the internal region of spherical geometry causes the formation of singularity. Nath et al. [11] examined the collapsing rate by employing matching criteria between quasi-spherical Szekeres and charged Vaidya spacetimes as interior and exterior geometries, respectively. They concluded that the formation of naked singularity is supported by electric charge. Herrera et al. [12] discussed self-gravitating viscous dissipative fluid and found that the dissipative parameters decrease the force of gravity that eventually decreases the collapsing rate.

Cylindrical gravitational waves exist and support cylindrically symmetric self-gravitating structures whose

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study yields significant consequences. Such geometrical objects prompted many researchers to explore their different fundamental characteristics. The study of these structures was pioneered by Bronnikov and Kovalchuk [13]. Wang [14] studied four-dimensional cylinder and determined exact solutions to the field equations corresponding to a massless scalar field. He found that collapse of such object may result in the black hole. Guha and Banerji [15] studied the dynamics of cylindrical anisotropic geometry, experiencing heat dissipation and undergoing the gravitational collapse, and derived the solutions for the matter source. In stellar evolution, the Weyl tensor plays a significant role that helps to measure the curvature of geometrical structure. The gravitational collapse of a sphere has been discussed by Penrose [16] by formulating a relation between state variables and the Weyl tensor. Sharif and Fatima [17] represented the Weyl tensor in terms of matter variables, anisotropy and coefficient of shear viscosity for cylindrical configuration and discussed the collapsing rate. They found conformal flatness condition corresponding to the homogeneous energy density.

The current accelerated expansion of the universe is considered as the most fascinating phenomenon in the field of cosmology and astrophysics for the last couple of years [18, 19]. This expansion was claimed to be triggered by an obscure form of force having immense repulsive effects, known as dark energy. The study of such cosmic nature in the theory of general relativity ($\mathbb{GR}$) faces some deficiencies like cosmic coincidence and fine-tuning problem. In this context, scientists developed several modifications to $\mathbb{GR}$ to address such issues appropriately. To study cosmological outcomes at large scales, the simplest possible generalization of $\mathbb{GR}$ was attained by replacing the Ricci scalar with its generic functional, named $f(\mathcal{R})$ theory [20]. Different modified $f(\mathcal{R})$ models have been investigated through multiple approaches and the obtained results are found to be physically feasible [21]–[24].

The idea of matter–geometry coupling was initially presented by Bertolami et al. [25] to study the appealing nature as well as composition of the universe. They analyzed the impact of such interaction in $f(\mathcal{R})$ framework by engaging the geometrical term in the fluid Lagrangian. This interaction was recently generalized by Harko et al. [26] at action level, who introduced $f(\mathcal{R},\mathcal{T})$ gravity, $\mathcal{T}$ being trace of the energy–momentum tensor (EMT). The gravitational theories involving such a matter term results in its non-vanishing divergence. This theory produces several astonishing results corresponding to self-gravitating structures [27–30]. However, the $f(\mathcal{R},\mathcal{T})$ gravity fails to entail the coupling effects on compact bodies at some point; thus, one needs to overcome this issue. Haghani et al. [31], in this regard, generalized the functional by inserting an additional term $\mathcal{Q}$, that represents contraction of EMT with the Ricci tensor $\mathcal{R}_{ab}$. They considered three different models in $f(\mathcal{R},\mathcal{T},\mathcal{Q})$ gravity and studied their respective cosmological applications for high-density regime as well as pressureless matter fluid case.

Sharif and Zubair [32] studied the thermodynamical laws for the black hole by adopting the models $\mathcal{R}+\lambda\mathcal{Q}$ and $\mathcal{R}(1+\lambda\mathcal{Q})$ along with matter Lagrangian in terms of energy density as well as pressure. The energy bounds are also addressed in this scenario, from which they concluded that only positive values of the model parameter λ satisfy weak energy conditions [33]. The flat FLRW spacetime was considered to check the behavior of this extended theory for different cosmological models [34]. They reconstructed the modified gravitational action and also described de Sitter universe solutions corresponding to the perfect fluid distribution. Baffou et al. [35] performed stability analysis in this modified gravity for two particular cases and concluded that both models present stability through some perturbation functions. Yousaf et al. [36] performed orthogonal decomposition of the Riemann tensor on the effective EMT corresponding to static/non-static spherical structures and computed some scalars to discuss the structural evolution of these bodies. The evolutionary patterns for cylindrical spacetime have also been discussed in modified scenario [37]. We also studied charged/uncharged sphere and obtained several physically acceptable anisotropic solutions through different schemes [38, 39].

The extensive discussion on the collapsing phenomenon has been done in various modified backgrounds. The numerical simulations were employed to study the collapse of spherical body in $f(\mathcal{R})$ framework from which an unusual increment in the density of fluid has been found [40]. In this context, Shamir and Fayyaz [41] analyzed the dynamics of a self-gravitational cylindrical geometry whose interior is filled with anisotropic dissipative fluid. They found the collapse as the crucial element to examine the rapid acceleration. The dynamical equations have been used to investigate the behavior of anisotropic gravitating source in $f(\mathcal{R},\mathcal{T})$ scenario [42]. Bhatti et al. [43] discussed the collapsing rate of dissipative anisotropic matter distribution with/without involving the effects of radiation density and coefficient of shear viscosity in $f(\mathcal{R},\mathcal{T},\mathcal{Q})$ framework. Sharif et al. [44] studied the celestial objects coupled with perfect/anisotropic configurations with/without the heat dissipation in different modified theories and examined the influence of correction terms on the collapsing rate.

This article addresses the dynamics of cylindrical geometry involving the impact of principal stresses and the dissipation flux in $f(\mathcal{R},\mathcal{T},\mathcal{R}_{ab}\mathcal{T}^{ab})$ theory. The paper is structured in the following format. We define some elementary terms related to the collapse and the extended theory and formulate the corresponding equations of motion and non-vanishing dynamical identities for $\mathcal{R}+\Phi\sqrt{\mathcal{T}}+\Psi\mathcal{Q}$ in Sect. 2. Moreover, the C-energy and the junction conditions are calculated through Darmois criteria. Section 3 formulates the dynamical equations and then couples them with the acceleration of the fluid. We further construct several dynamical forces in modified gravity in Sect.4 to study their impact on the collapsing rate. Section 5 explores some interesting relations between the effective state parameters and the Weyl scalar. The last section summarizes all of our findings.

2. $f(\mathcal{R},\mathcal{T},\mathcal{R}_{ab}\mathcal{T}^{ab})$ theory

The modified Einstein–Hilbert action for the $f(\mathcal{R},\mathcal{T},\mathcal{Q})$ gravity (with $\kappa=8\pi$) has the following form [34]

$$\mathbb{S}=\int\left\{\frac{f(\mathcal{R},\mathcal{T},\mathcal{Q})}{16\pi}+\mathbb{L}_{\mathcal{M}}\right\}\sqrt{-g}d^4x, \quad (1)$$

where $\mathbb{L}_{\mathcal{M}}$ is the Lagrangian corresponding to the matter density. Implementing the principle of least action in Eq. (1) provides

$$\mathcal{G}_{ab}=\mathcal{T}^{(\mathrm{EFF})}_{ab}=\frac{1}{f_{\mathcal{R}}-\mathbb{L}_{\mathcal{M}}f_{\mathcal{Q}}}\left(8\pi\mathcal{T}_{ab}+\mathcal{T}^{(D)}_{ab}\right), \quad (2)$$

where $\mathcal{T}^{(\mathrm{EFF})}_{ab}$ and $\mathcal{T}_{ab}$ are termed as the effective and usual anisotropic matter EMT. Also, $\mathcal{G}_{ab}$ is the Einstein tensor. The modified corrections are represented by $\mathcal{T}^{(D)}_{ab}$, which has the form

$$\begin{aligned}\mathcal{T}^{(D)}_{ab}=&\left(f_{\mathcal{T}}+\frac{1}{2}\mathcal{R}f_{\mathcal{Q}}\right)\mathcal{T}_{ab}\\&+\left\{\frac{\mathcal{R}}{2}\left(\frac{f}{\mathcal{R}}-f_{\mathcal{R}}\right)-\frac{1}{2}\nabla_c\nabla_d\left(f_{\mathcal{Q}}\mathcal{T}^{cd}\right)\right.\\&\left.-\mathbb{L}_{\mathcal{M}}f_{\mathcal{T}}\right\}g_{ab}-\frac{1}{2}\Box\left(f_{\mathcal{Q}}\mathcal{T}_{ab}\right)\\&-2f_{\mathcal{Q}}\mathcal{R}_{c(a}\mathcal{T}^{c}_{b)}+\nabla_c\nabla_{(a}[\mathcal{T}^{c}_{b)}f_{\mathcal{Q}}]\\&-\left(g_{ab}\Box-\nabla_a\nabla_b\right)f_{\mathcal{R}}\\&+2\left(f_{\mathcal{Q}}\mathcal{R}^{cd}+f_{\mathcal{T}}g^{cd}\right)\frac{\partial^2\mathbb{L}_{\mathcal{M}}}{\partial g^{ab}\partial g^{cd}},\end{aligned} \quad (3)$$

where $f_{\mathcal{R}}=\frac{\partial f(\mathcal{R},\mathcal{T},\mathcal{Q})}{\partial\mathcal{R}}$, $f_{\mathcal{T}}=\frac{\partial f(\mathcal{R},\mathcal{T},\mathcal{Q})}{\partial\mathcal{T}}$ and $f_{\mathcal{Q}}=\frac{\partial f(\mathcal{R},\mathcal{T},\mathcal{Q})}{\partial\mathcal{Q}}$. Also, the mathematical expression of the D'Alembert operator is $\Box\equiv\frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}g^{ab}\partial_b\right)$ and ∇_c indicates the covariant derivative. Generally, the matter Lagrangian can be taken in terms of energy density or pressure; thus, we consider it as $\mathbb{L}_{\mathcal{M}}=-\mu$ for the case of anisotropic matter distribution, which results in $\frac{\partial^2\mathbb{L}_{\mathcal{M}}}{\partial g^{ab}\partial g^{cd}}=0$ [31].

To discuss the collapse of dynamical cylinder, we take line element representing the interior geometry as

$$ds^2=-\mathcal{A}^2dt^2+\mathcal{B}^2dr^2+\mathcal{C}^2d\phi^2+dz^2, \quad (4)$$

where $\mathcal{A}=\mathcal{A}(t,r)$ and $\mathcal{B}=\mathcal{B}(t,r)$ are dimensionless, while $\mathcal{C}=\mathcal{C}(t,r)$ has the dimension of r. The EMT portraying anisotropic dissipative fluid is given as:

$$\begin{aligned}\mathcal{T}_{ab}=&\left(\mu+\mathrm{P_r}\right)\mathcal{U}_a\mathcal{U}_b+\mathrm{P_r}g_{ab}\\&+\left(\mathrm{P}_\phi-\mathrm{P_r}\right)\mathcal{K}_a\mathcal{K}_b+\left(\mathrm{P_z}-\mathrm{P_r}\right)\mathcal{S}_a\mathcal{S}_b\\&+\varsigma_a\mathcal{U}_b+\varsigma_b\mathcal{U}_a-\left(g_{ab}+\mathcal{U}_a\mathcal{U}_b\right)\alpha\Omega,\end{aligned} \quad (5)$$

where $\mathrm{P_r}$, P_ϕ and $\mathrm{P_z}$ are the principal pressures and μ is the energy density. Also, α and Ω are the coefficient of bulk viscosity and the expansion scalar, respectively. The four velocity ($\mathcal{U}_a$), four vectors ($\mathcal{K}_a$ and $\mathcal{S}_a$), heat flux ς_a and Ω are defined as:

$$\begin{aligned}&\mathcal{U}_a=-\mathcal{A}\delta^0_a,\quad\mathcal{K}_a=\mathcal{C}\delta^2_a,\quad\mathcal{S}_a=\delta^3_a,\\&\varsigma_a=\varsigma\mathcal{B}\delta^1_a,\quad\Omega=\mathcal{U}^a_{;a},\end{aligned} \quad (6)$$

satisfying the following relations

$$\begin{aligned}&\mathcal{U}_a\mathcal{U}^a=-1,\quad\mathcal{K}_a\mathcal{K}^a=1,\quad\mathcal{S}_a\mathcal{S}^a=1,\\&\mathcal{U}_a\mathcal{K}^a=0=\mathcal{S}_a\mathcal{K}^a=\mathcal{U}_a\mathcal{S}^a.\end{aligned} \quad (7)$$

Due to the interaction of matter components and geometry in this extended theory, the EMT has non-disappearing divergence, i.e., $\nabla_a\mathcal{T}^{ab}\neq0$. This exerts an extra force in the gravitational field that triggers the non-geodesic motion of test particles. Consequently, we have

$$\begin{aligned}\nabla^a\mathcal{T}_{ab}=&\frac{2}{2f_{\mathcal{T}}+\mathcal{R}f_{\mathcal{Q}}+16\pi}\Big[\nabla_a\left(f_{\mathcal{Q}}\mathcal{R}^{ca}\mathcal{T}_{cb}\right)\\&+\nabla_b\left(\mathbb{L}_{\mathcal{M}}f_{\mathcal{T}}\right)-\mathcal{G}_{ab}\nabla^a\left(f_{\mathcal{Q}}\mathbb{L}_{\mathcal{M}}\right)\\&-\frac{1}{2}\nabla_b\mathcal{T}^{cd}\left(f_{\mathcal{T}}g_{cd}+f_{\mathcal{Q}}\mathcal{R}_{cd}\right)\\&-\frac{1}{2}\left\{\nabla^a(\mathcal{R}f_{\mathcal{Q}})+2\nabla^af_{\mathcal{T}}\right\}\mathcal{T}_{ab}\Big].\end{aligned} \quad (8)$$

The trace of modified field equations is given by

$$\begin{aligned}&3\nabla^c\nabla_cf_{\mathcal{R}}-\mathcal{T}(f_{\mathcal{T}}+8\pi)\\&+\mathcal{R}\left(f_{\mathcal{R}}-\frac{\mathcal{T}}{2}f_{\mathcal{Q}}\right)+\frac{1}{2}\nabla^c\nabla_c(f_{\mathcal{Q}}\mathcal{T})\\&+\nabla_a\nabla_c(f_{\mathcal{Q}}\mathcal{T}^{ac})-2f+(\mathcal{R}f_{\mathcal{Q}}+4f_{\mathcal{T}})\mathbb{L}_{\mathcal{M}}\\&+2\mathcal{R}_{ac}\mathcal{T}^{ac}f_{\mathcal{Q}}-2g^{bd}(f_{\mathcal{T}}g^{ac}+f_{\mathcal{Q}}\mathcal{R}^{ac})\frac{\partial^2\mathbb{L}_{\mathcal{M}}}{\partial g^{bd}\partial g^{ac}}=0.\end{aligned}$$

The disappearance of $f_{\mathcal{Q}}$ from the field equations provides the gravitational effects of $f(\mathcal{R},\mathcal{T})$ theory, whereas the $f(\mathcal{R})$ gravity can be achieved for $f_{\mathcal{T}}=0=f_{\mathcal{Q}}$.

We adopt a standard model of the form

$$f(\mathcal{R},\mathcal{T},\mathcal{Q}) = f_1(\mathcal{R}) + f_2(\mathcal{T}) + f_3(\mathcal{Q}) = \mathcal{R} + \Phi\sqrt{\mathcal{T}} + \Psi\mathcal{Q}. \tag{9}$$

It is noteworthy that the gravitational model produces physically acceptable results by taking different choices of the model parameters (involving in that model) within their noticed range. For $\Phi = 0$, this model was used to analyze isotropic systems and some acceptable values of Ψ have been acquired for which the systems show stable behavior [32, 33]. The quantities $\mathcal{R}$, $\mathcal{T}$ and $\mathcal{Q}$ of the model (9) become

$$\mathcal{R} = -\frac{2}{\mathcal{A}^3\mathcal{B}^3\mathcal{C}}\Big[\mathcal{A}^3\mathcal{B}\mathcal{C}'' - \mathcal{A}\mathcal{B}^3\ddot{\mathcal{C}} - \mathcal{A}\mathcal{B}^2\mathcal{C}\ddot{\mathcal{B}} + \mathcal{A}^2\mathcal{B}\mathcal{C}\mathcal{A}'' - \mathcal{A}^3\mathcal{B}'\mathcal{C}' + \mathcal{B}^3\dot{\mathcal{A}}\dot{\mathcal{C}} + \mathcal{A}^2\mathcal{B}\mathcal{A}'\mathcal{C}' - \mathcal{A}\mathcal{B}^2\dot{\mathcal{B}}\dot{\mathcal{C}} + \mathcal{B}^2\mathcal{C}\dot{\mathcal{A}}\dot{\mathcal{B}} - \mathcal{A}^2\mathcal{C}\mathcal{A}'\mathcal{B}'\Big],$$

$$\mathcal{T} = -\mu + \mathrm{P_r} + \mathrm{P}_\phi + \mathrm{P_z} - 3\alpha\Omega,$$

$$\mathcal{Q} = -\frac{1}{\mathcal{A}^3\mathcal{B}^3\mathcal{C}}\Big[\mu\{\mathcal{A}\mathcal{B}^2\mathcal{C}\ddot{\mathcal{B}} - \mathcal{A}^2\mathcal{B}\mathcal{C}\mathcal{A}'' + \mathcal{A}^2\mathcal{C}\mathcal{A}'\mathcal{B}' - \mathcal{A}^2\mathcal{B}\mathcal{A}'\mathcal{C}' + \mathcal{A}\mathcal{B}^3\ddot{\mathcal{C}} - \mathcal{B}^2\mathcal{C}\dot{\mathcal{A}}\dot{\mathcal{B}} - \mathcal{B}^3\dot{\mathcal{A}}\dot{\mathcal{C}}\} + 2\varsigma\mathcal{A}\mathcal{B}\{\mathcal{A}\mathcal{B}\dot{\mathcal{C}}' - \mathcal{B}\mathcal{A}'\dot{\mathcal{C}} - \mathcal{A}\dot{\mathcal{B}}\mathcal{C}'\} + (\mathrm{P_r} - \alpha\Omega)\{\mathcal{B}^2\mathcal{C}\dot{\mathcal{A}}\dot{\mathcal{B}} - \mathcal{A}\mathcal{B}^2\mathcal{C}\ddot{\mathcal{B}} + \mathcal{A}^2\mathcal{B}\mathcal{C}\mathcal{A}'' - \mathcal{A}\mathcal{B}^2\dot{\mathcal{B}}\dot{\mathcal{C}} - \mathcal{A}^3\mathcal{B}'\mathcal{C}' - \mathcal{A}^2\mathcal{C}\mathcal{A}'\mathcal{B}' + \mathcal{A}^3\mathcal{B}\mathcal{C}''\} + (\mathrm{P}_\phi - \alpha\Omega)\{\mathcal{A}^3\mathcal{B}\mathcal{C}'' - \mathcal{A}^3\mathcal{B}'\mathcal{C}' + \mathcal{A}^2\mathcal{B}\mathcal{A}'\mathcal{C}' - \mathcal{A}\mathcal{B}^2\dot{\mathcal{B}}\dot{\mathcal{C}} - \mathcal{A}\mathcal{B}^3\ddot{\mathcal{C}} + \mathcal{B}^3\dot{\mathcal{A}}\dot{\mathcal{C}}\}\Big],$$

where $. = \frac{\partial}{\partial t}$ and $' = \frac{\partial}{\partial r}$.

The corresponding field equations are

$$\frac{1}{1+\Psi\mu}\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2}\right) = \frac{\mathcal{B}'\mathcal{C}'}{\mathcal{B}^3\mathcal{C}} - \frac{\mathcal{C}''}{\mathcal{B}^2\mathcal{C}} + \frac{\dot{\mathcal{B}}\dot{\mathcal{C}}}{\mathcal{A}^2\mathcal{B}\mathcal{C}}, \tag{10}$$

$$\frac{1}{1+\Psi\mu}\left(8\pi\mathrm{P_r} - \zeta\Omega + \frac{\mathrm{P_r^{(D)}}}{\mathcal{B}^2}\right) = \frac{\dot{\mathcal{A}}\dot{\mathcal{C}}}{\mathcal{A}^3\mathcal{C}} - \frac{\ddot{\mathcal{C}}}{\mathcal{A}^2\mathcal{C}} + \frac{\mathcal{A}'\mathcal{C}'}{\mathcal{A}\mathcal{B}^2\mathcal{C}}, \tag{11}$$

$$\frac{1}{1+\Psi\mu}\left(8\pi\mathrm{P}_\phi - \zeta\Omega + \frac{\mathrm{P}_\phi^{(D)}}{\mathcal{C}^2}\right) = \frac{\dot{\mathcal{A}}\dot{\mathcal{B}}}{\mathcal{A}^3\mathcal{B}} + \frac{\mathcal{A}''}{\mathcal{A}\mathcal{B}^2} - \frac{\ddot{\mathcal{B}}}{\mathcal{A}^2\mathcal{B}} - \frac{\mathcal{A}'\mathcal{B}'}{\mathcal{A}\mathcal{B}^3}, \tag{12}$$

$$\frac{1}{1+\Psi\mu}\left(8\pi\mathrm{P_z} - \zeta\Omega + \mathrm{P_z^{(D)}}\right) = \frac{\dot{\mathcal{A}}\dot{\mathcal{C}}}{\mathcal{A}^3\mathcal{C}} - \frac{\ddot{\mathcal{B}}}{\mathcal{A}^2\mathcal{B}} - \frac{\ddot{\mathcal{C}}}{\mathcal{A}^2\mathcal{C}} + \frac{\dot{\mathcal{A}}\dot{\mathcal{B}}}{\mathcal{A}^3\mathcal{B}} + \frac{\mathcal{A}''}{\mathcal{A}\mathcal{B}^2} + \frac{\mathcal{C}''}{\mathcal{B}^2\mathcal{C}} + \frac{\mathcal{A}'\mathcal{C}'}{\mathcal{A}\mathcal{B}^2\mathcal{C}} - \frac{\mathcal{A}'\mathcal{B}'}{\mathcal{A}\mathcal{B}^3} - \frac{\mathcal{C}'\mathcal{B}'}{\mathcal{B}^3\mathcal{C}} - \frac{\dot{\mathcal{B}}\dot{\mathcal{C}}}{\mathcal{A}^2\mathcal{B}\mathcal{C}}, \tag{13}$$

$$\frac{1}{1+\Psi\mu}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right) = \frac{\dot{\mathcal{C}}'}{\mathcal{A}\mathcal{B}\mathcal{C}} - \frac{\dot{\mathcal{B}}\mathcal{C}'}{\mathcal{A}\mathcal{B}^2\mathcal{C}} - \frac{\mathcal{A}'\dot{\mathcal{C}}}{\mathcal{A}^2\mathcal{B}\mathcal{C}}, \tag{14}$$

where $\zeta = 8\pi\alpha$. These equations describe how gravity and matter components bend spacetime. The second term on the left-hand side of the above equations $\left(\mu^{(D)},\ \mathrm{P_r^{(D)}},\ \mathrm{P_\phi^{(D)}},\ \mathrm{P_z^{(D)}}\text{ and }\varsigma^{(D)}\right)$ appears due to the modification of gravity, and their values are provided in "Appendix **A**". The quantities $\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2}\right)$, $\left(8\pi\mathrm{P_r} - \zeta\Omega + \frac{\mathrm{P_r^{(D)}}}{\mathcal{B}^2}\right)$, $\left(8\pi\mathrm{P}_\phi - \zeta\Omega + \frac{\mathrm{P}_\phi^{(D)}}{\mathcal{C}^2}\right)$, $\left(8\pi\mathrm{P_z} - \zeta\Omega + \mathrm{P_z^{(D)}}\right)$ and $\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)$ depict the effective energy density, effective principal pressures and the effective heat flux, respectively.

The C-energy within the interior geometrical structure (4) can be determined as [45]

$$\tilde{m}(t,r) = \mathfrak{L}\hat{\mathrm{E}} = \frac{\mathfrak{L}}{8}(1 - \mathfrak{L}^{-2}\nabla_{\mathrm{a}}\hat{r}\nabla^{\mathrm{a}}\hat{r}), \tag{15}$$

where $\hat{\mathrm{E}}$ is the total gravitational energy per specific length of the cylinder and $\hat{r} = \varrho\mathfrak{L}$ symbolizes the circumference radius. The terms ϱ and $\mathfrak{L}$ are the areal radius of the cylinder and specific length, respectively, whose mathematical expressions are $\varrho^2 = \eta_{(1)\mathrm{b}}\eta^{\mathrm{b}}_{(1)}$ and $\mathfrak{L}^2 = \eta_{(2)\mathrm{b}}\eta^{\mathrm{b}}_{(2)}$. Also, the Killing vectors are defined as $\eta_{(1)} = \frac{\partial}{\partial\phi}$, $\eta_{(2)} = \frac{\partial}{\partial z}$. Equation (15), after some manipulation, yields the mass as

$$\tilde{m} = \frac{\mathfrak{L}}{8}\left[1 - \left(\frac{\mathcal{C}'}{\mathcal{B}}\right)^2 + \left(\frac{\dot{\mathcal{C}}}{\mathcal{A}}\right)^2\right]. \tag{16}$$

The 3D hypersurface Σ splits the geometry into the interior and exterior regions. The interior region is defined in Eq. (4), while the exterior spacetime is taken as:

$$ds^2 = \frac{2\mathcal{M}(v)}{R}dv^2 - 2dvdR + R^2\left(d\phi^2 + \lambda^2dz^2\right), \tag{17}$$

where v is the retarded time. Also, $\mathcal{M}$ and R symbolize the mass and radius of the exterior region. We utilize Darmois junction conditions [46] whose fundamental forms are given by

- The continuity of the metric coefficients of the interior and exterior spacetimes holds at the hypersurface.
- There is no difference between the extrinsic curvature corresponding to both geometries at Σ that equals the radial pressure to the heat flux for the case of dynamical fluid distribution.

Since the collapse of a self-gravitating object is associated with the matter sector; thus, we only require to employ the second fundamental form that yields, after some manipulation, in this modified theory as

$$\mathcal{M} - \tilde{m} \overset{\Sigma}{=} \frac{\mathfrak{L}}{8}, \quad 8\pi\mathrm{P_r} - \zeta\Omega + \frac{\mathrm{P_r^{(D)}}}{\mathcal{B}^2} \overset{\Sigma}{=} 8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}. \tag{18}$$

It is observed that the least satisfactory definition of the C-energy produces the difference between masses of both regions by $\frac{\mathfrak{L}}{8}$, which disappears in the case of spherical spacetime. The other equation equals the effective radial pressure and effective heat flux at the boundary Σ. This guarantees the fulfillment of the condition of vanishing radial pressure at the boundary only if the heat flux along with modified corrections disappears, i.e., $8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}} = 0$.

3. Dynamics of the cylindrical star

Initially, Misner and Sharp formulated some dynamical quantities to study the evolution of spherical geometry. The proper radial and temporal derivatives were used to compute the velocity and acceleration of the considered collapsing source. These equations have later been used in the study of spherical as well as cylindrical spacetimes [47]. The dynamical equations in this scenario are:

$$\mathcal{T}^{(\mathrm{EFF})\mathrm{b}}_{\mathrm{a;b}}\mathcal{U}^{\mathrm{a}} = \left(8\pi\mathcal{T}^{\mathrm{b}}_{\mathrm{a}} + \mathcal{T}^{(D)\mathrm{b}}_{\mathrm{a}}\right)_{;\mathrm{b}}\mathcal{U}^{\mathrm{a}} = 0, \tag{19}$$

$$\mathcal{T}^{(\mathrm{EFF})\mathrm{b}}_{\mathrm{a;b}}\varsigma^{\mathrm{a}} = \left(8\pi\mathcal{T}^{\mathrm{b}}_{\mathrm{a}} + \mathcal{T}^{(D)\mathrm{b}}_{\mathrm{a}}\right)_{;\mathrm{b}}\varsigma^{\mathrm{a}} = 0. \tag{20}$$

Equations (19) and (20) yield, respectively, as

$$\begin{aligned}&\frac{1}{\mathcal{A}^2}\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2}\right)^{\cdot} \\ &\quad + \frac{\dot{\mathcal{B}}}{\mathcal{A}^2\mathcal{B}}\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi\mathrm{P_r} - \zeta\Omega + \frac{\mathrm{P_r^{(D)}}}{\mathcal{B}^2}\right) \\ &\quad + \frac{1}{\mathcal{A}\mathcal{B}}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)' \\ &\quad + \frac{\dot{\mathcal{C}}}{\mathcal{A}^2\mathcal{C}}\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi\mathrm{P}_\phi - \zeta\Omega + \frac{\mathrm{P}_\phi^{(D)}}{\mathcal{C}^2}\right) \\ &\quad + \frac{1}{\mathcal{A}\mathcal{B}}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)\left(\frac{2\mathcal{A}'}{\mathcal{A}} + \frac{\mathcal{C}'}{\mathcal{C}}\right) = 0,\end{aligned} \tag{21}$$

$$\begin{aligned}&\frac{\mathcal{B}}{\mathcal{A}}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)^{\cdot} \\ &\quad + \frac{\mathcal{A}'}{\mathcal{A}}\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi\mathrm{P_r} - \zeta\Omega + \frac{\mathrm{P_r^{(D)}}}{\mathcal{B}^2}\right) \\ &\quad + \left(8\pi\mathrm{P_r} - \zeta\Omega + \frac{\mathrm{P_r^{(D)}}}{\mathcal{B}^2}\right)' \\ &\quad + \frac{\mathcal{C}'}{\mathcal{C}}\left(8\pi\mathrm{P_r} + \frac{\mathrm{P_r^{(D)}}}{\mathcal{B}^2} - 8\pi\mathrm{P}_\phi - \frac{\mathrm{P}_\phi^{(D)}}{\mathcal{C}^2}\right) \\ &\quad + \frac{\mathcal{B}}{\mathcal{A}}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)\left(\frac{2\dot{\mathcal{B}}}{\mathcal{B}} + \frac{\dot{\mathcal{C}}}{\mathcal{C}}\right) = 0.\end{aligned} \tag{22}$$

These equations play a significant role in the study of variations arising in the stellar evolution. Our goal is to discuss the dynamics of the collapsing source; thus, the definitions of proper radial as well as temporal derivatives are [3, 47]

$$\mathfrak{D}_{\mathrm{r}} = \frac{1}{\mathcal{C}'}\frac{\partial}{\partial\mathrm{r}}, \quad \mathfrak{D}_{\mathrm{t}} = \frac{1}{\mathcal{A}}\frac{\partial}{\partial\mathrm{t}}. \tag{23}$$

The radius of an astrophysical object decreases continuously during the collapse as gravity dominates the outward pressure. Consequently, the velocity of interior fluid turns out to be negative, i.e.,

$$\mathbb{U} = \mathfrak{D}_{\mathrm{t}}(\mathcal{C}) = \frac{\dot{\mathcal{C}}}{\mathcal{A}} < 0. \tag{24}$$

Using this equation in C-energy (16), we have

$$\frac{\mathcal{C}'}{\mathcal{B}} = \left(1 + \mathbb{U}^2 - \frac{8\tilde{m}}{\mathfrak{L}}\right)^{\frac{1}{2}} = \omega. \tag{25}$$

The C-energy of the current cylindrical configuration yields after applying the definition of $\mathfrak{D}_{\mathrm{t}}$ as

$$\begin{aligned}\mathfrak{D}_{\mathrm{t}}(\tilde{m}) = &-\frac{\mathcal{C}\mathfrak{L}}{4(1+\Psi\mu)}\left\{\left(8\pi\mathrm{P_r} - \zeta\Omega + \frac{\mathrm{P_r^{(D)}}}{\mathcal{B}^2}\right)\mathbb{U}\right. \\ &\left. + \left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)\omega\right\},\end{aligned} \tag{26}$$

which demonstrates that how the total energy varies with time. This equation also indicates how the collapsing phenomenon is influenced by the radial pressure, the expansion scalar as well as heat flux and modified corrections. As $\mathbb{U}$ is negative; thus, the factor $\left(8\pi\mathrm{P_r} - \zeta\Omega + \frac{\mathrm{P_r^{(D)}}}{\mathcal{B}^2}\right)\mathbb{U}$ on the right-hand side of the above equation becomes positive, which guarantees that the total energy of the system increases. The other entity $\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)\omega$ confirms the reduction of total energy as heat dissipates from the source.

Next, in order to discuss the variation of energy between the adjoining cylindrical surfaces, we employ the definition of $\mathfrak{D}_{\mathrm{r}}$ in Eq.(16) and combine it with Eqs. (10) and (14) as

$$\begin{aligned}\mathfrak{D}_{\mathrm{r}}(\tilde{m}) = &\frac{\mathcal{C}\mathfrak{L}}{4(1+\Psi\mu)}\left\{\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2}\right)\right. \\ &\left. + \left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)\frac{\mathbb{U}}{\omega}\right\}.\end{aligned} \tag{27}$$

The collapse rate of the current setup is also affected by the effective energy density. The term $\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2}\right)$ in the above equation ultimately increases the total energy of the matter source. The next entity, $\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)\frac{\mathbb{U}}{\omega}$, reveals that the heat energy dissipates from the system, as the fluid has negative velocity. The acceleration of the collapsing source can be calculated by taking proper temporal derivative of $\mathbb{U}$ as

$$\mathfrak{D}_t(\mathbb{U}) = -\frac{\mathcal{C}}{1+\Psi\mu}\left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right) - \frac{\tilde{m}}{\mathcal{C}^2} + \frac{\omega\mathcal{A}'}{\mathcal{AB}} + \frac{\mathfrak{L}}{8\mathcal{C}^2}\left(1+\mathbb{U}^2-\omega^2\right). \quad (28)$$

One can get the value of $\frac{\mathcal{A}'}{\mathcal{A}}$ from Eq. (22) as

$$\frac{\mathcal{A}'}{\mathcal{A}} = -\frac{1}{\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)} \left\{\frac{\mathcal{B}}{\mathcal{A}}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right)^{\cdot} + \left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)' + \frac{\mathcal{C}'}{\mathcal{C}}\left(8\pi P_r + \frac{P_r^{(D)}}{\mathcal{B}^2} - 8\pi P_\phi - \frac{P_\phi^{(D)}}{\mathcal{C}^2}\right) + \frac{\mathcal{B}}{\mathcal{A}}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right)\left(\frac{2\dot{\mathcal{B}}}{\mathcal{B}} + \frac{\dot{\mathcal{C}}}{\mathcal{C}}\right)\right\}. \quad (29)$$

Inserting this value in Eq. (28), we have

$$\begin{aligned}
&\mathfrak{D}_t(\mathbb{U})\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right) \\
&= -\left\{\frac{\tilde{m}}{\mathcal{C}^2} - \frac{\mathfrak{L}}{8\mathcal{C}^2}\left(1+\mathbb{U}^2\right) + \frac{\mathcal{C}}{1+\Psi\mu}\left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)\right\} \\
&\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right) \\
&-\frac{\omega^2}{\mathcal{C}}\left\{\left(8\pi P_r + \frac{P_r^{(D)}}{\mathcal{B}^2} - 8\pi P_\phi - \frac{P_\phi^{(D)}}{\mathcal{C}^2}\right) + \frac{\mathfrak{L}}{8\mathcal{C}}\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)\right\} - \omega\left\{\frac{1}{B}\left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)' \right. \\
&\left. + \mathfrak{D}_t\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right) + \frac{1}{\mathcal{A}} \times \left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right)\left(\frac{2\dot{\mathcal{B}}}{\mathcal{B}} + \frac{\dot{\mathcal{C}}}{\mathcal{C}}\right)\right\}.
\end{aligned} \quad (30)$$

The left side of this equation represents the Newtonian force as the product of acceleration ($\mathfrak{D}_t\mathbb{U}$), and the term $\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)$ (refers to the inertial mass density) appears. On the other hand, the same term also arises in the first term on the right side, which now presents the gravitational mass density. Thus, the equivalence of these both masses leads to the fulfillment of the equivalence principle. The second curly bracket determines the impact of gravitational mass density and effective stresses in r as well as ϕ directions on the collapse rate. The role of gradient of effective radial pressure and the expansion scalar in this scenario can be manifested through the entity $\left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)'$. Also, the last two terms containing the heat flux and modified corrections can well describe hydrodynamics of the cylinder.

4. Transport equations

As the EMT (5) involves the heat flux, the transport equations in this regard are very useful tool to analyze the structural evolution of compact geometry. They also disclose how some physical quantities such as mass, heat and momentum are evaluated during the collapse. The diffusion process is supported by the following transport equation given as:

$$\varrho h^{ab}\mathcal{U}^c\bar{\varsigma}_{b;c} + \bar{\varsigma}^a = -\eta h^{ab}\left(\tau_{,b} + \tau a_b\right) - \frac{1}{2}\eta\tau^2\left(\frac{\varrho\mathcal{U}^b}{\eta\tau^2}\right)_{;b}\bar{\varsigma}^a, \quad (31)$$

where $\bar{\varsigma} = \left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right)$ and $h^{ab} = g^{ab} + \mathcal{U}^a\mathcal{U}^b$ are the projection tensor. Also, η, ϱ, τ and a_b (i.e., $a_1 = \frac{\mathcal{A}'}{\mathcal{A}}$) are mathematical symbols of the thermal conductivity, relaxation time, temperature and acceleration, respectively. After some simplification, Eq. (31) produces

$$\mathcal{B}\mathfrak{D}_t\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right) = -\frac{\eta\tau'}{\varrho} - \frac{\eta\tau}{\varrho}\left(\frac{\mathcal{A}'}{\mathcal{A}}\right) - \frac{\eta\tau^2\mathcal{B}}{2\mathcal{A}\varrho}\left(\frac{\varrho}{\eta\tau^2}\right)^{\cdot}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right) - \frac{\mathcal{B}}{2\mathcal{A}}\left(\frac{3\dot{\mathcal{B}}}{\mathcal{B}} + \frac{\dot{\mathcal{C}}}{\mathcal{C}} + \frac{2\mathcal{A}}{\varrho}\right)\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right). \quad (32)$$

This equation becomes after inserting the value of $\frac{\mathcal{A}'}{\mathcal{A}}$ as

$$\begin{aligned}
\mathcal{B}\mathfrak{D}_t\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right) &= -\frac{\eta\tau^2\mathcal{B}}{2\mathcal{A}\varrho}\left(\frac{\varrho}{\eta\tau^2}\right)^{\cdot}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right) - \frac{\mathcal{B}}{2\mathcal{A}}\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right) \\
&\times\left(\frac{3\dot{\mathcal{B}}}{\mathcal{B}} + \frac{\dot{\mathcal{C}}}{\mathcal{C}} + \frac{2\mathcal{A}}{\varrho}\right) - \frac{\eta\tau\mathcal{B}}{\varrho\omega}\left\{\mathfrak{D}_t(\mathbb{U}) - \frac{\mathfrak{L}}{8\mathcal{C}^2}\left(1+\mathbb{U}^2-\omega^2\right) + \frac{\tilde{m}}{\mathcal{C}^2} + \frac{\mathcal{C}}{1+\Psi\mu}\left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)\right\} - \frac{\eta\tau'}{\varrho},
\end{aligned} \quad (33)$$

which demonstrates how much variation takes place in heat energy with the passage of time. This equation also explains the impact of temperature, thermal conductivity and relaxation time on the self-gravitating systems. Eliminating $\mathfrak{D}_t\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right)$ from Eqs. (30) and (33), we obtain

$$
\begin{aligned}
&\mathfrak{D}_t(\mathbb{U})\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2} - \frac{\eta\tau}{\varrho}\right)\\
&\quad = -\left\{\frac{\tilde{m}}{\mathcal{C}^2} - \frac{\mathfrak{L}}{8\mathcal{C}^2}\left(1 + \mathbb{U}^2\right)\right.\\
&\quad \left. + \frac{\mathcal{C}}{1 + \Psi\mu}\left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)\right\}\\
&\quad \left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2} - \frac{\eta\tau}{\varrho}\right)\\
&\quad \times\left\{1 - \frac{\eta\tau}{\varrho}\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)^{-1}\right\}\\
&\quad + \omega^2\left\{\frac{\eta\tau\mathfrak{L}}{8\varrho\mathcal{C}^2} - \frac{\mathfrak{L}}{8\mathcal{C}^2}\right.\\
&\quad \times\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)\\
&\quad \left. - \frac{1}{\mathcal{C}}\left(8\pi P_r + \frac{P_r^{(D)}}{\mathcal{B}^2} - 8\pi P_\phi - \frac{P_\phi^{(D)}}{\mathcal{C}^2}\right)\right\}\\
&\quad\quad - \omega\left[-\frac{\eta\tau'}{\varrho\mathcal{B}} + \frac{1}{\mathcal{B}}\left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)'\right.\\
&\quad + \frac{1}{2}\left\{\frac{\dot{\mathcal{B}}}{\mathcal{AB}} + \frac{\dot{\mathcal{C}}}{\mathcal{AC}} - \frac{2}{\varrho} - \frac{\eta\tau^2}{\mathcal{A}\varrho}\left(\frac{\varrho}{\eta\tau^2}\right)^{\cdot}\right\}\\
&\quad \left. \times\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right)\right].
\end{aligned}
\tag{34}
$$

We can rearrange this equation as

$$
\begin{aligned}
&\mathfrak{D}_t(\mathbb{U})\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)(1 - \mathfrak{H})\\
&\quad = -\mathcal{F}_{grav}(1 - \mathfrak{H})\\
&\quad + \mathcal{F}_{hyd} + \omega^2\left\{\frac{\eta\tau\mathfrak{L}}{8\varrho\mathcal{C}^2} - \frac{\mathfrak{L}}{8\mathcal{C}^2}\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2}\right.\right.\\
&\quad \left. + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)\\
&\quad \left. - \frac{1}{\mathcal{C}}\left(8\pi P_r + \frac{P_r^{(D)}}{\mathcal{B}^2} - 8\pi P_\phi - \frac{P_\phi^{(D)}}{\mathcal{C}^2}\right)\right\},
\end{aligned}
\tag{35}
$$

where

$$
\mathfrak{H} = \frac{\eta\tau}{\varrho}\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)^{-1}, \tag{36}
$$

$$
\begin{aligned}
\mathcal{F}_{grav} = &\left(8\pi\mu + \frac{\mu^{(D)}}{\mathcal{A}^2} + 8\pi P_r - \zeta\Omega\right.\\
&\left. + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)\left\{\frac{\tilde{m}}{\mathcal{C}^2} - \frac{\mathfrak{L}}{8\mathcal{C}^2}\left(\mathbb{U}^2 + 1\right)\right.\\
&\left. s + \frac{\mathcal{C}}{1 + \Psi\mu}\left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)\right\},
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
\mathcal{F}_{hyd} = &-\omega\left[\frac{1}{\mathcal{B}}\left(8\pi P_r - \zeta\Omega + \frac{P_r^{(D)}}{\mathcal{B}^2}\right)'\right.\\
&+ \frac{1}{2}\left\{\frac{\dot{\mathcal{B}}}{\mathcal{AB}} + \frac{\dot{\mathcal{C}}}{\mathcal{AC}} - \frac{2}{\varrho} - \frac{\eta\tau^2}{\mathcal{A}\varrho}\left(\frac{\varrho}{\eta\tau^2}\right)^{\cdot}\right\}\\
&\left. \times\left(8\pi\varsigma - \frac{\varsigma^{(D)}}{\mathcal{AB}}\right) - \frac{\eta\tau'}{\varrho\mathcal{B}}\right].
\end{aligned}
\tag{38}
$$

Equation (35) explains how the collapse rate is affected by different forces, comprising Newtonian ($\mathcal{F}_{newtn}$), hydrodynamical ($\mathcal{F}_{hyd}$) and gravitational ($\mathcal{F}_{grav}$) forces. It is known that energy always dissipates (in the form of radiation, convection and conduction) from higher to lower energy state of the system. The energy of a star is dissipated through radiations, if photons acquire it from the higher phase of that object. On the other hand, when photons do not possess all the energy, it will be dissipated by convection. The hot gasses in this phenomenon move to the upper zone and thus radiate energy, whereas cooler gases attain energy by traveling toward the hot zone. There occur continuous collisions of atoms inside an object due to which every atom transfers its energy to the nearest one, and thus energy dissipates in the form of conduction.

Equation (35) involves an entity $(1 - \mathfrak{H})$ that acknowledges the equivalence principle, while the gravitational mass density and the term $(\mathfrak{H})$ [defined in Eq. (36)] are inversely proportional to each other. This relation provides the fact that the gravitational force and the quantity $(1 - \mathfrak{H})$ are strongly affected by each other, leading to the following different cases.

- The entity $(1 - \mathfrak{H})$ remains positive only for $\mathfrak{H} < 1$, which results in the negative gravitational force (i.e., repulsive force) due to the appearance of minus sign in the first term on the right side of Eq. (35). Consequently, the collapse rate diminishes.
- The rate of the cylindrical collapse increases for the case when $\mathfrak{H} > 1$, i.e., $(1 - \mathfrak{H}) < 0$.
- Finally, if we consider $\mathfrak{H} = 1$, the gravitational as well as inertial forces disappear and we have from Eq.(35) as

$$\omega^2\left\{\frac{\eta\tau\mathfrak{Q}}{8\varrho\mathcal{C}^2}-\frac{\mathfrak{Q}}{8\mathcal{C}^2}\left(8\pi\mu+\frac{\mu^{(D)}}{\mathcal{A}^2}+8\pi\mathrm{P_r}-\zeta\Omega+\frac{\mathrm{P_r}^{(D)}}{\mathcal{B}^2}\right)-\frac{1}{\mathcal{C}}\left(8\pi\mathrm{P_r}+\frac{\mathrm{P_r}^{(D)}}{\mathcal{B}^2}-8\pi\mathrm{P}_\phi-\frac{\mathrm{P}_\phi^{(D)}}{\mathcal{C}^2}\right)\right\}=\omega\left[-\frac{\eta\tau'}{\varrho\mathcal{B}}+\frac{1}{\mathcal{B}}\left(8\pi\mathrm{P_r}-\zeta\Omega+\frac{\mathrm{P_r}^{(D)}}{\mathcal{B}^2}\right)'+\frac{1}{2}\left(8\pi\varsigma-\frac{\varsigma^{(D)}}{\mathcal{A}\mathcal{B}}\right)\times\left\{\frac{\dot{\mathcal{B}}}{\mathcal{A}\mathcal{B}}+\frac{\dot{\mathcal{C}}}{\mathcal{A}\mathcal{C}}-\frac{2}{\varrho}-\frac{\eta\tau^2}{\mathcal{A}\varrho}\left(\frac{\varrho}{\eta\tau^2}\right)^{\cdot}\right\}\right]. \tag{39}$$

This equation expresses the involvement of temperature, thermal conductivity, the bulk viscosity and the modified corrections in the collapsing phenomenon. The equilibrium position of the collapsing cylinder is supported by the hydrodynamical force (given on left side of the above equation), and hence, the collapse rate is reduced.

5. Relation between the Weyl scalar and effective physical quantities

In this section, we develop some relations between effective physical variables and the Weyl scalar. ($\mathbb{C}^2=\mathfrak{C}_{\mathrm{cadb}}\mathfrak{C}^{\mathrm{cadb}}$, where $\mathfrak{C}_{\mathrm{cadb}}$ is the Weyl tensor.) This scalar can be expressed as a linear combination of the Kretchmann scalar ($\mathbb{R}=\mathcal{R}_{\mathrm{cadb}}\mathcal{R}^{\mathrm{cadb}}$, $\mathcal{R}_{\mathrm{cadb}}$ is the Riemann tensor.), the Ricci tensor ($\mathcal{R}_{\mathrm{ab}}$) and the Ricci scalar as [10]

$$\mathbb{C}^2=\mathbb{R}-2\mathcal{R}_{\mathrm{ab}}\mathcal{R}^{\mathrm{ab}}+\frac{1}{3}\mathcal{R}^2. \tag{40}$$

The scalar $\mathbb{R}$ can be manipulated as

$$\mathbb{R}=\frac{4}{\mathcal{A}^4\mathcal{B}^4\mathcal{C}^4}\left\{\mathcal{C}^4\left(\mathcal{R}^{0101}\right)^2+\mathcal{B}^4\left(\mathcal{R}^{0202}\right)^2+\mathcal{A}^4\left(\mathcal{R}^{1212}\right)^2-\frac{\mathcal{A}^2\mathcal{B}^2\left(\mathcal{R}^{1202}\right)^2}{2}\right\}. \tag{41}$$

For the considered spacetime (4), the Ricci scalar, nonzero components of the Riemann tensor and the Ricci tensor in terms of the Einstein tensor are

$$\mathcal{R}=-2\left(\frac{\mathcal{G}_{11}}{\mathcal{B}^2}+\frac{\mathcal{G}_{22}}{\mathcal{C}^2}-\frac{\mathcal{G}_{00}}{\mathcal{A}^2}\right),$$
$$\mathcal{R}^{0101}=\frac{\mathcal{G}_{22}}{(\mathcal{A}\mathcal{B}\mathcal{C})^2},\quad \mathcal{R}^{0202}=\frac{\mathcal{G}_{11}}{(\mathcal{A}\mathcal{B}\mathcal{C})^2},\quad \mathcal{R}^{1212}=\frac{\mathcal{G}_{00}}{(\mathcal{A}\mathcal{B}\mathcal{C})^2},$$
$$\mathcal{R}^{0212}=\frac{\mathcal{G}_{01}}{(\mathcal{A}\mathcal{B}\mathcal{C})^2},\quad \mathcal{R}_{00}=\mathcal{A}^2\left(\frac{\mathcal{G}_{11}}{\mathcal{B}^2}+\frac{\mathcal{G}_{22}}{\mathcal{C}^2}\right),\quad \mathcal{R}_{01}=\mathcal{G}_{01},$$
$$\mathcal{R}_{11}=\mathcal{B}^2\left(\frac{\mathcal{G}_{00}}{\mathcal{A}^2}-\frac{\mathcal{G}_{22}}{\mathcal{C}^2}\right),\quad \mathcal{R}_{22}=\mathcal{C}^2\left(\frac{\mathcal{G}_{00}}{\mathcal{A}^2}-\frac{\mathcal{G}_{11}}{\mathcal{B}^2}\right).$$

These values provide the scalar $\mathbb{R}$ (41) as

$$\mathbb{R}=\frac{4}{\mathcal{A}^4\mathcal{B}^4\mathcal{C}^4}\left\{\mathcal{B}^4\mathcal{C}^4\mathcal{G}_{00}^2+\mathcal{A}^4\mathcal{C}^4\mathcal{G}_{11}^2+\mathcal{A}^4\mathcal{B}^4\mathcal{G}_{22}^2-4\mathcal{A}^2\mathcal{B}^2\mathcal{C}^4\mathcal{G}_{01}^2\right\}. \tag{42}$$

Inserting these equations in Eq. (40), the Weyl scalar takes the form as

$$\mathbb{C}^2=\frac{4}{3\mathcal{A}^4\mathcal{B}^4\mathcal{C}^4}\left\{\mathcal{B}^4\mathcal{C}^4\mathcal{G}_{00}^2+\mathcal{A}^4\mathcal{C}^4\mathcal{G}_{11}^2+\mathcal{A}^4\mathcal{B}^4\mathcal{G}_{22}^2+\mathcal{A}^2\mathcal{B}^2\mathcal{C}^4\mathcal{G}_{00}\mathcal{G}_{11}+\mathcal{A}^2\mathcal{B}^4\mathcal{C}^2\mathcal{G}_{00}\mathcal{G}_{22}-\mathcal{A}^4\mathcal{B}^2\mathcal{C}^2\mathcal{G}_{11}\mathcal{G}_{22}\right\}. \tag{43}$$

Using this equation and the field Eqs. (10–12) yields

$$\frac{\sqrt{3}\mathbb{C}}{2}=\left[\left\{\frac{1}{1+\Psi\beta}\left(8\pi\mu+\frac{\mu^{(D)}}{\mathcal{A}^2}+8\pi\mathrm{P_r}+\frac{\mathrm{P_r}^{(D)}}{\mathcal{B}^2}-8\pi\mathrm{P}_\phi-\frac{\mathrm{P}_\phi^{(D)}}{\mathcal{C}^2}\right)\right\}^2-\frac{1}{1+\Psi\beta}\left\{\left(8\pi\mathrm{P_r}-\zeta\Omega+\frac{\mathrm{P_r}^{(D)}}{\mathcal{B}^2}\right)\left(8\pi\mathrm{P}_\phi-\zeta\Omega+\frac{\mathrm{P}_\phi^{(D)}}{\mathcal{C}^2}\right)+\left(8\pi\mathrm{P_r}+\frac{\mathrm{P_r}^{(D)}}{\mathcal{B}^2}+2\zeta\Omega-24\pi\mathrm{P}_\phi-\frac{3\mathrm{P}_\phi^{(D)}}{\mathcal{C}^2}\right)\left(8\pi\mu+\frac{\mu^{(D)}}{\mathcal{A}^2}\right)\right\}\right]^{\frac{1}{2}}. \tag{44}$$

The necessary and sufficient condition for a spacetime to be conformally flat is the energy density homogeneity. We check the validity of this result in the present modified gravity. For this, we have considered the standard model (9). We take the case when $\mathcal{R}=\mathcal{R}_0$ and $f_2(\mathcal{T})$ as well as $f_3(\mathcal{Q})$ are treated as constants; thus, Eq. (44) is left with

$$\frac{\sqrt{3}\mathbb{C}}{2}=\left[\left\{8\pi\left(\mu+\mathrm{P_r}-\mathrm{P}_\phi-\frac{\mathcal{C}_0}{16\pi}\right)\right\}^2-\left(8\pi\mathrm{P_r}+2\zeta\Omega-24\pi\mathrm{P}_\phi-\mathcal{C}_0\right)\times\left(8\pi\mu-\frac{\mathcal{C}_0}{2}\right)-\left(8\pi\mathrm{P_r}-\zeta\Omega+\frac{\mathcal{C}_0}{2}\right)\left(8\pi\mathrm{P}_\phi-\zeta\Omega+\frac{\mathcal{C}_0}{2}\right)\right]^{\frac{1}{2}}, \tag{45}$$

where $\mathcal{C}_0=\Phi\sqrt{\mathcal{T}_0}+\Psi\mathcal{Q}_0$ is a constant term. This equation depicts that inhomogeneity in the energy density of the fluid is induced due to the presence of the bulk viscosity and the principal pressures. The above relation also says that inhomogeneity in the system (during evolution) is increased by the tidal forces [16]. The only possibility to obtain conformally flat spacetime is the consideration of dust matter distribution, which gives in the absence of bulk viscosity as

$$\sqrt{3}\mathbb{C} = 16\pi\left(\mu - \frac{\mathcal{C}_0}{16\pi}\right) \quad \Rightarrow \quad \sqrt{3}\mathbb{C}' = 16\pi\mu'. \tag{46}$$

We observe from this equation that homogeneous energy density (i.e., $\mu' = 0$) implies conformally flat spacetime ($\mathbb{C} = 0$ through regular axis condition) and vice versa, and hence, the required condition is obtained.

6. Conclusions

Our cosmos comprises an abundance of astronomical systems whose structural formation is highly influenced by an appealing phenomenon, named as the gravitational collapse. The study of gravitational waves through multiple observations has prompted several astrophysicists to investigate the collapsing rate of self-gravitating geometries in $\mathbb{GR}$ and other extended theories [48]. This article is based on the formulation of dynamical description of the cylindrical fluid distribution to investigate the changes that are gradually produced within the system in the background of $f(\mathcal{R},\mathcal{T},\mathcal{R}_{ab}\mathcal{T}^{ab})$ gravity. The effect of heat dissipation and principal pressures (such as P_r, P_ϕ and P_z) on the interior geometry has been considered. We have formulated two dynamical equations through Misner–Sharp formalism to examine the variations in the total energy with respect to radial as well as temporal coordinates.

We have constructed the transport equation as well as some fundamental forces (such as the gravitational, hydrodynamical and Newtonian) and then coupled them with dynamical equations to analyze the impact of modified gravity on the collapse rate. The entity $\mathfrak{H}$ is found to be in direct relation with temperature as well as thermal conductivity and inversely related to gravitational mass density. In the following, we summarize our results.

- The entity $\mathfrak{H}$ will be less as compared to $\mathbb{GR}$ for positive effect of the modified corrections. This leads to the increment in the term $(1-\mathfrak{H})$ as well as the gravitational force. However, the appearance of minus sign ultimately diminishes the collapse rate.
- The rate of cylindrical collapse may increase for the case when the effect of correction terms is negative.
- One cannot say anything about the decrement/increment in the collapsing rate when the corrections of this modified gravity involved in $\mathfrak{H}$ have opposite signs.

The relevance of density inhomogeneity and the Weyl scalar has also been developed. By implying some constraints on the considered modified model, it has been shown that the homogenous density and the conformal flatness of the current setup imply each other. It is worth mentioning here that the tidal forces involving in the Weyl tensor produce more inhomogeneity in the fluid during the evolutionary process. All these results can be recovered in $\mathbb{GR}$ for $\Phi = 0 = \Psi$.

Appendix A

The modified corrections in Eqs. (10)–(14) are

$$\begin{aligned}
\mu^{(D)} = &-\frac{\mathcal{A}^2(\Phi\sqrt{\mathcal{T}}+\Psi\mathcal{Q})}{2}\\
&+\Psi\bigg\{\mu\bigg(\frac{4\dot{\mathcal{A}}^2}{\mathcal{A}^2}-\frac{\mathcal{A}'^2}{\mathcal{B}^2}+\frac{\mathcal{A}\mathcal{A}''}{\mathcal{B}^2}+\frac{3\dot{\mathcal{A}}\dot{\mathcal{B}}}{\mathcal{A}\mathcal{B}}-\frac{\mathcal{A}\mathcal{A}'\mathcal{B}'}{\mathcal{B}^3}\\
&+\frac{3\dot{\mathcal{A}}\dot{\mathcal{C}}}{\mathcal{A}\mathcal{C}}+\frac{\mathcal{A}\mathcal{A}'\mathcal{C}'}{\mathcal{B}^2\mathcal{C}}-\frac{2\ddot{\mathcal{C}}}{\mathcal{C}}-\frac{2\ddot{\mathcal{B}}}{\mathcal{B}}\bigg)\\
&+\dot{\mu}\bigg(\frac{\dot{\mathcal{C}}}{2\mathcal{C}}+\frac{\dot{\mathcal{B}}}{2\mathcal{B}}\bigg)-\mu'\bigg(\frac{2\mathcal{A}\mathcal{A}'}{\mathcal{B}^2}-\frac{\mathcal{A}^2\mathcal{B}'}{2\mathcal{B}^3}\\
&+\frac{\mathcal{A}^2\mathcal{C}'}{2\mathcal{B}^2\mathcal{C}}\bigg)\\
&-\frac{\mu''\mathcal{A}^2}{2\mathcal{B}^2}+P_r\bigg(\frac{4\mathcal{A}^2\mathcal{B}'^2}{\mathcal{B}^4}-\frac{\mathcal{A}^2\mathcal{B}''}{\mathcal{B}^3}+\frac{\dot{\mathcal{B}}^2}{\mathcal{B}^2}\bigg)-\frac{\dot{P}_r\dot{\mathcal{B}}}{2\mathcal{B}}\\
&-\frac{5P_r'\mathcal{A}^2\mathcal{B}'}{2\mathcal{B}^3}\\
&+\frac{P_r''\mathcal{A}^2}{2\mathcal{B}^2}-P_\phi\bigg(\frac{\dot{\mathcal{C}}^2}{\mathcal{C}^2}-\frac{\mathcal{A}^2\mathcal{C}'^2}{\mathcal{B}^2\mathcal{C}^2}\bigg)\\
&-\frac{\dot{P}_\phi\dot{\mathcal{C}}}{2\mathcal{C}}+\frac{P_\phi'\mathcal{A}^2\mathcal{C}'}{2\mathcal{B}^2\mathcal{C}}-\varsigma\bigg(\frac{2\mathcal{A}\dot{\mathcal{C}}'}{\mathcal{B}\mathcal{C}}-\frac{2\dot{\mathcal{A}}\mathcal{B}'}{\mathcal{B}^2}\\
&+\frac{2\dot{\mathcal{A}}'}{\mathcal{B}}-\frac{4\dot{\mathcal{A}}\mathcal{A}'}{\mathcal{A}\mathcal{B}}-\frac{2\mathcal{A}'\dot{\mathcal{B}}}{\mathcal{B}^2}-\frac{2\mathcal{A}\dot{\mathcal{B}}\mathcal{C}'}{\mathcal{B}^2\mathcal{C}}-\frac{2\mathcal{A}'\dot{\mathcal{C}}}{\mathcal{B}\mathcal{C}}-\frac{2\mathcal{A}'\dot{\mathcal{B}}}{\mathcal{B}^2}\bigg)\\
&-\frac{2\dot{\varsigma}\mathcal{A}'}{\mathcal{B}}-\frac{2\varsigma'\dot{\mathcal{A}}}{\mathcal{B}}\bigg\},
\end{aligned}$$

$$
\begin{aligned}
\mathrm{P}_{\mathrm{r}}^{(D)} &= \frac{\mathcal{B}^2}{2}\left\{\left(\frac{\Phi}{\sqrt{\mathcal{T}}}+\Psi\mathcal{R}\right)\mathrm{P_r}+\Phi\sqrt{\mathcal{T}}+\Psi\mathcal{Q}+\frac{\Phi\mu}{\sqrt{\mathcal{T}}}\right\} \\
&+\Psi\left\{\mu\left(\frac{\ddot{\mathcal{A}}\mathcal{B}^2}{\mathcal{A}^3}-\frac{\mathcal{A}'^2}{\mathcal{A}^2}\right.\right. \\
&\left.-\frac{4\dot{\mathcal{A}}^2\mathcal{B}^2}{\mathcal{A}^4}\right)+\frac{5\dot{\mu}\dot{\mathcal{A}}\mathcal{B}^2}{2\mathcal{A}^3}+\frac{\mu'\mathcal{A}'}{2\mathcal{A}}-\frac{\ddot{\mu}\mathcal{B}^2}{2\mathcal{A}^2} \\
&+\mathrm{P_r}\left(\frac{\mathcal{B}\dot{\mathcal{A}}\dot{\mathcal{B}}}{\mathcal{A}^3}-\frac{3\mathcal{A}'\mathcal{B}'}{\mathcal{A}\mathcal{B}}-\frac{\mathcal{B}\dot{\mathcal{B}}\dot{\mathcal{C}}}{\mathcal{A}^2\mathcal{C}}\right. \\
&\left.+\frac{\dot{\mathcal{B}}^2}{\mathcal{A}^2}-\frac{4\mathcal{B}'^2}{\mathcal{B}^2}-\frac{\mathcal{B}\ddot{\mathcal{B}}}{\mathcal{A}^2}-\frac{3\mathcal{B}'\mathcal{C}'}{\mathcal{B}\mathcal{C}}+\frac{2\mathcal{A}''}{\mathcal{A}}+\frac{2\mathcal{C}''}{\mathcal{C}}\right) \\
&+\dot{\mathrm{P}}_{\mathrm{r}}\left(\frac{\mathcal{B}^2\dot{\mathcal{C}}}{2\mathcal{A}^2\mathcal{C}}-\frac{\mathcal{B}^2\dot{\mathcal{A}}}{2\mathcal{A}^3}\right. \\
&\left.+\frac{2\mathcal{B}\dot{\mathcal{B}}}{\mathcal{A}^2}\right)-\mathrm{P}_{\mathrm{r}}'\left(\frac{\mathcal{A}'}{2\mathcal{A}}+\frac{\mathcal{C}'}{2\mathcal{C}}\right) \\
&+\frac{\ddot{\mathrm{P}}_{\mathrm{r}}\mathcal{B}^2}{2\mathcal{A}^2}-\mathrm{P}_\phi\left(\frac{\mathcal{B}^2\dot{\mathcal{C}}^2}{\mathcal{A}^2\mathcal{C}^2}-\frac{\mathcal{C}'^2}{\mathcal{C}^2}\right)-\frac{\dot{\mathrm{P}}_\phi\mathcal{B}^2\dot{\mathcal{C}}}{2\mathcal{A}^2\mathcal{C}} \\
&-\frac{\mathrm{P}_\phi'\mathcal{C}'}{2\mathcal{C}}+\varsigma\left(\frac{2\dot{\mathcal{B}}'}{\mathcal{A}}-\frac{2\dot{\mathcal{A}}\mathcal{B}'}{\mathcal{A}^2}\right. \\
&\left.-\frac{4\mathcal{A}'\dot{\mathcal{B}}}{\mathcal{A}^2}-\frac{4\dot{\mathcal{B}}\mathcal{B}'}{\mathcal{A}\mathcal{B}}+\frac{2\mathcal{B}\dot{\mathcal{C}}'}{\mathcal{A}\mathcal{C}}-\frac{2\dot{\mathcal{B}}\mathcal{C}'}{\mathcal{A}\mathcal{C}}-\frac{2\mathcal{B}\mathcal{A}'\dot{\mathcal{C}}}{\mathcal{A}^2\mathcal{C}}\right) \\
&\left.+\frac{2\dot{\varsigma}\mathcal{B}'}{\mathcal{A}}+\frac{2\varsigma'\dot{\mathcal{B}}}{\mathcal{A}}\right\},
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}_{\phi}^{(D)} &= \frac{\mathcal{C}^2}{2}\left\{\left(\frac{\Phi}{\sqrt{\mathcal{T}}}+\Psi\mathcal{R}\right)\mathrm{P}_\phi+\Phi\sqrt{\mathcal{T}}+\Psi\mathcal{Q}+\frac{\Phi\mu}{\sqrt{\mathcal{T}}}\right\} \\
&+\Psi\left\{\mu\left(\frac{\ddot{\mathcal{A}}\mathcal{C}^2}{\mathcal{A}^3}-\frac{\mathcal{A}'^2\mathcal{C}^2}{\mathcal{A}^2\mathcal{B}^2}\right.\right. \\
&\left.-\frac{4\dot{\mathcal{A}}^2\mathcal{C}^2}{\mathcal{A}^4}\right)+\frac{5\dot{\mu}\dot{\mathcal{A}}\mathcal{C}^2}{2\mathcal{A}^3}+\frac{\mu'\mathcal{A}'\mathcal{C}^2}{2\mathcal{A}\mathcal{B}^2}-\frac{\ddot{\mu}\mathcal{C}^2}{2\mathcal{A}^2} \\
&+\mathrm{P_r}\left(\frac{\mathcal{B}''\mathcal{C}^2}{\mathcal{B}^3}-\frac{4\mathcal{B}'^2\mathcal{C}^2}{\mathcal{B}^4}-\frac{\dot{\mathcal{B}}^2\mathcal{C}^2}{\mathcal{A}^2\mathcal{B}^2}\right) \\
&+\frac{\dot{\mathrm{P}}_{\mathrm{r}}\mathcal{C}^2\dot{\mathcal{B}}}{2\mathcal{A}^2\mathcal{B}}+\frac{5\mathrm{P}_{\mathrm{r}}'\mathcal{C}^2\mathcal{B}'}{2\mathcal{B}^3}-\frac{\mathrm{P}_{\mathrm{r}}''\mathcal{C}^2}{2\mathcal{B}^2} \\
&+\mathrm{P}_\phi\left(\frac{\dot{\mathcal{C}}^2}{\mathcal{A}^2}-\frac{\mathcal{C}\ddot{\mathcal{C}}}{\mathcal{A}^2}+\frac{\mathcal{C}\dot{\mathcal{A}}\dot{\mathcal{C}}}{\mathcal{A}^3}-\frac{\mathcal{C}\mathcal{B}'\mathcal{C}'}{\mathcal{B}^3}-\frac{\mathcal{C}\dot{\mathcal{B}}\dot{\mathcal{C}}}{\mathcal{A}^2\mathcal{B}}\right. \\
&\left.-\frac{\mathcal{C}'^2}{\mathcal{B}^2}-\frac{\mathcal{C}\mathcal{A}'\mathcal{C}'}{\mathcal{A}\mathcal{B}^2}+\frac{\mathcal{C}\mathcal{C}''}{\mathcal{B}^2}\right) \\
&+\dot{\mathrm{P}}_\phi\left(\frac{\mathcal{C}^2\dot{\mathcal{B}}}{2\mathcal{A}^2\mathcal{B}}-\frac{\mathcal{C}^2\dot{\mathcal{A}}}{2\mathcal{A}^3}+\frac{2\mathcal{C}\dot{\mathcal{C}}}{\mathcal{A}^2}\right)-\mathrm{P}_\phi'\left(\frac{\mathcal{C}^2\mathcal{A}'}{2\mathcal{A}\mathcal{B}^2}\right. \\
&\left.-\frac{\mathcal{C}^2\mathcal{B}'}{2\mathcal{B}^3}+\frac{2\mathcal{C}\mathcal{C}'}{\mathcal{B}^2}\right)+\frac{\ddot{\mathrm{P}}_\phi\mathcal{C}^2}{2\mathcal{A}^2}-\frac{\mathrm{P}_\phi''\mathcal{C}^2}{2\mathcal{B}^2} \\
&-\varsigma\left(\frac{3\mathcal{C}^2\dot{\mathcal{A}}\mathcal{A}'}{\mathcal{A}^3\mathcal{B}}+\frac{3\mathcal{C}^2\dot{\mathcal{B}}\mathcal{B}'}{\mathcal{A}\mathcal{B}^3}+\frac{3\mathcal{C}^2\mathcal{A}'\dot{\mathcal{B}}}{\mathcal{A}^2\mathcal{B}^2}\right. \\
&\left.-\frac{\mathcal{C}^2\dot{\mathcal{A}}'}{\mathcal{A}^2\mathcal{B}}-\frac{\mathcal{C}^2\dot{\mathcal{B}}'}{\mathcal{A}\mathcal{B}^2}+\frac{\mathcal{C}^2\dot{\mathcal{A}}\mathcal{B}'}{\mathcal{A}^2\mathcal{B}^2}\right) \\
&+\dot{\varsigma}\left(\frac{2\mathcal{C}^2\mathcal{A}'}{\mathcal{A}^2\mathcal{B}}+\frac{\mathcal{C}^2\mathcal{B}'}{\mathcal{A}\mathcal{B}^2}\right)+\varsigma'\left(\frac{\mathcal{C}^2\dot{\mathcal{A}}}{\mathcal{A}^2\mathcal{B}}+\frac{2\mathcal{C}^2\dot{\mathcal{B}}}{\mathcal{A}\mathcal{B}^2}\right) \\
&\left.-\frac{\dot{\varsigma}'\mathcal{C}^2}{\mathcal{A}\mathcal{B}}\right\},
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P}_{\mathrm{z}}^{(D)} &= \frac{1}{2}\left\{\left(\frac{\Phi}{\sqrt{\mathcal{T}}}+\Psi\mathcal{R}\right)\mathrm{P}_\phi+\Phi\sqrt{\mathcal{T}}+\Psi\mathcal{Q}+\frac{\Phi\mu}{\sqrt{\mathcal{T}}}\right\} \\
&+\Psi\left\{\mu\left(\frac{\ddot{\mathcal{A}}}{\mathcal{A}^3}-\frac{\mathcal{A}'^2}{\mathcal{A}^2\mathcal{B}^2}\right.\right. \\
&\left.-\frac{4\dot{\mathcal{A}}^2}{\mathcal{A}^4}\right)+\frac{5\dot{\mu}\dot{\mathcal{A}}}{2\mathcal{A}^3}+\frac{\mu'\mathcal{A}'}{2\mathcal{A}\mathcal{B}^2} \\
&-\frac{\ddot{\mu}}{2\mathcal{A}^2}+\mathrm{P_r}\left(\frac{\mathcal{B}''}{\mathcal{B}^3}-\frac{4\mathcal{B}'^2}{\mathcal{B}^4}-\frac{\dot{\mathcal{B}}^2}{\mathcal{A}^2\mathcal{B}^2}\right)+\frac{\dot{\mathrm{P}}_{\mathrm{r}}\dot{\mathcal{B}}}{2\mathcal{A}^2\mathcal{B}} \\
&+\frac{5\mathrm{P}_{\mathrm{r}}'\mathcal{B}'}{2\mathcal{B}^3}-\frac{\mathrm{P}_{\mathrm{r}}''}{2\mathcal{B}^2}+\mathrm{P}_\phi\left(\frac{\mathcal{C}'^2}{\mathcal{B}^2\mathcal{C}^2}\right. \\
&\left.-\frac{\dot{\mathcal{C}}^2}{\mathcal{A}^2\mathcal{C}^2}\right)+\frac{\dot{\mathrm{P}}_\phi\dot{\mathcal{C}}}{2\mathcal{A}^2\mathcal{C}}-\frac{\mathrm{P}_\phi'\mathcal{C}'}{2\mathcal{B}^2\mathcal{C}}+\dot{\mathrm{P}}_{\mathrm{z}}\left(\frac{\dot{\mathcal{B}}}{2\mathcal{A}^2\mathcal{B}}\right. \\
&\left.-\frac{\dot{\mathcal{A}}}{2\mathcal{A}^3}+\frac{\dot{\mathcal{C}}}{2\mathcal{A}^2\mathcal{C}}\right) \\
&+\mathrm{P}_{\mathrm{z}}'\left(\frac{\mathcal{B}'}{2\mathcal{B}^3}-\frac{\mathcal{A}'}{2\mathcal{A}\mathcal{B}^2}-\frac{\mathcal{C}'}{2\mathcal{B}^2\mathcal{C}}\right)+\frac{\ddot{\mathrm{P}}_{\mathrm{z}}}{2\mathcal{A}^2}-\frac{\mathrm{P}_{\mathrm{z}}''}{2\mathcal{B}^2} \\
&+\varsigma\left(\frac{\dot{\mathcal{A}}'}{\mathcal{A}^2\mathcal{B}}-\frac{3\dot{\mathcal{A}}\mathcal{A}'}{\mathcal{A}^3\mathcal{B}}-\frac{\dot{\mathcal{A}}\mathcal{B}'}{\mathcal{A}^2\mathcal{B}^2}\right. \\
&\left.+\frac{\dot{\mathcal{B}}'}{\mathcal{A}\mathcal{B}^2}-\frac{3\mathcal{A}'\dot{\mathcal{B}}}{\mathcal{A}^2\mathcal{B}^2}-\frac{3\dot{\mathcal{B}}\mathcal{B}'}{\mathcal{A}\mathcal{B}^3}\right)+\dot{\varsigma}\left(\frac{2\mathcal{A}'}{\mathcal{A}^2\mathcal{B}}\right. \\
&\left.\left.+\frac{\mathcal{B}'}{\mathcal{A}\mathcal{B}^2}\right)+\varsigma'\left(\frac{\dot{\mathcal{A}}}{\mathcal{A}^2\mathcal{B}}+\frac{2\dot{\mathcal{B}}}{\mathcal{A}\mathcal{B}^2}\right)-\frac{\dot{\varsigma}'\mathcal{C}^2}{\mathcal{A}\mathcal{B}}\right\},
\end{aligned}
$$

$$
\varsigma^{(D)} = -\frac{\varsigma\mathcal{A}\mathcal{B}}{2}\left(\frac{\Phi}{\sqrt{\mathcal{T}}}+\Psi\mathcal{R}\right)
$$

$$+\Psi\Bigg\{\mu\left(\frac{\mathcal{A}'\dot{\mathcal{C}}}{\mathcal{AC}}-\frac{\dot{\mathcal{C}}'}{\mathcal{C}}+\frac{\dot{\mathcal{B}}\mathcal{C}'}{\mathcal{BC}}\right)+\frac{\dot{\mu}\mathcal{A}'}{2\mathcal{A}}+\frac{\mu'\dot{\mathcal{A}}}{2\mathcal{B}}$$
$$-\frac{\dot{\mu}'}{2}+\mathrm{P_r}\left(\frac{\dot{\mathcal{C}}'}{\mathcal{C}}-\frac{\mathcal{A}'\dot{\mathcal{C}}}{\mathcal{AC}}-\frac{\dot{\mathcal{B}}\mathcal{C}'}{\mathcal{BC}}\right)-\frac{\dot{\mathrm{P}}_{\mathrm{r}}\mathcal{A}'}{2\mathcal{A}}-\frac{\mathrm{P}_{\mathrm{r}}'\dot{\mathcal{B}}}{2\mathcal{B}}$$
$$+\frac{\dot{\mathrm{P}}_{\mathrm{r}}'}{2}+\varsigma\Bigg(\frac{2\ddot{\mathcal{B}}}{\mathcal{A}}-\frac{\ddot{\mathcal{A}}\mathcal{B}}{\mathcal{A}^2}$$
$$-\frac{4\dot{\mathcal{A}}\dot{\mathcal{B}}}{\mathcal{A}^2}+\frac{2\dot{\mathcal{A}}^2\mathcal{B}}{\mathcal{A}^3}+\frac{\mathcal{A}'^2}{\mathcal{AB}}+\frac{4\mathcal{A}'\mathcal{B}'}{\mathcal{B}^2}-\frac{2\mathcal{A}''}{\mathcal{B}}+\frac{\mathcal{AB}''}{\mathcal{B}^2}$$
$$-\frac{\dot{\mathcal{B}}^2}{\mathcal{AB}}+\frac{\mathcal{B}\ddot{\mathcal{C}}}{\mathcal{AC}}-\frac{\mathcal{AC}''}{\mathcal{BC}}$$
$$-\frac{2\mathcal{AB}'^2}{\mathcal{B}^3}-\frac{3\mathcal{B}\dot{\mathcal{A}}\dot{\mathcal{C}}}{2\mathcal{A}^2\mathcal{C}}+\frac{\dot{\mathcal{B}}\dot{\mathcal{C}}}{2\mathcal{AC}}-\frac{\mathcal{A}'\mathcal{C}'}{2\mathcal{BC}}+\frac{3\mathcal{AB}'\mathcal{C}'}{2\mathcal{B}^2\mathcal{C}}\Bigg)$$
$$-\dot{\varsigma}\left(\frac{2\dot{\mathcal{A}}\mathcal{B}}{\mathcal{A}^2}+\frac{\mathcal{B}\dot{\mathcal{C}}}{2\mathcal{AC}}\right)$$
$$+\varsigma'\left(\frac{2\mathcal{AB}'}{\mathcal{B}^2}+\frac{\mathcal{AC}'}{2\mathcal{BC}}\right)\Bigg\}.$$

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