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ORIGINAL PAPER

Heavy-ion fusion reactions of astrophysical interest

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Abstract: The hindrance in fusion of heavy-ion reactions crops up in the region of extreme sub-barrier energies. This phenomenon can be analyzed effectively using an uncomplicated straightforward elegant mathematical formula gleaned presuming diffused barrier with a Gaussian distribution. The mathematical formula for cross section of nuclear fusion reaction has been obtained by folding together a Gaussian function representing the fusion barrier height distribution and the expression for classical cross section of fusion assuming a fixed barrier. The variation of fusion cross section as a function of energy, thus obtained, describes well the existing data on sub-barrier heavy-ion fusion for lighter systems of astrophysical interest. Using this simple formula, an analysis has been presented from ^{16}O + ^{18}O to ^{12}C + ^{198}Pt, all of which were measured down to < 10 μb. The agreement of the present theoretically calculated estimates with the measured values is excellent. The relatively smooth variation of the three parameters of this formula implies that it may be exploited to estimate the excitation function or to extrapolate cross sections for pairs of interacting nuclei which are yet to be measured. Possible extensions of the present methodology and its limitations have also been discussed.

Keywords: Fusion reactions; Excitation function; Barrier distribution; Nucleosynthesis

1. Introduction

The phenomenon of fusion hindrance in heavy-ion fusion reactions for lighter systems may have important consequences on the nuclear processes taking place in astrophysical scenarios. The hindrance can seriously affect the energy generation by heavy-ion fusion reactions occurring in the region of extreme sub-barrier energies, encompassing reactions involving lighter systems, such as the reactions occurring in the stage of carbon and oxygen burning in heavy stars [1–3] and their evolution and elemental abundances. It is well recognized that excitation functions for fusion of two colliding nuclei cannot be explained satisfactorily within the framework of a single barrier penetration that is well-defined in its total potential energy. While replicating the shapes of excitation functions for fusion reactions, particularly in the domain of low energies near-threshold, coexistence of different barriers must be assumed. Describing the fusion cross section calculations within the framework of any theory of nuclear reactions which involves coupling to several collective states [4–11], this condition is naturally accounted.

The intent of the current exercise is to estimate the cross sections for nuclear reactions in heavy-ion fusion of lighter systems of astrophysical interest. The fusion reactions between light heavy ions have a prominent role in the dynamics of stellar evolution. These fusion reactions have significant impact on the stellar burning and resulting nucleosynthesis. Such fusion reaction cross sections and reaction rates in stellar matter at temperatures $T \leq$ (3-10) $\times$ 10^8 K, especially at densities $\geq$ 10^9 g cm^{-3}, in a strongly screened or in the pycnonuclear burning regimes are important.

The phenomenological description of the dependence of nuclear fusion reactions on the collisional kinetic energy is accomplished by assuming the Gaussian form of fusion barrier distribution and considering the mean height of barriers and the variance as freely varying parameters. The cross sections for fusion reactions have been obtained by folding a Gaussian distribution for fusion barrier heights together with the classical expression of fixed single barrier cross section for fusion. The effective radius, which is the distance of the position of the potential barrier of interacting nuclei, is treated as an additional free parameter.

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These three parameters can be extracted distinctively for each reaction by fitting the theoretical predictions to the experimental data. The excitation functions for fusion reactions, thus obtained, describe well existing data of sub-barrier fusion and energy dependence of capture reactions for lighter heavy-ion systems of astrophysical interest.

2. Distribution of fusion barriers

It has been observed that the excitation functions for fusion of two colliding nuclei can not be well explained by invoking a well-defined single one-dimensional potential barrier penetration model. The explanation of fusion cross sections, particularly in case of heavy-ions, requires invoking of a potential barrier distribution [1]. The quantum mechanical barrier penetration smoothens out a set of distinct potential energy barriers to an effective barrier distribution which is continuous. In order to replicate the dependence of nuclear cross sections of fusion reaction on the collisional kinetic energy, specifically measured at low energies near fusion threshold, the assumption of a fusion barrier height distribution becomes necessary to simulate the effects resulting from the coupling to other channels. In the coupled-channel calculations, it is naturally achieved which involve, in both colliding nuclei, the coupling to collective states down to the lowest level. The nuclear structure effects influencing the distribution of potential energy barriers have been considered negligible and hence ignored in this work. A Gaussian form $D(B)$ simulating the shape of the diffused barrier has been conceptualized [12] for the fusion barrier height distribution. Therefore, the distribution of barriers is provided by

$$D(B) = \frac{1}{\sqrt{2\pi}\sigma_B} \exp\left[-\frac{(B-B_0)^2}{2\sigma_B^2}\right] \tag{1}$$

where for each individual reaction, the parameters B_0 (mean barrier height) and σ_B (width of barrier distribution) are to be determined exclusively.

3. Fusion cross-sectional calculation

In furtherance of providing a systematic analysis to the excitation function measurements of nuclear fusion reactions, a mathematical formula for the cross section can be derived [12] for surmounting the barrier arising due to interacting nuclei. The energy dependency of the cross sections for nuclear fusion reactions is accomplished by folding the Gaussian distribution for fusion barriers [12, 13] given by Eq. (1) together with the classical nuclear fusion reaction cross-sectional expression which is provided by

$$\sigma_f(B) \begin{array}{ll} = \pi R_B^2\left[1-\frac{B}{E}\right] & \text{for } B \le E \\ = 0 & \text{for } B \ge E \end{array} \tag{2}$$

where $E = \mathrm{E_{c.m.}}$ is the energy in the center of mass system, R_B marks, approximately, the position of the barrier (effective radius corresponding to the relative distance), which results in the following expression [12]

$$\begin{aligned}\sigma_c(E) &= \int_{E_0}^{\infty} \sigma_f(B)D(B)dB \\ &= \int_{E_0}^{B_0} \sigma_f(B)D(B)dB + \int_{B_0}^{E} \sigma_f(B)D(B)dB \\ &= \pi R_B^2 \frac{\sigma_B}{E\sqrt{2\pi}}\left[\xi\sqrt{\pi}\{\mathrm{erf}(\xi)+\mathrm{erf}(\xi_0)\}+e^{-\xi^2}-e^{-\xi_0^2}\right]\end{aligned} \tag{3}$$

where $E_0 = 0$ for positive Q value reactions and $E_0 = Q$ for negative Q value reactions, Q value being the sum of the rest masses of fusing nuclei minus rest mass of the resultant fused nucleus,

$$\begin{aligned}\xi &= \frac{E-B_0}{\sigma_B\sqrt{2}} \\ \xi_0 &= \frac{B_0-E_0}{\sigma_B\sqrt{2}}\end{aligned} \tag{4}$$

and $\mathrm{erf}(\xi)$ is the Gaussian error integral for argument ξ. The three parameters R_B, B_0 and σ_B have to be determined by a least square fitting of Eq. (3), while making use of Eq. (4), to the measured excitation functions for fusion reactions. While deriving the formula of Eq. (3), the quantal effects of barrier penetration have not been taken explicitly into account. The structure of a given excitation function for fusion reaction is, however, influenced by the sub-barrier tunneling which has been included effectively through the parameter σ_B which describes the width of barrier distribution.

The mathematical expression of Eq. (3), for the interaction cross section of overcoming the barrier arising due to potential-energy, is achieved by the use of a diffused-barrier. This formula provides a very elegant parametrization for such cross sections. Hence, for the predictions and analysis of excitation functions for fusion reactions, particularly in the span of energies below barrier, it may be used effectively for systems involving light, medium or moderately heavy ions.

In case of systems involving light or medium heavy ions, fusion is automatically guaranteed that leads to compound nucleus formation, once it surmounts the barrier of the colliding nuclei. The word 'capture' refers to the action of surmounting the potential barrier of colliding ions that follows a composite system formation. In general, there is a probability f that composite nucleus experiences fusion in an event of target nucleus capturing the projectile. This probability approaches unity for light and medium systems. Under this condition ($f \sim 1$), fusion ensues for most of the capture events leading to capture cross sections being practically identical to fusion cross sections. On the contrary, there is only a meager probability ($f < 1$) that events leading to capture would eventually proceed to fusion in case of very heavy systems. In these cases, most of the events remaining re-separate before equilibration. For such cases, distinguishing fusion from capture becomes necessary. Therefore, in the event of quite heavy systems, predictions based on Eq. (3) provide capture cross sections where fusion is not automatically guaranteed once the barrier penetration is complete.

4. Results and discussion

The excitation functions for nuclear reactions of heavy-ion fusion for lighter systems of astrophysical interest at sub-barrier energies have been analyzed theoretically. This has been facilitated through Eq. (3) which is obtained by folding an elegant diffused barrier distribution [provided in Eq. (1)] of Gaussian shape together with the classical mathematical fusion cross-sectional expression for single fixed barrier. The identical combinations of projectile-target system involved in heavy ion sub-barrier fusion reactions have been chosen which have been recently [14–17] studied. By using the method of least-square fitting, the values of three parameters B_0 (mean barrier height), σ_B (width) and R_B (effective radius) have been extracted. The values of these are tabulated in Table 1 arranging it in the ascending order of projectile masses.

The task of estimating cross section $\sigma_c(E)$ for a particular reaction rests upon predicting the parameter values for B_0 (mean barrier height), σ_B (width) and R_B (effective radius) reliably. Since B_0 is necessarily the mean barrier height, it should be a function of $z = Z_1Z_2/(A_1^{1/3} + A_2^{1/3})$ (Coulomb parameter) in the vicinity of the barrier. The third entity, the effective barrier radius R_B, unquestionably should depend upon $r_0(A_1^{1/3} + A_2^{1/3}) = r_0A_{12}$ where r_0 is the nuclear radius parameter. In this work, the parameters of the model are adjusted to reproduce the reaction observables itself. In order to retain certain predictive power in the present model, a reasonable extrapolation formula for B_0 may be expressed as a polynomial of z. The extrapolation of the tendencies of σ_B is extra difficult, which basically arise due to nuclear deformation, vibrations and quantum mechanical barrier tunneling probability. The σ_B is, therefore, a result of adding in quadratures σ_1, σ_2 arising out of the nuclear deformations of interacting nuclei and the σ_0 parameter which is approximately constant accounting for nuclear vibrations and quantal barrier penetrability. It may be observed from Table 1 that the mean barrier height B_0 increases with the Coulomb parameter z for all cases while the effective radius R_B also increases with A_{12} except for $^{16}O+^{18}O$ and $^{28}Si+^{64}Ni$ and interestingly, as may be seen from Figs. 1–7, for $^{16}O+^{18}O$ case, the theoretical calculations are slightly off at lower energies as well.

In Figs. 1–7, the fusion excitation functions which are measured experimentally have been depicted by full circles and compared with the predicted estimates obtained using diffused barrier formula represented by the solid lines. Results for the colliding $^{12}C+^{24}Mg$, $^{12}C+^{30}Si$, $^{12}C+^{198}Pt$,

Table 1 The extracted values of B_0 (mean barrier height), σ_B (width) and R_B (effective radius), obtained from the analyses of the measured fusion excitation functions. The table is arranged in order of increasing projectile mass

Reaction	Refs.	z	A_{12}	σ_B [MeV]	B_0 [MeV]	R_B [fm]
$^{12}C+^{24}Mg$	[15]	13.916	5.174	0.815	11.483	6.646
$^{12}C+^{30}Si$	[14]	15.565	5.397	1.090	13.540	8.300
$^{12}C+^{198}Pt$	[18]	57.650	8.118	1.749	55.140	11.179
$^{16}O+^{18}O$	[15]	12.450	5.141	0.859	9.797	7.743
$^{28}Si+^{64}Ni$	[17]	55.709	7.037	1.402	50.403	7.182
$^{58}Ni+^{58}Ni$	[18]	101.269	7.742	2.275	98.278	8.550
$^{64}Ni+^{64}Ni$	[18]	98.000	8.000	1.466	92.646	8.862

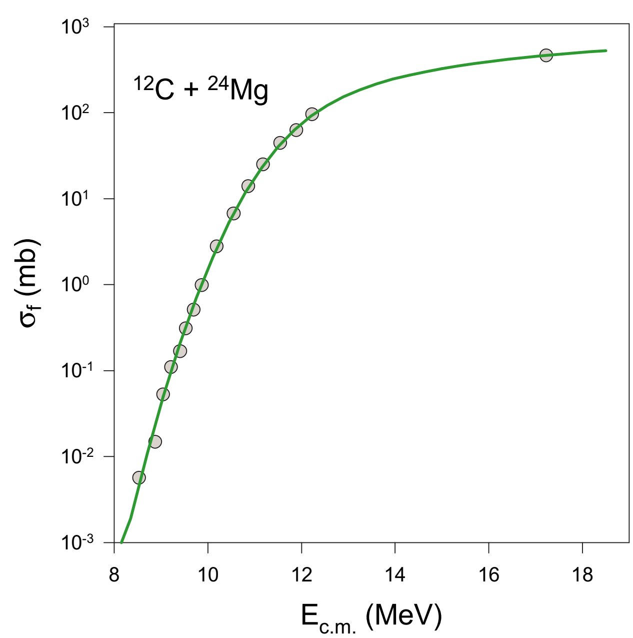

Fig. 1 Plots of the analytical estimates (solid line) and the measured values (full circles) of the capture excitation functions for $^{12}C+^{24}Mg$

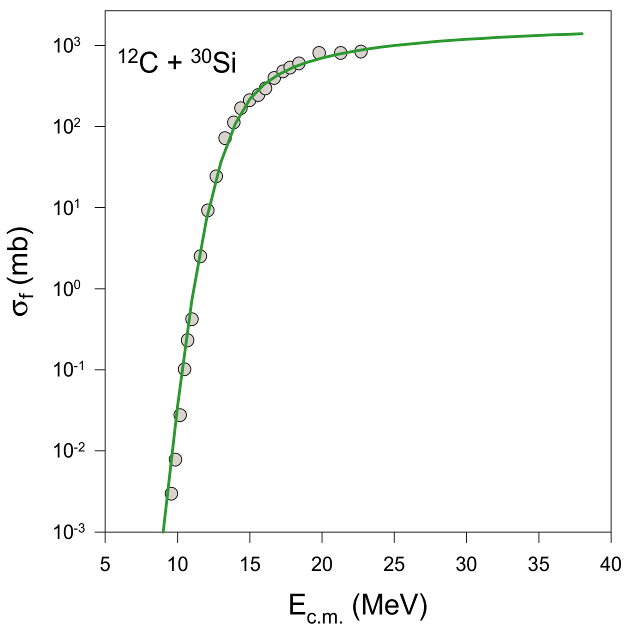

Fig. 2 Plots of the analytical estimates (solid line) and the measured values (full circles) of the capture excitation functions for $^{12}C+^{30}Si$

$^{16}O+^{18}O$, $^{28}Si+^{64}Ni$, $^{58}Ni+^{58}Ni$ and $^{64}Ni+^{64}Ni$ systems are illustrated in Figs. 1–7. It may be easily identified from the plots that accurately measured excitation functions for

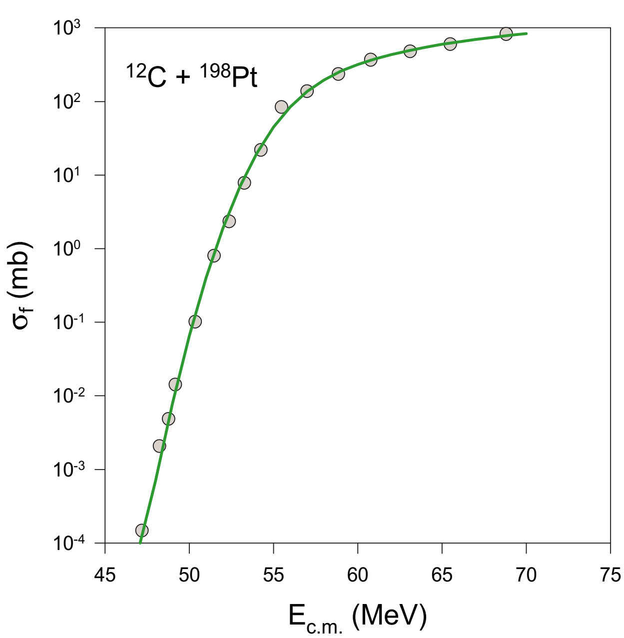

Fig. 3 Plots of the analytical estimates (solid line) and the measured values (full circles) of the capture excitation functions for $^{12}C+^{198}Pt$

fusion reactions yield systematic information on the cardinal attributes of the nucleus–nucleus interaction potential, *viz.* B_0 (mean barrier height) and σ_B (width) of its distribution for collisions between two nuclei. The capture or fusion cross sections for planning experiments can be also guessed using Eq. (3) along with the theoretically extracted values of B_0 and σ_B parameters.

As can be visualized from Figs. 1–7, the theoretical estimates facilitated by the diffused barrier formula described in this work resulted in good fits to the measured experimental data. This observation obviously infers that almost all the events leading to capture proceed to fusion for the chosen set of nuclei resulting capture cross sections to be essentially identical to the fusion cross sections. It may further imply that for the barrier distribution, the choice of Gaussian form describes quite well the nuclear cross sections for fusion reactions at energies below the barrier. This fact justifies the model 'beyond single barrier' which arises out of vibration and deformation of nuclei and more importantly tunneling. Whereas theoretically 'barrier distribution' is a valid concept under a few approximations, the fact that fits to the experimentally measured data is good and implies certainly that in fusion reactions involving heavy-ions, it remains a meaningful concept at least for lighter systems of astrophysical interest.

It is pertinent to mention here that although the single-Gaussian parametrization [Eq. (1)] for barrier distribution is reasonably successful in providing a good description of fusion process in general, neither the formula derived for fusion cross section nor the method using the barrier distribution can be put to use for all fusing systems. One can visualize from Eq. (3) that the excitation function is all the time a monotonically rising function of energy. This puts a limitation on Eqs. (1, 2) which cannot be used for describing fusion reactions at higher energies when incomplete fusion as well as deep-inelastic scattering can cause a lowering of the fusion cross section. Similar limitation arises for lighter systems (e.g., $^{12}C+^{12}C$, $^{12}C+^{16}O$, $^{16}O+^{16}O$, etc.) as well when excitation functions possess oscillations and resonance structures. The possibility of better agreement to data may further be explored by opting for a more intricate formula for the barrier distribution, which, nonetheless, will bring in more additional adjustable parameters than just three used in the present work. Such refinements for the barrier distribution of Eq. (1) may include distributions having different widths on lower or higher energy sides, certain moderation of the exponent in Gaussian distribution form or multi-component distributions.

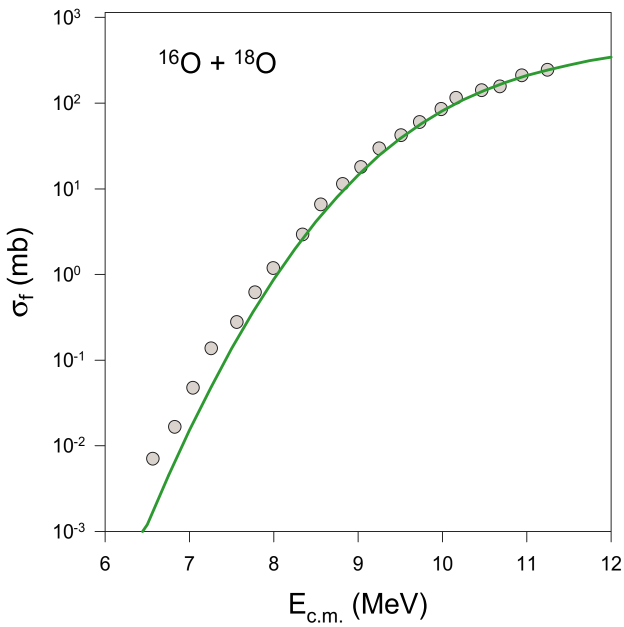

Fig. 4 Plots of the analytical estimates (solid line) and the measured values (full circles) of the capture excitation functions for $^{16}O+^{18}O$

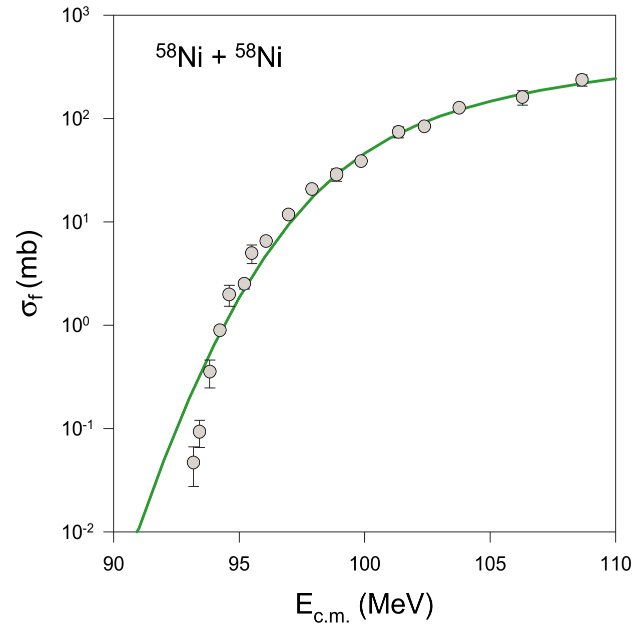

Fig. 6 Plots of the analytical estimates (solid line) and the measured values (full circles) of the capture excitation functions for $^{58}Ni+^{58}Ni$

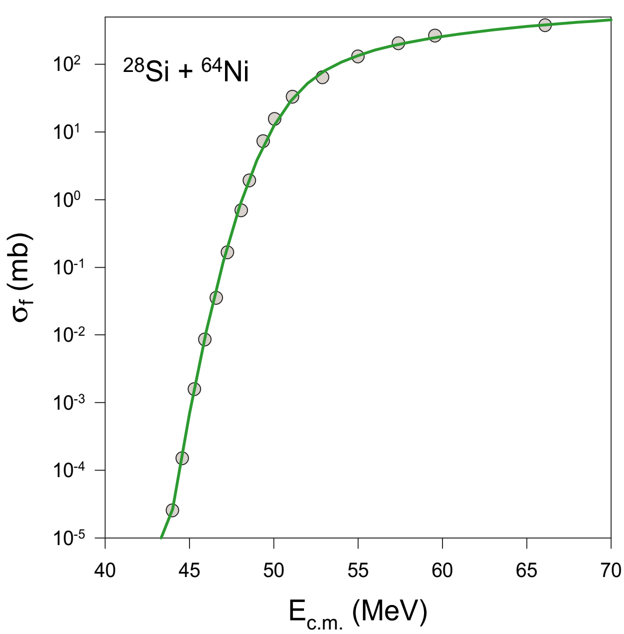

Fig. 5 Plots of the analytical estimates (solid line) and the measured values (full circles) of the capture excitation functions for $^{28}Si+^{64}Ni$

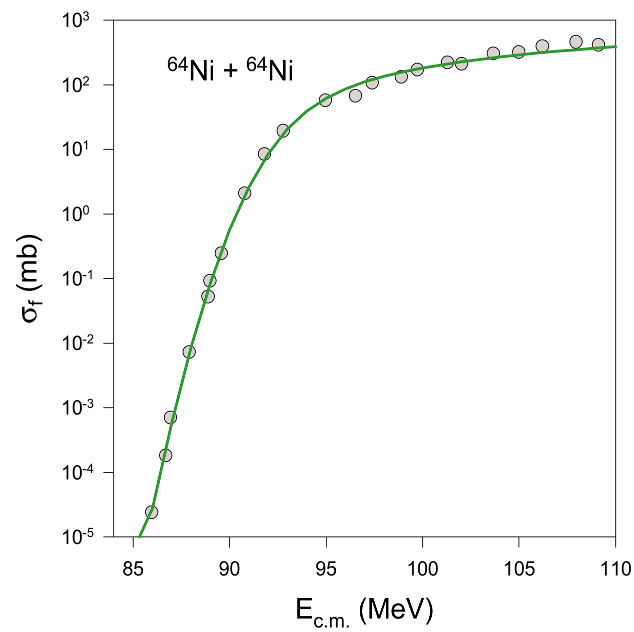

Fig. 7 Plots of the analytical estimates (solid line) and the measured values (full circles) of the capture excitation functions for $^{64}Ni+^{64}Ni$

5. Conclusions

In the region of sub-barrier energies, the fusion reaction cross sections have been estimated spanning a broad energy range. In order to envision the conditions of overcoming the potential barrier in nuclear collisions and to have a systematic knowledge on the essential characteristics, *viz.* B_0 (mean barrier height), σ_B (width) and R_B (effective radius), of the interacting potential, a set of accurately measured excitation functions of fusion reactions has been studied for two colliding nuclei. A Gaussian distribution function for the barrier heights is assumed to derive a simple diffused-barrier formula. The values of the essential parameters B_0 (mean barrier height), σ_B (width) and R_B (effective radius) are determined using the method of least-square fit.

The present formula of cross section for fusion reactions can be used to calculate the cross sections for surmounting the barrier in collisions of moderately heavy systems for a given projectile-target combination. For calculating the production cross sections of superheavy nuclei, the prediction of the capture excitation functions or sticking can be used in the sticking-diffusion-survival model [19] as one of three basic ingredients. The reasonably good fit to the experimental data provided by the theory described above implies two principal facts that for the investigated set of nucleus-nucleus systems almost all the events leading to capture ultimately proceeds to fusion and the idea of the Gaussian distribution of barrier provides excellent description of cross sections for fusion reactions in the domain of the sub-barrier energies. Although the single-Gaussian parametrization for barrier distribution is reasonably successful in providing a good description of fusion process in general, neither the formula derived for fusion cross section nor the method using the barrier distribution can be put to use for all fusing systems. One can visualize from Eq. (3) that the excitation function is all the time a monotonically rising function of energy. This puts a limitation on Eqs. (1,2) which cannot be used for describing fusion reactions at higher energies when incomplete fusion as well as deep-inelastic scattering can cause a lowering of the fusion cross section. Similar limitation can arise for lighter systems also when excitation functions possess oscillations and resonance structures. Possibility of better compliance to measured data may be explored by employing a more intricate formula for barrier distribution, which, however, will bring in more additional adjustable parameters than just three used in the present work. Such improvements for the barrier distribution can be realized through distributions having distinctive widths on the lower or higher energy sides, multi-component distributions or a modification of the exponent appearing in distribution represented by a Gaussian form.

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