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ORIGINAL PAPER

New exact solutions for the doubly dispersive equation using the improved Bernoulli sub-equation function method

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Abstract: In this study, the doubly dispersive equation is presented by the application of the improved Bernoulli sub-equation function method (IBSEFM). The doubly dispersive equation which is a nonlinear partial differential equation is transformed into nonlinear ordinary differential equation using a wave transformation and then is solved by IBSEFM. Some new solutions are successfully constructed. All the obtained solutions in this study have been satisfied the doubly dispersive equation. In the present study, we have used Wolfram Mathematica 9 software for all of the computations and graphic plottings.

Keywords: Doubly dispersive equation; Improved Bernoulli sub-equation function method; Singular soliton solutions; Travelling wave solutions

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1. Introduction

Nonlinear evolution equations (NLEEs) arise in many fields of sciences including physics, mechanics and material science [1–5]. As the NLEEs describe various aspects of our real life situations, the searching for solitary wave solutions plays an important fundamental role [6]. Various methods have been developed for obtaining the analytical solutions of different kinds of partial differential equations, such as the improved Bernoulli sub-equation function [7–10], the modified extended tanh [11], the simplest equation [12–14], the modified simplest equation [15, 16], the extended sinh-Gordon equation expansion [17–20], the fractional variation iteration [21–24], the hybrid differential transform [25], the exponential function [26], the modified exp $(-\phi(\eta))$-expansion function [27], the q-homotopy analysis transform [28], homotopy perturbation Sumudu transform [29], the implicit finite difference scheme, the Dufort–Frankel finite difference scheme [30] and the difference schemes method [31]. In addition, the improved Bernoulli sub-equation method (IBSEFM) is used by constructing variety of solitary wave solutions to a nonlinear model for describing the wave propagation in the nonlinearity elastic inhomogeneous Murnaghan's rod, called the doubly dispersive equation [32].

The doubly dispersive equation [32] is given by

$$\emptyset_{tt} - \left(\frac{1}{\rho}(E\emptyset)_x\right)_x = \frac{\epsilon}{2}\left(\frac{1}{\rho}\left(p\beta\emptyset^2 + pv^2\emptyset_{tt} - (b\alpha v^2\emptyset_x)_x\right)_x\right)_x \tag{1}$$

where $b = \frac{M}{E} < 1$ and $p = \frac{B}{E}$ are combinations of the constant scale factors.

2. The IBSEFM

Improved Bernoulli sub-equation function method (IBSEFM) which is formed by modifying the Bernoulli sub-equation function method will be given in this part.

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Step 1 Let us consider the following partial differential equation

$$P(u, u_x, u_t, u_{xt}, \ldots) = 0, \tag{2}$$

and take the wave transformation

$$u(x,t) = U(\gamma), \gamma = x - ct, \tag{3}$$

where c is constant and will be determined later. Substituting Eq. (3) into Eq. (2), we obtain the following nonlinear ordinary differential equation

$$N(U, U', U'', U''', \ldots) = 0. \tag{4}$$

Step 2 Considering the trial equation of solution in Eq. (4), it can be written as follows:

$$U(\eta) = \frac{\sum_{i=0}^{n} a_i F^i(\eta)}{\sum_{j=0}^{m} b_j F^j(\eta)} = \frac{a_0 + a_1 F(\eta) + a_2 F^2(\eta) + \ldots + a_n F^n(\eta)}{b_0 + b_1 F(\eta) + b_2 F^2(\eta) + \ldots + b_m F^m(\eta)}. \tag{5}$$

According to the Bernoulli theory, we can consider the general form of Bernoulli differential equation for F' as follows:

$$F' = wF + dF^M,\ w \neq 0,\ d \neq 0,\ M \in R - \{0,1,2\}, \tag{6}$$

where $F = F(\eta)$ is Bernoulli differential polynomial. Substituting the above relations in Eq. (4), it yields equations of polynomial $\Omega(F)$ of F as follows:

$$\Omega(F) = \rho_s F^s + \ldots + \rho_1 F + \rho_0 = 0. \tag{7}$$

According to the balance principle, we can determine the relationship between n, m and M.

Step 3 Setting all of the coefficients of $\Omega(F)$ to zero will yield us an algebraic system of equations:

$$\rho_i = 0,\ i = 0, \ldots, s. \tag{8}$$

Solving this system, we will specify the values of $a_0, \cdots, a_n$ and $b_0, \cdots, b_m$.

Step 4 When we solve nonlinear Bernoulli differential equation given by Eq. (6), we obtain the following two situations in terms of w and d:

$$F(\eta) = \left[\frac{-d}{w} + \frac{E}{e^{w(M-1)\eta}}\right]^{\frac{1}{1-M}},\ w \neq d, \tag{9}$$

$$F(\eta) = \left[\frac{(E-1) + (E+1)\tanh(w(1-M)\eta/2)}{1 - \tanh(w(1-M)\eta/2)}\right]^{\frac{1}{1-M}}, \quad w = d, \tag{10}$$

3. Application

In this section, the DDE is presented by the application of the IBSEFM. Using the wave transformation given by Eq. (1),

$$\emptyset(x,t) = U(\gamma), \gamma = \mu(x - \vartheta t) \tag{11}$$

Substituting Eq. (11) into Eq. (1) yields the following NODE:

$$2(E - \vartheta^2\rho)U + p\beta\epsilon U^2 + \epsilon\mu^2 v^2(\vartheta^2\rho - b\alpha)U'' = 0 \tag{12}$$

Balancing Eq. (12) by considering the highest derivative and the highest power, we obtain $n - m = 2M - 2$.

Choosing $M = 2, m = 1$, gives $n = 3$. Thus, the trial solution to Eq. (1) takes the following form:

$$U(\gamma) = \frac{a_0 + a_1 F(\gamma) + a_2 F^2(\gamma) + a_3 F^3(\gamma)}{b_0 + b_1 F(\gamma)}, \tag{13}$$

where $F' = wF + dF^3, w \neq 0, d \neq 0$. Substituting Eq. (13), its second derivative along with $F' = wF + dF^3, w \neq 0, d \neq 0$ into Eq. (12) yields a polynomial in F. Solving the system of the algebraic equations yields the values of the parameter involved. Substituting the obtained values of the parameters into Eq. (13) yields the solutions to Eq. (1).

For $w \neq d$, we can find the following coefficients:

Case 1

$$a_0 = 0,\ a_2 = \frac{12w\mu^2 v^2\sigma(\mathrm{E} - \alpha F)b_0}{p\beta(-2 + \epsilon\mu^2 v^2\sigma^2)},$$
$$a_3 = \frac{12w^2\mu^2 v^2(\mathrm{E} - \alpha F)b_1}{p\beta(-2 + \epsilon\mu^2 v^2\sigma^2)},\ \rho = \frac{-2\mathrm{E} + \alpha\epsilon\mu^2 v^2\sigma^2 F}{\vartheta^2(-2 + \epsilon\mu^2 v^2\sigma^2)} \tag{14}$$

Case 2

$$a_0 = -\frac{2\mu^2 v^2\sigma^2(\mathrm{E} - \alpha F)b_0}{p\beta(2 + \epsilon\mu^2 v^2\sigma^2)};$$
$$a_1 = -\frac{2\mu^2 v^2\sigma(\mathrm{E} - \alpha F)(6wb_0 + \sigma b_1)}{p\beta(2 + \epsilon\mu^2 v^2\sigma^2)};$$
$$a_2 = -\frac{12w\mu^2 v^2(\mathrm{E} - \alpha F)(wb_0 + \sigma b_1)}{p\beta(2 + \epsilon\mu^2 v^2\sigma^2)};$$
$$a_3 = -\frac{12w^2\mu^2 v^2(\mathrm{E} - \alpha F)b_1}{p\beta(2 + \epsilon\mu^2 v^2\sigma^2)};\ \rho = \frac{2\mathrm{E} + \alpha\epsilon\mu^2 v^2\sigma^2}{\vartheta^2(2 + \epsilon\mu^2 v^2\sigma^2)}; \tag{15}$$

Case 3

$$a_0 = 0;\ a_1 = 0;\ a_2 = \frac{12w\mu^2 v^2\sigma(\mathrm{E} - \alpha F)b_1}{p\beta(-2 + \epsilon\mu^2 v^2\sigma^2)};\ a_3 = \frac{12w^2\mu^2 v^2(\mathrm{E} - \alpha F)b_1}{p\beta(-2 + \epsilon\mu^2 v^2\sigma^2)};\ b_0 = 0;\ \rho = \frac{-2\mathrm{E} + \alpha\epsilon\mu^2 v^2\sigma^2 F}{\vartheta^2(-2 + \epsilon\mu^2 v^2\sigma^2)}; \tag{16}$$

Case 4

$$a_0 = 0;\ a_1 = \frac{(-\mathrm{E} + \alpha F)b_1}{p\beta\epsilon};\ a_2 = -\frac{6w(\mathrm{E} - \alpha F)b_1}{p\beta\epsilon\sigma};$$
$$a_3 = -\frac{6w^2(\mathrm{E} - \alpha F)b_1}{p\beta\epsilon\sigma^2};\ b_0 = 0; \rho = \frac{\mathrm{E} + \alpha F}{2\vartheta^2};\ \mu = -\frac{\sqrt{2}}{\sqrt{\epsilon}v\sigma}; \tag{17}$$

Case 5

$$a_0 = 0;\ a_1 = 0;\ a_2 = \frac{6w(\mathrm{E} - \alpha F)b_1}{p\beta\epsilon\sigma};$$
$$a_3 = \frac{6w^2(\mathrm{E} - \alpha F)b_1}{p\beta\epsilon\sigma^2};\ b_0 = 0;\ \rho = \frac{\mathrm{E} + \alpha F}{2\vartheta^2};\ \mu = -\frac{\mathrm{i}\sqrt{2}}{\sqrt{\epsilon}v\sigma}; \tag{18}$$

Case 6

$$a_0 = 0;\ a_1 = \frac{(-\mathrm{E} + \alpha F)b_1}{p\beta\epsilon};\ a_2 = -\frac{6w(\mathrm{E} - \alpha F)b_1}{p\beta\epsilon\sigma};$$
$$a_3 = -\frac{6w^2(\mathrm{E} - \alpha F)b_1}{p\beta\epsilon\sigma^2};\ b_0 = 0;\ \rho = \frac{\mathrm{E} + \alpha F}{2\vartheta^2};\ \mu = \frac{\sqrt{2}}{\sqrt{\epsilon}v\sigma}; \tag{19}$$

$$\begin{aligned} &a_0 = 0;\ a_1 = \frac{6w\mu^2v^2\sigma(-\vartheta^2\rho + \alpha F)b_0}{p\beta};\\ &a_2 = -\frac{6w\mu^2v^2(\vartheta^2\rho - \alpha F)(wb_0 + \sigma b_1)}{p\beta};\\ &a_3 = \frac{6w^2\mu^2v^2(-\vartheta^2\rho + \alpha F)b_1}{p\beta};\\ &\mathrm{E} = \frac{1}{2}\left(\vartheta^2\rho\left(2 - \epsilon\mu^2v^2\sigma^2\right) + \alpha\epsilon\mu^2v^2\sigma^2F\right); \end{aligned} \tag{20}$$

Case 8

$$\begin{aligned} &a_0 = \frac{\mu^2v^2\sigma^2(-\vartheta^2\rho + \alpha F)b_0}{p\beta};\\ &a_1 = -\frac{\mu^2v^2\sigma(\vartheta^2\rho - \alpha F)(6wb_0 + \sigma b_1)}{p\beta};\\ &a_2 = -\frac{6w\mu^2v^2(\vartheta^2\rho - \alpha F)(wb_0 + \sigma b_1)}{p\beta};\\ &a_3 = \frac{6w^2\mu^2v^2(-\vartheta^2\rho + \alpha F)b_1}{p\beta};\\ &\mathrm{E} = \frac{1}{2}\left(\vartheta^2\rho\left(2 + \epsilon\mu^2v^2\sigma^2\right) - \alpha\epsilon\mu^2v^2\sigma^2F\right); \end{aligned} \tag{21}$$

Substituting Eq. (14) into Eq. (13) gives

$$u_1(x,t) = \frac{\frac{12w\mu^2v^2\sigma(\mathrm{E}-\alpha F)b_0}{p\beta\left(\mathrm{e}^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)(-2+\epsilon\mu^2v^2\sigma^2)} + \frac{12w^2\mu^2v^2(\mathrm{E}-\alpha F)b_1}{p\beta\left(\mathrm{e}^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^3(-2+\epsilon\mu^2v^2\sigma^2)} + \frac{12w\mu^2v^2(\mathrm{E}-\alpha F)(wb_0+\sigma b_1)}{p\beta\left(\mathrm{e}^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^2(-2+\epsilon\mu^2v^2\sigma^2)}}{b_0 + \frac{b_1}{\mathrm{e}^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}}} \tag{22}$$

Case 7

Substituting Eq. (15) into Eq. (13) gives

$$u_2(x,t) = \frac{-\frac{2\mu^2v^2\sigma^2(\mathrm{E}-\alpha F)b_0}{p\beta(2+\epsilon\mu^2v^2\sigma^2)} - \frac{12w^2\mu^2v^2(\mathrm{E}-\alpha F)b_1}{p\beta\left(e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^3(2+\epsilon\mu^2v^2\sigma^2)} - \frac{12w\mu^2v^2(\mathrm{E}-\alpha F)(wb_0+\sigma b_1)}{p\beta\left(e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^2(2+\epsilon\mu^2v^2\sigma^2)} - \frac{2\mu^2v^2\sigma(\mathrm{E}-\alpha F)(6wb_0+\sigma b_1)}{p\beta\left(e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)(2+\epsilon\mu^2v^2\sigma^2)}}{b_0 + \frac{b_1}{e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}}} \tag{23}$$

Substituting Eq. (16) into Eq. (13) gives

$$u_3(x,t) = \frac{\left(e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)\left(\frac{12w^2\mu^2v^2(\mathrm{E}-\alpha F)b_1}{p\beta\left(e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^3(-2+\epsilon\mu^2v^2\sigma^2)} + \frac{12w\mu^2v^2\sigma(\mathrm{E}-\alpha F)b_1}{p\beta\left(e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^2(-2+\epsilon\mu^2v^2\sigma^2)}\right)}{b_1} \tag{24}$$

Substituting Eq. (17) into Eq. (13) gives

$$u_4(x,t) = \frac{\left(\mathrm{e}^{\frac{\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon}v}}k-\frac{w}{\sigma}\right)\left(-\frac{6w^2(\mathrm{E}-\alpha F)b_1}{p\beta\epsilon\left(\mathrm{e}^{\frac{\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon}v}}k-\frac{w}{\sigma}\right)^3\sigma^2} - \frac{6w(\mathrm{E}-\alpha F)b_1}{p\beta\epsilon\left(\mathrm{e}^{\frac{\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon}v}}k-\frac{w}{\sigma}\right)^2\sigma} + \frac{(-\mathrm{E}+\alpha F)b_1}{p\beta\epsilon\left(\mathrm{e}^{\frac{\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon}v}}k-\frac{w}{\sigma}\right)}\right)}{b_1} \tag{25}$$

Substituting Eq. (18) into Eq. (13) gives

$$u_5(x,t)=\frac{\left(e^{\frac{i\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon v}}}k-\frac{w}{\sigma}\right)\left(\frac{6w^2(\mathrm{E}-\alpha F)b_1}{p\beta\epsilon\left(e^{\frac{i\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon v}}}k-\frac{w}{\sigma}\right)^3\sigma^2}+\frac{6w(\mathrm{E}-\alpha F)b_1}{p\beta\epsilon\left(e^{\frac{i\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon v}}}k-\frac{w}{\sigma}\right)^2\sigma}\right)}{b_1} \tag{26}$$

Substituting Eq. (19) into Eq. (13) gives

$$u_6(x,t)=\frac{\left(e^{-\frac{\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon v}}}k-\frac{w}{\sigma}\right)\left(-\frac{6w^2(\mathrm{E}-\alpha F)b_1}{p\beta\epsilon\left(e^{-\frac{\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon v}}}k-\frac{w}{\sigma}\right)^3\sigma^2}-\frac{6w(\mathrm{E}-\alpha F)b_1}{p\beta\epsilon\left(e^{-\frac{\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon v}}}k-\frac{w}{\sigma}\right)^2\sigma}+\frac{(-\mathrm{E}+\alpha F)b_1}{p\beta\epsilon\left(e^{-\frac{\sqrt{2}(x-t\vartheta)}{\sqrt{\epsilon v}}}k-\frac{w}{\sigma}\right)}\right)}{b_1} \tag{27}$$

Substituting Eq. (20) into Eq. (13) gives

$$u_7(x,t)=\frac{\frac{6w\mu^2v^2\sigma\left(-\vartheta^2\rho+\alpha F\right)b_0}{p\beta\left(e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)}+\frac{6w^2\mu^2v^2\left(-\vartheta^2\rho+\alpha F\right)b_1}{p\beta\left(e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^3}-\frac{6w\mu^2v^2\left(\vartheta^2\rho-\alpha F\right)(wb_0+\sigma b_1)}{p\beta\left(e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^2}}{b_0+\frac{b_1}{e^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}}} \tag{28}$$

Substituting Eq. (21) into Eq. (13) gives

$$u_8(x,t)\frac{\frac{\mu^2v^2\sigma^2\left(-\vartheta^2\rho+\alpha F\right)b_0}{p\beta}+\frac{6w^2\mu^2v^2\left(-\vartheta^2\rho+\alpha F\right)b_1}{p\beta\left(\mathrm{e}^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^3}-\frac{6w\mu^2v^2\left(\vartheta^2\rho-\alpha F\right)(wb_0+\sigma b_1)}{p\beta\left(\mathrm{e}^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)^2}-\frac{\mu^2v^2\sigma\left(\vartheta^2\rho-\alpha F\right)(6wb_0+\sigma b_1)}{p\beta\left(\mathrm{e}^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}\right)}}{b_0+\frac{b_1}{\mathrm{e}^{-(x-t\vartheta)\mu\sigma}k-\frac{w}{\sigma}}} \tag{29}$$

4. Results and discussion

In this study, the improved Bernoulli sub-equation function method is used in finding wave solutions to the doubly dispersive equation. Exponential function solutions are successfully constructed. Cattani et al. [17] reported some solutions to the doubly dispersive equation. When the reported results in this paper and the results in [17] are compared, we observed that the reported results in this study are new and presented for the first time. The reported results in this study may be useful in explaining the physical meaning of the studied nonlinear model.

Remark 1 Solution (26) is travelling wave solution and the other solutions are singular soliton solutions.

Choosing the suitable values of parameters, we performed the numerical simulations of the obtained solutions for (24–26–28) equation by plotting their 2D and 3D (Figs. 1, 2, and 3).

5. Conclusions

In this article, new solutions are obtained for the doubly dispersive equation using the IBSEFM method. We have seen that the results we obtained are new solutions when we compare them with previous ones. Our results might be useful in explaining the physical meaning of various nonlinear models arising in the field of nonlinear sciences. IBSEFM is a powerful and efficient mathematical tool that can be used to handle various nonlinear mathematical models.

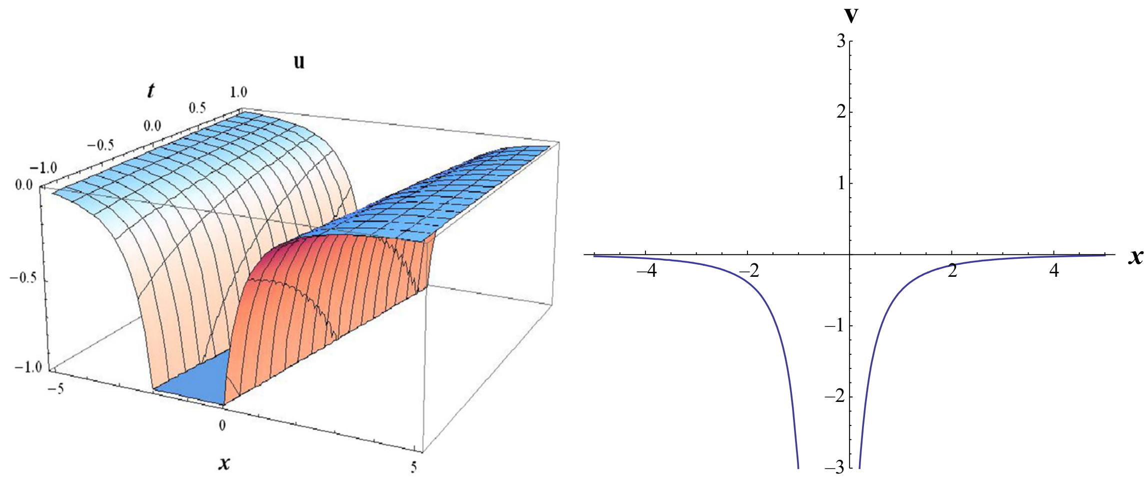

Fig. 1 The 3D and 2D surfaces of the solution Eq. (24) for suitable values

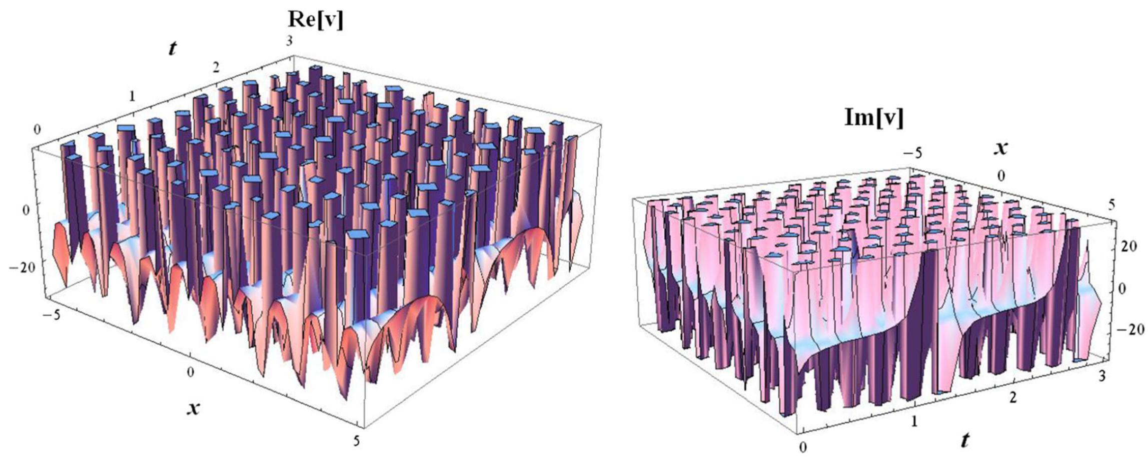

Fig. 2 The 3D surface of the solution Eq. (26) for suitable values

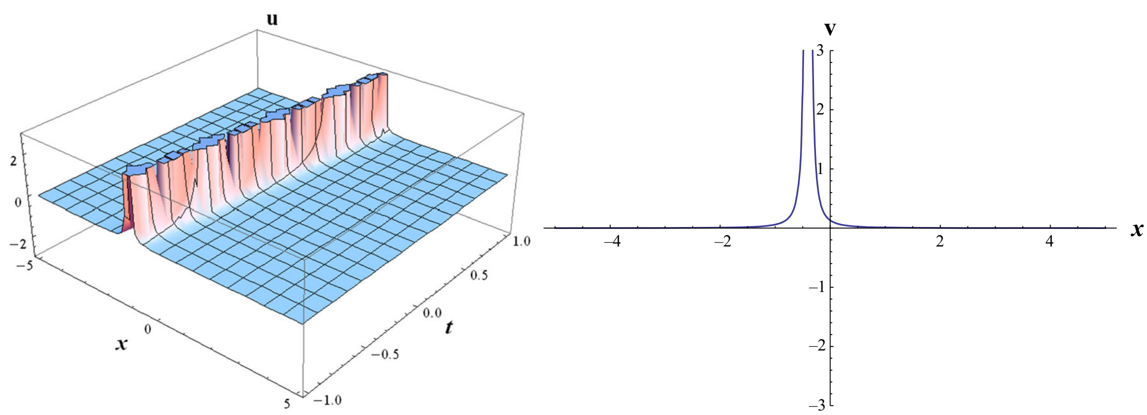

Fig. 3 The 3D and 2D surfaces of the solution Eq. (28) for suitable values

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