Indian J Phys (May 2020) 94(5):725–736
https://doi.org/10.1007/s12648-019-01505-8

ORIGINAL PAPER

# Exact solutions of generalized thermoelastic medium with double porosity under L–S theory

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Received: 18 January 2019 / Accepted: 22 March 2019 / Published online: 22 May 2019

**Abstract:** In this paper, a general solution to the field equations of a generalized thermoelastic medium with double porosity has been obtained. To investigate the problem, we use the Lord–Shulman theory in the thermoelasticity. The half-space of an isotropic homogeneous thermoelastic material is considered. Using the normal mode analysis and the numerical inversion technique, the analytic expressions of the physical quantities are obtained. Numerically, computed results for these quantities and its depicted graphically lead to study the effect of porosity. Comparisons in the presence and absence of double porosity, in two different times, are obtained. Although the problem has been solved theoretically, it is possible for researchers to benefit from their results in many different sciences, for example, in the field of geophysics, earthquake engineering, along with seismologist working in the field of mining tremors and drilling into the crust of the earth.

**Keywords:** L–S theory; Thermoelastic medium; Normal mode; Double porosity

**PACS Nos.:** 44.05. +e; 81.40.Jj; 62.20. fq; 62.20.Dc; 62.40.+i

**List of symbols**

| | |
|---|---|
| $\lambda$, $\mu$ | Lame' parameters |
| $\delta_{ij}$ | Kronecker delta |
| $c_e$ | The specific heat at constant strain |
| $T_0$ | The reference temperature |
| $\sigma_i$ | The equilibrated stress corresponding to $v_1$ |
| $\tau_i$ | The equilibrated stress corresponding to $v_2$ |
| $b$, $d$, $b_1$, $\gamma$, $\gamma_1$, $\gamma_2$ | The constitutive coefficients |
| $v_1$ | The volume fraction field corresponding to pores and $v_2$ is the volume fraction field corresponding to fissures |
| $\Psi$, $\Phi$ | The volume fraction fields corresponding to $v_1$ and $v_2$, respectively |
| $K_1$ and $K_2$ | The coefficients of equilibrated inertia |
| $T$ | The temperature change measured form the absolute temperature $T_0$ |
| $u_i$ | The displacement vector |
| $\rho$ | The mass density |
| $\tau_{ij}$ | The stress tensor |
| $\tau_0$ | The relaxation time |
| $K \geq 0$ | The thermal conductivity |

## 1. Introduction

The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity. In order to remove the paradox of physical impossible phenomena of an infinite velocity of thermal signals in the classical coupled thermoelasticity, the generalized theory of thermoelasticity has been developed. The coupled theory of thermoelasticity is explained in [1]. In [2] generalized thermoelasticity theory involving one thermal relaxation time is studied. Lord–Shulman has been applied, in [3], to study the effect of dependence of the modulus of elasticity on the reference temperature in

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two-dimensional generalized thermoelasticity. Generalized electro-magneto-thermo-viscoelastic in the case of 2-D thermal shock problem in a finite conducting medium with one relaxation time was studied in [4]. Marin [5] explained an approach of a heat-flux-dependent theory for micropolar porous media. The deformation of a rotating two-temperature generalized magneto-thermoelastic medium with internal heat source due to hydrostatic initial stress was studied by Said [6]. The weak solutions in elasticity of dipolar bodies with stretch were studied by Marin and Stan [7]. In [8], two-temperature generalized magneto-thermoelastic medium, for dual-phase-lag model, under the effect of the gravity field and hydrostatic initial stress, is studied. Wave propagation in a magneto-micropolar thermoelastic medium with two temperatures for three-phase-lag model was studied by Said [9]. The origin of the linear theory of elastic materials with double porosity goes back to papers of Barenblatt et al. [10]. Wilson and Aifantis, in [11], discussed the theory of consolidation with double porosity. Khalili and Valliappan, in [12], used the theory of flow and deformation in double porous media. In [13], Masters et al. studied coupling temperature to a double-porosity model of deformable porous media. Berryman and Wang [14] investigated the elastic wave propagation and attenuation in a double-porosity dual-permeability medium. In [15], Khalili and Selvadurai studied the fully coupled constitutive model for thermo-hydro-mechanical analysis in elastic media with double porosity. Linear dynamics of double-porosity dual-permeability materials-I was discussed by Pride and Berryman [16]. Zhao and Chen [17] introduced the fully coupled dual-porosity model for anisotropic formations. Svanadze [18] studied the dynamical problems of the theory of elasticity for solids with double porosity. In [19], Ainouz investigated the homogenized double-porosity models for poro-elastic media with interfacial flow Barrier. Plane waves and boundary value problems in the theory of elasticity for solids with double porosity were studied by Svanadze [20]. Straughan [21] studied the stability and uniqueness in double-porosity elasticity. The so-called double-porosity model allows the body to have a double porous structure: macroporosity connected to pores in the body and a microporosity connected to fissures in the skeleton. Moreover, the generalized theory, with the help of Darcy's law, is established to obtain the basic equations for elastic materials with double porosity involve the displacement vector field, a pressure associated with the pores, and a pressure associated with the fissures (see [17, 20, 21]). The materials with double porosity are of interest in geophysics [22, 23] and mechanics of bone [21]. The theory is established with the help of Darcy's law. The basic equations for elastic materials with double porosity involve the displacement vector field, a pressure associated with the pores, and pressure associated with the issues [20–26]. Othman and Marin [27] studied the effect of thermal loading due to laser pulse on thermoelastic porous media under G-N theory. The plane waves in magneto-thermoelastic solids with voids and microtemperatures due to hall current and rotation were investigated by Othman et al. [28].

In the present paper, we have studied the equations of generalized thermoelastic material with double-porosity structure with one relaxation time. Effect of porosity and different times is shown graphically.

## 2. Formulation of the problem and basic equations

Consider a homogeneous thermoelastic half-space with double-porosity structure in the undeformed state at uniform temperature $T_0$. It follows from the description of the problem that all the considered functions will depend upon $(x, z, t)$. We thus obtain the displacement vector $u$ of the form $\boldsymbol{u} = (u_1, 0, u_3)$. The field equations and constitutive relations for a homogeneous isotropic thermo-elastic solid with double-porosity structure in the absence of incremental body forces and heat source by L–S model are:

Stress–strain equation (see [29])

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\Phi + d\delta_{ij}\Psi - \beta\delta_{ij}(T - T_0). \quad (1)$$

In case of isotropic solids, the constitutive equations for double porosity [29]

$$\sigma_i = \alpha\ \Phi_{,i} + b_1\ \Psi_{,i}. \quad (2)$$

$$\tau_i = b_1\ \Phi_{,i} + \gamma\ \Psi_{,i}. \quad (3)$$

The equation of motion in the absence of body force

$$t_{ij} = \rho\ddot{u}_{,i}. \quad (4)$$

Using (1) in (4), we get the equation of motion in the two dimensions

$$\mu\nabla^2 u_1 + (\lambda + \mu)\frac{\partial e}{\partial x} + b\frac{\partial \Phi}{\partial x} + d\frac{\partial \Psi}{\partial x} - \beta\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u_1}{\partial t^2}, \quad (5)$$

$$\mu\nabla^2 u_3 + (\lambda + \mu)\frac{\partial e}{\partial z} + b\frac{\partial \Phi}{\partial z} + d\frac{\partial \Psi}{\partial z} - \beta\frac{\partial T}{\partial z} = \rho\frac{\partial^2 u_3}{\partial t^2}. \quad (6)$$

Equilibrated stress equations of motion

$$\alpha\nabla^2\Phi + b_1\nabla^2\Psi - be - \alpha_1\Phi - \alpha_3\Psi + \gamma_1 T = K_1\frac{\partial^2\Phi}{\partial t^2}, \tag{7}$$

$$b_1\nabla^2\Phi + \gamma\nabla^2\Psi - de - \alpha_3\Phi - \alpha_2\Psi + \gamma_2 T = K_2\frac{\partial^2\Psi}{\partial t^2}. \tag{8}$$

Equation of heat

$$K\nabla^2 T = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\rho C^*\frac{\partial T}{\partial t} + \beta T_0\frac{\partial e}{\partial t} + \gamma_1 T_0\frac{\partial\Phi}{\partial t} + \gamma_2 T_0\frac{\partial\Psi}{\partial t}\right). \tag{9}$$

For the purpose of numerical evaluation, we introduce dimensionless variables

$$(x',z') = \frac{\omega_1}{c_1}(x,z), \quad (u_1',u_3') = \frac{\omega_1}{c_1}(u_1,u_3),$$
$$\{\sigma_1',\tau_1'\} = \frac{c_1}{\alpha\omega_1}\{\sigma_1,\tau_1\}, \quad (t',\tau_o') = \omega_1(t,\tau_0),$$
$$[\Phi',\Psi'] = \frac{K_1\omega_1^2}{\alpha_1}[\Phi,\Psi], \quad c_1^2 = \frac{\lambda+2\mu}{\rho},$$
$$\omega_1 = \frac{\rho c^* c_1^2}{K}, \quad \nabla^2 = \frac{\omega_1^2}{c_1^2}\nabla'^2, \quad \gamma = (3\lambda+2\mu)\alpha_t,$$
$$t_{ij}' = \left(\frac{1}{\beta T_0}\right)t_{ij}, \quad T' = \frac{T}{T_0}.$$

Using the above dimensionless quantities, Eqs. (5)–(9) become:

$$\left(\frac{\lambda+\mu}{\rho c_1^2}\right)\frac{\partial e}{\partial x} + \left(\frac{\mu}{\rho c_1^2}\right)\nabla^2 u_1 + a_1\frac{\partial\Phi}{\partial x} + a_2\frac{\partial\Psi}{\partial x} - a_3\frac{\partial T}{\partial x} = \frac{\partial^2 u_1}{\partial t^2}, \tag{10}$$

$$\left(\frac{\lambda+\mu}{\rho c_1^2}\right)\frac{\partial e}{\partial z} + \left(\frac{\mu}{\rho c_1^2}\right)\nabla^2 u_3 + a_1\frac{\partial\Phi}{\partial z} + a_2\frac{\partial\Psi}{\partial z} - a_3\frac{\partial T}{\partial z} = \frac{\partial^2 u_3}{\partial t^2}, \tag{11}$$

$$a_4\nabla^2\Phi + a_5\nabla^2\Psi - a_6 e - a_7\Phi - a_8\Psi + a_9 T = \frac{\partial^2\Phi}{\partial t^2}, \tag{12}$$

$$a_{10}\nabla^2\Phi + a_{11}\nabla^2\Psi - a_{12}e - a_{13}\Phi - a_{14}\Psi + a_{15}T = \frac{\partial^2\Psi}{\partial t^2}, \tag{13}$$

$$a_{16}\nabla^2 T = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial T}{\partial t} + a_{17}\frac{\partial e}{\partial t} + a_{18}\frac{\partial\Phi}{\partial t} + a_{19}\frac{\partial\Psi}{\partial t}\right). \tag{14}$$

where

$$a_1 = \frac{b\alpha_1}{\rho c_1^2 K_1\omega_1^2}, \quad a_2 = \frac{d\alpha_1}{\rho c_1^2 K_1\omega_1^2}, \quad a_3 = \frac{\beta T_0}{\rho c_1^2}, \quad a_4 = \frac{\alpha}{K_1 c_1^2},$$
$$a_5 = \frac{b_1}{K_1 c_1^2}, \quad a_6 = \frac{b}{\alpha_1},$$
$$a_7 = \frac{\alpha_1}{K_1\omega_1^2}, \quad a_8 = \frac{\alpha_3}{K_1\omega_1^2}, \quad a_9 = \frac{\gamma_1 T_0}{\alpha_1}, \quad a_{10} = \frac{b_1}{c_1^2 K_2},$$
$$a_{11} = \frac{\gamma}{c_1^2 K_2}, \quad a_{12} = \frac{dK_1}{\alpha_1 K_2},$$
$$a_{13} = \frac{\alpha_3}{\omega_1^2 K_2}, \quad a_{14} = \frac{\alpha_2}{\omega_1^2 K_2}, \quad a_{15} = \frac{\gamma_2 T_0 K_1}{\alpha_1 K_2},$$
$$a_{16} = \frac{K\omega_1}{\rho c^* c_1^2}, \quad a_{17} = \frac{\beta}{\rho c^*}, \quad a_{18} = \frac{\gamma_1\alpha_1}{\rho c^* K_1\omega_1^2},$$
$$a_{19} = \frac{\gamma_2\alpha_1}{\rho c^* K_1\omega_1^2}.$$

Define displacement potentials $\phi_1$ and $\psi_1$ that relate to displacement components $u_1$ and $u_3$ as,

$$u_1 = \frac{\partial\phi_1}{\partial x} - \frac{\partial\psi_1}{\partial z}, \quad u_3 = \frac{\partial\phi_1}{\partial z} + \frac{\partial\psi_1}{\partial x}. \tag{15}$$

Using Eq. (15) in Eqs. (10)–(14), to obtain:

$$\nabla^2\phi_1 + a_1\Phi + a_2\Psi - a_3 T = \frac{\partial^2\phi_1}{\partial t^2}, \tag{16}$$

$$\left(\frac{\mu}{\rho c_1^2}\right)\nabla^2\psi_1 = \frac{\partial^2\psi_1}{\partial t^2}, \tag{17}$$

$$a_4\nabla^2\Phi + a_5\nabla^2\Psi - a_6\nabla^2\phi_1 - a_7\Phi - a_8\Psi + a_9 T = \frac{\partial^2\Phi}{\partial t^2}, \tag{18}$$

$$a_{10}\nabla^2\Phi + a_{11}\nabla^2\Psi - a_{12}\nabla^2\phi_1 - a_{13}\Phi - a_{14}\Psi + a_{15}T = \frac{\partial^2\Psi}{\partial t^2}, \tag{19}$$

$$a_{16}\nabla^2 T = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial T}{\partial t} + a_{17}\nabla^2\frac{\partial\phi_1}{\partial t} + a_{18}\frac{\partial\Phi}{\partial t} + a_{19}\frac{\partial\Psi}{\partial t}\right). \tag{20}$$

Dimensionless variables of the stress components take the form,

$$\tau_{xx} = \left(\frac{\lambda}{\beta T_0}\right)e + \left(\frac{2\mu}{\beta T_0}\right)\frac{\partial u_1}{\partial x} - T + \left(\frac{b\alpha_1}{K_1\omega_1^2\beta T_0}\right)\Phi + \left(\frac{d\alpha_1}{K_1\omega_1^2\beta T_0}\right)\Psi, \tag{21}$$

$$\tau_{zz} = \left(\frac{\lambda}{\beta T_0}\right)e + \left(\frac{2\mu}{\beta T_0}\right)\frac{\partial u_3}{\partial z} - T + \left(\frac{b\alpha_1}{K_1\omega_1^2\beta T_0}\right)\Phi + \left(\frac{d\alpha_1}{K_1\omega_1^2\beta T_0}\right)\Psi, \tag{22}$$

$$\tau_{xz} = \left(\frac{2\mu}{\beta T_0}\right) e_{xz}. \tag{23}$$

## 3. Normal mode analysis

The solution of the considered physical variables can be decomposed in terms of normal mode as the following form

$$\begin{aligned}&[u_1, u_3, e, T, \phi_1, \psi_1, \Phi, \psi, t_{ij}](x, z, t)\\ &\quad = [u_1^*, u_3^*, e^*, T^*, \phi_1^*, \psi_1^*, \Phi^*, \psi^*, t_{ij}^*](z) \exp\,[i(\omega t + ax)].\end{aligned} \tag{24}$$

where $\omega$ is the complex time constant (frequency), $i$ is the imaginary unit, and $a$ is the wave number in the $x$-direction.

Using (24) in Eqs. (16)–(20), we obtain

$$(D^2 - n_1)\phi_1^* + a_1 \Phi^* + a_2 \Psi^* - a_3 T^* = 0, \tag{25}$$

$$(D^2 - m^2)\psi_1^* = 0, \tag{26}$$

$$\begin{aligned}&(a_4 D^2 - n_2)\Phi^* + (a_5 D^2 - n_3)\Psi^* - (a_6 D^2 - n_4)\phi_1^* + a_9 T^*\\ &\quad = 0,\end{aligned} \tag{27}$$

$$(a_{16} D^2 - n_8)T^* + n_9(D^2 - a^2)\phi_1^* + n_{10}\Phi^* + n_{11}\Psi^* = 0. \tag{29}$$

where

$$D = \frac{\partial}{\partial z}, \quad m^2 = a^2 + \frac{\omega^2 \rho c_1^2}{\mu}, \quad n_1 = a^2 - \omega^2,$$

$$n_2 = a_4 a^2 + a_7 - \omega^2, \quad n_3 = a_5 a^2 + a_8,$$

$$n_4 = a_6 a^2, \quad n_5 = a_{10} a^2 + a_{13}, \quad n_6 = a_{11} a^2 + a_{14} - \omega^2,$$

$$n_7 = a_{12} a^2, \quad n_8 = a_{16} a^2 + i\omega(1 + i\tau_0 \omega),$$

$$n_9 = -i\omega a_{17}(1 + i\tau_0\omega), \quad n_{10} = -i\omega a_{18}(1 + i\tau_0\omega),$$

$$n_{11} = -i\omega a_{19}(1 + i\tau_0\omega).$$

Put the above Eqs. (25), (27), (28), (29) in the matrix, we find that the differential equation takes the form:

$$\begin{aligned}&[\mathrm{D}^8 - \mathrm{A}\mathrm{D}^6 + \mathrm{B}\,\mathrm{D}^4\\ &\quad - \mathrm{C}\,\mathrm{D}^2 + E]\{\phi_1^*(z), \Phi^*(z), \psi^*(z), \psi_1^*(z), \mathrm{T}^*\} = 0.\end{aligned} \tag{30}$$

where

$$A = \frac{a_{11}(a_4 n_8 + a_4 a_{16} n_6 + a_{16} n_2 - a_1 a_6 a_{16} + a_4 a_{16} n_1 - a_4 a_3 n_9) + a_{16}(a_{10} a_2 a_6 - a_{10} a_5 n_1 - a_{10} n_3 - a_5 n_5 - a_4 a_2 a_{12}) + a_5(a_1 a_{12} n_{16} + a_{10} a_3 n_9)}{(a_4 a_{11} a_{16} - a_{10} a_5 a_{16})},$$

$$B = \frac{\begin{array}{c} n_8(a_4 n_6 + a_{11} n_2 - a_{10} n_3 + n_5 a_5 + a_1 a_5 a_{12} - a_1 a_{11} a_6) + a_{16}(n_5 n_3 + n_2 n_6 + a_1 a_{12} n_3 + a_1 a_5 n_7 - a_1 a_6 n_6 - a_1 a_{11} n_4) \\ + n_{11}(-a_4 a_{15} + a_{10} a_9) + n_{10}(-a_{15} a_5 + a_{11} a_9) - a_1(a_5 a_{15} n_9 + a_{11} a_9 n_9) - a_4(a_2 a_{12} n_8 + a_2 a_{16} n_7 - a_2 a_{15} n_9 - a_{11} n_1 n_8 \\ - a_3 a_{11} n_9 n_8 - a_{16} n_1 n_6 + a_3 n_6 n_9 - a_3 n_{11} a_{12}) - n_2(a_2 a_{16} a_{12} - a_{16} a_{11} n_1 + a_{11} a_3 n_9) + n_{10}(a_5 a_3 a_{12} - a_{11} a_3 a_6) \\ + a_{10}(a_2 a_6 n_8 + a_2 a_{16} n_4 - a_2 a_9 n_9 - a_5 n_1 n_8 + a_5 a_3 n_9 n_8 - a_{16} n_1 n_3 + a_3 n_9 n_3 - a_6 a_3 n_{11}) + n_5(a_2 a_6 a_{16} - a_5 a_{16} n_1 - a_5 a_3 n_9) \end{array}}{a_{16}(a_4 a_{11} - a_{10} a_5)},$$

$$C = \frac{\begin{array}{c} n_2(n_6 n_8 - n_{11} a_{15}) + n_{10}(n_3 n_{15} - a_9(-a_{11} n_1 + a_9 + a_2 a_{12}) - a_2 n_2(a_{12} n_8 - a_{16} n_7 - a_{15} n_9) + a_{15}(a_6 a_2 + a_5 n_1) \\ - a_3(a_5 n_7 + a_{12} n_3 + a_{11} n_4 + a_6 n_6)) + a_1 n_8(a_5 n_7 + a_{12} n_3 - a_{11} n_4 - a_6 n_6 - a_5 a_{15} n_9 + a_{11} a_9 n_9) \\ + a_1(a_{16} n_7 n_3 - a_{15} n_9 n_3 - a_{16} n_4 n_6 + a_9 n_6 n_9 + a_6 n_{11} a_6 - a_9 n_{11} a_{12}) - a_4 n_8(a_2 n_7 + a_2 a_{15} n_9 + n_6\, n_1 - a_3 n_6 n_9 - a_{15} n_{11} n_1) \\ + n_8(+a_3 n_7 n_{11} + a_{11} n_2 n_1 - a_3 a_{11} n_2 n_9 + a_{10} a_2 n_4 - a_{10} a_2 a_9 n_9 - a_5 n_1 n_5 - a_3 a_5 n_5 n_9 - a_{10} n_3 n_1 + a_{10} a_3 n_3 n_9 + a_2 a_6 n_5) \\ + n_2 n_6(a_{16} n_1 - 2a_3 n_9) + n_{11}(a_3 a_{12} n_2 + a_{10}(a_9 n_1 - a_3 n_4)) + n_5(a_{16} a_2 n_4 - a_2 a_9 n_9 - a_{16} n_3 n_1 - a_3 n_3 n_9 + a_3 a_6 n_{11}\ + n_3 n_8 - n_{11} n_9) \end{array}}{a_{16}(a_4 a_{11} - a_{10} a_5)},$$

$$E = \frac{\begin{array}{c} - a_1(-a_9 n_{11} n_7 - a_{15} n_{11} n_4 - n_8(n_3 n_7 + a_{15} n_9 n_3 + n_6 n_4 \\ - a_9 n_9 n_6)) - n_2(n_{11}(a_{15} n_1 + a_3 n_7) + n_9 n_8(a_2 a_{15} - a_3 n_6)) \\ + n_8 n_5(a_2 n_4 - n_3 n_1 - a_2 a_9 n_9 + a_3 n_3 n_9) + n_{11} n_5(a_9 n_1 -_3\, n_4) \\ - n_{10}(a_2(a_{15} n_4 + a_9 n_7) - n_3(a_{15} n_1 + a_3 n_7) + n_6(a_9 n_1 - a_3 n_4)) \end{array}}{a_{16}(a_4 a_{11} - a_{10} a_5)}.$$

$$\begin{aligned}&(a_{10} D^2 - n_5)\Phi^* + (a_{11} D^2 - n_6)\Psi^* - (a_{12} D^2 - n_7)\phi_1^*\\ &\quad + a_{15} T^*\\ &\quad = 0,\end{aligned} \tag{28}$$

The solution of Eq. (30) has the form

$$\Phi^* = \sum_{n=1}^{4} M_n e^{-k_n z}, \tag{31}$$

$$\Psi^* = \sum_{n=1}^{4} H_{1n} M_n e^{-k_n z}, \tag{32}$$

$$\phi_1^* = \sum_{n=1}^{4} H_{2n} M_n e^{-k_n z}, \tag{33}$$

$$T^* = \sum_{n=1}^{4} H_{3n} M_n e^{-k_n z}, \tag{34}$$

$$\psi_1^* = M_5 e^{-mz}. \tag{35}$$

After substituting Eqs. (33)–(35) into (15), the displacements take the form:

$$\sigma_3 = \eta_1 \Phi_{,z} + \eta_2 \Psi_{,z}, \tag{41}$$

$$\tau_3 = \eta_3 \Phi_{,z} + \eta_4 \Psi_{,z}. \tag{42}$$

where $\eta_1 = \frac{\alpha_1}{k_1 \omega_1^2}, \eta_2 = \eta_3 = \frac{b_1 \alpha_1}{\alpha k_1 \omega_1^2}, \eta_4 = \frac{\gamma \alpha_1}{\alpha k_1 \omega_1^2}$.

Moreover, substituting from Eqs. (31), (32) into (41) and (42), we get the solution of $\sigma_3$ and $\tau_3$, as:

$$\sigma_3 = \sum_{n=1}^{4} H_{9n} M_n e^{-k_n z} e^{i(\omega t + ax)}, \tag{43}$$

$$\tau_3 = \sum_{n=1}^{4} H_{8n} M_n e^{-k_n z} e^{i(\omega t + ax)}. \tag{44}$$

where

$$H_{1n} = \frac{[a_1 a_9 + a_3(a_4 k_n^2 - n_2)][a_{15} n_9 (k_n^2 - a^2) - (a_{12} k_n^2 - n_7)(a_{16} k_n^2 - n_8)] - [a_9(k_n^2 - n_1) - a_3(a_6 k_n^2 - n_4)][n_{10} a_{15} + (a_{10} k_n^2 - n_5)(a_{16} k_n^2 - n_8)]}{[a_9(k_n^2 - n_1) - a_3(a_6 k_n^2 - n_4)][(a_{11} k_n^2 - n_6)(a_{16} k_n^2 - n_8) + n_{11} a_{15}]},$$

$$H_{2n} = \frac{-[a_2 a_9 + a_3(a_5 k_n^2 - n_3)][a_{15} n_9 (k_n^2 - a^2) - (a_{12} k_n^2 - n_7)(a_{16} k_n^2 - n_8)] - [n_{10} a_{15} + (a_{10} k_n^2 - n_5)(a_{16} k_n^2 - n_8)] + H_{1n}[(a_{11} k_n^2 - n_6)(a_{16} k_n^2 - n_8) + n_{11} a_{15}]}{[-a_{15} n_9 (k_n^2 - a^2) + (a_{12} k_n^2 - n_7)(a_{16} k_n^2 - n_8)]},$$

$$H_{3n} = \frac{[H_{2n} a_9 (k_n^2 - a^2) + n_{10} + H_{1n} n_{11}]}{(a_{16} k_n^2 - n_8)}.$$

$$H_{4n} = \left(\frac{\lambda}{\beta T_0}(k_n^2 - a^2) - \frac{2\mu a^2}{\beta T_0}\right) H_{2n} - H_{3n} + \left(\frac{b\alpha_1}{k_1 \omega_1^2 \beta T_0}\right) + \left(\frac{d\alpha_1}{k_1 \omega_1^2 \beta T_0}\right) H_{1n}, \quad H_5 = \left(\frac{2iam\mu}{\beta T_0}\right)$$

$$H_{6n} = \frac{-2iak_n \mu}{\beta T_0} H_{2n}, \quad H_7 = m^2 + a^2, \quad H_{8n} = -\eta_3 k_n - \eta_4 k_n H_{1n}, \quad H_{9n} = -\eta_1 k_n - \eta_2 k_n H_{1n}.$$

$$u_1 = \sum_{i=1}^{4} iaH_{2n} M_n e^{-k_n z} e^{i(\omega t + ax)} + mM_5 e^{-mz} e^{i(\omega t + ax)}, \tag{36}$$

$$u_3 = \sum_{i=1}^{4} -k_n H_{2n} M_n e^{-k_n z} e^{i(\omega t + ax)} + iaM_5 e^{-mz} e^{i(\omega t + ax)}. \tag{37}$$

In addition, substituting from Eqs. (34) and (36–37) into (21)–(23), the stress displacements become

$$\tau_{xx} = \sum_{i=1}^{4} H_{4n} M_n e^{-k_n z} e^{i(\omega t + ax)} + H_5 M_5 e^{-mz} e^{i(\omega t + ax)}, \tag{38}$$

$$\tau_{zz} = \sum_{i=1}^{4} H_{5n} M_n e^{-k_n z} e^{i(\omega t + ax)} - H_5 M_5 e^{-mz} e^{i(\omega t + ax)}, \tag{39}$$

$$\tau_{xz} = \sum_{i=1}^{4} H_{6n} M_n e^{-k_n z} e^{i(\omega t + ax)} - H_7 M_5 e^{-mz} e^{i(\omega t + ax)}. \tag{40}$$

Dimensionless variables for the components of $\sigma_i$, $\tau_i$

## 4. Boundary conditions

We apply five boundary conditions for present problem at the plane surface $z = 0$.

$$\tau_{zz} = P_1 e^{i(\omega t + ax)}, \tag{45}$$

$$\tau_{xz} = 0, \tag{46}$$

$$\tau_3 = 0, \tag{47}$$

$$\sigma_3 = 0, \tag{48}$$

$$T = P_2 e^{i(\omega t + ax)}. \tag{49}$$

Applying Eqs. (45–49) in (39), (40), (43), (44) and (34), we get

$$\sum_{n=1}^{4} H_{5n} M_n - H_5 M_5 = P_1, \tag{50}$$

$$\sum_{n=1}^{4} H_{6n} M_n - H_7 M_5 = 0, \tag{51}$$

$$\sum_{n=1}^{4} H_{8n} M_n = 0, \quad (52)$$

$$\sum_{n=1}^{4} H_{9n} M_n = 0, \quad (53)$$

$$\sum_{n=1}^{4} H_{3n} M_n = P_2. \quad (54)$$

We can put Eqs. (50)–(54) in matrix and using MATLAB program to get $M_1, M_2, \ldots..., M_5$,

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} H_{51} & H_{52} & H_{53} & H_{54} & -H_5 \\ H_{61} & H_{62} & H_{63} & H_{64} & -H_7 \\ H_{81} & H_{82} & H_{83} & H_{84} & 0 \\ H_{91} & H_{92} & H_{93} & H_{94} & 0 \\ H_{31} & H_{32} & H_{33} & H_{34} & 0 \end{pmatrix}^{-1} \begin{pmatrix} P_1 \\ 0 \\ 0 \\ 0 \\ P_2 \end{pmatrix}. \quad (55)$$

## 5. Special cases

Case (i) when we neglect the presence of double porosity, the problem turns into a generalized thermoelastic medium.

Case (ii) If $\tau_0 = 0$ in Eq. (9),the corresponding expressions for thermoelastic medium with double porosity in the context of the coupled theory of thermoelasticity were yielded.

## 6. Results and discussion

To discuss, numerically, the effect of double porosity, the copper is considered as the thermoelastic material for which we take the following values of the different physical constants as Othman et al. [30].

$$\lambda = 7.7 \times 10^{10}\,\mathrm{N\,M^{-2}}, \quad \mu = 3.86 \times 10^{10}\,\mathrm{N\,m^{-2}},$$
$$K = 3.86 \times 10^{3}\,\mathrm{N\,s^{-1}\,K^{-1}}, \quad \omega = 0.01,$$
$$\alpha_t = 1.78 \times 10^{-5}\,\mathrm{K^{-1}}, \quad \rho = 8954\,\mathrm{kg\,m^{-3}},$$
$$C^* = 383.1\,\mathrm{J\,kg^{-1}\,K^{-1}}, \quad T_0 = 293\,\mathrm{K}, \quad a = 1, \quad \tau_0 = 0.7,$$
$$x = 0.5, \quad \xi = -1, \quad p_1 = 1 \times 10^{-2}, \quad p_2 = 10 \times 10^{-2}.$$

Following Khalili [31], the double porous parameters are taken as

$$\alpha = 1.3 \times 10^{-5}\,\mathrm{N}, \quad b_1 = 0.12 \times 10^{-5}\,\mathrm{N},$$
$$\gamma = 1.1 \times 10^{-5}\,\mathrm{N\,m^{-2}}, \quad \gamma_1 = 0.16 \times 10^{5}\,\mathrm{N\,m^{-2}},$$
$$\gamma_2 = 0.219 \times 10^{5}\,\mathrm{N\,m^{-2}}, \quad d = 0.1 \times 10^{10}\,\mathrm{N\,m^{-2}},$$
$$b = 0.9 \times 10^{10}\,\mathrm{N\,m^{-2}}, \quad K_2 = 0.1546 \times 10^{-12}\,\mathrm{N\,m^{-2}},$$
$$K_1 = 0.1456 \times 10^{-12}\,\mathrm{N\,m^{-2}}.$$

The numerical technique, outlined above, was used for the distribution of the real part of the temperature $T$, the displacement components $u_1, u_3$ the stress components $\tau_{xx}$, $\tau_{xz}$, $\tau_{zz}$ the components of double porosity $\sigma$ and $\tau$ for the problem. All the variables are taken in non-dimensional form from the result.

Figures 1 and 2 explain the comparison of the stress component $\tau_{xx}$ in the presence and absence of double porosity at two different times. We find that in Figs. 1 and 2 the stress $\tau_{xx}$ increases a small shift in the presence of double porosity and then decreases at two different values of time in the presence and absence of double porosity and take the form of the wave and try to return to zero. Figures 3 and 4 show the comparison of the stress component $\tau_{zz}$ in the presence and absence of double porosity at two different times. We find that in Fig. 3 the stress $\tau_{zz}$ increases to a maximum value at $t$ = 1 and then decreases to a minimum value at $t$ = 2 in the presence of double porosity and take the form of a wave and try to return to

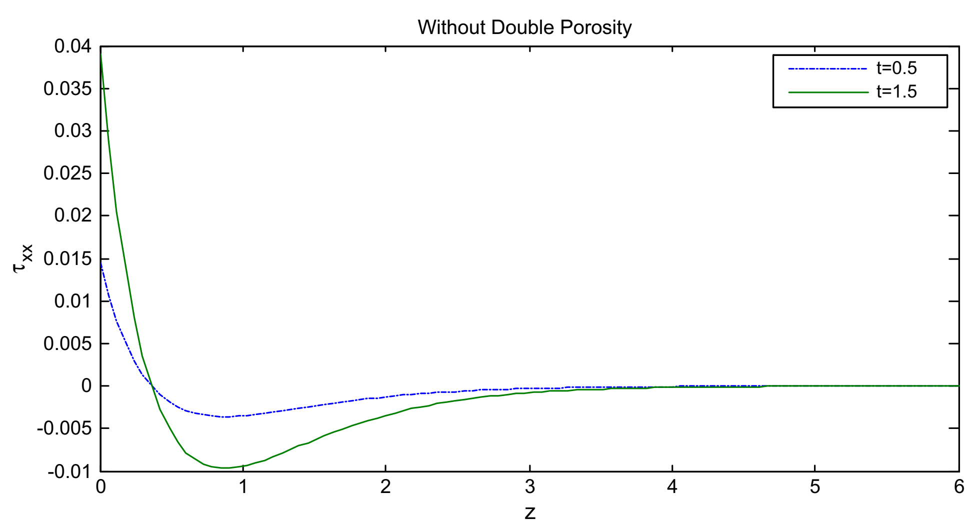



**Fig. 1** Distribution of the stress component $\tau_{xx}$

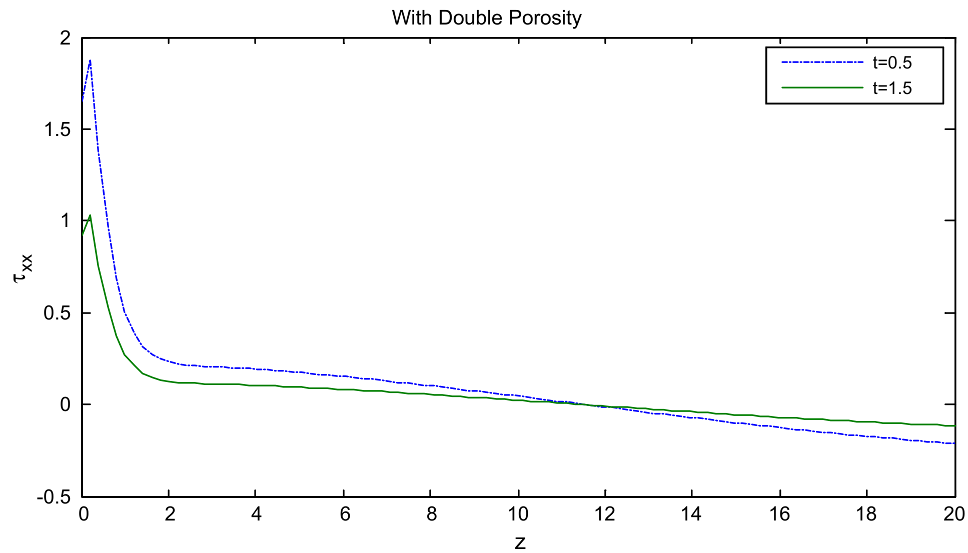



**Fig. 2** Distribution of the stress component $\tau_{xx}$

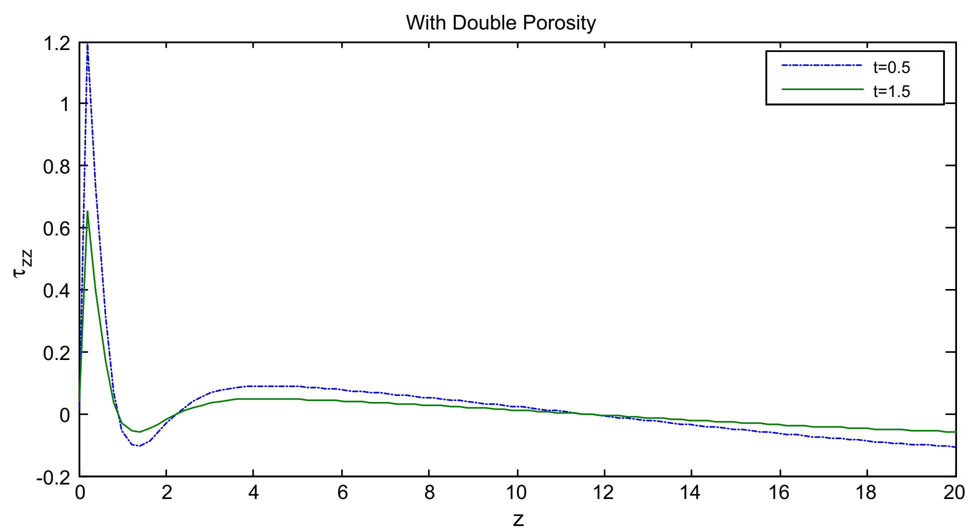



**Fig. 3** Distribution of the stress component $\tau_{zz}$

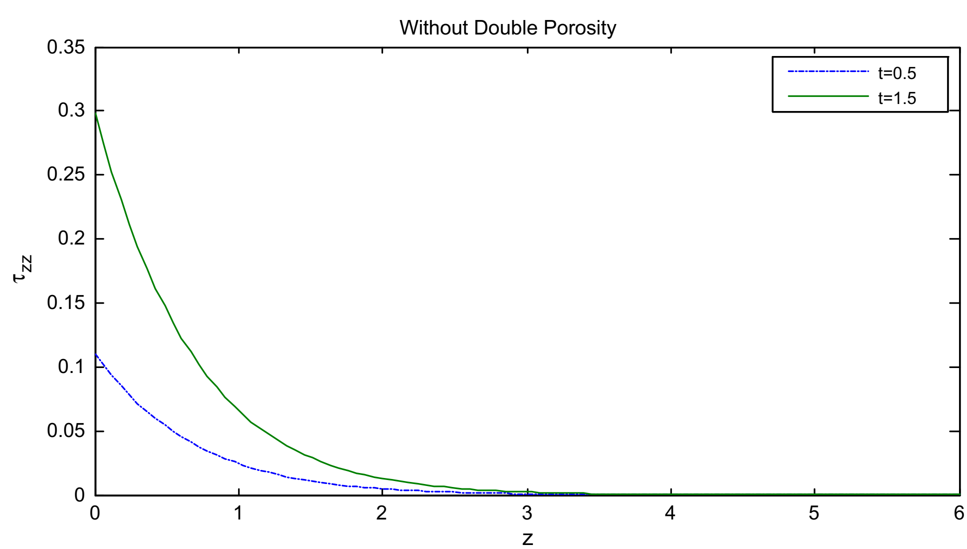



**Fig. 4** Distribution of the stress component $\tau_{zz}$

**Fig. 5** Distribution of the stress component $\tau_{xz}$

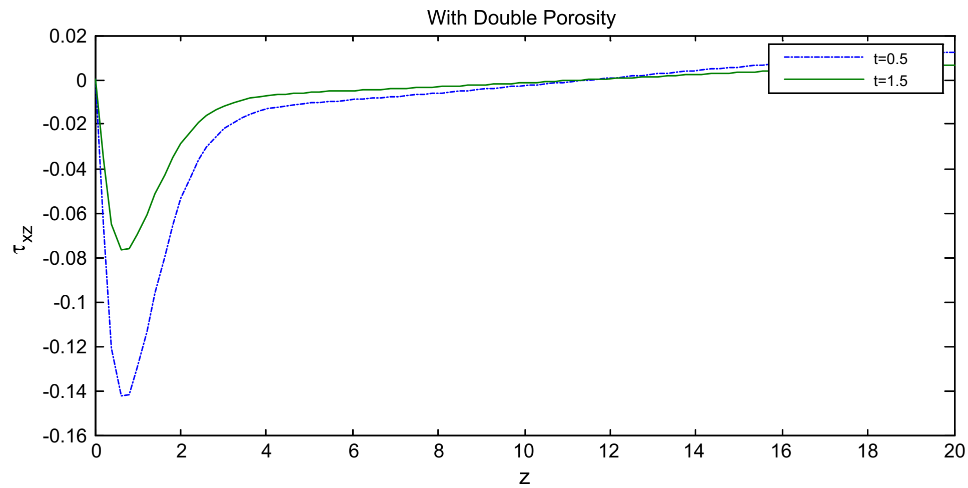



**Fig. 6** Distribution of the stress component $\tau_{xz}$

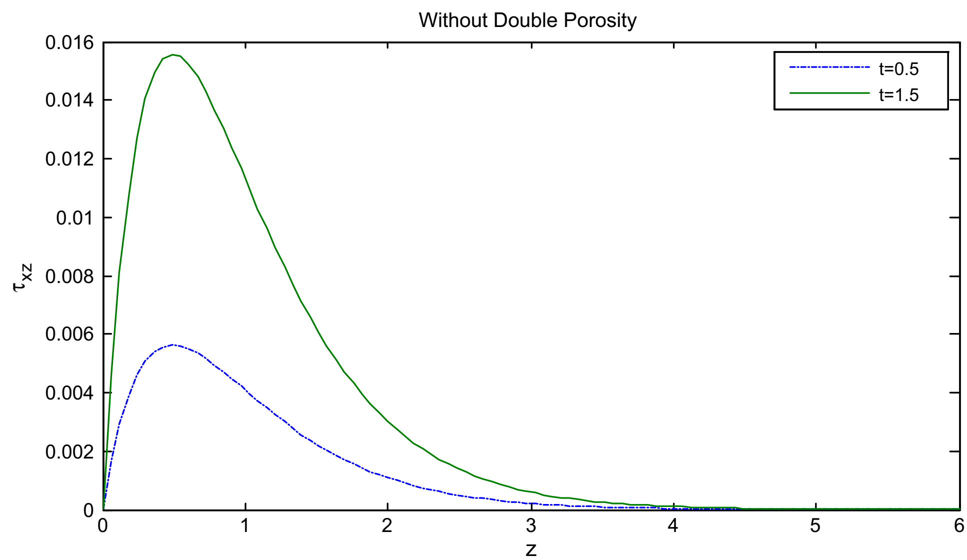
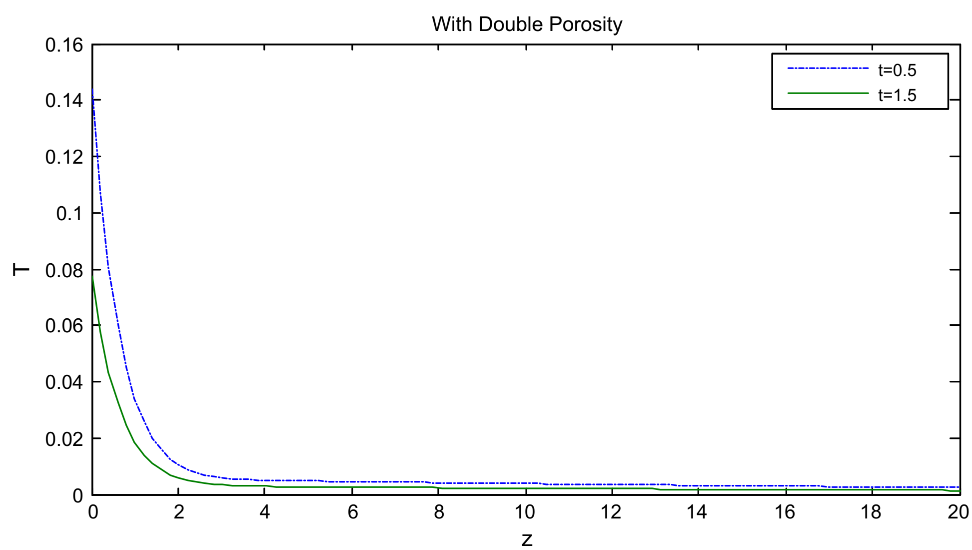



**Fig. 7** Distribution of the temperature $T$



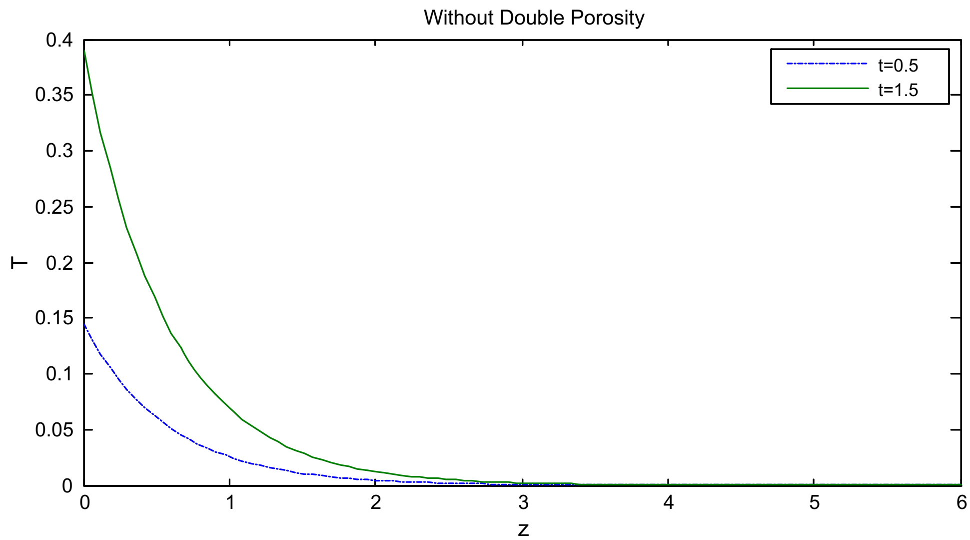



**Fig. 8** Distribution of the temperature $T$

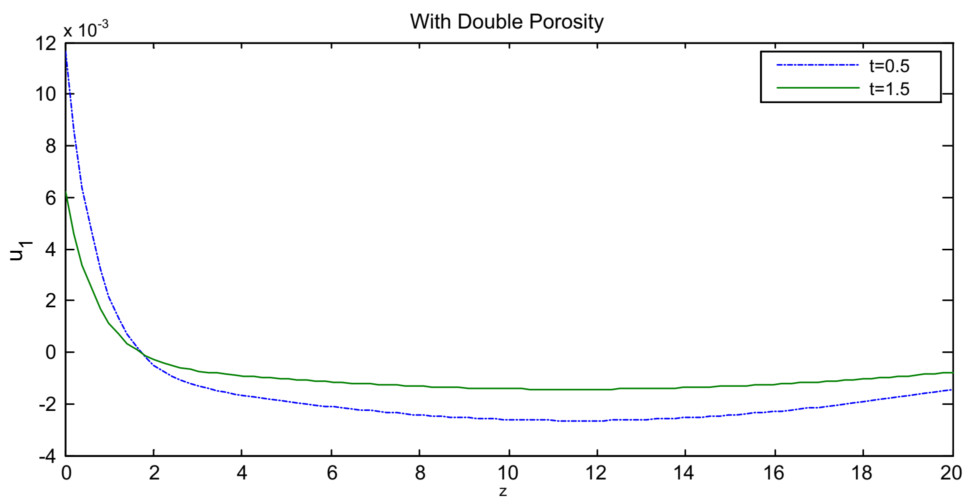



**Fig. 9** Distribution of the displacement $u_1$

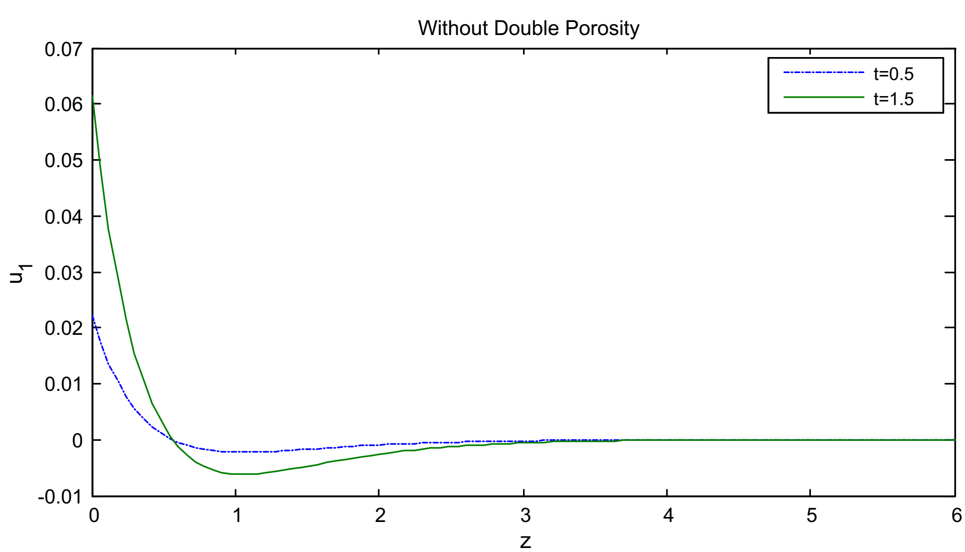



**Fig. 10** Distribution of the displacement $u_1$

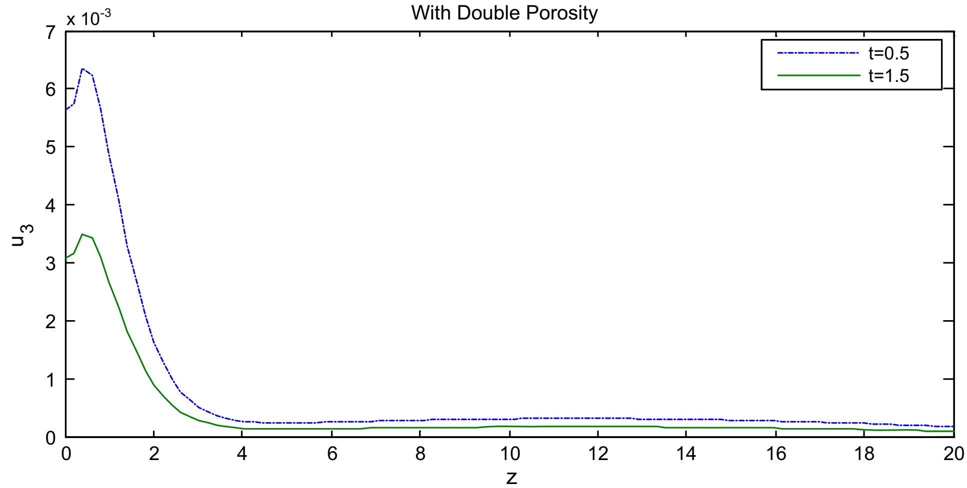



**Fig. 11** Distribution of the displacement $u_3$

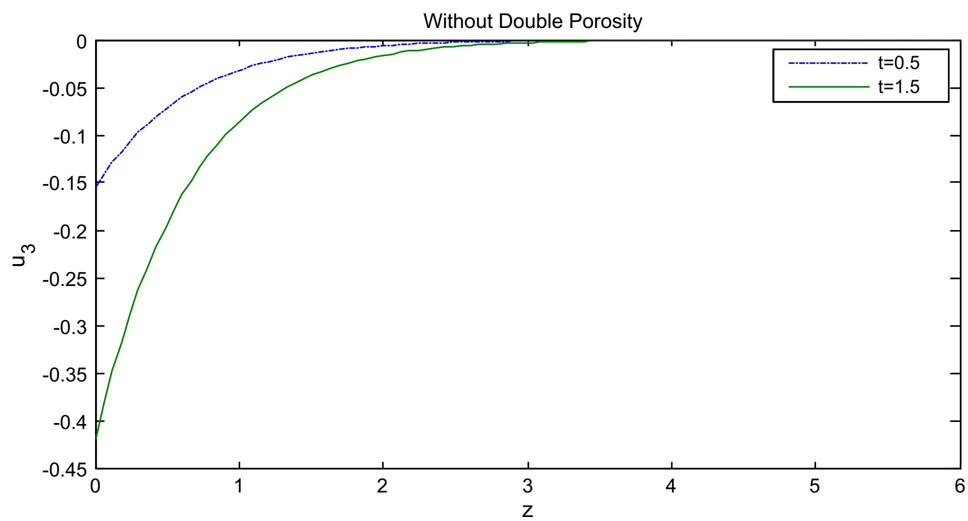



**Fig. 12** Distribution of the displacement $u_3$

zero. Figure 4 explains that the stress $\tau_{zz}$ decreases to a minimum value in the absence of double porosity at two different times and then decays until it returns to zero. Figures 5 and 6 demonstrate the comparison of the stress component $\tau_{xz}$ in the presence and absence of double porosity at two different times. We find that in Fig. 5 the stress $\tau_{xz}$ decreases and then increases to a maximum value at $t = 2$ in the presence of double porosity and take the form of the wave and try to return to zero. Figure 6 shows that the stress $\tau_{xz}$ increases to a maximum value at $z = 0.5$ in the absence of double porosity at two different times and then decreases until it returns to zero. Figures 7 and 8 explain the comparison of the temperature $T$ in the presence and absence of double porosity at two different times. We find that in Figs. 7 and 8 the temperature $T$ decreases in the two cases (with and without double porosity) at two different times and then decays to zero. Figures 9 and 10 show the comparison of the displacement $u_1$ in the presence and absence of double porosity at two different times. We find that in Fig. 9 the displacement $u_1$ decreases at $t = 0.5$ more than at $t = 1.5$ and then decreases until it decays to zero in the positive direction of z, but in Fig. 10 the displacement $u_1$ decreases to minimum value at $t = 1.5$ more than at $t = 0.5$ and takes the form of the wave until it decays to zero. Figures 11 and 12 illustrate the comparison of the displacement $u_3$ in the presence and absence of double porosity at two different times. We find that in Fig. 11 the displacement $u_3$ increases a small shift in the begging at $t = 0.5$ more than at $t = 1.5$ and then begins to decrease until it decays to zero, but in Fig. 12 the displacement $u_3$ increases at $t = 0.5$ more than at $t = 1.5$ and then begins to decrease until it decays to zero. Figures 13 and 14 demonstrate the comparison of the equilibrated stresses $\sigma$ and $\tau$ in the presence of double porosity at two different times. We find that in Figs. 13 and 14 the equilibrated stresses $\sigma$ and $\tau$ increase with the increase in time to a minimum value at 1 and then begin to decrease and take the form of the wave and try to return to zero.

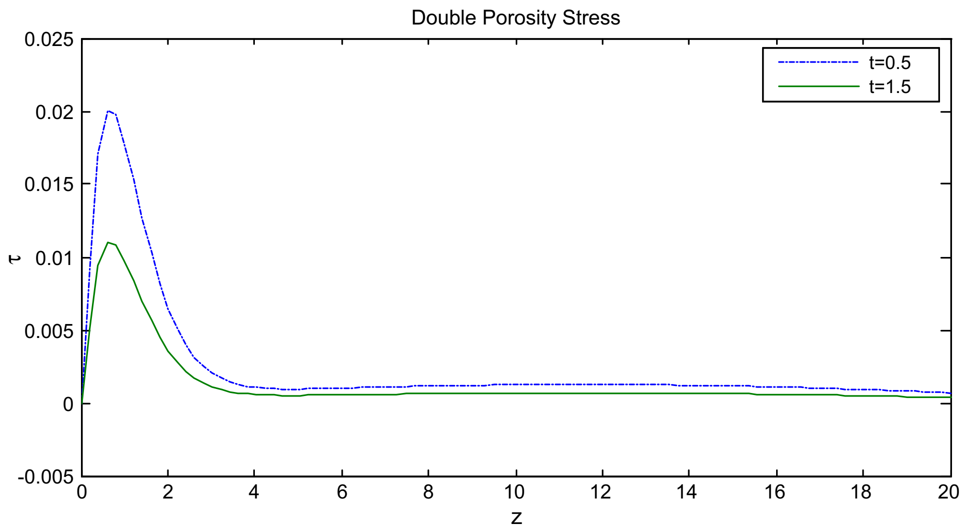



**Fig. 13** Distribution of the equilibrated stress $\tau$

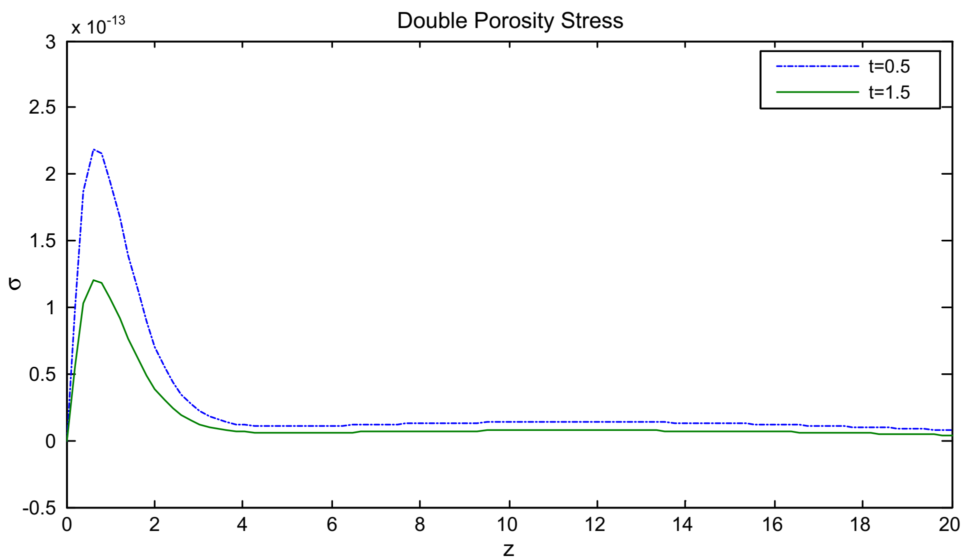



**Fig. 14** Distribution of the equilibrated stress $\sigma$

## 7. Conclusion

From the figures obtained by comparing the functions in the presence and absence of double porosity at two different times, important phenomena are observed:

Analytic solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed, which used in the present article is applicable to a wide range of problems in hydrodynamics and thermoelasticity. There are significant differences in the presence and absence of double porosity under two different times.

All the physical quantities satisfy the boundary conditions. The value of all the physical quantities converges to zero, and all the functions are continuous. Though the problem is theoretical, it can provide useful information for experimental researchers working in the field of geophysics, earthquake engineering, along with seismologist working in the field of mining tremors and drilling into the crust of the earth. The numerical treatment of the general system of equations and conditions governing the phenomenon may be useful in getting rid of the limitations of the method of normal modes' technique, and this task is in progress.

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