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ORIGINAL PAPER

Fractal study on Saraswati supercluster

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Abstract: The study of galaxies and superclusters is a major field of interest of astrophysicists. With the development of modern advanced telescopes and data acquisition systems, scientists could gather more information about the universe. Investigations using Sloan 2.5-m telescope, the multi-fiber spectrograph and the galaxy redshift data recorded enabled the discovery of Saraswati—an extremely massive and giant supercluster. Fractal analysis is an established powerful mathematical technique for the analysis and study of complex systems extending from microcosm to macrocosm. Fractal dimension is related to the complexity of the system. In the present work, an attempt has been made to analyze the recently reported supercluster Saraswati by box-counting method. It is found that fractal dimension is very high for the supercluster and low for the voids. The fractal dimension of 1.75–2 in the Saraswati region suggests a sheet-like morphology to it. Complexity mapping is carried out by plotting contour plot of fractal dimension and percentage occupancy plot, which enable greater understanding of the distribution of galaxies in the universe.

Keywords: Fractals; Saraswati supercluster; Box-counting method

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1. Introduction

Clusters of galaxies are the bound system of gravitationally interacting particles with typical masses ranging from 10^{14} to 10^{15} solar masses, which act as a potential tool for the detailed study of galaxy formation, dark matter, large-scale structure, cosmology, etc. Until the discovery of superclusters in 1980, these galaxy clusters were considered as the largest known systems in the universe. Superclusters are large group of galaxy clusters which highlight the large-scale structure of the universe that reflects the initial conditions existed in the early universe and gives strong evidence for the different cosmological models of galaxy formation [1, 2]. Superclusters usually span around 100 megaparsecs (Mpc) across and can have mass greater than 10^{16} solar masses [1, 3–7]. The position of superclusters in the hierarchical formation is above clusters with a difference in virialization. Superclusters, a collection of clusters with a spatial density enhancement [8], have not attained a quasi-equilibrium configuration unlike clusters with equilibrium configuration. Hence, superclusters are considered as transition objects reflecting their initial state [1, 2]. Superclusters are surrounded by regions with very low or zero mass density called voids [9, 10]. The connection between the dark matter and dark energy is responsible for the formation of large-scale structures like superclusters in the universe [11]. Superclusters show both filamentary and sheet-like morphology, and those with voids are commonly called cosmic web [12]. The spectroscopic analysis of the redshift data recorded from different regions of the cosmic web helps in identifying a supercluster. Friends of Friends (FoF) algorithm [6, 13] and smoothed density field method [11] are the two popularly used techniques for the identification of superclusters. From the literature, it can be seen that wavelet [14, 15] and minimal spanning tree (MST) [16, 17] approaches are extensively used in cluster identification.

The statistical properties of the large-scale structure of the universe can be analyzed using the data from the redshift survey. The galaxy redshift data from the Stripe 82 region of the Sloan Digital Sky Survey (SDSS-III D12) enabled Bagchi et al. [1] to discover the massive

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superclusters of galaxies with the calculated mass of approximately 2×10^{16} solar masses, called Saraswati [1, 18, 19]. The SDSS uses a multi-fiber spectrograph mounted on the Sloan 2.5 m telescope to record the redshift data. The redshift data are converted into a plot on comoving coordinates, and then the Saraswati supercluster is identified by using the FoF algorithm. Saraswati spans around 200 Mpc across and is found at a mean redshift $z = 0.28$. Saraswati consists of about 43 massive galaxy clusters, with extremely massive one at the central core region [1].

A number of quantitative methods are recently introduced to study the distribution of galaxies [20]. Wavelet transform is one such mathematical tool that can be used in the analysis of images. Fractal analysis introduced by Gomory [21] and Mandelbrot [22] is another tool having the potential to analyze microcosm as well as macrocosm [12, 23–27]. Hence, the technique is widely used in the study of galactic distribution and images. Fractals are self-similar objects having the same details in different scales [22, 27]. Fractal analysis enables us to measure the complexities and self-similarities of the patterns by a quantity called fractal dimension. Fractal dimension is a non-integer value which is greater than the topological dimension and less than the euclidean dimension of the pattern or the data under study. The deviation of fractal dimension from this topological and euclidean dimension features to the complexity and self-similarity of the pattern [28].

Fractal dimension does not change with different transformations applied to the image like changing the intensity, adding multiplicative noise and locally scaling the gray values. In cosmology, fractal analysis enables us (i) to study the complexities and self-similarities of structures in the universe, (ii) to identify the morphology of the structures and (iii) to classify the structures based on their distribution [12, 29]. The various methods commonly used for finding the fractal dimension are power spectrum, walking divider, prism counting, epsilon bracket, correlation and box-counting method [25, 30–32]. Of these methods, box counting is the most commonly used technique for finding the fractal dimension of highly complex structures [33]. Box-counting method is superior to other fractal dimension-finding techniques in cosmology like correlation dimension because it does not require any prior assumption about the homogeneity of the universe [34, 35].

Fractal analysis is employed by many scientists to understand the large-scale structures in the universe such as galaxies, clusters and superclusters [29, 36–39]. Jones et al. [27] effectively used box-counting and multifractal methods to characterize the large-scale clustering of the universe. Klypin et al. [12] used the box-counting and cluster analysis technique to find the fractal dimension of the Virgo supercluster. They have reported a fractal dimension of about 1.3 in the inter-Virgo-Coma supercluster, which is assigned to the filamentary morphology, and a fractal dimension of 1.9 in the Virgo supercluster which is assigned to the sheet-like morphology. The fractal studies carried out by Conde-Saavedra et al. [28] revealed a high fractal dimension in the low redshift region and low fractal dimension in the high redshift region.

In the present work, the recently reported Saraswati supercluster is subjected to fractal analysis and also an attempt has been made to classify regions of identical complexities. The data used for the study are obtained from the galaxy redshift data provided by the Sloan Digital Sky Survey (SDSS-III DR12). This is obtained by combining the photometric data of the galaxies taken from "LEGACY," "BOSS" and "SOUTHERN" (LBS) programs of the SDSS-III DR12 database [1, 40, 41]. The Stripe 82 region which spans along the celestial equator with right ascension (RA) values between 310° and 59° and declination between − 1.25° and 1.25° is explored. The galaxies which give clean spectra with redshift errors < 1% are selected. The magnitudes of the galaxies are k-corrected using K-CORRECT version 4.3 software for converting them into comoving coordinates [42]. The sample is volume limited by using the cModel r-band absolute magnitude $M_r \leq -21.53$ and the redshift $z \leq 0.33$ [43]. Fractal analysis is done on a volume-limited region from the central part of Stripe 82 that contains about 3016 galaxies, the span of which can be described with RA ranging from 336° to 16°, declination ranging from − 1.25° to 1.25° and redshift data ranging from 0.23 to 0.33. The cosmological parameters used for the data collection and analysis are based on the 5-year Wilkinson Microwave Anisotropy Probe (WMAP) results [44]. The parameters used are Hubble constant $H_0 = 70.1$ km s^{-1} -Mpc^{-1}, matter density parameter $\Omega_M = 0.279$, dark energy density parameter $\Omega_\Lambda = 0.721$, the radiation density parameter $\Omega_R = 8.493 \times 10^{-15}$ and the state of the dark energy $\omega_0 = -1$ [1].

2. Fractal analysis

Among the various methods for calculating the fractal dimension, the simplest and widely used technique is the box-counting method. In this method, the pattern to be analyzed is superimposed by grids (boxes) of side $\in$ and the number of boxes $N(\in)$ required to cover the fractal pattern is counted. The process of counting $N(\in)$ is repeated for different values of box size $\in$. The number of boxes is related to the box size by the following equation:

$$N(\in) \propto \in^{-d} \qquad (1)$$

where d is the fractal dimension; taking logarithm, we get

$$\ln N(\in) = d \ln\left(\frac{1}{\in}\right) + \text{constant} \quad (2)$$

such that $\lim_{\in \to 0} \frac{\ln(N(\in))}{\ln(1/\in)} = \text{d}$.

The slope of the plot of log(N(∈)) versus the log(1/∈) corresponds to the fractal dimension (d) of the pattern [23, 31]. Since fractal analysis is based on the principle of self-similarity and scale invariance of fractal dimension, one can determine the fractal dimension by taking any set of box sizes ∈. Since the choice of maximum and minimum value of ∈ does not change the slope of the straight line fitted, the fractal dimension remains unchanged.

The data used for the fractal analysis are the 2D projection of the 3D data, which is obtained by suppressing the small depth of 2.5° in the declination direction. The depth of the sample used is very small compared to the RA value (45°), and it can be approximated to a laminar 2D structure. This transformation of the 3D into 2D equivalence does not affect the information derived from the sample under study. The mathematical theorem about projection of a 3D object into 2D and the estimation of fractal dimension say that the fractal dimension of 2D photograph of a 3D object and that of the 3D object are related. Thus, if the dimension of the 3D object is greater than two, then its 2D picture will have dimension of two and if the dimension of the 3D object is less than two, then the dimension of the 2D photograph will be the same as that of 3D object [32, 45].

3. Results and discussion

The recently discovered massive large supercluster Saraswati found in the central part of Stripe 82 region is shown in Fig. 1 (plotted using the data from SDSS III DR 12 database [18, 19]) which is subjected to fractal analysis using box-counting method. Figure 1 is then divided into 783 grids with sides 18.5 Mpc, which is comparable to the linking length used in Friends of Friends algorithm (19 Mpc) by Bagchi et al. [1]. For analyzing the variation of fractal dimension over this region, box-counting method is applied to each grid by varying the box size from 0.6 to 9.8 Mpc as described in Sect. 2. The portion of Fig. 1 occupying one such grid of 18.5 Mpc × 18.5 Mpc size is shown in the inset of its log N(∈) vs log 1/∈ plot shown in Fig. 2. The log N(∈) and log 1/∈ show a strong correlation with R^2 value equal to 0.9995. For complexity mapping, the fractal dimension of each grid is determined and the contour surface plot of the fractal dimension in the XY plane is plotted as shown in Fig. 3a. The contour plot gives information about the regions of identical complexity.

From the contour plot, it is observed that the fractal dimension of the grids varies between zero and two. The central part spanning around 200 Mpc is found to show a high fractal dimension indicating a higher level of complexity or high-density clustering of galaxies. This high fractal dimensional area in the contour plot at the center region is the same as the position of the Saraswati supercluster obtained by FoF algorithm reported by Bagchi et al. [1]. Thus, the fractal analysis provides a means for complexity analysis giving information regarding clustering of galactic points.

In the contour plot, there are regions with very low fractal dimension (< 0.25). The position of voids in the comoving data is characterized by fractal dimensions with low value and evident from Fig. 3a. Also, it is observed that while going radially outward from any void, the fractal dimension is increasing which is an indication of the increase of complexity and greater crowding of galaxies in that region. Most of the regions in the contour plot have a fractal dimension ranging from 1.5 to 1.75. It is evident from Fig. 3a that the majority of regions in the Stripe 82 region show similar complexities and self-similarities with the same number density of galaxies. The variation of fractal dimension across the Saraswati supercluster is shown in Fig. 3b. The percentage occupancy plot is another way of expressing the complexity of the system. The percentage occupancy of each grid is calculated by finding the percentage of area occupied by the image in a

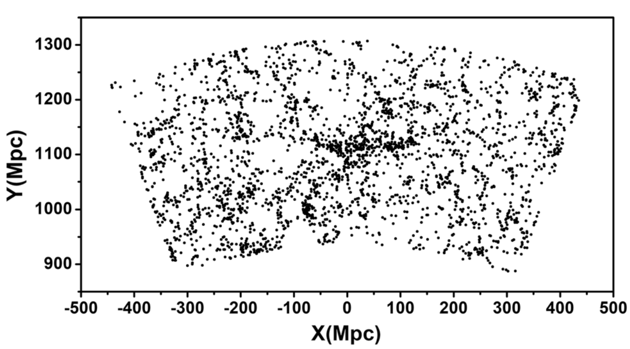

Fig. 1 Central part of Stripe 82 containing Saraswati supercluster

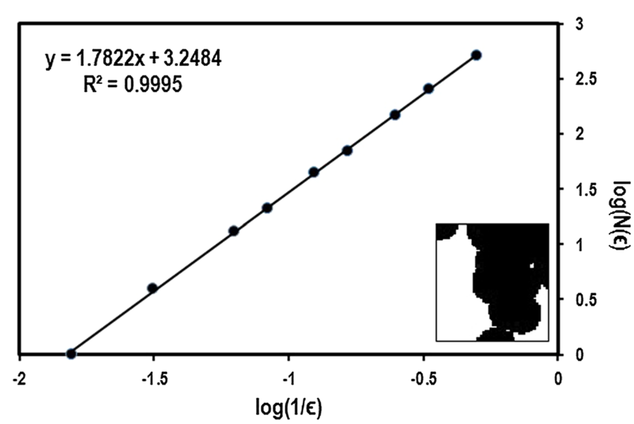

Fig. 2 Log N(∈) vs log 1/∈ plot of one representative grid

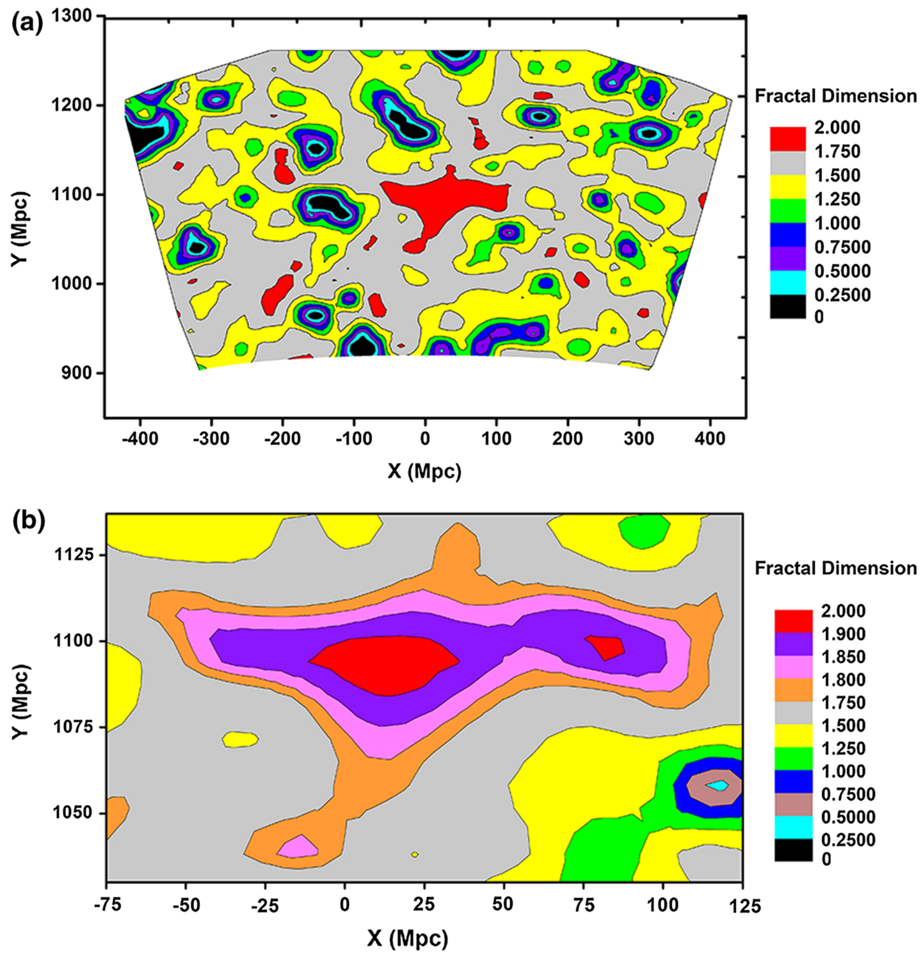

Fig. 3 (**a**) Contour plot of fractal dimension in central part of Stripe 82 and (**b**) variation of fractal dimension across the Saraswati supercluster

grid to the area of one grid. Figure 4 shows the contour plot of the percentage area occupancy of each grid across the central part of Stripe 82. From this plot (Fig. 4), a higher percentage occupancy ($> 50\%$) can be observed in the region called Saraswati. The lower value of percentage occupancy indicates voids or lower number of galactic points. This helps in understanding the nature of the distribution of galaxy points and morphology. From the literature, it can be seen that in the low redshift region, the fractal dimension will be high [28]. The present study is in agreement with the literature showing a fractal dimension of 1.75–2 in the Saraswati region having a low redshift value of ~ 0.28. Klypin et al. [12] reported sheet-like morphology for the Virgo supercluster with a fractal dimension of 1.9. Since the fractal dimension of Saraswati

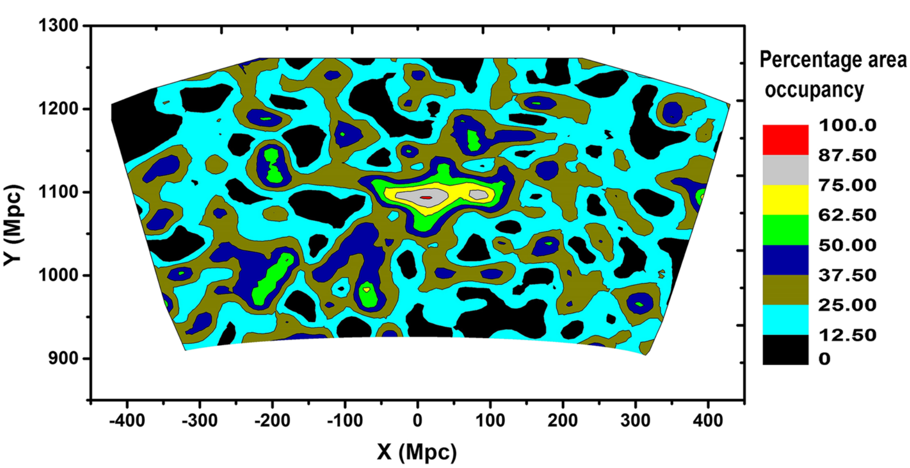

Fig. 4 Percentage area occupancy of the central part of Stripe 82

supercluster also is found to be about 1.75–2, we propose a sheet-like morphology to it.

4. Conclusion

The results of the study reveal that fractal analysis using box-counting method can be used as a potential tool for studying the distribution of galaxies in the universe. High fractal dimension is related to the presence of the clustered distribution of galaxies, and low fractal dimension is attributed to low mass density distributions called voids. The results of the study are in agreement with original report, and the region Saraswati shows a fractal dimension of 1.75–2. The study reveals sheet-like morphology to the Saraswati supercluster in comparison with the Virgo supercluster. The Saraswati region at a low redshift value of $\sim$ 0.28 shows a higher fractal dimension which is also in agreement with the literature. The distribution of galactic points is also analyzed by plotting contour plot of fractal dimension and percentage occupancy plot.

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