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ORIGINAL PAPER

# Anisotropic Bianchi-V dark energy model under the new perspective of accelerated expansion of the universe in Brans–Dicke theory of gravitation

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**Abstract:** In this paper, we have proposed a cosmological model, which is consistent with the new findings of 'The Supernova Cosmology project' headed by Saul Perlmutter, and the 'High-Z Supernova Search team', headed by Brian Schimdt. According to these new findings, the universe is undergoing an expansion with an increasing rate, in contrast to the earlier belief that the rate of expansion is constant or the expansion is slowing down. We have considered spatially homogeneous and anisotropic Bianchi-V dark energy model in Brans–Dicke theory of gravitation. We have taken the scale factor $a(t) = kt^{\alpha}e^{\beta t}$, which results into variable deceleration parameter (DP). The graph of DP shows a transition from positive to negative, which shows that universe has passed through the past decelerated expansion to the current accelerated expansion phase. In this context, we have also calculated and plotted various parameters and observed that these are in good agreement with physical and kinematic properties of the universe and are also consistent with recent observations.

**Keywords:** Bianchi type-V universe; Dark energy; Brans–Dicke theory

**PACS No.:** 98.80.-k

## 1. Introduction

At the early stage in the study of cosmological models, the universe was assumed to be static. But as the time progressed, it was found that the universe is not static but expanding. Before 1998, almost every researcher in the field of relativity and cosmology assumed that the universe is expanding, but the rate of expansion is slowing down, due to gravity acting on the matter. In 1990, two research teams, one 'The Supernova Cosmology project' headed by Saul Perlmutter based at Lawrence-Berkeley National Laboratory, U.S.A. and the second 'High-Z Supernova Search team', headed by Brian Schimdt at Mount Stromlo and Siding Spring Observatories, Australia, while looking for distant Type Ia supernovae for measuring the expansion rate of the universe, found that the rate of expansion is increasing instead of slowing down. Both the teams were in international collaborations with researchers in England, France, Germany, and Sweden etc. The findings of these two teams are presented in the following papers: Garnavich et al. [1, 2], Perlmutter et al. [3–5], Riess et al. [6], and Schmidt et al. [7]. These teams have measured the distances on the basis of observations of type Ia supernovae and they predicted that the expansion of the universe is accelerating, and thus, it is likely to go on expanding forever. These measurements, combined with red-shift data for the supernovae lead to the conclusion that the universe is expanding.

In addition, measurements of the cosmic microwave background (CMB) [8] indicate that the universe has a flat geometry on large scales. As the universe has neither enough ordinary matter nor enough dark matter to produce this flatness, the difference must be attributed to a 'dark energy'. This same dark energy (DE) causes the accelerated expansion of the universe. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment suggests, 73% content of the universe is in the form of dark energy, 23% in the form of non-baryonic dark matter and the rest 4% is in the form of usual baryonic matter as well as radiation.

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Although our knowledge about nature and properties of dark energy is limited, but alternative gravity theories provide, certainly a way of understanding the problem of DE and the possibility of reconstructing the gravitational field theory that would be capable to explain the late-time accelerated expansion of the universe. One of such theories was proposed by Brans and Dicke [9]. Brans and Dicke in their alternative theory, introduced an additional scalar field $\phi$ along with the metric tensor $g_{ij}$ and a dimensionless coupling constant $w$. In terms of this new scalar field, the Einsteins term $\frac{1}{G}R$ from the Lagrangian, takes the form $\phi R$. To get a full description of scaler–tensor theory, they need to add, to the Lagrangian, the term $\frac{\phi_{,i}\phi^{,i}}{\phi}$, describing the effective energy density of the scalar field. As a result, they arrive at the following Lagrangian:

$$L_{BDT} = \phi(R - w\frac{\phi_{,i}\phi^{,i}}{\phi^2}) + 16\pi L_{mat}$$

Varying over $g_{ij}$ and $\phi$, following field equations are found:

$$R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi}{\phi}T_{ij} - \frac{w}{\phi^2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) - \frac{1}{\phi}(\phi_{;ij} - g_{ij}\Box\phi)$$

and

$$\Box\phi = \phi^{;k}_{;k} = \frac{8\pi}{(2w+3)}T$$

where $T$ is the trace $T^i_i$ of the energy momentum tensor $T_{ij}$.

Pradhan et al. [10] studied accelerating Bianchi type-V cosmology with perfect fluid and heat flow in the Saez–Ballester theory. String cosmological models in a scalar–tensor theory and in the Brans–Dicke theory of gravitation were proposed by Reddy [11, 12]. String cosmological models in Bianchi type III and LRS Bianchi type I with time-dependent bulk viscosity were studied by Bali et al. [13, 14].

Recently, Venkateshwarlu et al. [15] have found solutions of the Brans–Dicke field equations for anisotropic string cosmological models with constant deceleration parameter. Rao et al. [16] have investigated Bianchi type-I dark energy model in Saez–Ballester theory. Rao et al. [17] have found the solutions of Bianchi type-V dark energy model in Brans–Dicke theory of gravitation under constant deceleration parameter. Maurya et al. [18] have investigated anisotropic string cosmological model in Brans–Dicke theory of gravitation under the new perspective of time dependent deceleration parameter.

Motivated by the above investigations, in this paper, we have considered Bianchi type-V dark energy model in Brans–Dicke theory of gravitation, but under the new perspective of time dependent deceleration parameter.

The out line of the paper is, as follows: Sect. 1 is introductory in nature. In Sect. 2, we have defined the metric and the Brans–Dicke field equations and then, the solutions of the field equations are derived. Also, in this section, we have defined and calculated the physical and kinematic parameters of the model by considering the time dependent scale factor $a(t) = kt^{\alpha}e^{\beta t}$. In Sect. 3, we have discussed the physical and kinematic features of the model with the help of the calculations, obtained in Sect. 2. We have discussed the energy conditions also in Sect. 3. Finally, conclusions are summarized in the last Sect. 4.

## 2. Theoretical calculations

We consider a homogeneous and anisotropic Bianchi type-V space–time for which the metric is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}\left[B^2(t)dy^2 + C^2(t)dz^2\right], \tag{1}$$

where $A$, $B$ and $C$ are metric functions and $m$ is a constant.

The field equations for Brans–Dicke theory are

$$R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi}{\phi}T_{ij} - \frac{w}{\phi^2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) - \frac{1}{\phi}(\phi_{;ij} - g_{ij}\Box\phi) \tag{2}$$

where

$$\Box\phi = \phi^{;k}_{;k} = \frac{8\pi}{(2w+3)}T \tag{3}$$

and $\phi$ is the scalar field, $w$ is dimensionless coupling constant, $T_{ij}$ is the energy momentum tensor, $R_{ij}$ is the Ricci tensor, $R$ is the Ricci scalar and $T$ is the trace of the energy momentum tensor.

The simplest generalization of EoS parameter of perfect fluid may be to determine the EoS parameter separately on each spacial axis by preserving the diagonal form of the energy momentum tensor in a consistent way with the considered metric. Therefore, the energy momentum tensor of fluid is taken as

$$T^i_j = diag\left[T^0_0, T^1_1, T^2_2, T^3_3\right] \tag{4}$$

The energy momentum tensor for anisotropic dark energy is

$$T^i_j = diag\left[\rho, -p_x, -p_y, -p_z\right], \tag{5}$$

where $\rho$ is the energy density of the fluid and $p_x$, $p_y$, $p_z$ are the pressures along the $x$, $y$, $z$ axes respectively.

We define the equation of state (EOS) parameter $\omega = p/\rho$. So, the energy momentum tensor may also be written as $diag\left[1, -\omega_x, -\omega_y, -\omega_z\right]\rho$ where $\omega_x, \omega_y, \omega_z$ are EoS parameters in the direction of $x$, $y$, $z$ axes respectively.

If we parameterize the deviation from isotropy by setting $\omega_x = \omega$ and then, introduce skewness parameters $r$ and $\delta$ (which are deviations from $\omega$ along the $y$ and $z$ axes respectively), the energy momentum tensor takes the form

$$diag[1, -\omega, -(\omega + r), -(\omega + \delta)]\rho \tag{6}$$

For the energy-momentum tensor (6) and Bianchi type-V space–time (1), the Brans–Dicke field equations (2) yield the following six independent equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{1}{A^2} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -8\pi\phi^{-1}\rho\omega, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -8\pi\phi^{-1}\rho(\omega + r), \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -8\pi\phi^{-1}\rho(\omega + \delta), \tag{9}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3}{A^2} + \frac{w}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 8\pi\phi^{-1}\rho, \tag{10}$$

$$\left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0, \tag{11}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 8\pi(3 + 2w)^{-1}\rho[1 - (3\omega + r + \delta)]. \tag{12}$$

The law of energy-conservation equation $T^{ij}_{;j} = 0$ gives

$$\dot{\rho} + \rho\left(\frac{\dot{A}}{A}(1 + \omega) + \frac{\dot{B}}{B}(1 + \omega + r) + \frac{\dot{C}}{C}(1 + \omega + \delta)\right) = 0. \tag{13}$$

where an over head dot denotes differentiation with respect to t.

The field equations (7)–(12) are a system of six independent equations in eight unknowns A, B, C, $\rho$, $\omega$, $r$, $\delta$ and $\phi$. To get a deterministic solution of highly nonlinear field equations we use the following two plausible physical conditions:

(i) The scalar expansion $\theta$ is proportional to the shear scalar $\sigma^2$ so, we can take

$$B = C^m \tag{14}$$

where m $\neq 0$ is a positive constant.

(ii) The trace of energy momentum tensor vanishes, so we have

$$T = T^1_1 + T^2_2 + T^3_3 + T^4_4 = \rho[1 - (3\omega + r + \delta)] = 0 \tag{15}$$

Now, integration of Eq. (11) gives

$$A^2 = \ell BC \tag{16}$$

where $\ell$ is a constant of integration (which can be taken as unity without any loss of generality), so that we have

$$A^2 = BC \tag{17}$$

Next, the average scale factor $a(t)$ is defined as

$$a(t) = (ABC)^{\frac{1}{3}} \tag{18}$$

Since we are representing a model, which was decelerating in the past and is accelerating in the present, the deceleration parameter (DP) must be time dependent and must show signature flipping. Various authors have used the different ansatz for the average scale factor in different contexts. Here we consider the hybrid expansion law (HEL) for the average scale factor given by

$$a(t) = kt^{\alpha}e^{\beta t} \tag{19}$$

where $k > 0$, $\alpha \geq 0$ and $\beta \geq 0$ are constants. HEL is a hybrid of power law and exponential expansion law, as if we put $\beta = 0$ in HEL we get $a(t) = kt^{\alpha}$ (ie. power law) and if we put $\alpha = 0$ we get $a(t) = k.e^{\beta t}$ (ie. exponential law).

Using equations (17) (18) and (19) we get

$$A = kt^{\alpha}e^{\beta t} \tag{20}$$

and then using equations (14), (17) and (20) we get

$$B = (kt^{\alpha}e^{\beta t})^{\frac{2m}{m+1}} \tag{21}$$

$$C = (kt^{\alpha}e^{\beta t})^{\frac{2}{m+1}} \tag{22}$$

Using equation (12) and (15), we get the scalar field given by

$$\phi(t) = \int \frac{1}{(kt^{\alpha}e^{\beta t})^3}dt = \frac{1}{k^3 t^{(1-3\alpha)} e^{3\beta t}} \sum_{n=0}^{\infty} \frac{(3\alpha t)^n}{(1 - 3\alpha)(2 - 3\alpha)(n + 1 - 3\alpha)} \tag{23}$$

For the convergence of the integral $1 - 3\alpha > 0$, ie. $\alpha < \frac{1}{3}$. $\beta \geq 0$

Now, we calculate the expressions for spatial volume $V$, the deceleration parameter $q$, the Hubble parameter $H$, expansion scalar $\theta$, shear scalar $\sigma$, and the average anisotropy parameter $A_m$ for the model.

The spatial volume $V$ is defined and calculated to be

$$V = a^3 = ABC = (kt^{\alpha}e^{\beta t})^3. \tag{24}$$

The deceleration parameter $q$ is defined and estimated as

$$q(t) = \frac{a\ddot{a}}{\dot{a}^2} = \frac{\alpha}{(\alpha+\beta t)^2} - 1. \tag{25}$$

The generalized mean Hubble's parameter $H$ is defined and calculated as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_x + H_y + H_z) = \frac{1}{3}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] = \frac{\alpha}{t} + \beta \tag{26}$$

where $H_x$, $H_y$ and $H_z$ are the directional Hubble's parameters in the directions of $x$, $y$ and $z$ axes respectively.

The scalar expansion $\theta$ is defined and calculated as

$$\theta = 3H = 3\left(\frac{\alpha}{t} + \beta\right) \tag{27}$$

The shear scalar $\sigma^2$ is defined and calculated to be

$$\sigma^2 = \frac{1}{2}\left[(H_x^2 + H_y^2 + H_z^2) - \frac{\theta^2}{3}\right] = \left(\frac{m-1}{m+1}\right)\left(\frac{\alpha}{t} + \beta\right) \tag{28}$$

The average anisotropy parameter $A_m$ are defined and estimated to be

$$A_m = \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{H_i - H}{H}\right)^2 = \frac{2}{3}\left(\frac{m-1}{m+1}\right)^2 \tag{29}$$

The energy density, for the model is found by using Eqs. (20)–(23) in Eq. (10)

$$\rho = \frac{1}{8\pi\phi^{-1}}\left[\frac{(2m^2+8m+2)}{(m+1)^2}\left(\frac{\alpha}{t} + \beta\right)^2 - \frac{3}{(kt^\alpha e^{\beta t})^2} - \frac{w}{2}\frac{(kt^\alpha e^{\beta t})^{-6}}{[\phi(t)]^2} + \frac{3(kt^\alpha e^{\beta t})^{-3}}{\phi(t)}\left(\frac{\alpha}{t} + \beta\right)\right] \tag{30}$$

From (30), we observe that energy density of the fluid $\rho(t)$ is a decreasing function of time and $\rho > 0$ under condition

$$\frac{(2m^2+8m+2)}{(m+1)^2}\left(\frac{\alpha}{t} + \beta\right)^2 + \frac{3(kt^\alpha e^{\beta t})^{-3}}{\phi(t)}\left(\frac{\alpha}{t} + \beta\right) > \frac{3}{(kt^\alpha e^{\beta t})^2} + \frac{w}{2}\frac{(kt^\alpha e^{\beta t})^{-6}}{[\phi(t)]^2} \tag{31}$$

The skewness parameter $r$ (i.e. deviation from $\omega$ along $y$ axis) is obtained with the help of Eqs. (7) and (8) and using Eqs. (20)–(22),

$$r = \frac{1}{8\pi\phi^{-1}}\left[\left(\frac{m-1}{m+1}\right)\frac{\alpha}{t^2} - \frac{3(m-1)}{m+1}\left(\frac{\alpha}{t} + \beta\right)^2 + \frac{(kt^\alpha e^{\beta t})^{-3}}{\phi(t)}\frac{(m-1)}{(m+1)}\left(\frac{\alpha}{t} + \beta\right)\right] \tag{32}$$

The skewness parameter $\delta$ (ie. deviation from $\omega$ along $z$ axis) is computed with the help of Eqs. (7) and (9) and using Eqs. (20)–(22),

$$\delta = \frac{1}{8\pi\phi^{-1}}\left[\left(\frac{m-1}{m+1}\right)\frac{\alpha}{t^2} - \frac{2(m^2-2m-4)}{(m+1)^2}\left(\frac{\alpha}{t} + \beta\right)^2 + \frac{(kt^\alpha e^{\beta t})^{-3}}{\phi(t)}\frac{(m-1)}{(m+1)}\left(\frac{\alpha}{t} + \beta\right)\right] \tag{33}$$

The equation of state (EoS) parameter is obtained by using Eq. (20)–(23) in Eq. (7)

$$\omega = \frac{1}{8\pi\phi^{-1}\rho}\left[\frac{(2m+1)}{(m+1)}\frac{\alpha}{t^2} - \frac{4(m^2+m+1)}{(m+1)^2}\left(\frac{\alpha}{t} + \beta\right)^2 + \frac{1}{(kt^\alpha e^{\beta t})^2} - \frac{w}{2}\frac{(kt^\alpha e^{\beta t})^{-6}}{[\phi(t)]^2}. + \frac{3(kt^\alpha e^{\beta t})^{-3}}{\phi(t)}\frac{(m-1)}{(m+1)}\left(\frac{\alpha}{t} + \beta\right)\right] \tag{34}$$

The expression for matter pressure is obtained by Eqs. (30) and (34)

$$p = \frac{1}{8\pi\phi^{-1}}\left[\frac{(2m+1)}{(m+1)}\frac{\alpha}{t^2} - \frac{4(m^2+m+1)}{(m+1)^2}\left(\frac{\alpha}{t} + \beta\right)^2 + \frac{1}{(kt^\alpha e^{\beta t})^2} - \frac{w}{2}\frac{(kt^\alpha e^{\beta t})^{-6}}{[\phi(t)]^2}. + \frac{3(kt^\alpha e^{\beta t})^{-3}}{\phi(t)}\frac{(m-1)}{(m+1)}\left(\frac{\alpha}{t} + \beta\right)\right] \tag{35}$$

## 3. Results and discussions

In this paper, we have considered anisotropic Bianchi type-V dark energy model in Brans–Dicke theory of gravitation. We have found the exact solution of the model by considering the scale factor $a(t) = kt^\alpha e^{\beta t}$. Our special choice of scale factor yields a time dependent deceleration parameter $q = -1 + \frac{\alpha}{(\alpha+\beta t)^2}$. It's graph, represented by Fig. 1, shows that at the early stage, the DP $q$ was +ve which represents the decelerated expansion phase of the universe. As the time progressed, DP crosses d through zero and at the current time, it is −ve, which shows that the universe is going through an accelerated expansion at the current time. This scenario is consistent with recent observations [3–7].

The plot of energy density $\rho$ with time, shown in Fig. 2, corresponding to the Eq. (30), shows that $\rho$ is a positive decreasing function of time. It is infinite at $t = 0$, and remains positive as $t \to \infty$.

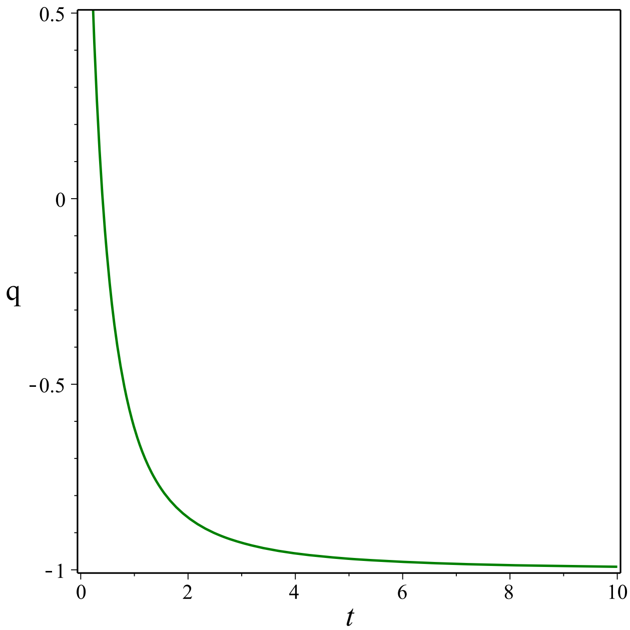


**Fig. 1** The plot of deceleration parameter $q$ versus cosmic time $t$. For $\alpha = 0.33, \beta = 0.6$

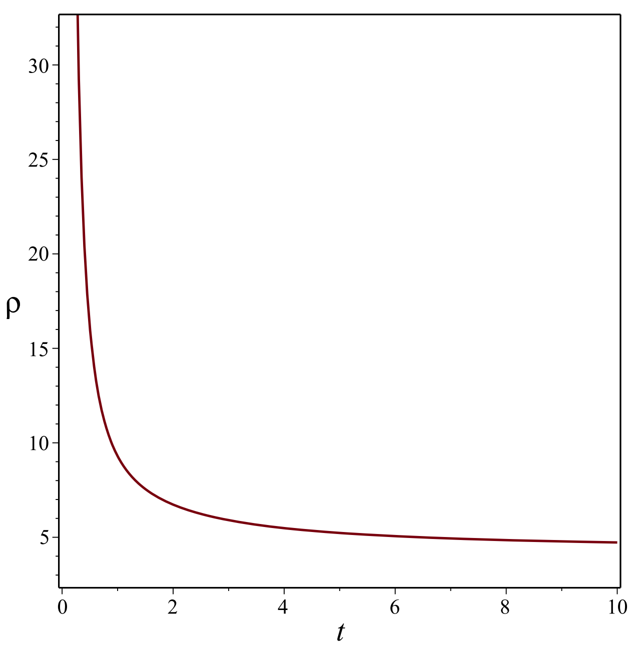


**Fig. 2** The plot of energy density $\rho$ versus cosmic time $t$. For $\alpha = 0.33, \beta = 0.6, m = 1, w = 1, k = 1$

The plot of matter pressure, shown in Fig. 4, indicates that it was very high at the early stage. But, it attains a negative constant value as $t \rightarrow \infty$, which shows that the universe is dominated by dark energy at late time, causing the late time accelerated expansion of the universe.

The dark energy has conventionally been characterized by **EoS** parameter $\omega(t) = \frac{p}{\rho}$. Now, if the present work is to be compared with experimental results, then one can conclude that the limit of $\omega$, given by Eq. (34), must be consistent with acceptable range of EoS parameter.

Here, it is observed that for $t = t_c$, $\omega$ vanishes, where $t_c$ is a critical time given by

$$\frac{\alpha(2m+1)}{(m+1)t_c^2} - \frac{4(m^2+m+1)}{(m+1)^2}\left(\frac{\alpha}{t_c}+\beta\right)^2 + \frac{1}{(kt_c^{\alpha}e^{\beta t_c})^2} - \frac{3(m-1)}{(m+1)\phi(t)(kt_c^{\alpha}e^{\beta t_c})^3}\left(\frac{\alpha}{t_c}+\beta\right) - \frac{w}{2[\phi(t)]^2(kt_c^{\alpha}e^{\beta t_c})^6} = 0 \tag{36}$$

Thus, for this particular time our model represents dusty universe. Earlier to that (i.e. at $t \leq t_c$) real matter dominates, where $\omega \geq 0$. Later on at $t > t_c$, where $\omega < 0$, we arrive at the dark energy dominated phase of the universe, as shown in Fig. 3.

For the value of $\omega$ to be consistent with observation [19], we have the following general condition

$$t_1 < t < t_2, \tag{37}$$

where $t_1$ and $t_2$ are given by the relations

$$\frac{(1.34m^2 - 9.36m + 1.34)}{(m+1)^2}\left(\frac{\alpha}{t_1}+\beta\right)^2 + \frac{4}{(kt_1^{\alpha}e^{\beta t_1})^2} - \frac{\alpha(2m+1)}{(m+1)t_1^2} - \frac{(8.67m+2.01)}{(m+1)(kt_1^{\alpha}e^{\beta t_1})^3\phi}\left(\frac{\alpha}{t_1}+\beta\right) + \frac{1.33w}{(kt_1^{\alpha}e^{\beta t_1})^6\phi^2} = 0 \tag{38}$$

and

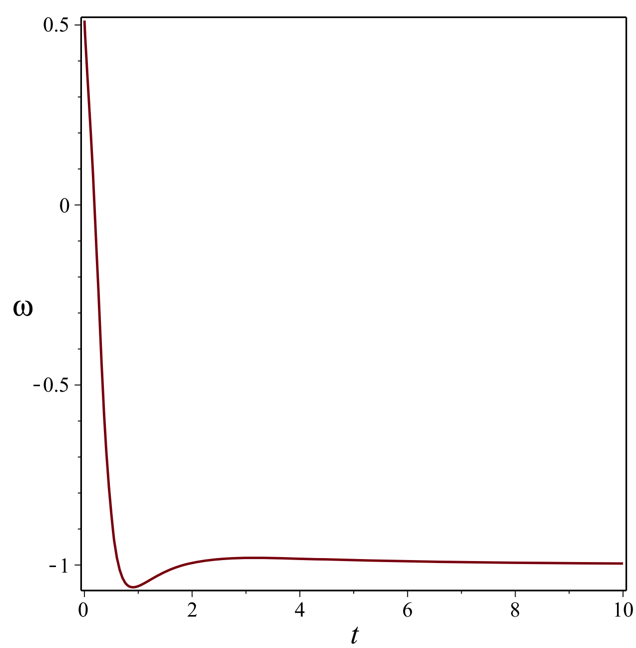


**Fig. 3** The plot of EoS parameter $\omega$ versus cosmic time $t$. For $\alpha = 0.33, \beta = 0.6, m = 1, w = 1, k = 1$

$$\frac{(2.67m^2-0.96m+2.67)}{(m+1)^2}\left(\frac{\alpha}{t_2}+\beta\right)^2+\frac{0.86}{(kt_2^{\alpha}e^{\beta t_2})^2}-\frac{\alpha(2m+1)}{(m+1)t_2^2}-\frac{1.14(m-1)}{(m+1)(kt_2^{\alpha}e^{\beta t_2})^3\phi}\left(\frac{\alpha}{t_2}+\beta\right)-\frac{0.81w}{(kt_2^{\alpha}e^{\beta t_2})^6\phi^2}=0 \quad (39)$$

For this constraint, the EoS parameter $\omega$ is restricted to the limit $-1.67<\omega<-0.62$ which is in a good agreement with the limit obtained from observational results of SN Ia data [19].

For the value of $\omega$ to be consistent with observations from SNe Ia data with CMB anisotropy and galaxy clustering statistics [20], we have the following general condition

$$t_3<t<t_4, \quad (40)$$

where $t_3$ and $t_4$ are given by

$$\frac{(1.34m^2-6.64m+1.34)}{(m+1)^2}\left(\frac{\alpha}{t_3}+\beta\right)^2+\frac{2.99}{(kt_3^{\alpha}e^{\beta t_3})^2}-\frac{\alpha(2m+1)}{(m+1)t_3^2}-\frac{(6.99m+0.99)}{(m+1)(kt_3^{\alpha}e^{\beta t_3})^3\phi}\left(\frac{\alpha}{t_3}+\beta\right)+\frac{1.65w}{(kt_3^{\alpha}e^{\beta t_3})^6\phi^2}=0 \quad (41)$$

and

$$\frac{(2.42m^2-3.76m+2.42)}{(m+1)^2}\left(\frac{\alpha}{t_4}+\beta\right)^2-\frac{0.21}{(kt_4^{\alpha}e^{\beta t_4})^2}-\frac{\alpha(2m+1)}{(m+1)t_4^2}-\frac{(5.37m-0.63)}{(m+1)(kt_4^{\alpha}e^{\beta t_4})^3\phi}\left(\frac{\alpha}{t_4}+\beta\right)-\frac{0.89w}{(kt_4^{\alpha}e^{\beta t_4})^6\phi^2}=0 \quad (42)$$

For this constraint we obtain $-1.33<\omega<-0.79$ which is in good agreement with the limit obtained from observational results coming from SNe Ia data with CMB anisotropy and galaxy clustering statistics [20].

For the value of $\omega$ to be consistent with latest observations of WMAP [21, 22], we have the following general condition

$$t_5<t<t_6, \quad (43)$$

where $t_5$ and $t_6$ are given by

$$\frac{(1.12m^2-7.52m+1.12)}{(m+1)^2}\left(\frac{\alpha}{t_5}+\beta\right)^2+\frac{3.32}{(kt_5^{\alpha}e^{\beta t_5})^2}-\frac{\alpha(2m+1)}{(m+1)t_5^2}-\frac{(7.32m+1.32)}{(m+1)(kt_5^{\alpha}e^{\beta t_5})^3\phi}\left(\frac{\alpha}{t_5}+\beta\right)-\frac{1.22w}{(kt_5^{\alpha}e^{\beta t_5})^6\phi^2}=0 \quad (44)$$

and

$$\frac{(2.16m^2-3.36m+2.16)}{(m+1)^2}\left(\frac{\alpha}{t_6}+\beta\right)^2+\frac{1.76}{(kt_6^{\alpha}e^{\beta t_6})^2}-\frac{\alpha(2m+1)}{(m+1)t_6^2}-\frac{(5.76m+0.24)}{(m+1)(kt_6^{\alpha}e^{\beta t_6})^3\phi}\left(\frac{\alpha}{t_6}+\beta\right)+\frac{0.96w}{(kt_6^{\alpha}e^{\beta t_6})^6\phi^2}=0 \quad (45)$$

For this constraint, we obtain the dark energy EoS to be $-1.44<\omega<-0.92$ which is in good agreement with the limit of latest observational result of WMAP, at 68% confidence level.

We also observe that if $t=t_0$ then $\omega=-1$, where $t_0$ is given by

$$\frac{(2m^2-4m+2)}{(m+1)^2}\left(\frac{\alpha}{t_0}+\beta\right)^2+\frac{2}{(kt_0^{\alpha}e^{\beta t_0})^2}-\frac{(2m+1)}{(m+1)}\left(\frac{\alpha}{t_0^2}\right)-\frac{6m}{(m+1)(kt_0^{\alpha}e^{\beta t_0})^3\phi}\left(\frac{\alpha}{t_0}+\beta\right)+\frac{w}{(kt_0^{\alpha}e^{\beta t_0})^6\phi^2}=0 \quad (46)$$

for $t=t_0$, $\omega=-1$ (i.e. cosmological constant dominated universe), and when $t<t_0$, $\omega>-1$ (i.e. quintessence), and for $t>t_0$, $\omega<-1$ (i.e supper quintessence or phantom fluid dominated universe) [23].

From Fig. 3, corresponding to the Eq. (34), we observe that at the early stage of the universe the EoS parameter $\omega>0$ (ie. matter dominated universe). It crosses the value zero (dusty universe), and at the present time $\omega<0$, which represents the dark energy dominated phase of the universe. We also observe that the range of $\omega$ is in good agreement with the acceptable range by the recent observations [19–21].

From Eq. (24), we observed that the spatial volume $V$ is zero at $t=0$, which shows that our model exhibits a point-type singularity at $t=0$. At this epoch the expansion scalar $\theta$ is infinite (Eq. 27), which shows that the universe started evolving with zero volume at $t=0$ which is a Big-Bang scenario. Also, $V$ increases with time and $V\to\infty$ as $t\to\infty$, hence the universe will keep on expanding forever, with the dominance of dark energy.

From Eq. 29, we observe that, the anisotropic parameter $A_m$ is constant and depends on the value of $m$. We can see

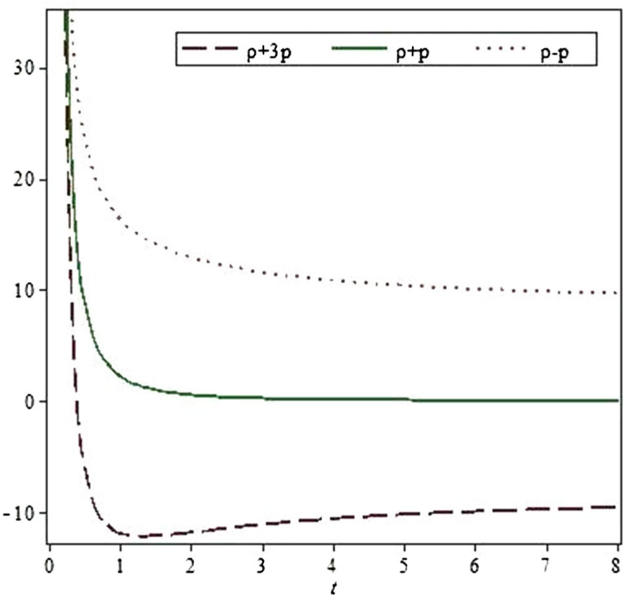


**Fig. 4** The plot of energy conditions $\rho + p$, $\rho - p$ and $\rho + 3p$ versus cosmic time $t$. For $\alpha = 0.33, \beta = 0.6, m = 1, w = 1, k = 1$

that for $m = 1$, $A_m = 0$. Thus the observed isotropy of the universe can be achieved in our model.

We have also studied the energy conditions for our derived model.

The weak energy conditions (WEC) are given by $\rho \geq 0$, and $\rho + p \geq 0$

Dominant energy conditions (DEC) are given by $\rho \geq |p|$ ie. $\rho + p \geq 0$. and $\rho - p \geq 0$, and the strong energy conditions (SEC) are given by $\rho + p \geq 0$ and $\rho + 3p \geq 0$.

Using our calculations of $\rho$ and $p$, we have plotted the graph of $\rho$ in Fig 2, $\rho + p$, $\rho - p$ and $\rho + 3p$ in Fig. 4.

From Figs. 2 and 4, we observe that the WEC's and DEC's for the derived model are satisfied. But the model violates the SEC $\rho + 3p \geq 0$, which is acceptable in case of dark energy cosmological models. The violation is due to a negative pressure attributed to dark energy. This violation gives a reverse gravitational effect, due to which, the universe undergoes a transition from earlier deceleration phase to the recent acceleration phase.

## 4. Conclusion

Our derived model represents a universe, which was decelerating at past and is accelerating at present. Energy density is a positive decreasing function of time and remains positive through out. The matter pressure was very high at the early stage, but it attains a negative constant value as $t \to \infty$, which shows that the universe is dominated by dark energy at late time, causing the late time accelerated expansion of the universe. We also observe that, at the early stage of the universe the EoS parameter is positive, which represents matter dominated universe. It crossed the value zero (dusty universe), and at the present time it is negative, which represents the dark energy dominated phase of the universe. We also observe that the range of EoS parameter is in good agreement with the acceptable range, found in the recent observations. We have also found that, the weak energy conditions (WEC's), dominant energy conditions (DEC's) are satisfied, but strong energy condition (SEC) is violated. So, our derived solutions are physically acceptable in concordance with the fulfillment of WEC's, DEC's and SEC's.

Therefore, we can say that, our derived model is capable of explaining various physical and kinematical phenomena of the universe and also is consistent with recent observations.

## References

[1] P M Garnavich, *et al. Astrophys. J.* **493** 53 (1998)
[2] P M Garnavich, *et al. Astrophys. J.* **509** 74 (1998)
[3] S Perlmutter, *et al. Astrophys. J.* **483** 565 (1997)
[4] S Perlmutter, *et al. Nature* **391** 51 (1998)
[5] S Perlmutter, *et al. Astrophys. J.* **517** 565 (1999)
[6] A G Riess, *et al. Astron. J.* **116** 1009 (1998)
[7] B P Schmidt, *et al. Astrophys. J.* **507** 46 (1998)
[8] C L Bennett, *et al. Astrophys. J. Suppl. Ser.* **148** 1 (2003)
[9] C H Brans, R H Dicke *Phys. Rev. A* **124** 925 (1961)
[10] A Pradhan, A K Singh, D S Chauhan . *Int. J. Theor. Phys.* **52** 266 (2013)
[11] D R K Reddy *Astrophys. Space Sci.* **286** 365 (2003)
[12] D R K Reddy *Astrophys. Space Sci.* **286** 359 (2003)
[13] R Bali, A Pradhan *Chin. Phys. Lett.* **24** 585 (2007)
[14] R Bali, R D Upadhaya *Astrophys. Space Sci.* **283** 97 (2003)
[15] R Venkateswarlu, J Satish *Journal of Gravity* **2014** 909374 (2014)
[16] V U M Rao, G K Sreedevi, D Neelima *Astrophys. Space Sci.* **357** 76 (2015)
[17] V U M Rao, V J Sudha *Astrophys. Space. Sci.* **337** 499 (2012)
[18] D C Maurya, R Zia, A Pradhan . *J. Exp. Theor. Phys.* **150** 716 (2016)
[19] R K Knop, *et al. Astrophys. J.* **598** 102 (2003)
[20] M Tegmark, *et al.* (SDSS Collaboration) *Phys. Rev. D* **69** 103501 (2004)
[21] G Hinshaw, *et al.* (WMAP Collaboration) *Astrophys. J. Suppl. Ser.* **180** 225 (2009)
[22] N Komatsu, *et al. Astrophys. J. Suppl. Ser.* **180** 330 (2009)
[23] R R Caldwell, *Phys. Lett.B* **545** 23 (2002)