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ORIGINAL PAPER

Nuclear constraints on the core-crust transition and crustal fraction of moment of inertia of neutron stars

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Abstract: The crustal fraction of moment of inertia in neutron stars is calculated using β-equilibrated nuclear matter obtained from density dependent M3Y effective interaction. The transition density, pressure and proton fraction at the inner edge separating the liquid core from the solid crust of the neutron stars are determined from the thermodynamic stability conditions. The crustal fraction of the moment of inertia can be extracted from studying pulsar glitches. This fraction is highly dependent on the core-crust transition pressure and corresponding density. These results for pressure and density at core-crust transition together with the observed minimum crustal fraction of the total moment of inertia provide a limit for the radius of the Vela pulsar: $R \geq 4.10 + 3.36 M/M_{\odot}$ km.

Keywords: Nuclear EoS; Neutron Star; Core-crust transition; Crustal Moment of Inertia

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1. Introduction

The pulsar glitches, which are discontinuities in rotational frequency during the pulsars spin-down, involve from an isolated component (consisting of superfluid neutrons in crust) a sudden transfer of angular momentum to the entire star through vortex unpinning. The sudden jumps in rotational frequencies ω which may be as large as $\frac{\Delta\omega}{\omega} \sim 10^{-6} - 10^{-9}$ have been observed for many pulsars. The hypothesis of experience of glitches [1] by all radio pulsars is substantiated by observation of glitches. A glitch is a sudden increase in the frequency rotation of a rotation-powered pulsar, which, due to braking provided by the emission of radiation and high-energy particles, generally decreases steadily. This sudden increase in the rotational frequency of pulsar is due to a short time coupling of the faster-spinning superfluid core of the pulsar to its crust, which are usually decoupled. The transfer of angular momentum from core to the surface caused by this brief coupling decreases the measured time period. It is envisaged that the breaking of the magnetic dipole of pulsar ensues coupling which applies a twisting force to the crust causing a brief coupling. The inner crust consists of a crystal lattice of nuclei immersed in a neutron superfluid [2] where core to crust transition occurs. With a regular array of rotational vortices created due to rotation of the pulsar, the superfluid consisting of neutrons (both deeper inside the star and within the inner crust) is entangled. The reason that the rotational frequency of a superfluid is proportional to the density of vortices, as the pulsar slows down these vortices need to gradually move outwards. Although in the crust the vortices are pinned by their interaction with the nuclear lattice, in the star's deep inside this process is freely allowed. Various theoretical models [3–7] differ in important aspects of the stress release mechanism of glitch which are associated with pinned vortices. The crust may get rearranged due to the breaking of vortices or a cluster of vortices may move macroscopically outward by overcoming the pinning force suddenly. This phenomenon results in a glitch due to sudden decrease in the angular momentum of the superfluid within the crust causing a sudden increase in angular momentum of the rigid crust itself. The common feature of all the models is that they agree that the fundamental requirement is the presence of a rigid structure which impedes the motion of rotational vortices present in

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a superfluid and which encompasses enough of the volume of the pulsar to contribute significantly to the total moment of inertia.

In the present work, the equation of state (EoS) used is obtained from the density dependent M3Y effective nucleon-nucleon (NN) interaction (DDM3Y) for which the incompressibility K_∞ for the symmetric nuclear matter (SNM), nuclear symmetry energy $E_{sym}(\rho_0)$ at saturation density ρ_0, the isospin dependent part K_τ of the isobaric incompressibility and the slope L are in excellent agreement with the constraints extracted from measured isotopic dependence of the giant monopole resonances in even-A Sn isotopes, from the neutron skin thickness of nuclei recently, and from analyses of experimental data on isospin diffusion and isotopic scaling in intermediate energy heavy-ion collisions [8, 9]. The core-crust transition in neutron stars is determined [10] by analyzing the stability of the β-equilibrated dense nuclear matter with respect to the thermodynamic stability conditions [11–15]. The mass–radius relation for neutron stars is obtained by solving the Tolman–Oppenheimer–Volkoff Equation (TOV) [16, 17] and then the crustal fraction of moment of inertia is determined using pressure and density at core-crust transition. Since in the Vela pulsar the angular momentum requirements of glitches indicate that 1.4 % of the star's moment of inertia drives these events, the allowed region for masses and radii for Vela pulsar is determined from the condition that the crustal fraction of the total moment of inertia $\frac{\Delta I}{I} > 0.014$ which sets a limit for its radius.

2. Core-crust transition in β-equilibrated neutron star matter

The nuclear matter EoS is obtained using DDM3Y effective interaction. For asymmetric nuclear matter EoS both the isoscalar and the isovector [18, 19] components are needed to be taken into consideration. The nuclear matter calculations uniquely determine the density dependence of this interaction. This is done by solving simultaneously for fixed values of the saturation energy per nucleon ϵ_0 and the saturation density ρ_0 of the cold SNM, the equations that $\epsilon = \epsilon_0$ at $\rho = \rho_0$ together with vanishing of pressure $\frac{\partial\epsilon}{\partial\rho}|_{\rho=\rho_0} = 0$ at this density. The zero range potential is allowed to vary freely with the kinetic energy part ϵ^{kin} of the energy per baryon ϵ over the entire range of ϵ implying that the effective interaction is momentum dependent. This is not merely more logical, but also provides excellent result for the SNM incompressibility K_∞ of the infinite nuclear matter that does not suffer from the superluminosity problem [20]. In a Fermi gas model of interacting baryons with isospin asymmetry $X = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$, $\rho = \rho_n + \rho_p$, where ρ_n, ρ_p and ρ are the neutron, proton and baryonic number densities respectively, the energy per baryon for isospin asymmetric nuclear matter can be derived as [20]

$$\epsilon(\rho, X) = \left[\frac{3\hbar^2 k_F^2}{10 m_b}\right] F(X) + \left(\frac{\rho J_v C}{2}\right)(1 - \beta\rho^n) \tag{1}$$

where m_b is the baryonic rest mass, $k_F = (1.5\pi^2\rho)^{\frac{1}{3}}$ which is equal to the Fermi momentum for SNM, the kinetic energy per baryon $\epsilon^{kin} = \left[\frac{3\hbar^2 k_F^2}{10 m_b}\right] F(X)$ with $F(X) = \left[\frac{(1+X)^{5/3} + (1-X)^{5/3}}{2}\right]$ and $J_v = J_{v00} + X^2 J_{v01}$, J_{v00} and J_{v01} represent the volume integrals of the isoscalar and the isovector parts of the M3Y effective interaction. The isoscalar t_{00}^{M3Y} and the isovector t_{01}^{M3Y} components of M3Y interaction potential are given by

$$\begin{aligned} t_{00}^{M3Y}(s, \epsilon^{kin}) &= +7999\frac{\exp(-4s)}{4s} - 2134\frac{\exp(-2.5s)}{2.5s} \\ &\quad + J_{00}(1 - \alpha\epsilon^{kin})\delta(s) \\ t_{01}^{M3Y}(s, \epsilon^{kin}) &= -4886\frac{\exp(-4s)}{4s} + 1176\frac{\exp(-2.5s)}{2.5s} \\ &\quad + J_{01}(1 - \alpha\epsilon^{kin})\delta(s) \end{aligned} \tag{2}$$

where s represents the relative distance between two interacting baryons, $J_{00} = -276$ MeV fm^3, $J_{01} = +228$ MeV fm^3 and the energy dependence parameter $\alpha = 0.005\,\text{MeV}^{-1}$. While the density dependence is determined from the nuclear matter calculations, the strengths of the Yukawas are extracted by fitting its matrix elements in an oscillator basis to those elements of G-matrix obtained with Reid–Elliott soft core NN interaction. The ranges of the Yukawas are selected to ensure OPEP tails in the relevant channels as well as a short-range part which simulates the σ-exchange process [21]. The density dependence of the interaction accounts for the Pauli blocking effects and the higher order exchange effects [22]. Hence the DDM3Y effective NN interaction is given by $v_{0i}(s, \rho, \epsilon^{kin}) = t_{0i}^{M3Y}(s, \epsilon^{kin}) g(\rho)$ where the density dependence is given by $g(\rho) = C(1 - \beta\rho^n)$ [20] with the constants of density dependence as C and β.

It is worthwhile to mention here that other density dependent forms [23] of the M3Y-Reid effective NN interaction can also provide description of nuclear matter with different values of incompressibility. The density, the pressure and the proton fraction at the inner edge separating the liquid core from the solid crust of the neutron stars determined in the present work are in close agreement with those obtained with other forms of density dependence which correspond to high nuclear incompressibility.

The β-equilibrated nuclear matter EoS is obtained by evaluating the asymmetric nuclear matter EoS at the isospin asymmetry $X = 1 - 2x_p$ determined from the β-equilibrium proton fraction x_p $[= \frac{\rho_p}{\rho}]$, obtained by solving $\hbar c(3\pi^2\rho x_p)^{1/3} = -\frac{\partial\epsilon(\rho, x_p)}{\partial x_p} = +2\frac{\partial\epsilon}{\partial X}$. The requirement of thermodynamical method is that the system must obey the intrinsic stability condition $V_{thermal} > 0$ which is given by [24]

$$V_{thermal} = \rho^2 \left[2\rho \frac{\partial \epsilon}{\partial \rho} + \rho^2 \frac{\partial^2 \epsilon}{\partial \rho^2} - \rho^2 \frac{\left(\frac{\partial^2 \epsilon}{\partial \rho \partial x_p} \right)^2}{\frac{\partial^2 \epsilon}{\partial x_p^2}} \right] \tag{3}$$

and obviously, it goes to zero at the inner edge separating the the solid crust from the liquid core since it corresponds to a phase transition from the inhomogeneous matter at low densities to the homogeneous matter at high densities. The core-crust transition density ρ_t, pressure P_t and proton fraction $x_{p(t)}$ of the neutron stars are obtained [10] by setting $V_{thermal} = 0$ which goes to negative with decreasing density.

3. Crustal fraction of moment of inertia in neutron stars

The crustal fraction of the moment of inertia $\frac{\Delta I}{I}$ can be expressed as a function of gravitational mass of the star M and its radius R by the following approximate expression [2]

$$\frac{\Delta I}{I} \approx \frac{28\pi \mathrm{P}_t R^3}{3Mc^2} \left(\frac{1 - 1.67\xi - 0.6\xi^2}{\xi} \right) \times \left(1 + \frac{2\mathrm{P}_t}{\rho_t m_b c^2} \frac{(1 + 7\xi)(1 - 2\xi)}{\xi^2} \right)^{-1} \tag{4}$$

where $\xi = \frac{GM}{Rc^2}$, ρ_t and P_t are the density and the pressure, respectively, at the core to crust transition. As obvious from the above equation the major dependence is on the value of P_t, since ρ_t enters only as a correction term. The fact that from the observations of pulsar glitches the crustal fraction of the moment of inertia can be inferred, makes it particularly interesting [25].

It has been shown that the glitches show a self-sustaining instability for which the star prepares over a waiting time [2]. The glitches in the Vela pulsar suggests that the angular momentum should be such that more than 1.4 % of the moment of inertia drives these events. Therefore, if glitches originate in the liquid of the inner crust, it would imply that $\frac{\Delta I}{I} > 1.4\,\%$.

4. Tolman–Oppenheimer–Volkoff equation and mass–radius relation

In general relativity, the structure of a spherically symmetric body of isotropic material which is in static gravitational equilibrium is given by the Tolman–Oppenheimer–Volkoff (TOV) equation [16, 17]

$$\frac{dP(r)}{dr} = -\frac{G}{c^4} \frac{[\varepsilon(r) + P(r)][m(r)c^2 + 4\pi r^3 P(r)]}{r^2 [1 - \frac{2Gm(r)}{rc^2}]} \tag{5}$$

where $\varepsilon(r) = (\epsilon + m_b c^2)\rho(r)$, $m(r)c^2 = \int_0^r \varepsilon(r') d^3 r'$

where $\varepsilon(r)$ and $P(r)$ are the energy density and pressure at a radial distance r from the centre whereas $m(r)$ is the mass of the star contained inside radius r. The TOV equation can be easily solved numerically for masses and radii using Runge–Kutta method. The $\varepsilon(r)$ and $P(r)$ are provided by the EoS. The size of the star is determined by the boundary condition $P(r) = 0$ at the surface R of the star and the total mass M of the star integrated up to R is given by $M = m(R)$ [26]. Being the initial value problem, the numerical solution of TOV equation requires single integration constant, the pressure P_c at the center $r = 0$ of the star calculated at a given central density ρ_c. The masses of slowly rotating neutron stars are very close [27–29] to those obtained by solving TOV equation.

The moment of inertia of a neutron star can be calculated by assuming it to be rotating slowly with a uniform angular velocity Ω [30, 31]. The angular velocity $\bar{\omega}(r)$ of a point in the star measured with respect to the angular velocity of the local inertial frame is determined by the equation

$$\frac{1}{r^4} \frac{d}{dr} \left[r^4 j \frac{d\bar{\omega}}{dr} \right] + \frac{4}{r} \frac{dj}{dr} \bar{\omega} = 0 \tag{6}$$

where

$$j(r) = e^{-\phi(r)} \sqrt{1 - \frac{2Gm(r)}{rc^2}}. \tag{7}$$

The function $\phi(r)$ is constrained by the condition

$$e^{\phi(r)} \mu(r) = \text{constant} = \mu(R) \sqrt{1 - \frac{2GM}{Rc^2}} \tag{8}$$

where the chemical potential $\mu(r)$ is defined as

$$\mu(r) = \frac{\varepsilon(r) + P(r)}{\rho(r)}. \tag{9}$$

Using these relations, Eq. (6) can be solved subject to the boundary conditions that $\bar{\omega}(r)$ is regular as $r \to 0$ and $\bar{\omega}(r) \to \Omega$ as $r \to \infty$. The moment of inertia of the star can then be calculated using the definition $I = J/\Omega$, where the total angular momentum J is given by

$$J = \frac{c^2}{6G} R^4 \frac{d\bar{\omega}}{dr}\Big|_{r=R}. \tag{10}$$

5. Theoretical calculations

The values of the saturation density ρ_0 and the saturation energy per baryon ϵ_0 of SNM used in the calculations are

0.1533 fm^{-3} [32] and -15.26 MeV [33], respectively. The co-efficient of the volume term a_v of the liquid drop model mass formula represents the saturation energy per baryon and can be determined by fitting the atomic mass excesses (experimental and estimated) from Audi–Wapstra–Thibault atomic mass table [34] by minimizing the mean square deviation. In such calculations the corrections for the electronic binding energy [35] are included. In a recent work that includes surface symmetry energy term, Wigner term, shell correction and proton form factor correction to Coulomb energy also, a_v turns out to be 15.4496 MeV and when A^0 and $A^{1/3}$ terms are also included [36] it turns out to be 14.8497 MeV. Using the usual value of $\alpha = 0.005$ MeV^{-1} for the parameter of energy dependence of the zero range potential and $n = \frac{2}{3}$, the values obtained for the constants of density dependence C and β and the SNM incompressibility K_∞ are 2.2497, 1.5934 fm^2 and 274.7 MeV, respectively. The value of -15.26 ± 0.52 MeV of the saturation energy per baryon, more or less, covers the entire range for which the values of $C = 2.2497 \pm 0.0420$, $\beta = 1.5934 \pm 0.0085$ fm^2 and the SNM incompressibility $K_\infty = 274.7 \pm 7.4$ MeV [20] are obtained.

The stability of the β-equilibrated dense matter in neutron stars is investigated and the location of the inner edge of their crusts and core-crust transition density and pressure are determined using the DDM3Y effective NN interaction. The results for the transition density, pressure and proton fraction at the inner edge separating the liquid core from the solid crust of neutron stars are calculated and presented in Table 1 for $n = \frac{2}{3}$. The symmetric nuclear matter incompressibility K_∞, nuclear symmetry energy at saturation density $E_{sym}(\rho_0)$, the slope L and isospin dependent part K_τ of the isobaric incompressibility are also tabulated since these are all in excellent agreement with the recently extracted constraints from the measured isotopic dependence of the giant monopole resonances in even-A Sn isotopes [37], from the neutron skin thickness of nuclei, and from analyses of experimental data on isospin diffusion and isotopic scaling in intermediate energy heavy-ion collisions.

The calculations for masses and radii are performed using the EoS covering the crustal region of a compact star which are Feynman–Metropolis–Teller (FMT) [38], Baym–Pethick–Sutherland (BPS) [39] and Baym–Bethe–Pethick (BBP) [40] upto number density of 0.0582 fm^{-3} and β-equilibrated neutron star matter beyond. The values of I obtained by solving Eq. (6) subject to the boundary conditions stated earlier are listed in Table 2 along with masses M, radii R and crustal thickness ΔR of neutron stars. Once masses and radii are determined, $\frac{\Delta I}{I}$ are obtained from Eq. (4) and listed in Table 2. In Fig. 1, variation of mass with central density is plotted for slowly rotating neutron stars for the present nuclear EoS. In Fig. 2, the mass–radius relation of slowly rotating neutron stars is shown. Using Eq. (4) again the mass–radius relation is obtained for fixed values of $\frac{\Delta I}{I}$, ρ_t and P_t. This is then plotted in the same figure for $\frac{\Delta I}{I}$ equal to 0.014. For Vela pulsar, the constraint $\frac{\Delta I}{I} > 1.4\,\%$ implies that allowed mass-radius lie to the right of the line defined by $\frac{\Delta I}{I} = 0.014$ (for $\rho_t = 0.0938$ fm^{-3}, $P_t = 0.5006$ MeV fm^{-3}). This condition is given by the inequality $R \geq 4.10 + 3.36 M/M_\odot$ km.

The calculations are performed for five different n values that correspond to SNM incompressibility ranging from $\sim$180 to 330 MeV. For each case, the constants C and β obtained by reproducing the ground state properties of SNM become different leading to five different sets of these three parameters. We certainly cannot change strengths and ranges of the M3Y interaction. In Table 3, the variations of the core-crust transition density, pressure and proton fraction for β-equilibrated neutron star matter, symmetric nuclear matter incompressibility K_∞, isospin dependent part K_τ of isobaric incompressibility, neutron star's maximum mass with corresponding radius and crustal thickness with parameter n are listed along with corresponding Vela pulsar constraints. It is important to mention here that recent observations of the binary millisecond pulsar J1614-2230 by Demorest et al. [41] suggest that the masses lie within 1.97 ± 0.04 $M_\odot$ where $M_\odot$ is the solar mass. Recently the radio timing measurements of the pulsar PSR J0348 + 0432 and its white dwarf companion have confirmed the mass of the pulsar to be in the range 1.97–2.05 $M_\odot$ at 68.27 % or 1.90–2.18 $M_\odot$ at 99.73 % confidence [42]. The observed 1.97 ± 0.04 $M_\odot$ neutron star rotates with 3.1 ms and results quoted in Table 2 are for non-rotating case. Similar work using M3Y effective interaction using the so called CDM3Y6 [43] density

Table 1 Results of present calculations for $n = \frac{2}{3}$ of symmetric nuclear matter incompressibility K_∞, nuclear symmetry energy at saturation density $E_{sym}(\rho_0)$, the slope L and isospin dependent part K_τ of the isobaric incompressibility (all in MeV) [9] are tabulated along with the density, pressure and proton fraction at the core-crust transition for β-equilibrated neutron star matter and corresponding Vela pulsar constraint

K_∞	$E_{sym}(\rho_0)$	L	K_τ
274.7 ± 7.4	30.71 ± 0.26	45.11 ± 0.02	-408.97 ± 3.01
ρ_t (fm^{-3})	P_t (MeV fm^{-3})	$x_{p(t)}$	Vela pulsar R (km)
0.0938	0.5006	0.0308	$R \geq 4.10 + 3.36 M/M_\odot$

Table 2 Radii, masses, total and crustal fraction of moment of inertia and crustal thickness as functions of central density ρ_c

ρ_c fm^{-3}	R km	M $M_\odot$	I $M_\odot$ km^2	$\frac{\Delta I}{I}$ Fraction	ΔR km
2.00	8.6349	1.8277	70.88	0.0055	0.2462
1.90	8.7598	1.8467	73.83	0.0057	0.2523
1.80	8.8957	1.8651	77.00	0.0060	0.2598
1.70	9.0444	1.8824	80.38	0.0063	0.2696
1.60	9.2052	1.8980	83.97	0.0067	0.2806
1.50	9.3810	1.9109	87.70	0.0072	0.2951
1.40	9.5710	1.9197	91.52	0.0079	0.3121
1.39	9.5911	1.9203	91.91	0.0080	0.3144
1.38	9.6109	1.9208	92.29	0.0080	0.3161
1.37	9.6314	1.9213	92.67	0.0081	0.3185
1.36	9.6514	1.9217	93.05	0.0082	0.3203
1.35	9.6718	1.9220	93.43	0.0083	0.3222
1.34	9.6928	1.9223	93.81	0.0084	0.3248
1.33	9.7141	1.9225	94.18	0.0085	0.3275
1.32	9.7349	1.9226	94.55	0.0085	0.3296
1.31	9.7559	1.9227	94.93	0.0086	0.3318
1.30	9.7770	1.9226	95.30	0.0087	0.3340
1.20	9.9995	1.9173	98.85	0.0098	0.3620
1.10	10.2371	1.9004	101.88	0.0112	0.3970
1.00	10.4902	1.8675	103.94	0.0132	0.4441
0.90	10.7544	1.8127	104.42	0.0158	0.5066
0.80	11.0239	1.7285	102.47	0.0197	0.5929
0.70	11.2865	1.6064	97.04	0.0255	0.7148
0.60	11.5245	1.4369	87.06	0.0344	0.8952
0.59	11.5456	1.4170	85.78	0.0356	0.9175
0.58	11.5666	1.3965	84.44	0.0368	0.9411
0.57	11.5874	1.3753	83.04	0.0381	0.9663
0.56	11.6073	1.3536	81.58	0.0394	0.9924
0.55	11.6262	1.3313	80.07	0.0408	1.0193
0.50	11.7135	1.2104	71.65	0.0492	1.1792
0.45	11.7830	1.0734	61.88	0.0602	1.3897
0.40	11.8388	0.9206	51.00	0.0752	1.6801
0.30	12.0129	0.5808	28.54	0.1249	2.7618
0.25	12.3703	0.4103	19.24	0.1686	3.9149
0.24	12.5113	0.3779	17.73	0.1805	4.2542
0.23	12.6944	0.3464	16.35	0.1942	4.6511
0.22	12.9314	0.3159	15.14	0.2103	5.1189
0.21	13.2434	0.2867	14.12	0.2296	5.6802
0.20	13.6576	0.2587	13.31	0.2537	6.3643
0.19	14.2131	0.2323	12.74	0.2847	7.2125
0.18	14.9725	0.2075	12.47	0.3265	8.2904
0.17	16.0398	0.1845	12.59	0.3863	9.7057
0.16	17.5771	0.1634	13.25	0.4767	11.6254
0.15	19.8913	0.1445	14.77	0.6254	14.3634
0.14	23.5740	0.1278	17.88	0.8972	18.5215

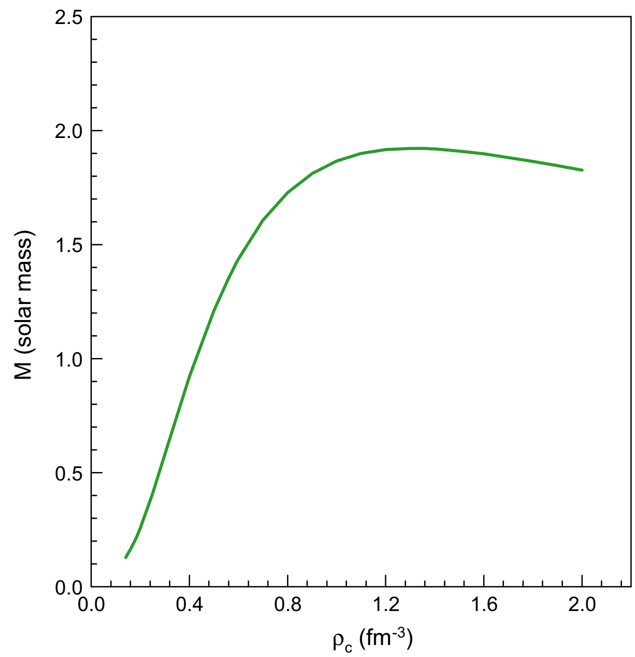

Fig. 1 Variation of mass with central density for slowly rotating neutron stars for the present nuclear EoS

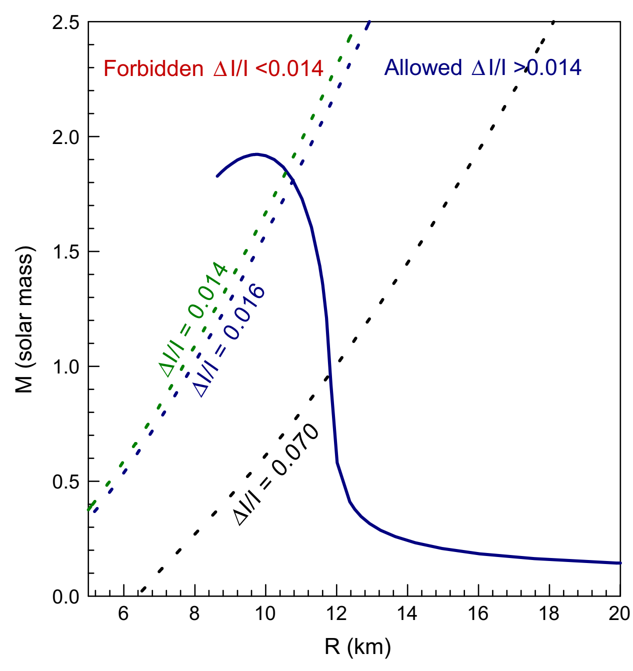

Fig. 2 The mass–radius relation of slowly rotating neutron stars for the present nuclear EoS. The constraint of $\frac{\Delta I}{I} > 1.4\,\%\,(1.6, 7\,\%)$ for the Vela pulsar implies that to the right of the line defined by $\frac{\Delta I}{I} = 0.014\,(0.016, 0.07)$ (for $\rho_t = 0.0938$ fm^{-3}, $P_t = 0.5006$ MeV fm^{-3}), allowed masses and radii lie

dependence can predict ~ 2 M$_\odot$ neutron stars. For rotating stars [29] present EoS predict masses higher than the lower limit of 1.93 M$_\odot$ for maximum mass of neutron stars. We

Table 3 Variations of the core-crust transition density, pressure and proton fraction for β-equilibrated neutron star matter, symmetric nuclear matter incompressibility K_∞ and isospin dependent part K_τ of isobaric incompressibility with parameter n

n Expt.	ρ_t fm^{-3} Values	P_t MeV fm^{-3} - - →	$x_{p(t)}$ →	K_∞ MeV 250–270	K_τ MeV -370 ± 120	Maximum mass $M_\odot$ 1.97 ± 0.04	Radius km	Crustal thickness km
1/6	0.0797	0.4134	0.0288	182.13	−293.42	1.4336	8.5671	0.4009
				$R(\mathrm{km}) \geq$	$4.48 +$	$3.37M/M_\odot$		
1/3	0.0855	0.4520	0.0296	212.98	−332.16	1.6002	8.9572	0.3743
				$R(\mathrm{km}) \geq$	$4.31 +$	$3.36M/M_\odot$		
1/2	0.0901	0.4801	0.0303	243.84	−370.65	1.7634	9.3561	0.3515
				$R(\mathrm{km}) \geq$	$4.19 +$	$3.36M/M_\odot$		
2/3	0.0938	0.5006	0.0308	274.69	−408.97	1.9227	9.7559	0.3318
				$R(\mathrm{km}) \geq$	$4.10 +$	$3.36M/M_\odot$		
1	0.0995	0.5264	0.0316	336.40	−485.28	2.2335	10.6408	0.3088
				$R(\mathrm{km}) \geq$	$3.99 +$	$3.36M/M_\odot$		

have used the same value of $\rho_0 = 0.1533$ fm^{-3} since we want to keep consistency with all our previous works on nuclear matter. We would like to mention that if instead we would have used the value of 0.16 fm^{-3} [44] for ρ_0, the value of K_∞ would have been slightly higher by ~ 2 MeV and correspondingly maximum mass of neutron stars by ~ 0.01 M$_\odot$.

6. Results and discussion

Recently, it is conjectured that the observed glitches in the Vela pulsar require an additional storage of angular momentum and to explain the phenomenon [45] the crust may not be enough. Large pulsar frequency glitches can be interpreted as sudden transfers of angular momentum between the neutron superfluid permeating the inner crust and the rest of the star. In spite of the absence of viscous drag, the neutron superfluid is strongly coupled to the crust due to non-dissipative entrainment effects. It is often argued that these effects may put a constraint on the maximum amount of angular momentum that during glitches can possibly be transferred [46]. We find that the present EoS can accommodate large crustal moments of inertia and that large enough transition pressures can be generated to explain the large Vela glitches without invoking an additional angular-momentum reservoir beyond that confined to the solid crust. Our results suggest that the crust may be enough [47] which can be substantiated from Table 2 that $\frac{\Delta I}{I} > 0.014$ for pulsars with masses 1.8 M$_\odot$ or less. Newer observational data [48] claims slightly higher estimate (1.6 %) based on glitch activity. This minute change neither affects the conclusions nor warrants any new idea of the neutron superfluidity extending partially into the core. However, if the phenomenon of crustal entrainment due to the Bragg reflection of unbound neutrons by the lattice ions is taken into account then [45, 46] a much higher fraction of the moment of inertia (7 % instead of 1.4–1.6 %) has to be associated to the crust. This causes drastic modification of the moment of inertia of the superfluid component. If $\frac{\Delta I}{I} > 0.07$ is considered, then the corresponding allowed masses and radii will be given by $R \geq 7.60 + 3.71M/M_\odot$ instead of $R \geq 4.10 + 3.36M/M_\odot$ which is shown in Fig. 2. But from Table 2 this would mean maximum mass $\sim 1.M_\odot$ which contradicts the observed mass of Vela pulsar [49] and suggests that this fraction can be at most 3.6 % due to crustal entrainment.

The results listed in Table 3 suggest that SNM incompressibility do have some effect in determining the crustal fraction of moment of inertia and on the Vela Pulsar Radius Constraint like some other recent studies [50]. But the incompressibility values of about 15 MeV window around 274.7 MeV corresponding to $n = \frac{2}{3}$ is experimentally supported. The present status of experimental determination of the SNM incompressibility from the compression modes of isoscalar giant dipole resonance (ISGMR) and isoscalar giant dipole resonance (ISGDR) of nuclei suggests [51] that due to lack of self consistency in HF-RPA calculations of the strength functions of ISGMR result in shifts in the calculated values of the centroid energies which may be larger in magnitude than the present experimental uncertainties. It is important to mention here that the predictions of low values of $K_\infty \sim 210$–220 MeV) are due to the use of a not fully self-consistent Skyrme calculations [51]. The Skyrme parameterizations of SLy4 type with this drawback corrected, predict higher values of $K_\infty \sim 230$–240 MeV [51]. Furthermore, it is possible to build bona fide Skyrme interactions for which SNM incompressibility is in the range of

250–270 MeV that is close to the theoretical values obtained using relativistic EoS. Thus, from the ISGMR experimental data, it may be concluded that $K_\infty \approx 240 \pm 20$ MeV. The constant of density dependence β has the dimension of cross section for $n = \frac{2}{3}$ and can be interpreted as the isospin averaged effective NN interaction cross section in ground state SNM. For a nucleon in ground state of SNM, $k_F \approx 1.3$ fm^{-1} and $q_0 \sim \hbar k_F c \approx 260$ MeV which yields 'in medium' effective cross section of ≈12 mb from the Dirac–Brueckner–Hartree–Fock [52] calculations and the present result for $\beta = 1.5934 \pm 0.0085$ fm^2 is reasonably close. This value of β corresponding to the value of the parameter $n = \frac{2}{3}$ and the baryonic density of 0.1533 fm^{-3} provides nuclear mean free path of about 4 fm which agrees well [53] with that obtained from other method.

The rigorous way of dealing with core-crust transition is producing a unified EoS and evaluating the density where the clustered phase becomes energetically disfavored with respect to the homogeneous solution [54]. It should be clarified here that the crustal region of the compact star in the present work consists of FMT+BPS+BBP up to number density of 0.0582 fm^{-3} and β-equilibrated neutron star matter up to core-crust transition number density of 0.0938 fm^{-3} which is far beyond 0.0582 fm^{-3}, otherwise we would have taken a unified EoS. The three different methods to calculate the transition density are the thermodynamical spinodal (the method used in this work), the dynamical spinodal within the Vlasov formalism and the relativistic random phase approximation. It is shown that the last two methods [55] give similar results, confirming previous studies [56, 57]. The thermodynamical method also gives a good estimate of the transition density [55, 58] and involves simpler calculations.

7. Conclusions

In summary, the DDM3Y effective interaction which provides a unified description of elastic and inelastic scattering, proton-, α-, cluster- radioactivities and nuclear matter properties, also provides an excellent description of the β-equilibrated neutron star matter which is stiff enough at high densities to reconcile with the recent observations of the massive compact stars [27–29] while the corresponding symmetry energy is supersoft as preferred by the FOPI/GSI experimental data [59, 60]. The experimental range of values quoted in Table 3 along with discussions provided above justifies the parameter set of $n = \frac{2}{3}$, $C = 2.2497 \pm 0.0420$ and $\beta = 1.5934 \pm 0.0085$ fm^2. The neutron star core-crust transition density, pressure and proton fraction determined from the thermodynamic stability condition to be $\rho_t = 0.0938$ fm^{-3}, $P_t = 0.5006$ MeV fm^{-3} and $x_{p(t)} = 0.0308$, respectively, along with observed minimum crustal fraction of the total moment of inertia of the Vela pulsar provide a limit for its radius. It is somewhat different from the other estimates [2, 61] and imposes a constraint $R \geq 4.10 + 3.36M/M_\odot$ km for the mass–radius relation of Vela pulsar like neutron stars.

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