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ORIGINAL PAPER

Stochastic resonance in FizHugh-Nagumo model driven by multiplicative signal and non-Gaussian noise

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Abstract: The stochastic resonance phenomenon in FizHugh-Nagumo neural system induced by a multiplicative periodic signal and non-Gaussian noise is studied. Based on path integral approach and two-state theory, the Fokker–Planck equation and signal-to-noise ratio are derived. By analyzing the influence of different parameters in the optimization of signal-to-noise ratio, we observe that the conventional stochastic resonance and double stochastic resonance occur in FizHugh-Nagumo neural model under different values of system parameters. Furthermore, there is a critical value of non-Gaussian noise intensity D, above which the increase of D weakens the resonant effect and below which it enhances the resonant effect.

Keywords: Non-Gaussian noise; Multiplicative periodic signal; FizHugh-Nagumo neural system; Stochastic resonance

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1. Introduction

Since stochastic resonance (SR) phenomenon is first found in a study of periodic changes of the ancient climate [1, 2], it has been extensively studied both theoretically and experimentally in many scientific fields, including biophysics, due to its potential application [3–9]. SR is the name coined for the rather counterintuitive fact that response of a nonlinear system to a periodic signal may be enhanced through the addition of an optimal amount of noise. And the typical signature of SR is the existence of a maximum of the signal-to-noise ratio (SNR) as a function of noise intensity.

FizHugh-Nagumo (FHN) neural model is one of the simplified modifications of the widely known Hodgkin-Huxley model [10], which describes neuron dynamics and in general the dynamics of excitable systems in different fields, such as the kinetics of chemical reactions and solid state physics [11–13]. A vast majority of studies on SR have been investigated in FHN neural system. For example, Lindner et al. [14] have studied FHN system under the influence of white Gaussian noise in the excitable regime, where coherence resonance phenomenon is observed. Tora et al. [15] have shown the existence of a system size coherence resonance effect in the coupled FHN models. The SR effect has been investigated in a FHN neural model driven by colored noise by Nozaki and Yamamoto [16]. SR in a FHN system with time-delayed feedback has also been analyzed by Wu and Zhu [17]. They have revealed that SR of the system is a non-monotonic function of the noise intensity and the signal period and variation of the time-delayed feedback can induce periodic SR in the system. Previous research is in the case of Gaussian noise, but an experimental research shows that some noise in the nervous, biological and physical systems tend to non-Gaussian distribution. Because non-Gaussian noise leads to a non-markov process and the mathematical expression is complex, studies of non-Gaussian noise are less. Zhang and Jin [18] have investigated resonant effect in FHN neural system driven by non-Gaussian noise and an additive periodic signal. They have revealed that addition of non-Gaussian noise is conductive to the enhancement of the response to the output signal of FHN neural system. However, SR induced by the multiplicative signal and non-Gaussian noise has not been studied in FHN model. Therefore, SR phenomenon induced by a multiplicative signal and non-Gaussian noise in FHN model needs to be investigated.

In this paper, we introduce a multiplicative periodic signal to the reduced one-dimensional FHN model with

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non-Gaussian noise. Our goal is to reveal some new features of SR due to presence of multiplicative signal.

2. FHN neural model

2.1. Stationary properties of the model

FHN neural model can be written as follows [19].

$$\frac{dv}{dt} = v(a-v)(v-1) - w, \tag{1}$$

$$\frac{dw}{dt} = bv - rw. \tag{2}$$

Where, in neural context, v is a fast variable denoting neuron membrane voltage and w is a slow or recovery variable, which is related to the time dependent conductance of the potassium channels in the membrane; $0 < a < 1$ is essentially the threshold value; b and r are positive constants. For the sake of simplicity, we take $r = 1$. By means of the adiabatic elimination method, the one-dimensional Langevin equation for FHN model can be obtained as [19].

$$\frac{dv}{dt} = v(a-v)(v-1) - bv + \xi(t). \tag{3}$$

The potential function,

$$U(v) = \frac{1}{4}v^4 - \frac{a+1}{3}v^3 + \frac{a+b}{2}v^2. \tag{4}$$

Equation (3) has two stable states: $v_1 = 0$, which represents the neurons of cells in the resting state. $v_2 = \frac{a+1+\sqrt{(a-1)^2-4b}}{2}$, which represents the neurons of cells in the excited state and an unstable state: $v_3 = \frac{a+1-\sqrt{(a-1)^2-4b}}{2}$. In its bistable regime, i.e., $b < \frac{(a-1)^2}{4}$. If we consider the external environmental fluctuation (i.e., temperature, ionic strength, etc.) and the intrinsic thermal fluctuation, the dimensionless form of the one-dimensional Langevin equation [Eq. (3)] reads

$$\frac{dv}{dt} = v(a-v)(v-1) - bv + vA\cos(\omega t) + v\eta(t) + \xi(t). \tag{5}$$

Where A and ω are amplitude and frequency of the multiplicative periodic signal, respectively. And all variables are normalized. $\eta(t)$ is non-Gaussian white noise and its statistical properties are described by:

$$\frac{d\eta(t)}{dt} = -\frac{1}{\tau}\frac{dV_q(\eta)}{d\eta} + \frac{1}{\tau}\varepsilon(t). \tag{6}$$

Here

$$V_q(\eta) = \frac{D}{\tau(q-1)}\ln\left(1 + \frac{\tau}{D}(q-1)\frac{\eta^2}{2}\right), \langle\eta(t)\rangle = 0, \tag{7}$$

$$\langle\eta^2(t)\rangle = \begin{cases} \frac{2D}{\tau(5-3q)}, & q < \frac{5}{3} \\ \infty, & \frac{5}{3} \le q < 3 \end{cases}$$

where $\varepsilon(t)$ and $\xi(t)$ are Gaussian white noise and their statistical properties are given by:

$$\langle\varepsilon(t)\rangle = \langle\xi(t)\rangle = 0,$$
$$\langle\varepsilon(t)\varepsilon(t')\rangle = 2D\delta(t-t'),$$
$$\langle\xi(t)\xi(t')\rangle = 2Q\delta(t-t'). \tag{8}$$

Here Q is the intensity of the additive noise $\xi(t)$; D is the intensity of $\varepsilon(t)$; and τ is the correlation time of non-Gaussian noise $\eta(t)$. Using the path integral approach [20], the non-Gaussian white noise can be written as

$$\frac{d\eta(t)}{dt} = -\frac{1}{\tau_1}\eta(t) + \frac{1}{\tau_1}\varepsilon_1(t), \tag{9}$$

Here, $\varepsilon_1(t)$ is Gaussian white noise.

$$\langle\varepsilon_1(t)\rangle = 0, \langle\varepsilon_1(t)\varepsilon_1(t')\rangle = 2D_1\delta(t-t'),$$

where, τ_1 is an effective correlation time of noise and D_1 is an effective intensity of noise:

$$\tau_1 = \frac{2(2-q)}{5-3q}\tau, D_1 = \frac{2(2-q)^2}{5-3q}D. \tag{10}$$

Parameter q is the deviation of non-Gaussian noise from Gaussian behavior. When $q \to 1$, $\eta(t)$ approximates as colored Gaussian noise that its associated time is τ_1 and noise intensity is D_1. Using the unified colored noise approximation [21], Eq. (8) takes the form:

$$\frac{\partial\rho(v,t)}{\partial t} = -\frac{\partial}{\partial v}[A(v)\rho(v,t)] + \frac{\partial^2}{\partial v^2}[B(v)\rho(v,t)]. \tag{11}$$

In which

$$C(v,\tau) = 1 - \tau_1[-2v^2 + (a+1)v],$$

$$f(v) = \frac{v(a-v)(v-1) - bv + vA\cos(\omega t)}{C(v,\tau)},$$

$$B(v) = \frac{D_1V^2 + Q}{C^2(v,\tau)},$$

$$A(v) = f(v) + \frac{1}{2}B'(v).$$

Thus, quasi-stationary probability distribution function (SPDF) $\rho_{st}(v)$ can be derived and Eq. (11) in the adiabatic limit as

$$\rho_{st}(v) = \frac{N}{\sqrt{B(v)}}e^{-\frac{V(v)}{D_1}}. \tag{12}$$

where N is normalization constant and generalized potential reads:

$$V(v) = -\int \frac{[v(a-v)(v-1) - bv + vA\cos(\omega t)][1-\tau_1(-2v^2+(a+1)v)]}{v^2+\frac{Q}{D_1}}dv$$
$$= \frac{k_1}{2}v^4 - \frac{k_2}{3}v^3 + (k_3 - k_1 h)v^2 + (k_1 h^2 - k_3 h + k_5)\ln(v^2 + \frac{Q}{D_1})$$
$$+ \sqrt{h}(k_4 - k_2 h)\arctan\frac{D_1}{Q}v + \left[-k_1 v^2 + \frac{k_2}{3}v + \left(k_1 h - \frac{1}{2}\right)\ln\left(v^2 + \frac{Q}{D_1}\right) - \frac{k_3}{3}\sqrt{h}\arctan\sqrt{\frac{D_1}{Q}v}\right]A\cos(\omega t), \tag{13}$$

In which

$$k_1 = \tau_1, \quad k_2 = 3(a+1)\tau_1,$$
$$k_3 = \frac{1}{2}[1 + (a+1)^2\tau_1 + 2\tau_1(a+b)],$$
$$k_4 = (a+1)[(a+b)\tau_1 + 1], \quad k_5 = \frac{1}{2}(a+b), h = \frac{Q}{D_1}. \tag{14}$$

2.2. SNR of the model

In order to calculate rate of transition out of the state $v_{1,\,2}$, we consider the mean first-passage time (MFPT) T_1 of the system from v_1 to reach v_2 and T_2 from v_2 to v_1. By means of the steepest-descent approximation [21, 22], the modified MFPT can be obtained as

$$T_1 = \frac{2\pi}{\sqrt{|U''(v_1)U''(v_u)|}}e^{\frac{V(v_u)-V(v_1)}{D_1}},$$
$$T_2 = \frac{2\pi}{\sqrt{|U''(v_2)U''(v_u)|}}e^{\frac{V(v_u)-V(v_2)}{D_1}}. \tag{15}$$

Then, the transition rates are given by

$$W_1 = \frac{1}{T_1} = \frac{1}{2\pi}\sqrt{|[3v_1^2 - 2(a+1)v_1 + a + b][3v_u^2 - 2(a+1)v_u + a + b]|}e^{\frac{V(v_1)-V(v_u)}{D_1}},$$
$$W_2 = \frac{1}{T_2} = \frac{1}{2\pi}\sqrt{|[3v_2^2 - 2(a+1)v_2 + a + b][3v_u^2 - 2(a+1)v_u + a + b]|}e^{\frac{V(v_2)-V(v_u)}{D_1}}. \tag{16}$$

Within the framework of the theory of SR [23, 24], the expression for SNR of the system can be given by:

$$SNR = \frac{A^2\pi(\mu_1\beta_2 + \mu_2\beta_1)^2}{4\mu_1\mu_2(\mu_1 + \mu_2)}. \tag{17}$$

In which $\mu_1 = W_1|_{A\cos(\omega t)=0}$, $\mu_2 = W_2|_{A\cos(\omega t)=0}$, $\beta_1 = -\frac{dW_1}{dA\cos(\omega t)}|_{A\cos(\omega t)=0}$ and $\beta_2 = -\frac{dW_2}{dA\cos(\omega t)}|_{A\cos(\omega t)=0}$. By virtue of the expression of Eq. (15) for the SNR, the effects of parameters on the SNR can be analyzed by numerical calculations. Here, we take $a = 0.5$, $b = 0.01$.

3. Results and discussion

In Fig. 1, we present the effect of parameter q on SPDF. As we can see, curves show an asymmetric double-peak structure and height of left peak is higher than right peak, which illustrates that probability of distribution of membrane variable voltage is bigger in $v_1 = 0$. Meanwhile, left peak emerges in $v_1 = 0$ and right peak appears near $v_2 = 1.0$, which are the positions of two states in potential function $U(v)$. More interestingly, as parameter q increasing, height of the two peaks are reduced and left peak reduces more faster than right peak, implying that there is a transfer of probability from v_1 to v_2. However, the double-peak structure of the curves always remains the same. It can be said that parameter q can not induce phase transition.

The influence of parameter q on SNR as a function of multiplicative noise intensity D is plotted in Fig. 2. The existence of the maximum in these curves is the identifying characteristic of the SR phenomenon induced by the multiplicative noise. One can clearly see form Fig. 2 that position of the maximum shifts to a smaller value of D as q increases. But maximum of the SNR almost remains the same. The influence of parameter q on the SNR as a function of additive noise intensity Q is also plotted in Fig. 3 when the other parameters are fixed. The results are opposite from Fig. 2. From Fig. 3, one can see that, as q increases, position of the maximum practically remain the same and the maximum of the SNR is decreased, namely the increase of q can weakens the resonant effect.

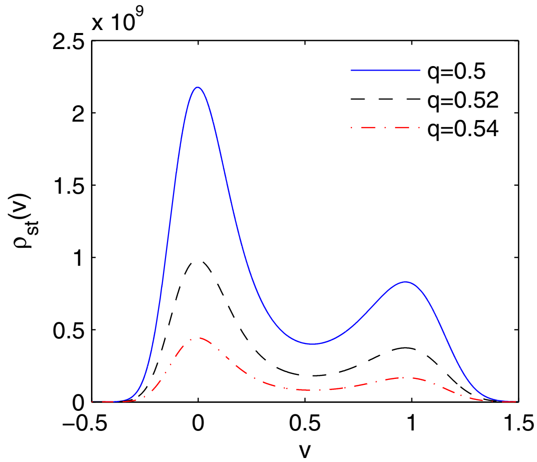

Fig. 1 Stationary probability distribution $\rho_{st}(v)$ versus normalized v for $D = 0.01$, $Q = 0.001$, $A = 0$, and $\tau = 0.1$

Compared with the case of an additive signal in which the increase of q strengthens the resonant effect [18], one can find that increasing q plays an opposite role on resonant effect for the cases of multiplicative and additive signals.

Here we take $q = 1.5$. We analyze the effect of correlation time τ of the non-Gaussian noise $\eta(t)$ on the SNR as a function of the multiplicative noise intensity D, when all other parameters are fixed in Fig. 4. One can see that maximum of SNR decreases as the correlation time τ increases. That is, the increase of τ weakens the resonant effect. Figure 5 shows the effect of τ on the SNR as a function of additive noise intensity Q. From Fig. 5, one can see that the SNR shows double-peak structure. We call this effect doubly stochastic resonance (DSR), which has been studied by Zaikin et al. [25]. They have reported the effect of DSR with Gaussian noise in nonlinear extended systems

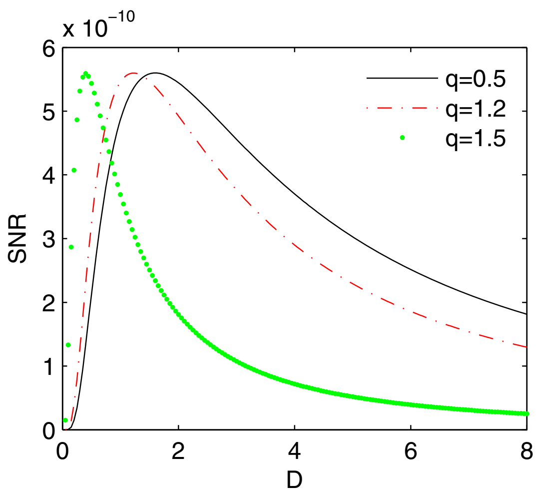

Fig. 2 Curves of SNR versus normalized multiplicative noise intensity D for $Q = 0.001$, $A = 1$, $\tau = 0.01$

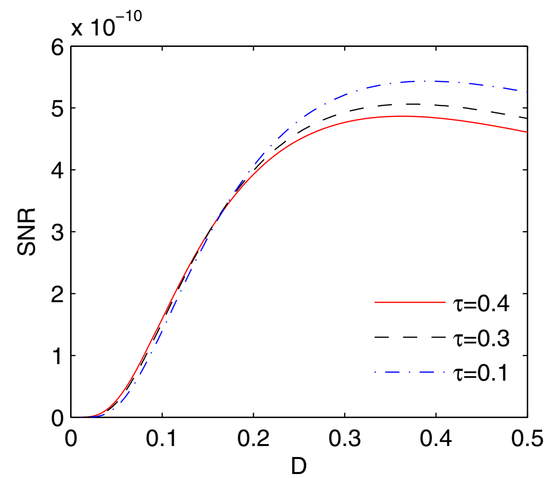

Fig. 4 Curves of SNR versus normalized multiplicative noise intensity D for $Q = 0.001$, $A = 1$, $q = 1.5$

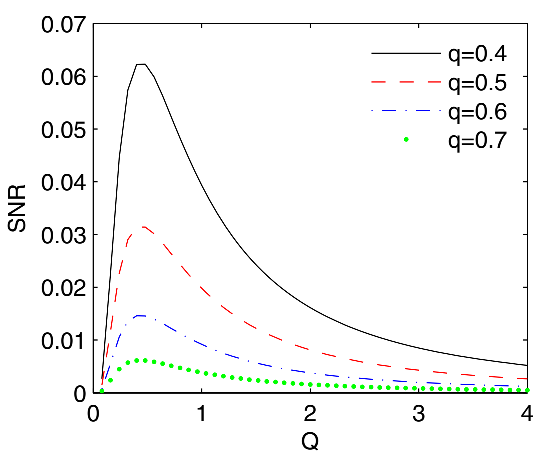

Fig. 3 Curves of SNR versus normalized additive noise intensity Q for $D = 0.1$, $A = 1$, $\tau = 0.01$

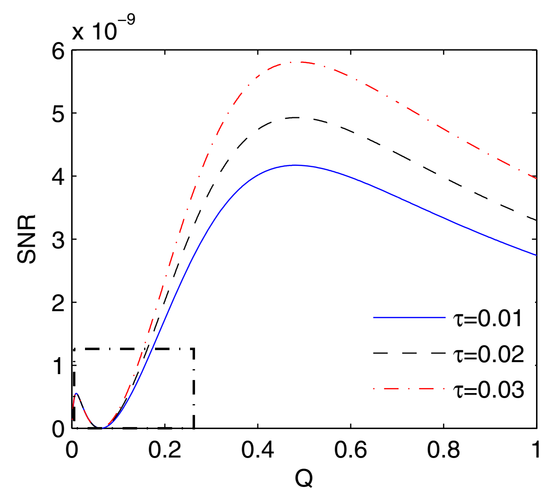

Fig. 5 Curves of SNR versus normalized addictive noise intensity Q for $D = 0.1$, $A = 1$, $q = 1.5$

and rerealed that an independent additive noise governs the dynamic behavior in response to small periodic driving. Figure 6 is an amplification of the dotted box in Fig. 5. From Fig. 6, we can see that with the increase of τ, the height of the left peak nearly remains the same and the right peak increases.

To see the further effects of multiplicative noise intensity D and the additive noise intensity Q, we plot the SNR as a function of correlation time τ of non-Gaussian noise in Figs. 7 and 8, respectively. In Fig. 7, we present the effects of multiplicative noise intensity D on the SNR as a function of correlation time τ. One can clearly see from Fig. 7 that there is multiplicative noise intensity causing critical effect on the SR phenomenon induced by the correlation timeτ. That is, there is a marginal value of D, below which the maximum of the SNR decreases as D increases, implying that the increase of D weakens the resonant effect and above which the increase of D enhances the resonant effect. Figure 8 shows the effect of Q on the SNR as a function of τ. We can see that the increase of Q enhances the resonant effect.

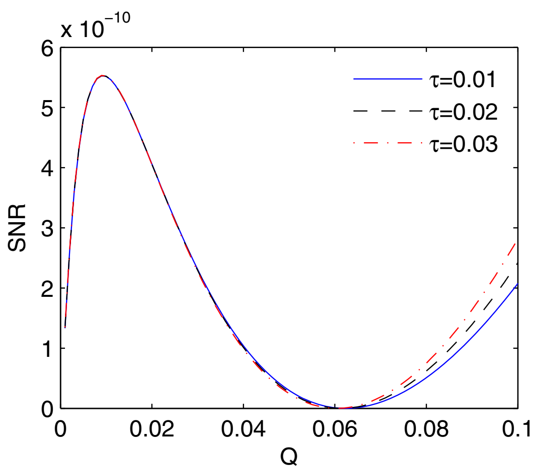

Fig. 6 Curves of SNR versus normalized addictive noise intensity Q for $D = 0.1$, $A = 1$, $q = 1.5$

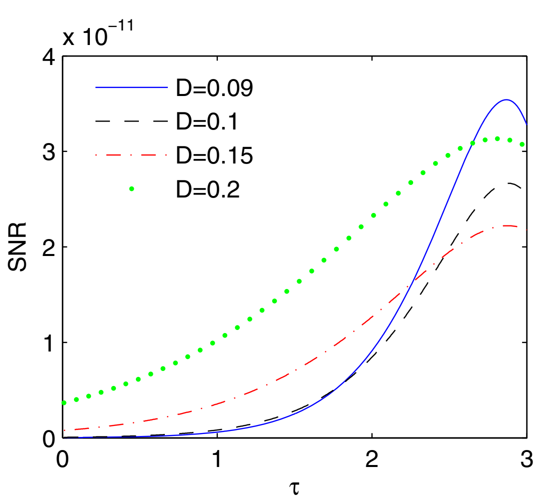

Fig. 7 Curves of SNR versus normalized cross-correlation time τ for the non-Guassian noise for $Q = 0.001$, $A = 1$, $q = 0.5$

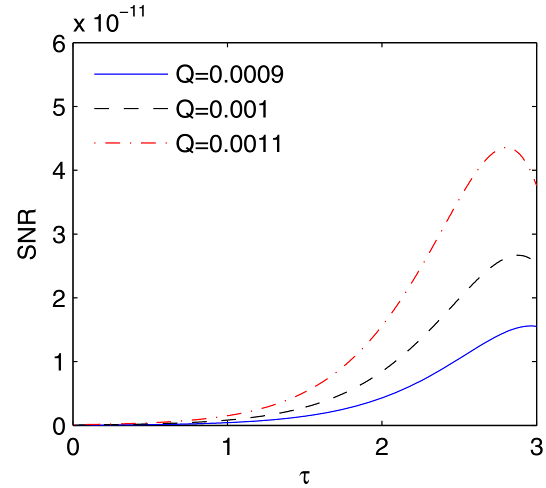

Fig. 8 Curves of SNR versus normalized cross-correlation time τ for the non-Guassian noise for $D = 0.1$, $A = 1$, $q = 0.5$

4. Conclusions

In this paper, in terms of SNR, we have theoretically analyzed SR phenomenon induced by a multiplicative periodic signal in FHN model under non-Gaussian noise. It is revealed that resonant effect in this system shows some new features due to the presence of multiplicative signal and non-Gaussian noise. Our study shows that the SNR as a function of τ exhibits a resonant effect, and there is a critical value of the multiplicative noise intension D, above which the increase of D enhances the resonant effect and below which it weaken the resonant effect. Moreover, the decreasing parameter q weakens the resonant effect induced by the addition noise. More interestingly, we observe that SNR as a function of Q shows double-peak structure. The results of the present study manifestly show that the combination of SR effect with multiplicative signal and non-Gaussian noise has more interesting and complex influences on the stochastic response of FHN. We believe that these new findings further deepen the understanding for the dynamics of various systems described by the FHN model in presence of multiplicative signal and non-Gaussian noise.

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