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ORIGINAL PAPER

Bound state solutions of Klein–Gordon equation with Mobius square plus Yukawa potentials

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Abstract: We have solved approximately Klein–Gordon equation with equal scalar and vector Mobius square plus Yukawa potentials in D-dimensions using the parametric form of Nikiforov–Uvarov method. Energy eigenvalues and corresponding wave functions in terms of Jacobi polynomials are obtained. We have also discussed some special cases of our potential.

Keywords: Klein–Gordon equation; Mobius potential; Yukawa potential; Bound state and NU method

PACS Nos.: 03.65.Ge; 03.65.Pm; 03.65.Ca

1. Introduction

In relativistic quantum mechanics the particle motions are commonly described using either Klein–Gordon equation or Dirac equation [1, 2] depending on spin character of the particle. Klein Gordon equation is one of the most frequently used wave equation that describes spin-zero particles such as mesons. On the other hand, however, electrons are described satisfactorily by Dirac equation.

The solution of Klein–Gordon equation under different potentials plays an important role because one can understand physics that can be brought by such solutions. Among the most successful methods that have been used to solve Schrödinger, Dirac and Klein–Gordon equation, Nikiforov–Uvarov (NU) and supersymmetric quantum mechanics (SUSYQM) methods have great importance [3–13].

Recently Klein–Gordon equation with different potentials have been solved and investigated by different researchers [14–24]. For instance, Egrifes and Sever [25] have obtained bound state solutions of Klein–Gordon equation for generalized PT-symmetric Hulthen potential, Soylu et al. [26] considered Klein–Gordon equation under Rosen–Morse type potentials.

Aim of this paper is to obtain approximate solutions of Klein–Gordon equation with equal scalar and vector Mobius square plus Yukawa potentials in D-dimensional space.

2. Nikiforov–Uvarov (NU) method

Nikiforov–Uvarov method [8–11] is based on the solution of a generalized second order linear differential equation with special orthogonal function. Schrödinger equation

$$\Psi''(x) + (E - V(x))\Psi(x) = 0 \quad (1)$$

can be solved by this method. This can be done by transforming this equation into equation of hypergeometric type with appropriate transformation, $S = S(x)$

$$\Psi''(s) + \frac{\overline{\tau}(s)}{\sigma(s)}\Psi'(s) + \frac{\overline{\sigma}(s)}{\sigma^2(s)}\Psi(s) = 0 \quad (2)$$

where $\sigma(s)$ and $\overline{\sigma}(s)$ must be polynomials of at most second-degree and $\overline{\tau}(s)$ is a polynomial with at most first-degree. In order to find exact solution to Eq. (2) we set wave function as

$$\Psi(s) = \Phi(s)\chi(s) \quad (3)$$

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On substituting Eq. (3) in Eq. (2), then Eq. (2) reduces to hypergeometric type,

$$\sigma(s)\chi''(s) + \tau(s)\chi'(s) + \lambda\chi(s) = 0 \tag{4}$$

where wave function $\Phi(s)$ is defined as logarithmic derivative [8]

$$\frac{\Phi'(s)}{\Phi(s)} = \frac{\pi(s)}{\sigma(s)} \tag{5}$$

where $\pi(s)$ is at most first order polynomial. Likewise, hypergeometric type function $\chi(s)$ in Eq. (4) for a fixed n is given by Rodriques relation as

$$\chi_n(s) = \frac{B_n}{\rho(s)}\frac{d^n}{ds^n}[\sigma^n(s)\rho(s)] \tag{6}$$

where B_n is normalization constant and weight functions $\rho(s)$ must satisfy the condition

$$\frac{d}{ds}[\sigma(s)\rho(s)] = \tau(s)\rho(s) \tag{7}$$

with

$$\tau(s) = \overline{\tau}(s) + 2\pi(s) \tag{8}$$

In order to accomplish the condition imposed on weight function $\rho(s)$, it is necessary that classical orthogonal polynomials $\tau(s)$ be equal to zero at some point of an interval (a, b) and its derivatives within this interval at $\sigma(s) > 0$ is negative, i.e.,

$$\frac{d}{ds}\tau(s) < 0 \tag{9}$$

Therefore, function $\pi(s)$ required for NU method are defined as follows:

$$\pi(s) = \frac{\sigma' - \overline{\tau}}{2} \pm \sqrt{\left(\frac{\sigma' - \overline{\tau}}{2}\right)^2 - \overline{\sigma} + k\sigma} \tag{10}$$

$$\lambda = k + \pi'(s) \tag{11}$$

k-value in Eq. (10) is possible to evaluate if the expression under square root is square of polynomials. This is possible if and only if its discriminant is zero. With this, new eigenvalues equation becomes

$$\lambda = \lambda_n = -n\frac{d\tau}{ds} - \frac{n(n-1)}{2}\frac{d^2\sigma}{ds^2}, \quad n = 0, 1, 2, \ldots \tag{12}$$

On comparing Eq. (11) with Eq. (12), we obtain energy eigenvalues.

Parametric generalization of NU method is given by generalized hypergeometric type equation as [27]

$$\Psi''(s) + \frac{(c_1 - c_2 s)}{s(1 - c_3 s)}\Psi'(s) + \frac{1}{s^2(1 - c_3 s)^2}\left[-\xi_1 s^2 + \xi_2 s - \xi_3\right]\Psi(s) = 0 \tag{13}$$

Eq. (13) is solved by comparing it with Eq. (20) and following polynomials are obtained

$$\overline{\tau}(s) = (c_1 - c_2 s), \quad \sigma(s) = s(1 - c_3 s), \quad \overline{\sigma}(s) = -\xi_1 s^2 + \xi_2 s - \xi_3 \tag{14}$$

where

$$c_4 = \frac{1}{2}(1 - c_1), \quad c_5 = \frac{1}{2}(c_2 - 2c_3), \quad c_6 = c_5^2 + \xi_1, \quad c_7 = 2c_4 c_5 - \xi_2, \quad c_8 = c_4^2 + \xi_3 \tag{15}$$

The resulting value of k in Eq. (15) is obtained from condition that function under square root must be square of a polynomials and it yields,

$$k_{\pm} = -(c_7 + 2c_3 c_8) \pm 2\sqrt{c_8 c_9}, \tag{16}$$

where

$$c_9 = c_3 c_7 + c_3^2 c_8 + c_6. \tag{17}$$

The new $\pi(s)$ for each k becomes

$$\pi(s) = c_4 + c_5 s - [(\sqrt{c_9} + c_3\sqrt{c_8})s - \sqrt{c_8}], \tag{18}$$

For k_- value as

$$k_- = -(c_7 + 2c_3 c_8) - 2\sqrt{c_8}c_9 \tag{19}$$

Using Eq. (8), we obtain

$$\tau(s) = c_1 + 2c_4 - (c_2 - 2c_5)s - 2[(\sqrt{c_9} + c_3\sqrt{c_8})s - \sqrt{c_8}]. \tag{20}$$

The physical condition for bound state solution is $\tau' < 0$ and thus

$$\tau' = -2c_3 - 2(\sqrt{c_9} + +c_3\sqrt{c_8}) < 0 \tag{21}$$

With aid of Eqs. (11) and (12), we derive energy equation as

$$(c_2 - c_3)n + c_3 n^2 - (2n+1)c_5 + (2n+1)(\sqrt{c_9} + c_3\sqrt{c_8}) + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0 \tag{22}$$

The weight function $\rho(s)$ is obtained from e.g. Eq. (7) as

$$\rho(s) = s^{c_{10}-1}(1 - c_3 s)^{\frac{c_{11}}{c_3} - c_{10} - 1} \tag{23}$$

together with e.g. Eq. (6), we have

$$\chi_n(s) = P_n^{\left(c_{10}-1, \frac{c_{11}}{c_3} - c_{10} - 1\right)}(1 - 2c_3 s) \tag{24}$$

where

$$c_{10} = c_1 + 2c_4 + 2\sqrt{c_8} \tag{25}$$

$$c_{11} = c_2 - 2c_5 + 2(\sqrt{c_9} + c_3\sqrt{c_8}) \tag{26}$$

$$P_n^{(\alpha,\beta)}(x) = \frac{\Gamma(\alpha+n+1)}{n!\Gamma(\alpha+\beta+n+1)} \times \sum_{m=0}^{n} \binom{n}{m} \frac{\Gamma(\alpha+\beta+n+m+1)}{\Gamma(\alpha+m+1)} \left(\frac{x-1}{2}\right)^m$$

and $P_n^{(\alpha,\beta)}$ is Jacobi polynomial.

Second part of wave function is obtained from Eq. (5) as

$$\Phi(s) = s^{c_{12}}(1-c_3 s)^{-c_{12}-\frac{c_{13}}{c_3}} \tag{27}$$

where

$$c_{12} = c_4 + \sqrt{c_8}, c_{13} = c_5 - (\sqrt{c_9} + c_3\sqrt{c_8}) \tag{28}$$

Thus, the wave function is

$$\Psi(s) = s^{c_{12}}(1-c_3 s)^{c_{12}-\frac{c_{13}}{c_3}} P_n^{\left(c_{10}-1, \frac{c_{11}}{c_3}-c_{10}-1\right)}(1-2c_3 s) \tag{29}$$

3. Radial part of Klein–Gordon equation in D-dimensions

The radial part of Klein–Gordon equation in presence of vector and scalar potentials in D-dimensional space is written as [28–30]

$$\left[\frac{d^2}{dr^2} + E_{nl}^2 + V^2(r) - 2E_{nl}V(r) - m^2 - S^2(r) - 2mS(r) - \frac{(D+2l-1)(D+2l-3)}{4r^2}\right] U_{n,l}(r) = 0 \tag{30}$$

For equal scalar and vector potentials Eq. (30) reduces to

$$\left[\frac{d^2}{dr^2} + E_{nl}^2 - m^2 - 2(E_{nl}+m)V(r) - \frac{(D+2l-1)(D+2l-3)}{4r^2}\right] U_{nl}(r) = 0 \tag{31}$$

Here, we consider Mobius square plus Yukawa potentials defined as [31, 32]

$$V(r) = V_0\left(\frac{A+Be^{-\alpha r}}{C+D'e^{-\alpha r}}\right)^2 - \frac{V_1 e^{-\alpha r}}{r}, \tag{32}$$

where V_0, V_1, A, B, C, D' and α are constant coefficients.

A good approximation for centrifugal barrier is taken as [33] (see Fig. 1)

$$\frac{1}{r^2} \approx \alpha^2\left(\frac{C}{C+D'e^{-\alpha r}}\right)^2 = \lim_{\alpha\to 0}\left(\frac{1}{r^2} + \frac{\alpha}{r} + \frac{5}{12}\alpha^2 + \frac{1}{12}\alpha^3 r + \frac{1}{240}\alpha^4 r^2 - \frac{1}{720}\alpha^5 r^3 - \frac{1}{6045}\alpha^6 r^4 + O(r^5)\right) \tag{33}$$

which is valid for $\alpha r \le 1$ when $C = 1$ and $D' = -1$ [33–36], similar to other related works [37, 38]. In addition, when performing a power series expansion and setting $\alpha \to 0$, Eq. (33) gives the desired r^{-2} suggested by Greene and Aldrich [39]. By substituting Eqs. (32) and (33) into Eq. (31) we obtain

$$\left\{\frac{d^2}{dr^2} + E_{nl}^2 - m^2 - 2(E_{nl}+m)V_0\left(\frac{A+Be^{-\alpha r}}{C+D'e^{-\alpha r}}\right)^2 + \frac{2\alpha C(E_{nl}+m)V_1 e^{-\alpha r}}{(C+D'e^{-\alpha r})} - \frac{\alpha^2 C^2(D+2l-1)(D+2l-3)}{4(C+D'e^{-\alpha r})^2}\right\} \times U_{nl}(r) = 0. \tag{34}$$

By a change of variable of the form

$$s = e^{-\alpha r} \tag{35}$$

Eq. (34) is written as

$$\frac{d^2U_{nl}}{ds^s} + \frac{\left(1+\frac{D'}{C}s\right)}{s\left(1+\frac{D'}{C}s\right)}\frac{dU_{nl}}{ds} + \frac{1}{s^2\left(1+\frac{D'}{C}s\right)^2}\left[-Q_1 s^2 + Q_2 s - Q_3\right]U_{nl}(r) = 0. \tag{36}$$

where

$$\begin{aligned} \varepsilon^2 &= \frac{E_{nl}^2 - m^2}{\alpha^2}, \\ Q_1 &= \frac{-D'^2}{C^2}\varepsilon^2 + \frac{2(E_{nl}+m)}{\alpha^2 C^2}V_0 B^2 - \frac{2D'}{C}\frac{(\mathrm{E}_{nl}+m)}{\alpha}V_1, \\ Q_2 &= \frac{2D'}{C}\varepsilon^2 - 4\frac{(\mathrm{E}_{nl}+m)}{\alpha^2 C^2}V_0 AB + 2\frac{(\mathrm{E}_{nl}+m)}{\alpha}V_1, \\ Q_3 &= -\varepsilon^2 + \frac{2(E_{nl}+m)}{\alpha^2 c^2}V_0 A^2 + \frac{(D+2l-1)(D+2l-3)}{4}. \end{aligned} \tag{37}$$

By comparing Eq. (36) with Eq. (13), we obtain energy spectrum for Mobius square plus Yukawa potentials as

$$\begin{aligned}&-\frac{D'}{c}n^2-(2n+1)\frac{D'}{2C}+(2n+1)\left(\begin{array}{l}\sqrt{\frac{D'^2}{4C^2}-\frac{4D'}{C}\frac{(E_{nl}+m)V_0AB}{\alpha^2C^2}+\frac{2D'^2}{C^2}\frac{(E_{nl}+m)V_0A^2}{\alpha^2C^2}+\frac{2(E_{nl}+m)V_0B^2}{\alpha^2C^2}+\frac{D'^2}{C^2}\frac{(D+2l-1)(D+2l-3)}{4}}\\ -\frac{D'}{c}\sqrt{-\left(\frac{E_{nl}^2-m^2}{\alpha^2}\right)+\frac{2(E_{nl}+m)V_0A^2}{\alpha^2C^2}+\frac{(D+2l-1)(D+2l-3)}{4}}\end{array}\right)\\&+\frac{4(E_{nl}+m)V_0AB}{\alpha^2C^2}-\frac{2(E_{nl}+m)V_1}{\alpha}-2\frac{D'}{C}\left[\frac{2(E_{nl}+m)V_0A^2}{\alpha^2C^2}+\frac{(D+2l-1)(D+2l-3)}{4}\right]\\&+2\sqrt{\left[-\left(\frac{E_{nl}^2-m^2}{\alpha^2}\right)+\frac{2(E_{nl}+m)V_0A^2}{\alpha^2C^2}+\frac{(D+2l-1)(D+2l-3)}{4}\right]\left[\frac{D'^2}{4C^2}-\frac{4D'}{C}\frac{(E_{nl}+m)V_0AB}{\alpha^2C^2}+\frac{2D'^2}{C^2}\frac{(E_{nl}+m)V_0A^2}{\alpha^2C^2}+\frac{2(E_{nl}+m)V_0B^2}{\alpha^2C^2}+\frac{D'^2}{C^2}\frac{(D+2l-1)(D+2l-3)}{4}\right]}=0\end{aligned} \tag{38}$$

or more explicitly, we have,

$$\mathrm{E}_m^2-m^2=\frac{-\alpha^2}{4}\left[\frac{\gamma+\frac{D'}{C}\left(n+\frac{1}{2}-\beta\right)^2}{\frac{D'}{C}\left(n+\frac{1}{2}-\beta\right)}\right]^2+\frac{2(E_{nl}+m)}{C^2}V_0A^2+\alpha^2\frac{(D+21-1)(D+21-3)}{4} \tag{39}$$

where

$$\begin{aligned}\gamma&=\frac{2D'}{C}\frac{(E_{nl}+m)V_oA^2}{\alpha^2C^2}+2\frac{(E_{nl}+m)V_1}{\alpha}-\frac{2C}{D'}\frac{(E_{nl}+m)V_0B^2}{\alpha^2C^2}+\frac{D'}{C}\frac{(D+21-1)(D+21-3)}{4}\\ \beta&=\frac{C}{D'}\sqrt{\frac{D'^2}{4C^2}-\frac{4D'}{C}\frac{(E_{nl}+m)}{\alpha^2C^2}V_oAB+\frac{2D'^2}{C^2}\left(\frac{E_{nl}+m}{\alpha^2C^2}\right)V_0A^2+2\frac{(E_{nl}+m)}{\alpha^2C^2}V_oB^2+\frac{D'^2}{C^2}\frac{(D+21-1)(D+21-3)}{4}}\end{aligned} \tag{40}$$

wave function of the system is obtained as

$$U_{nl}(r)=N_{nl}(e^{-\alpha r})^{c_{12}}\left(1+\frac{D'}{C}e^{-\alpha r}\right)^{-c_{12}+\frac{C}{D'}c_{13}}\times P_n^{\left(c_{10}-1,-\frac{C}{D'}c_{11}-c_{10}-1\right)}\left(1+2\frac{D'}{C}e^{-\alpha r}\right) \tag{41}$$

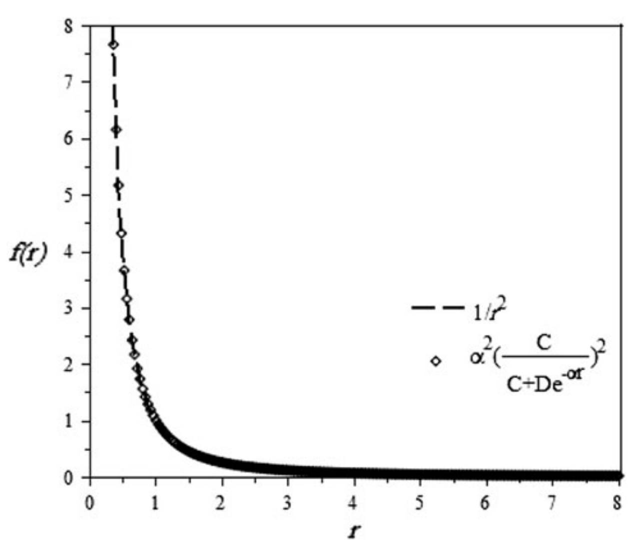

Fig. 1 $\frac{1}{r^2}$ and its approximation for $\alpha = 0.01$

4. Results and discussion

In order to test accuracy of our work, we compute numerical values for energy spectrum and graphical solutions. In Table 1, we have reported numerical values of energy for various states. Also, we have reported behaviour of energy in Fig. 2, We see that energy increases with increasing D. In Fig. 3, we show behaviour of energy versus alpha. It reveals that energy decreases as alpha decreases and tends to a constant value. Finally, energy has also been plotted versus the potential coefficients in Figs. 4 and 5. It shows well how energy increases for increasing V_0 and decreases for increasing V_1. Now a few special cases are discussed below. By adjusting some potential parameters, some well kwon potentials can be obtained. Setting $V_0 = 0$, $C = 1$, $D' = -1$ into Eq. (32), Yukawa potential [32]

$$V(r)=\frac{-V_1e^{-\alpha r}}{r}, \tag{42}$$

is obtained. Corresponding energy eigenvalues become

$$E_{nl}^2 - m^2 = -\frac{-\alpha^2}{4}\left[\frac{\frac{(D+2l-1)(D+2l-3)}{4} - \frac{2(E_{nl}+m)V_1}{\alpha} + \left(n+\frac{1}{2}-\sqrt{\frac{1}{4}+\frac{(D+2l-1)(D+2l-3)}{4}}\right)^2}{\left(n+\frac{1}{2}-\sqrt{\frac{1}{4}+\frac{(D+2l-1)(D+2l-3)}{4}}\right)}\right]^2 + \frac{\alpha^2}{4}(D+2l-1)(D+2l-3). \tag{43}$$

Similarly we set $V_o = 0$, $C = 1$, $D' = -1$ and $\alpha \to 0$ into Eq. (32), the potential reduces to Coulomb potential [37]

$$V(r) = \frac{-V_1}{r}, \tag{44}$$

Corresponding energy eigenvalues become

$$E_{nl}^2 - m^2 = -\frac{(E_{nl}+m)^2 V_1^2}{\left(n+\frac{1}{2}-\sqrt{\frac{1}{4}+\frac{(D+21-1)(D+21-3)}{4}}\right)^2} \tag{45}$$

where $V_1 = 0$, potential reduces to Mobius square potential. The generalized form of Mobius square potential can be written as

$$V(r) = \frac{V_0}{C^2}\left[\frac{A^2 + 2ABe^{-\alpha r} + B^2 e^{-2\alpha r}}{\left(1+\frac{D'}{C}e^{-\alpha r}\right)^2}\right] \tag{46}$$

Comparing Eq. (46) with Deng–Fan potential [38]

$$V(r) = D_e\left[\frac{1-2(1+b)e^{-\alpha r} + (1+b)^2 e^{-2\alpha r}}{(1-e^{-\alpha r})^2}\right] \tag{47}$$

for $C = 1$ and $D' = -1$, we have $V_0 A^2 = D_e$, $V_0 AB = -D_e\,(1+b)$, $V_0 B^2 = D_e\,(1+b)^2$, where $b = e^{\alpha r_d} - 1$ and r_d is equilibrium inter-nuclear distance. Substitution of these parameters in Eq. (39) gives energy spectrum of Deng–Fan potential as

$$E_{nl}^2 - m^2 = \frac{-\alpha^2}{4}\left[\frac{\gamma_1 + \left(n+\frac{1}{2}+\beta_1\right)^2}{\left(n+\frac{1}{2}+\beta_1\right)}\right]^2 + 2D_e(E_{nl}+m) + \frac{\alpha^2(D+2l-1)(D+2l-3)}{4}, \tag{48}$$

Table 1 Energy for $D = 3, \alpha = 0.01$, $A = 1, B = -2, m = 1, C = 1, D' = -1, V_0 = 0.2$

$\vert n, l\rangle$	$E_{n,l}$ (1/fm)	
	$V_1 = 0.1$	$V_1 = -0.1$
$\vert 0, 0\rangle$	1.006897567	1.010822408
$\vert 0, 1\rangle$	1.007290610	1.011212746
$\vert 0, 2\rangle$	1.008075656	1.011992392
$\vert 0, 3\rangle$	1.009250634	1.013159302
$\vert 1, 0\rangle$	1.024029797	1.027811039
$\vert 1, 1\rangle$	1.024407462	1.028186166
$\vert 1, 2\rangle$	1.025161830	1.028935469
$\vert 1, 3\rangle$	1.026290984	1.030057055
$\vert 2, 0\rangle$	1.040385455	1.044030733
$\vert 2, 1\rangle$	1.040748852	1.044391743
$\vert 2, 2\rangle$	1.041474753	1.045112881
$\vert 2, 3\rangle$	1.042561382	1.046192388
$\vert 3, 0\rangle$	1.056015689	1.059531917
$\vert 3, 1\rangle$	1.056365808	1.059879786
$\vert 3, 2\rangle$	1.057065216	1.060574701
$\vert 3, 3\rangle$	1.058112258	1.061615025

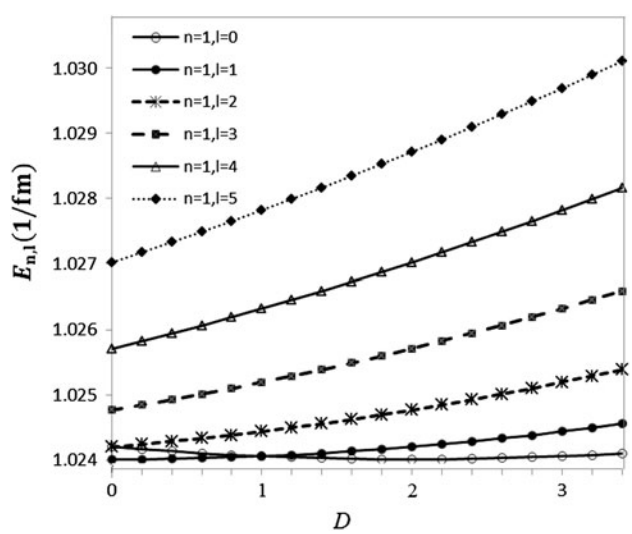

Fig. 2 Energy versus D for $\alpha = 0.01$, $A = 1, B = -2, m = 1, C = 1$, $D' = -1, V_0 = 0.2, V_1 = 0.1$

where

$$\gamma_1 = \frac{2D_e(E_{nl}+m)}{\alpha^2} - 2D_e(1+b)^2\frac{(E_{nl}+m)}{\alpha^2} + \frac{(D+2l-1)(D+2l-3)}{4},$$
$$\beta_1 = \sqrt{\frac{1}{4} - 4D_e(1+b)\frac{(E_{nl}+m)}{\alpha^2} + \frac{2D_e(E_{nl}+m)}{\alpha^2} + 2D_e(1+b)^2\frac{(E_{nl}+m)}{\alpha^2} + \frac{(D+2l-1)(D+2l-3)}{4}}, \tag{49}$$

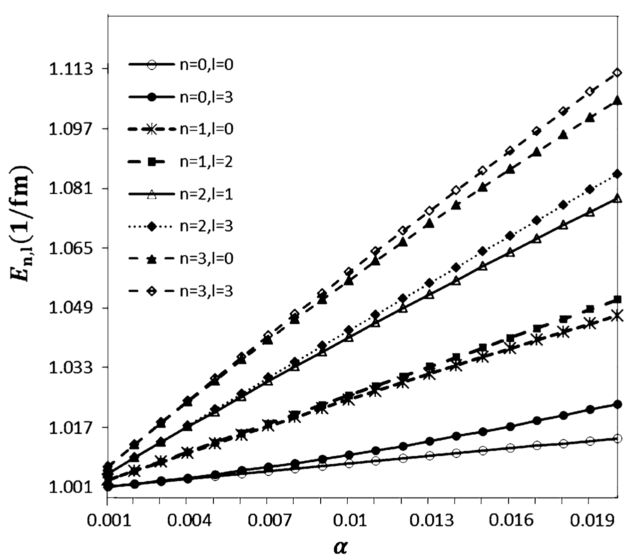

Fig. 3 Energy versus α for $D = 3$, $A = 1, B = -2, m = 1, C = 1, D' = -1, V_0 = 0.2, V_1 = 0.1$

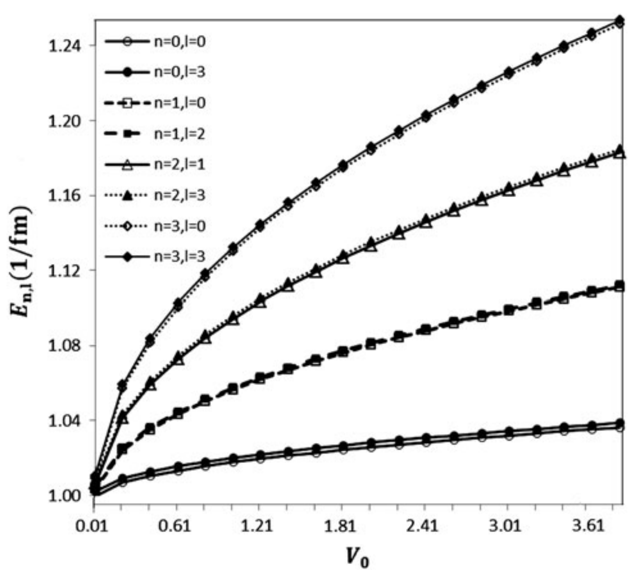

Fig. 4 Energy versus V_0 for $D = 3, \alpha = 0.01$, $A = 1, B = -2, m = 1$, $C = 1, D' = -1, V_1 = 0.1$

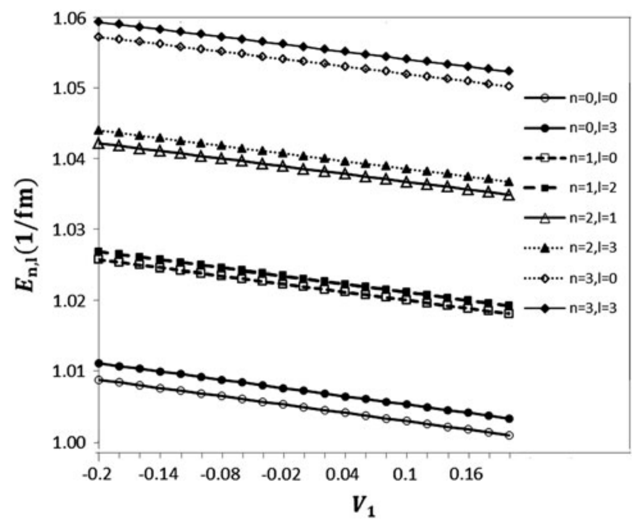

Fig. 5 Energy versus V_1 for $D = 3, \alpha = 0.01$, $A = 1, B = -2, m = 1$, $C = 1, D' = -1, V_0 = 0.2$

This result is consistent with that of Dong [38] when $D = 3$.

5. Conclusions

Approximate solutions of Klein–Gordon equation in case of equal scalar and vector Mobius square plus Yukawa potentials have been obtained using parametric form of NU method. By adjusting some parameters of potential in Eq. (3), three well known potentials are obtained. With a good approximation to centrifugal term, we have obtained energy eigenvalues and unnormalized wave function in terms of Jacobi polynomials. Numerical data for the energy spectrum are discussed indicating usefulness for other physical systems [40].

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