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ORIGINAL PAPER

Relativistic treatment of spinless particle subject to generalized Tiez-Wei oscillator

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Abstract: The solution of Klein–Gordon equation with equal scalar and vector generalized Tiez-Wei potentials is presented for arbitrary l-wave. The energy bound states and unnormalized wave functions are obtained using the Nikiforov–Uvarov method.

Keywords: Klein–Gordon equation; Nikiforov–Uvarov method; Generalized Tiez-Wei potential

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1. Introduction

The study of bound state solutions of the Schrödinger, Klein–Gordon (KG) and Dirac equations for spherically symmetric potentials plays a vital role in various fields of physics. The bound state solutions of such potentials can effectively be used to model some physics problems. KG equation is the well-known relativistic wave equation describing spin-zero particles due to its square terms [1]. However, the analytical solutions of KG equation are possible only for a few simple cases such as the hydrogen atom, the harmonic oscillator and others [2, 3]. In recent years, many researchers are interested in searching for the solutions of KG equation for spherically symmetric potential in D-dimensional Hilbert space [4]. Different potential models have been examined in D-dimensional space. These potentials include Posch-Teller [5], Hulthen [6], Tietz [7], Hylleraas [8], Hua Plus Modified Eckart potential [9], Yukawa potential [10] and others [11].

Various methods have been used to solve these quantum mechanical problems exactly or approximately. These methods include the Nikiforov–Uvarov (NU) method [12], asymptotic iteration method (AIM) [13], supersymmetric quantum mechanics (SUSYQM) [14] and others [15, 16]. The Tiez-Wei (TW) potential mostly called TH potential is one of the best potential model for vibrational energy of diatomic molecules defined in [17],

$$V(r) = D\left(\frac{1 - e^{-\alpha r}}{1 - qe^{-\alpha r}}\right)^2, \quad 0 < q < 1, \tag{1}$$

where D, α and q are potential parameters. The TW potential is much more realistic than Morse potential in describing molecular dynamics at moderate and high rotational and vibrational quantum number [17]. Also, the TW fits the experimental Rydberg–Klein–Rees curves closely than the Morse function, especially when the potential domain extends to near to the dissociation limit. The analytical expression of the TH rotating oscillator with framework of Hamilton–Jacobi theory and Bohr-Sommerfeld quantization rule has been derived by Kunc and Gordillo-Vazquez [18]. In this work we consider a more generalized Tietz-Wei (GTW) potential defined by the Mobius square potential as [19],

$$V(r) = D_e\left(\frac{A + Be^{-2\alpha r}}{C + D'e^{-2\alpha r}}\right)^2 \tag{2}$$

where A, B, C and D' are the potential parameters. The Mobius square potential is reduced to the TW potential for $A = 1$, $B = -1$, $C = 1$ and $D' = -q$. We plot the TW potential, Morse and GTW as function of r for $\alpha = 0.01$ in Fig. 1.

The main aim of the present paper is to investigate the KG equation in D-dimensional Hilbert space for this potential. Therefore, we attempt to study this potential with the

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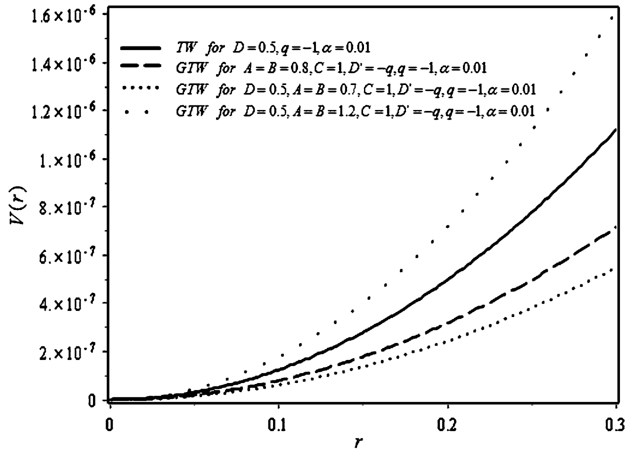

Fig. 1 Shapes of TW and GTW potentials

centrifugal term in D-dimension using the NU method and present the analytically approximate solutions to this system.

2. Parametric NU method

NU method [12] and its parametric form [20] were proposed to solve second order differential equation of the form

$$\psi''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\psi'(s) + \frac{\sigma(s)}{\sigma^2(s)}\psi(s) = 0 \quad (3)$$

$$\frac{d^2\psi}{ds^2} + \frac{(\alpha_1 - \alpha_2 s)}{s(1-\alpha_3 s)}\frac{d\psi}{ds} + \frac{1}{s^2(1-\alpha_3 s)^2}\left[-\xi_1 s^2 + \xi_2 s - \xi_3\right]\psi(s) = 0 \quad (4)$$

with appropriate coordinate transformation $s = s(r)$, where $\sigma(r)$ and $\tilde{\sigma}(s)$ are polynomials at most a second degree and $\tilde{\tau}(s)$ is a first degree polynomial. The eigenfunction and corresponding energy eigenvalues to the equation becomes

$$\psi(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}} P_n^{\left(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s) \quad (5)$$

$$(\alpha_2-\alpha_3)n + \alpha_3 n^2 - (2n+1)\alpha_5 + (2n+1)[\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}] + \alpha_7 + 2\alpha_3\alpha_8 + 2\sqrt{\alpha_8\alpha_9} = 0 \quad (6)$$

where

$$\alpha_4 = \frac{1}{2}(1-\alpha_1), \quad \alpha_5 = \frac{1}{2}(\alpha_2 - 2\alpha_3),$$
$$\alpha_6 = \alpha_5^2 + \xi_1, \quad \alpha_7 = 2\alpha_4\alpha_5 - \xi_2,$$
$$\alpha_8 = \alpha_4^2 + \xi_3, \quad \alpha_9 = \alpha_3\alpha_7 + \alpha_3^2\alpha_8 + \alpha_6$$
$$\alpha_{10} = \alpha_1 + 2\alpha_4 + 2\sqrt{\alpha_8}$$
$$\alpha_{11} = \alpha_2 - 2\alpha_5 + 2(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8})$$
$$\alpha_{12} = \alpha_4 + \sqrt{\alpha_8}, \quad \alpha_{13} = \alpha_5 - (\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}) \quad (7)$$

3. Radial part of KG equation in D-dimensions

KG equation with vector *V(r)* and scalar *S(r)* potentials in spherical coordinate can be written as (in relativistic units $\hbar = c = 1$) [21–30],

$$\left[\frac{d^2}{dr^2} - (m^2 - E_{nl}^2) - 2(E_{nl}V(r) + mS(r)) + V^2(r) - S^2(r) - \frac{(D+2l-1)(D+21-3)}{4r^2}\right]R_{nl}(r) = 0. \quad (8)$$

Substituting Eq. (1) into Eq. (7) for equal scalar and vector potential, we have,

$$\left[\frac{d^2}{dr^2} + (E_{nl}^2 - m^2) - 2D_e(E_{nl}+m)\left(\frac{A+Be^{-2\alpha r}}{C+D'e^{-2\alpha r}}\right)^2 - \frac{(D+2l-1)(D+21-3)}{4r^2}\right]R_{nl}(r) = 0 \quad (9)$$

This equation cannot be solved analytically for $l \neq 0$. Then we must use an approximation to the centrifugal term similar to the one suggested by other authors [22, 23]. The approximation for a short range potential takes the form [24],

$$\frac{1}{r^2} \approx 4\alpha^2 \frac{C^2}{(C+D'e^{-2\alpha r})^2}, \quad (10)$$

which is valid for $\alpha r \leq 1$ when $C = 1$ and $D' = -1$ [24]. When performing a power series expansion and setting $\alpha \to 0$, it gives the desired r^{-2} suggested by Greene and Aldrich [23] and others [28–30]. In order to test the

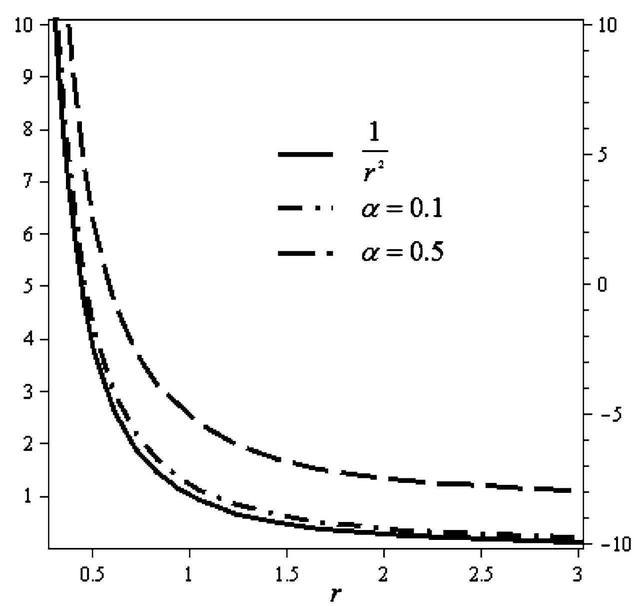

Fig. 2 $\frac{1}{r^2}$ and its approximation ($C = 1$, $D' = -1$)

accuracy and validity of this approximation, we plot variation of the approximation of Eq. (10) and centrifugal term $\frac{1}{r^2}$ as a function of r for $C = 1$, $D' = -1$ with various parameters of α in Fig. 2, which shows that for a short range potential, Eq. (10) is a good approximation to $\frac{1}{r^2}$. Now substituting Eq. (10) into Eq. (9), we obtain

$$\left[\frac{d^2}{dr^2} + \left(E_{nl}^2 - m^2\right) - 2D_e(E_{nl} + m)\left(\frac{A + Be^{-2\alpha r}}{C + D'e^{-2\alpha r}}\right)^2 - \frac{C^2\alpha^2(D + 2l - 1)(D + 2l - 3)}{\left(C + D'e^{-2\alpha r}\right)^2}\right]R_{nl}(r) = 0 \tag{11}$$

If we defined the new variable $s = e^{-2\alpha r}$, Eq. (11) becomes,

$$\frac{d^2R_{nl}}{ds^2} + \frac{\left(1 + \frac{D'}{C}s\right)}{s\left(1 + \frac{D'}{C}s\right)}\frac{dR_{nl}}{ds} + \frac{1}{s^2\left(1 + \frac{D'}{C}s\right)^2}\left[-A_1s^2 + A_2s - A_3\right]R_{nl}(s) = 0 \tag{12}$$

Comparing Eq. (12) with Eq. (3), we get,

$$\begin{aligned} &\alpha_1 = 1, \quad \xi_1 = A_1, \\ &\alpha_2 = -\frac{D'}{C}, \quad \xi_2 = A_2, \\ &\alpha_3 = -\frac{D'}{C}, \quad \xi_3 = A_3 \end{aligned} \tag{14}$$

and using Eq. (6) we can obtain the following coefficients:

$$\begin{aligned} &\alpha_4 = 0, \quad \alpha_5 = \frac{D'}{2C}, \\ &\alpha_6 = \frac{D'^2}{4C^2} + \beta^2B^2 - \varepsilon^2\frac{D'^2}{C^2}, \\ &\alpha_7 = 2\beta^2AB - 2\varepsilon^2\frac{D'}{C}, \\ &\alpha_8 = \left(\beta^2A^2 + \gamma^2 - \varepsilon^2\right), \\ &\alpha_9 = \left(\frac{1}{4} + \gamma^2 + \beta^2A^2\right)\frac{D'^2}{C^2} - \left(2AB\beta^2\right)\frac{D'}{C} + B^2\beta^2 \end{aligned} \tag{15}$$

Other coefficients are determined as,

$$\begin{aligned} &\alpha_{10} = 1 + 2\sqrt{\left(\beta^2A^2 + \gamma^2 - \varepsilon^2\right)}, \\ &\alpha_{11} = \left[-\frac{2D'}{C} + 2\left(\sqrt{\left(\gamma^2 + \frac{1}{4} + \beta^2A^2\right)\frac{D'^2}{C^2} - \left(2AB\beta^2\right)\frac{D'}{C} + B^2\beta^2} - \frac{D'}{C}\sqrt{\left(\beta^2A^2 + \gamma^2 - \varepsilon^2\right)}\right)\right], \\ &\alpha_{12} = \sqrt{\left(\beta^2A^2 + \gamma^2 - \varepsilon^2\right)}, \\ &\alpha_{13} = \left[\frac{D'}{2C} - \left(\sqrt{\left(\gamma^2 + \frac{1}{4} + \beta^2A^2\right)\frac{D'^2}{C^2} - \left(2AB\beta^2\right)\frac{D'}{C} + B^2\beta^2} - \frac{D'}{C}\sqrt{\left(\beta^2A^2 + \gamma^2 - \varepsilon^2\right)}\right)\right] \end{aligned} \tag{16}$$

where,

$$\begin{aligned} &A_1 = \left[\beta^2B^2 - \varepsilon^2\frac{D'^2}{C^2}\right], \\ &A_2 = \left(2\varepsilon^2\frac{D'}{C} - 2\beta^2AB\right) \\ &A_3 = (\gamma^2 + \beta^2A^2 - \varepsilon^2), \\ &\varepsilon^2 = \frac{E_{nl}^2 - m^2}{4\alpha^2}, \quad \beta^2 = \frac{D_e(E_{nl} + m)}{2\alpha^2C^2}, \\ &\gamma^2 = \frac{(D + 2l - 1)(D + 2l - 3)}{4} \end{aligned} \tag{13}$$

By using Eq. (5) and constants in Eq. (15), one can easily find the energy formula for the GTW potential as,

$$-\frac{D'}{C}n^2 - \frac{D'}{2C}(2n + 1) + (2n + 1)\left(\sqrt{\alpha_9} - \frac{D'}{C}\sqrt{\alpha_8}\right) - A_2 - \frac{2D'}{C}A_3 + 2\sqrt{\alpha_8\alpha_9} = 0, \tag{17}$$

Furthermore, we calculate the radial wave function to be,

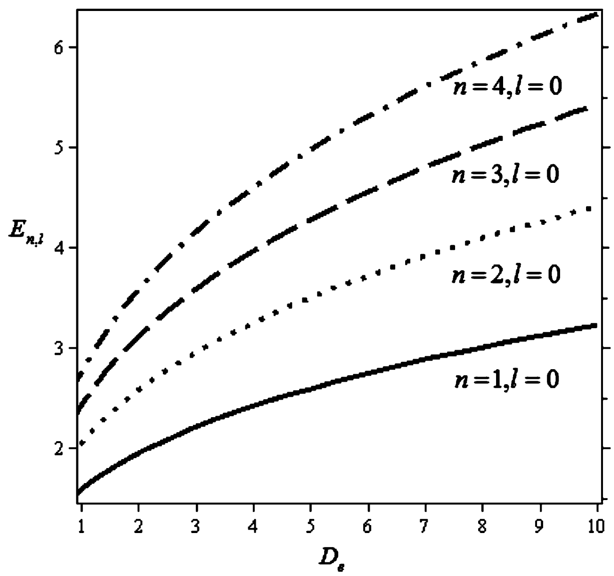

Fig. 3 The energy of the system versus D_e for $A=-2$, $B=3$, $C=1$, $D'=-1$, $m=\frac{1}{2}$, $D=3$, $\alpha=0.1$

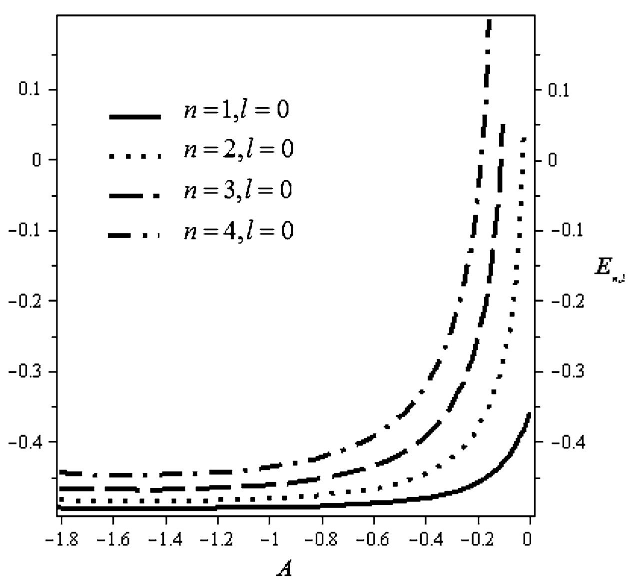

Fig. 4 The energy of the system versus A for $D_e=5$, $B=3$, $C=1$, $D'=-1$, $m=\frac{1}{2}$, $D=3$, $\alpha=0.1$

$$R_{nl}(r)=N_{nl}e^{-2\alpha\sqrt{A_3}r}\left(1+\frac{D'}{C}e^{-2\alpha r}\right)^{\frac{1}{2}-\frac{C}{D'}\sqrt{\alpha_9}}P_n^{\left(2\sqrt{A_3},-\frac{2C}{D'}\sqrt{\alpha_9}\right)}\times\left(1+2\frac{D'}{C}e^{-2\alpha r}\right) \tag{18}$$

where N_{nl} is the normalization constant. The behavior of $E_{n,l}$ versus D_e, A, B and α is plotted in Figs. 3, 4, 5 and 6. From Figs. 3, 4 and 6, we obtain that the energy of the system has an increasing behavior as D_e, α and A increase. Figure 5 shows that as B increases the energy of the system decreases and tends to a constant value. The wave function of the system is plotted in Fig. 7.

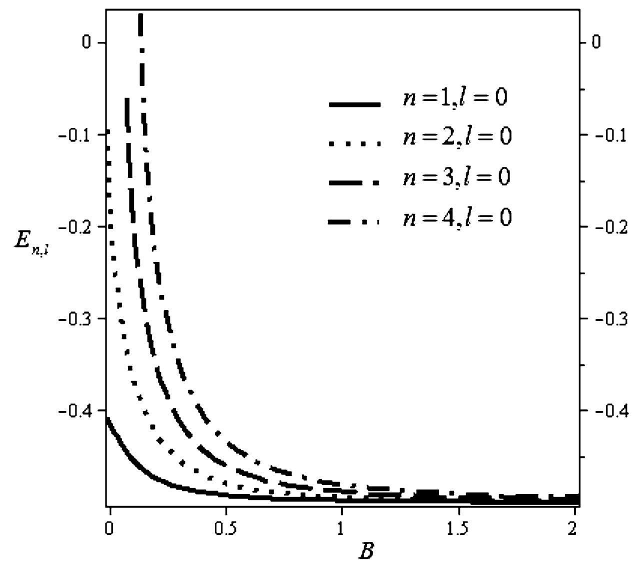

Fig. 5 The energy of the system versus B for $D_e=5$, $A=-2$, $C=1$, $D'=-1$, $m=\frac{1}{2}$, $D=3$, $\alpha=0.1$

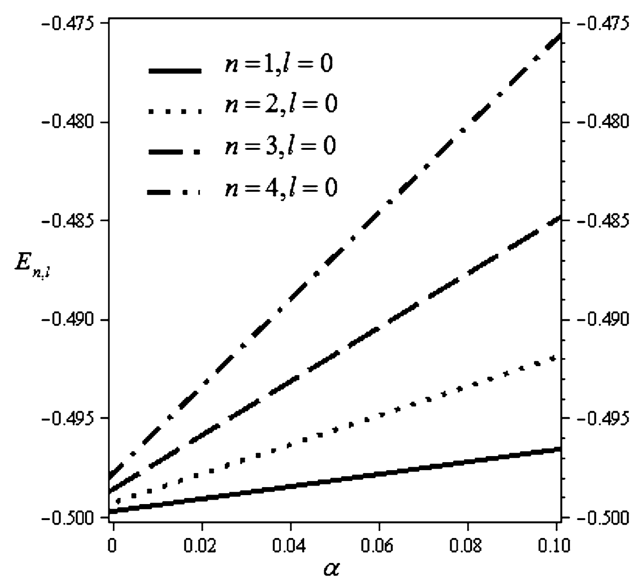

Fig. 6 The energy of the system versus α for $D_e=5$, $A=-2$, $C=1$, $D'=-1$, $m=\frac{1}{2}$, $D=3$, $B=3$

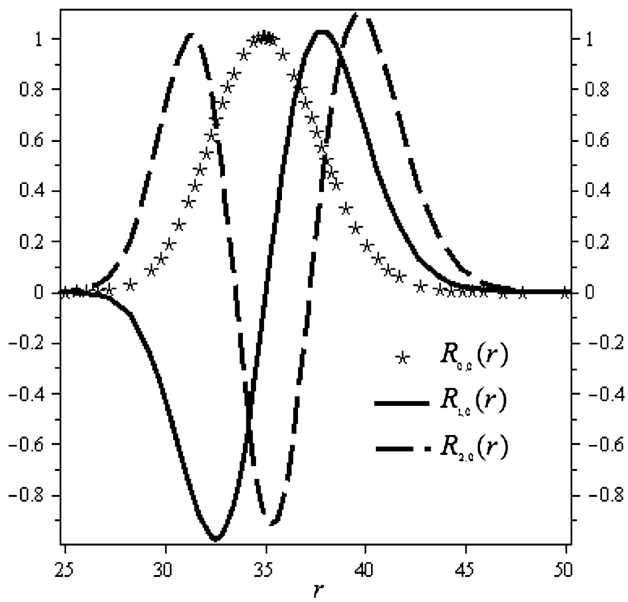

Fig. 7 The wave function of the system for $D_e = 3, A = -1, C = 1, D' = -1, m = 1, D = 3, B = 2, \alpha = 0.01$

4. Conclusions

We have obtained explicitly the analytical solutions of Klein–Gordon equation with a generalized GTW potential. GTW will be the better candidate for the study of molecular dynamics than the Morse and the TW potentials.

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