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ORIGINAL PAPER

Ion acoustic solitary waves in electron-positron-ion plasmas with q-nonextensive electrons and high relativistic ions

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Abstract: Propagation of small amplitude ion acoustic solitary waves in plasmas containing q-nonextensive electrons, thermal positrons and high relativistic ions is addressed in this paper. Our results show that the Korteweg-de Vries equation describes the nonlinear waves in such plasmas. The amplitude and the width of the solitons are derived and the effects of relativistic ions and q-nonextensive distribution of electrons on these quantities are discussed.

Keywords: Relativistic plasma; Nonlinear; Ion acoustic soliton

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1. Introduction

In the last few years, there has been increasing attention on the different types of localized wave structures in the multispecies plasmas [1–4]. The ion acoustic soliton (IAS) is one of the most aspects of nonlinear phenomena in modern plasma physics research. Such nonlinear wave structures arise from the competition between nonlinearity, dispersion and dissipation behaviours. Investigation of such nonlinear structures is usually carried out by employing of perturbation techniques. In small amplitude approximation of the equations, one can derive some forms of nonlinear differential equations for one spatial dimension situations like Korteweg-de Vries (KdV), modified Korteweg-de Vries (m-KdV) or nonlinear Schrodinger equation, etc. Such equations have well known extended solutions, like solitary waves or solitons. A great number of authors have studied ion-acoustic solitary solutions using the reductive perturbation technique in different plasmas [5, 6]. In contrast to the usual plasmas consisting of electrons and positive ions, it has been observed that the nonlinear waves in plasmas which contain additional components such as positrons have different characters [7]. The behaviour of the electron-positron-ion plasmas helps us to find better knowledge about the early universe which assumes to be a kind of plasma [8, 9], describing the active galactic nuclei [10], pulsar magnetospheres [11] and also the solar atmosphere [12]. Positrons can be used to probe particle transport in tokomaks, since they have sufficient lifetime. In this case, two-component electron-ion (e-i) plasmas become a three-component electron-ion-positron (e-i-p) medium [13]. During the last decade, e-p-i plasmas have attracted the attention of several authors [14–19]. It is known that the propagation of ion acoustic solitary wave is modified when the ion velocity approaches the speed of light. Relativistic plasmas occur in a variety of situations, such as, space–plasmas [20], laser–plasma interaction [21], plasma sheet boundary layer of the earth's magnetosphere [22]. This situation is also used for describing the Van Allen radiation belts [23]. Das and Paul [24] have investigated the weakly relativistic effects on ion-acoustic wave propagation in one dimension using the KdV equation for cold plasmas but without electron inertia effects. Nejoh [25] has investigated the same situation in the warm plasmas. El-Labany and Shaaban [26] have investigated nonlinear ion-acoustic waves in weakly relativistic plasmas consisting of warm ion-fluid with non-isothermal electrons through the modified equations. Nejoh and Sanuki [27] have studied the large amplitude Langmuir and ion-acoustic waves in relativistic two fluid plasmas with deriving the pseudo potential. The relativistic effects may be induced by the fluid velocity of the relativistic particles having speed

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near the light velocity. Also it is possible that the relativistic effects are induced by thermal effects of the particles under concern. In this case, the ratio $\frac{T}{mc^2}$ (where T is the particle temperature, m is its mass and c is the light velocity) cannot be neglected. Moreover, one can consider production of pair of electrons and positrons by relativistic ions [28]. In most of the studies, the authors worked in the weak relativistic limit, which assumes that the relativistic effect of flow speed of plasma is so small that the Lorentz relativistic factor γ can be approximated as $\gamma = 1/\sqrt{1-u^2/c^2} \cong 1 + u^2/2c^2$ in which u is the particle celcity and c is the light speed. The weak relativistic approximation itself is a convenient tool and can be refined more accurately by including higher order terms such as $\gamma \cong 1 + u^2/2c^2 + 3u^4/8c^4$. In this situation, relativistic effects are usually taken into account by using the equation of motion which can be conjectured by the simple replacement of the momentum u with the relativistic momentum $\gamma\, u$. In fact, the approximations in these studies are applied partially and inconsistently, considering relativistic effects only for the equation of motion. Recently, a fully relativistic set of two-fluid plasma equations derived from the covariant formulation of relativistic fluid equations has been outlined [29]. This new approach is consistent with the relativistic principle and consequently leads to a more general set of equations which is valid for fully relativistic plasmas with arbitrary Lorentz relativistic factor. The understanding of the behaviour of multi species plasmas containing cold or warm ions with Boltzmann's distribution has been studied for the last few years. More recently, it has been found that the distribution function of electrons and ions play a crucial role in characterizing the physics of the nonlinear waves [30–34]. In the last few years, a great deal of attention has been paid to the nonextensive statistical mechanics based on the deviations of Boltzmann–Gibbs–Shannon (B-G-S) entropic measure. Gougam et al. [35] have shown that the presence of a nonextensive distribution of electrons changes the nature of ion acoustic solitary structures. Astrophysical electron-nuclear plasmas are properly described by nonextensive distributions of metastable states. Leubner [36] has shown that distributions very close to kappa-distributions are a consequence of the generalized entropy favoured by nonextensive statistics, which provides the missing link for power-law models of nonthermal features from fundamental physics. Nonextensive statistics has been successfully applied to a number of astrophysical and cosmological scenarios. Those include stellar polytropes [37], the solar neutrino problem [38], peculiar velocity distributions of galaxies [39] and generally systems with long range interactions and fractal like space-times. Cosmological implications have been discussed [40] and recently an analysis of plasma oscillations in collisionless thermal plasmas has been provided from q-statistics in [41]. On the other hand, kappa-distributions are highly favoured in any kind of space plasma modelling [42] where a reasonable physical background was not apparent. A comprehensive discussion of kappa distributions in view of experimentally favoured non-thermal tail formations has been provided by Leubner et al. [43] (where typical values of the index κ are quoted and referenced for different space plasma environments). In the present analysis the missing link to fundamental physics is provided within the framework of an entropy modification consistent with nonextensive statistics. The family of kappa distribution is obtained from the positive definite part $12 \leq \kappa \leq \infty$, corresponding to $-1 \leq q \leq 1$ of the general statistical formalism where in analogy the spectral index kappa is a measure of the degree of nonextensivity. Since the main theorems of the standard Maxwell–Boltzmann statistics admit profound generalizations within nonextensive statistics [44–48], a justification for the use of kappa distributions in astrophysical plasma modeling is provided from fundamental physics. In recent years, several authors [49–51] have theoretically investigated the properties of ion and dust acoustic solitary waves in plasmas. The aim of the present paper is therefore to study the effects of nonextensive q-parameter on the electron acoustic solitary waves in plasmas consisting of high relativistic ions. Our investigation may be of wide relevance to astronomers and space scientists working on interstellar and space plasmas.

2. Basic equations

Let us consider one-dimensional, collisionless, unmagnetized high relativistic plasmas with thermal positrons and q-nonextensive electrons. Charge neutrality at equilibrium gives $n_{0e} = n_{0p} + n_0$, where n_0, n_{0e} and $n_{0\ p}$ are unperturbed ion, electron and positron number densities respectively. The nonlinear dynamics of the low frequency ion-acoustic solitons in the three component plasmas are governed by the following set of equations [30].

$$\begin{aligned} &\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0 \\ &\frac{\partial (\gamma u)}{\partial t} + u\frac{\partial (\gamma u)}{\partial x} + \frac{\partial \phi}{\partial x} = 0 \\ &\frac{\partial^2 \phi}{\partial x^2} = n_e - n - n_p \end{aligned} \tag{1}$$

where n and u are ion number density and ion fluid velocity respectively. Also ϕ and γ are electrostatic potential and Lorentz relativistic factor. For high relativistic plasmas parameter γ is approximated by its expansion up to term $\frac{u^4}{c^4}$ as

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} \cong 1 + \frac{u^2}{2c^2} + \frac{3u^4}{8c^4} \tag{2}$$

Effects of electron nonextensivity can be modelled using the following q-distribution function given by Lima et al. [41].

$$f_e(v_e) = C_q\left\{1 + (1-q)\left[\frac{m_e v_e^2}{2T_e} - \frac{e\psi}{T_e}\right]\right\}^{\frac{1}{q-1}} \tag{3}$$

where ψ stands for the electrostatic potential and the remaining variables/parameters have their usual meaning. It may be useful to note that $f_e(v_e)$ is the particular distribution that maximizes the Tsallis entropy and therefore conforms to the laws of thermodynamics. The constant of normalization C_q is given by

$$C_q = n_{e0}\frac{\Gamma(\frac{1}{q-1})}{\Gamma(\frac{1}{q-1}-\frac{1}{2})}\sqrt{\frac{m_e(1-q)}{2\pi T_e}} \quad \text{for } -1<q<1 \tag{4}$$

$$C_q = n_{e0}\left(\frac{1+q}{2}\right)\frac{\Gamma(\frac{1}{q-1}+\frac{1}{2})}{\Gamma(\frac{1}{q-1})}\sqrt{\frac{m_e(q-1)}{2\pi T_e}} \quad \text{for } q>1 \tag{5}$$

The parameter q stands for the strength of nonextensivity. It may be useful to note that for $q < -1$, the q-distribution is unnormalizable. In the extensive limiting case ($q = 1$), the q-distribution reduces to the well-known Maxwell–Boltzmann distribution. Also for $q > 1$, the q-distribution function exhibits a thermal cutoff on the maximum value allowed for the velocity of the particles, which is given by

$$v_{\max} = \sqrt{\frac{2T_e}{m_e}\left(\frac{e\psi}{T_e} + \frac{1}{q-1}\right)} \tag{6}$$

Integrating the q-distribution over all values of the velocity space, one obtains the following nonextensive hot electron number density [35].

$$n_e = \frac{1}{1-p}[1 + (q-1)\phi]^{\frac{q+1}{2(q-1)}} \tag{7}$$

where $p = \frac{n_{p0}}{n_{e0}}$ and q is a parameter quantifying the degree of nonextensivity and it is larger than -1 ($q > -1$).

Positrons are assumed to be in thermal equilibrium such that

$$n_p = \frac{p}{1-p}e^{-\sigma\phi}, \tag{8}$$

where $\sigma = \frac{T_e}{T_p}$ is the ratio of electron temperature to positron temperature. In Eq. (1), densities of the plasma species are normalized by unperturbed electron density n_{e0}. Ion velocity is normalized by the ion acoustic speed $c_i = \sqrt{T_e/m}$, space variables are normalized by electron Debye length $\lambda_D = \sqrt{T_e/4\pi n_0 e^2}$, time variable is normalized by electron plasma period $T = \sqrt{m_e/4\pi n_{e0}e^2}$ and electrostatic potential is normalized by $\frac{T_e}{e}$.

3. Derivation of the KdV equation

As mentioned before, reductive perturbation method has been used to investigate the behaviour of nonlinear ion acoustic waves in this plasma medium. The stretched coordinates are defined as follows [30]

$$\xi = \varepsilon^{1/2}\,(x - \lambda t), \quad \tau = \varepsilon^{3/2} t \tag{9}$$

where ε is a small parameter which characterizes the strength of the nonlinearity and λ is the phase velocity of propagated wave. Dependent variables are expanded as follows.

$$\begin{aligned} n &= 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \varepsilon^3 n_3 + \ldots, \\ u &= u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \ldots, \\ \phi &= \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \ldots \end{aligned} \tag{10}$$

By substituting Eq. (10) into Eq. (1), using Eq. (9) and collecting the terms with different powers of ε, one can derive the following equations in the lowest order of ε

$$\begin{aligned} & n_1 = \frac{(1-p)u_1}{\lambda - u_0}, \quad u_1 = \frac{(\lambda-u_0)(1+p\sigma)\phi_1}{(1-p)}, \\ & n_1 = (1+p\sigma)\phi_1 \\ & (\lambda-u_0)^2\gamma_1 = \frac{(1-p)}{\frac{q+1}{2} + p\sigma} \end{aligned} \tag{11}$$

For the higher orders of ε, we have

$$\begin{aligned} & -(\lambda-u_0)\frac{\partial n_2}{\partial\xi} + \frac{\partial n_1}{\partial\tau} + \frac{\partial(n_1u_1)}{\partial\xi} + (1-p)\frac{\partial u_2}{\partial\xi} = 0 \\ & \frac{\partial n_2}{\partial\tau} + u_2\frac{\partial n_1}{\partial\xi} + u_1\frac{\partial n_2}{\partial\xi} + n_1\frac{\partial u_2}{\partial\xi} + n_2\frac{\partial u_1}{\partial\xi} = 0 \\ & \left[\gamma_0 + \frac{u_0^2}{c^2} + \frac{3u_0^4}{2c^4} - (\lambda-u_0)\left(\frac{3u_0}{c^2} + \frac{15u_0^3}{2c^4}\right)\right]u_1\frac{\partial u_1}{\partial\xi} \\ & \quad -(\lambda-u_0)\left(\gamma_0 + \frac{u_0^2}{c^2} + \frac{3u_0^4}{2c^4}\right)\frac{\partial u_2}{\partial\xi} \\ & \quad + \left(\gamma_0 + \frac{u_0^2}{c^2} + \frac{3u_0^4}{2c^4}\right)\frac{\partial u_1}{\partial\tau} + \frac{\partial\phi_2}{\partial\xi} = 0 \\ & \frac{\partial^2\phi_2}{\partial\xi^2} + \left(p\sigma^2 - 1\right)\ \phi_1\phi_2 - \frac{1}{2}\left(\frac{1}{3} + \frac{1}{3}p\sigma^3\right)\phi_1^3 = 0 \end{aligned} \tag{12}$$

where $\gamma_0 = \ 1 + \frac{u_0^2}{2c^2} + \frac{3u_0^4}{8c^4}$.

Finally the KdV equation is derived from Eqs. (11) and (12) as

$$\frac{\partial\phi_1}{\partial\tau} + A\phi_1\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3} = 0 \tag{13}$$

with

$$\begin{aligned} A &= \frac{1}{2}\left[\frac{3}{(\lambda-u_0)\gamma_1} - \frac{\gamma_2}{\gamma_1^2} - \frac{\gamma_1(\lambda-u_0)^3\left(\frac{(q+1)(3-q)}{4} - p\sigma^2\right)}{(1-p)}\right], \\ B &= \frac{\gamma_1(\lambda-u_0)^3}{2} \end{aligned} \tag{14}$$

in which $\gamma_1 = 1 + \frac{3u_0^2}{2c^2} + \frac{15u_0^4}{8c^4}$, $\gamma_2 = \frac{3u_0}{c^2} + \frac{15u_0^3}{2c^4}$.

The stationary solution of Eq. (13) is given by

$$\phi_1 = \phi_0 \sec h^2 \frac{(\xi - U\tau)}{w} \tag{15}$$

where U is constant velocity of solitary wave. The ion acoustic wave amplitude (ϕ_0) and its width (w) are given as

$$\phi_0 = \frac{3U}{A}, \quad w = 2\sqrt{\frac{B}{U}} \tag{16}$$

Parameters A and B can be compared with the results in [44] for nonplanar ion-acoustic solitary waves in electron-positron-ion plasmas with electrons following a q-nonextensive distribution. Our results (with $\gamma_1 = 1$and $\gamma_2 = 0$) reduce to the results of [44] (with $m = 0$) correctly. It may be noted that plasma components in [44] are not relativistic.

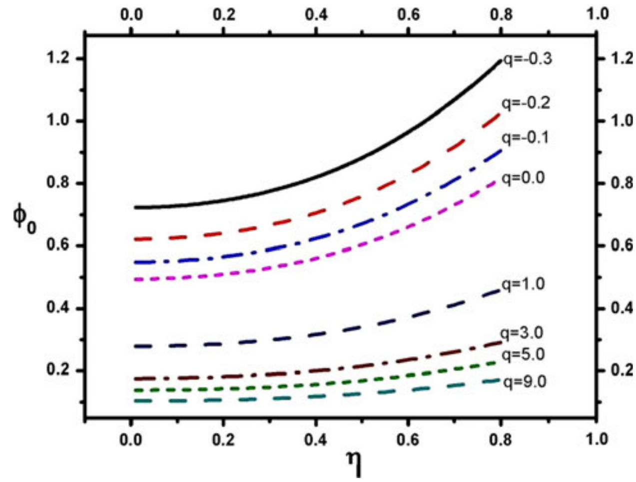

Fig. 1 Soliton amplitude shown as a function of η for different values of q. The other parameters are $\sigma = 0.1$, $p = 0.6$ and $u = 0.2$

4. Discussion

We have two important parameters which affect the behaviour of propagated solitons in the above described plasma. These are relativistic parameter η which is characterises by $\eta = \frac{u_0}{c}$ and nonextensive parameter q. Equation (14) shows that the effects of these parameters on the behaviour of soliton are complicated. Therefore, some plots can help us to find the general influence of these parameters on the soliton characters. It may be noted that our calculations are valid for $\eta < 0.65$ with acceptable precision.

Figure 1 presents the soliton amplitude as a function of η with different values of q. The other parameters have been chosen as $\sigma = 0.1$, $p = 0.6$ and $u = 0.2$. This figure also shows that soliton amplitude increases when η increases and also the increasing rate is higher for larger values of q. The increase of the parameter η means that the ions velocity becomes larger and therefore the soliton energy (read the soliton amplitude) increases. But one can find from the Fig. 1 that the soliton amplitude decreases when q increases. Therefore, the behaviour of these two parameters is opposite.

Figure 2 demonstrates the soliton width as a function of η for different values of q. This figure shows that the width of the soliton decreases with an increasing η, and also with increasing values of q too. However, the effects of the nonextensive parameter q is more significant.

Figures 1 and 2 show that our results in the interval $0 < \eta$<0.2 (for weakly relativistic plasmas) confirm the results reported by Pakzad [30]. It is observed that the changing ratio of ϕ_0 in the high relativistic limit is more than that of in weakly relativistic situation.

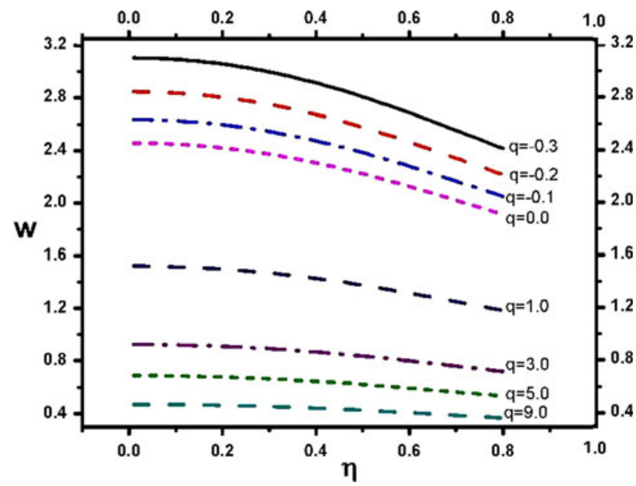

Fig. 2 Soliton width as a function of η for different values of q. Other parameters are $\sigma = 0.1$, $p = 0.6$ and u $= 0.2$

The soliton profile is presented in Fig. 3 for different values of η. This figure also indicates that solitons have higher energy with larger values of η. It can be seen that as η increases, i.e., the relativistic character of the plasma becomes important, the soliton amplitude increases, while its width becomes narrower. This means that an increase in η makes the solitary structure more spiky.

Figure 4 shows that soliton amplitude and its width decrease when q increases. In other words, soliton energy decreases when nonextensive character of the plasma becomes dominant i.e. the electrons evolve far away from their Maxwell–Boltzmann thermodynamic equilibrium. It is noted that nonextensivity contributes to the change in *Ne* in the region of soliton lump.

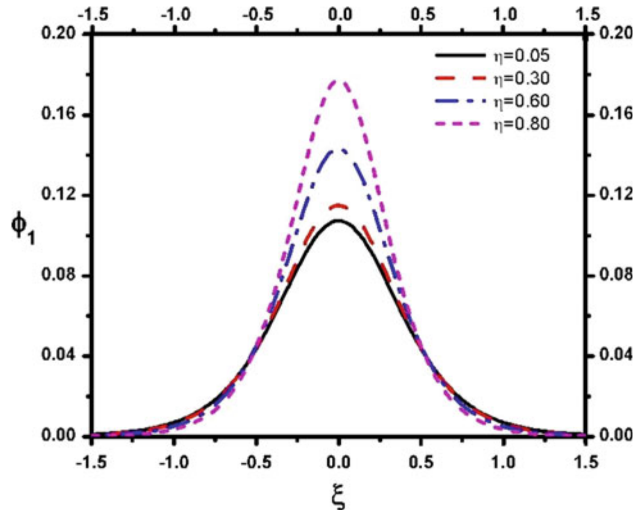

Fig. 3 Soliton profiles shown as a function of ξ for different values of η. Other parameters are $\sigma = 0.1$, $p = 0.6$, $u = 0.2$ and $q = 8.5$

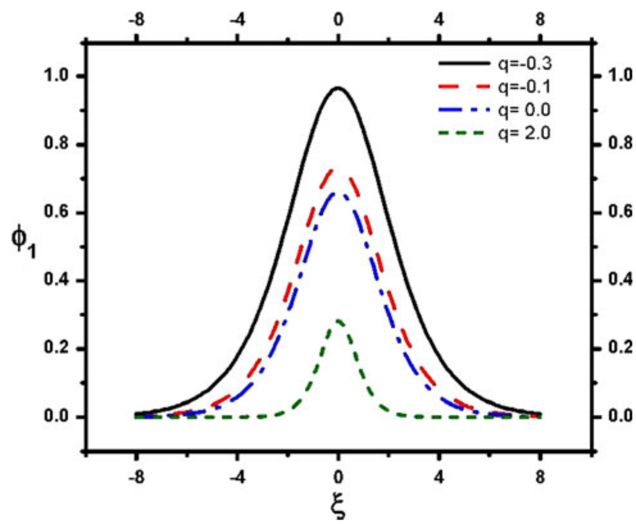

Fig. 4 Soliton profile shown as functions of ξ for different values of q. The other parameters are $\sigma = 0.1$, $p = 0.6$, $u = 0.2$ and $\eta = 0.6$

5. Conclusions

The propagation of Ion acoustic solitary waves in electron-positron-ion plasmas with q-nonextensive electrons and high relativistic ions has been investigated. Results are in agreement with the results for weakly relativistic situation which have been presented before. However, some new results have been found relative to non-realistic or weakly relativistic situations. It is shown that the soliton energy and its amplitude increase when the ions take relativistic velocities while the width of the soliton decreases. This means that the soliton becomes spiky when ions contribute in the plasma with high relativistic speed. On the other hand, the effects of the nonextensive parameter are in opposite direction of the effect of the relativistic parameter. Therefore the solitary waves may propagate in such plasmas with better stability.

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