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ORIGINAL PAPER

Quantum field theory from an exponential action functional

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Abstract: In recent years plentiful research papers have been advocated to the study of analytical techniques in calculus of variations. Many important applications are found in such fields as quantum field theories and many new properties have been raised, studied and explicit solutions have been achieved by many authors. In this work we derive a modification of the Klein–Gordon and Dirac equations of quantum field theories starting from an exponential action functional recently introduced by the author of the present work. Both standard and non-standard Lagrangians are considered. It was observed that some quantum field equations which appear in many field theories dealing with quantum gravitational corrections are raised.

Keywords: Exponential action functional; Modified Klein–Gordon equation; Modified Dirac equation; Non-standard Lagrangians

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1. Introduction

In recent years overflowing research papers have been advocated to the study of generalized and modified quantum field theories that help to solve divergence problems [1]. Some of them include higher-derivative theories [2, 3], quantum field theory on noncommutative spacetime [4], Galilei-invariant version of a field theory [5], fractional quantum field theories [6–18] and so on. Besides, some of basic equations of quantum field theory are modified at very high energies due to quantum gravity effects [19–22]. On the other hand, it is strongly believed that the variational approach is one of the corner-stones of nonperturbative methods in quantum mechanics and quantum field theory where many applications can be found in the literature with growing attractiveness [23, 24]. The main purpose of this communication is to construct some of the basic equations in a modified quantum field theory mainly the modified Klein–Gordon and the modified Dirac equations starting from the exponential action function $S = \int e^{L} dt$ recently introduced by the author [25]. Here L is the Lagrangian of the theory which could be standard and non-standard as well. It is noteworthy that the non-standard Lagrangian formalism is a new approach introduced recently in literature to formulate many hidden properties of a given dynamical system [26–28]. We will demonstrate how this new functional will lead to numerous original attractive properties of the quantum field theory. Herein, units where $\hbar = c = 1$ are used. The metric is diagonal and its entries are $(1, -1, -1, -1)$. Greek indices run from 0 to 3. Before we do so however, let us reexamine some well-known aspects of the exponential action function (EAF). In fact, the EAF is basically defined by $S = \int_a^b e^{L(t,\dot{q}(t),q(t))} dt$ where $(t, \dot{q}(t), q(t)) \to L(t, \dot{q}(t), q(t))$ is assumed to be a C^2 functions with $q(t) \in C^1([a,b]; \mathbf{R}^n)$ the generalized coordinate and $L(t, \dot{q}(t), q(t)) \in C^2([a,b] \times \mathbf{R}^n \times \mathbf{R}^n; \mathbf{R})$ is the Lagrangian of the theory and $\dot{q}(t) = dq/dt$. Any admissible function $q \in C^1[a,b]$ subject to given boundary conditions $q(a) = q_a$ and $q(b) = q_b$ for which the action has an extremum satisfies the subsequent Euler–Lagrange equation:

$$\frac{\partial L}{\partial q(t)} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}(t)}\right) = \frac{\partial L}{\partial \dot{q}(t)}\left(\frac{\partial L}{\partial t} + \dot{q}(t)\frac{\partial L}{\partial q(t)} + \ddot{q}(t)\frac{\partial L}{\partial \dot{q}(t)}\right). \tag{1}$$

In two dimensions, this equation is generalized as follows: we consider a smooth 2-dimensional manifold M and we

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let $q:\Omega\subset\mathbf{R}^2\to M$ be the admissible paths satisfying fixed Dirichlet conditions on $\partial\Omega$. The 2-dimensional EAF is defined by $S=\iint\Omega e^{L(x,\,y,\,q,\,q_x,\,q_y)}dxdy$ where $q_x=\partial q/\partial x$, $q_y=\partial q/\partial y$, and $q=q(x,\,y)$. The Lagrangian is supposed to be an adequately smooth function of its five arguments. It is an easy exercise to prove that if $q=q(x,\,y)$ makes the two-dimensional action stationary, then it satisfies the following two-dimensional modified Euler–Lagrange equation:

$$\begin{aligned}\frac{\partial L}{\partial q}-\frac{d}{dx}\left(\frac{\partial L}{\partial q_x}\right)-\frac{d}{dy}\left(\frac{\partial L}{\partial q_y}\right)&=\frac{dL}{dx}\frac{\partial L}{\partial q_x}+\frac{dL}{dy}\frac{\partial L}{\partial q_y},\\&=\left(\frac{\partial L}{\partial x}+q_x\frac{\partial L}{\partial q}+q_{xx}\frac{\partial L}{\partial q_x}\right)\frac{\partial L}{\partial q_x}\\&\quad+\left(\frac{\partial L}{\partial y}+q_y\frac{\partial L}{\partial q}+q_{yy}\frac{\partial L}{\partial q_y}\right)\frac{\partial L}{\partial q_y}.\end{aligned}\tag{2}$$

Here $q_{xx}=\partial^2q/\partial x^2$ and $q_{yy}=\partial^2q/\partial y^2$. The proof is obtained by considering the following scalar function $S(\varepsilon)=\iint_\Omega Ldxdy$ for any $\varepsilon\in\mathbf{R}$ where $L=e^{L(x,\,y,\,q+\varepsilon Q,\,q_x+\varepsilon Q_x,\,q_y+\varepsilon Q_y)}$. Here the variation $Q(x,\,y)$ is assumed to satisfy homogeneous Dirichlet boundary conditions $Q(x,y)=0$ for $(x,\,y)\in\Omega$. If $q=q(x,\,y)$ is a minimizer then the EAF will have a minimum at $\varepsilon=0$ and accordingly $S'(0)=0$. We assume naturally that the functions involved are adequately smooth so as to permit us to bring the derivative inside the integral, and then apply the chain rule. Accordingly, following the standard procedure found in any "Calculus of Variations" textbooks, we obtained the required result.

All the previous arguments can be repeated to higher dimensions and it is an easy exercise to prove that for the following 4-dimensional EAF $S=\iiiint_\Omega e^{L(x,\,q(x),\,\dot q(x))}dx$ with $x=(x_1,\,x_2,\,x_3,\,x_4)$ and where the admissible paths are smooth functions $q:\Omega\subset\mathbf{R}^4\to M$ satisfying giving Dirichlet boundary conditions on $\partial\Omega$, the following modified Euler–Lagrange equation holds accordingly:

$$\frac{\partial L}{\partial q_i}-\frac{d}{dx_i}\left(\frac{\partial L}{\partial q_{x_i}}\right)=\left(\frac{\partial L}{\partial x_i}+q_{x_i}\frac{\partial L}{\partial q}+q_{x_ix_i}\frac{\partial L}{\partial q_{x_i}}\right)\frac{\partial L}{\partial q_{x_i}},\tag{3}$$

where the Einstein summation is used. We expect naturally that all these arguments may be applied successfully to both scalar and spinor fields. Our main aim afterward is to construct some of the basic equations of quantum field theory based on the EAF, mainly the Klein–Gordon and the Dirac equations, and to explore some of their consequences.

2. Modified Klein–Gordon equation

In order to derive the corresponding Klein–Gordon equation from the EAF, we replace the generalized coordinate q_i by the scalar field $\phi(x)$where $x=(t,\vec{x})$, i.e. the generalized coordinate q_i has been replaced by the field variable $\phi(x)$ and the discrete index i has been replaced by a continuously varying index x. A covariant form of the EAF may be obtained from the non-covariant EAF $S=\int e^Ldt$ by simply considering $S=\int e^Ld^4x=\int e^Ld^3xdt$ where $e^L=\int e^Ld^3x$ and $e^L=e^{L(\phi,\partial_\mu\phi)}$ where Roman letters $(i,\,j,\,k,\,l,\,m,\,n)$ run from 1 to 3, Greek letters $(\alpha,\,\beta,\,\gamma,\,\delta,\,\mu,\,v,\,\eta,\,\xi)$ run from 0 to 3, 4-vector $(t,\,x,\,y,\,z)\to(x^0,\,x^1,\,x^2,\,x^3)$, contravariant vectors transform as $A'^\alpha=(\partial x'^\alpha/\partial x^\beta)A^\beta$ and covariant vectors transforms as $A'_\alpha=(\partial x^\beta/\partial x'^\alpha)A_\beta$. For a given scalar field ϕ, $\partial\phi_a\equiv\partial\phi/\partial a$. We can perform the following replacement into Eq. (2) [1]:

$$\frac{\partial L}{\partial q_i}\leftrightarrow\frac{\partial L}{\partial\phi(x)},\qquad \frac{d}{dt}\leftrightarrow\partial_\mu\equiv\frac{\partial}{\partial x^\mu},\qquad \frac{\partial L}{\partial\dot q_i}\leftrightarrow\frac{\partial L}{\partial(\partial_\mu\phi(x))}.$$

It is notable that ∂_μ contains space and time derivatives as well. Accordingly, the modified covariant generalization of the point particle Euler–Lagrange equation is:

$$\begin{aligned}\frac{\partial L}{\partial\phi(x)}-\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\phi(x))}\right)&=\left(\frac{\partial L}{\partial x^\mu}+\partial_\mu\phi(x)\left(\frac{\partial L}{\partial\phi(x)}\right)\right.\\&\left.+\partial_\mu\partial_\mu\phi(x)\left(\frac{\partial L}{\partial(\partial_\mu\phi(x))}\right)\right)\frac{\partial L}{\partial(\partial_\mu\phi(x))}.\end{aligned}\tag{4}$$

If there is more than one scalar field, then we replace simply $\phi(x)$ by $\phi(x_i)\equiv\phi_i$ in Eq. (4). To illustrate, we choose the Lagrangian density $L=\partial_\mu\phi(x)-M\phi(x)-Mx^\mu$ where $M$is a free parameter. The field equation is then easily derived and takes the form $\partial_\mu\partial_\mu\phi(x)-M\partial_\mu\phi(x)=0$ or after contracting by the inverse Minkowski metric tensor $g^{\mu v}$ and letting $\mu=v$, we get $\Box\phi(x)-M\partial^\mu\phi(x)=0$ where $\Box=\partial^\mu\partial_\mu$. This could be interpreted as the modified damped massless Klein–Gordon equation [29]. If for instance, the Lagrangian is function of scalar field and its derivative through the invariant d'Alembertian operator $\Box$, i.e. $L=L(\phi(x),\Box\phi(x))$, then from the principle of least action, Eq. (1) takes the special form:

$$\begin{aligned}\frac{\partial L}{\partial\phi(x)}+\Box\left(\frac{\partial L}{\partial(\Box\phi(x))}\right)&=\left(\Box\phi(x)\left(\frac{\partial L}{\partial\phi(x)}\right)\right.\\&\left.+\Box\Box\phi(x)\left(\frac{\partial L}{\partial(\Box\phi(x))}\right)\right)\frac{\partial L}{\partial(\Box\phi(x))}.\end{aligned}\tag{5}$$

To illustrate, we consider the well-know massive Klein–Gordon Lagrangian density given by $L=-\frac{1}{2}\phi(x)\Box\phi(x)-\frac{1}{2}m^2\phi^2(x)$ where m is the mass of the field. Then the corresponding Euler–Lagrange equation is derived from Eq. (4):

$$\frac{1}{2}\phi^2(x)\Box\Box\phi(x)+\frac{1}{2}\phi(x)\Box\phi(x)\Box\phi(x) + (1+m^2\phi^2(x))\Box\phi(x)+m^2\phi(x)=0. \quad (6)$$

This is a modified Klein–Gordon equation for a free massive scalar field and is somewhat mathematically complicated. For a massless scalar field, this equation is reduced to $\phi^2(x)\Box\Box\phi(x)+\phi(x)\Box\phi(x)\Box\phi(x)+2\Box\phi(x)=0$. If, for instance, we choose $L=\Box\phi(x)+m^2\phi(x)$, then the equation of motion is $m^2\Box\phi(x)+\Box\Box\phi(x)=m^2$. One more illustration concerns the non-standard Lagrangian $L=\Box\phi(x)+U(x)+m^2\phi(x)$ where $U(x)$ is a potential. Hence, Eq. (4) gives $m^2\Box\phi(x)+\Box\Box\phi(x)=m^2-dU(x)/dx$.

More generally, we can consider $L=L(\phi(x),(\Box+m_1^2)\phi(x))$ with m_1 a real parameter. It is easy to check that Eq. (5) is replaced by:

$$\frac{\partial L}{\partial\phi(x)}+(\Box+m_1^2)\left(\frac{\partial L}{\partial((\Box+m_1^2)\phi(x))}\right) = \left((\Box+m_1^2)\phi(x)\left(\frac{\partial L}{\partial\phi(x)}\right)+(\Box+m_1^2)(\Box+m_1^2)\phi(x)\left(\frac{\partial L}{\partial\left((\Box+m_1^2)\phi(x)\right)}\right)\right)\frac{\partial L}{\partial\left((\Box+m_1^2)\phi(x)\right)}. \quad (7)$$

Accordingly, let us consider the Lagrangian $L=(\Box+m_1^2)\phi(x)(\Box+m_2^2)\phi(x)$ where m_2 is another real parameter. Eq. (7) gives easily:

$$(\Box+m_1^2)(\Box+m_2^2)\phi(x)[1-L]=0. \quad (8)$$

As $L\neq 1$, then the field equation is $(\Box+m_1^2)(\Box+m_2^2)\phi(x)=0$ or $\Box\Box\phi(x)+(m_1^2+m_2^2)\Box\phi(x)+m_1^2m_2^2\phi(x)=0$. This equation is obtained long time ago by Pais and Uhlenbeck in their quantum approach to field theory [30] and recently by the authors of [19, 20] in their Lorentz-covariant deformed Quesne–Tkachuk algebra formulation of quantum field theory in the presence of a minimal length due to gravitational corrections. One more interesting example is obtained if we choose the simple Lagrangian $L=(\Box+m_1^2)\phi(x)$ from which we derive $(\Box+m_1^2)(\Box+m_1^2)\phi(x)=0$. These equations are interesting as they are derived from the EAF without any quantum arguments.

To have a naïve idea about the approximate solutions of some of the modified Klein–Gordon equations obtained previously, we follow the standard and usual approach and we separate the equation into the space and time parts. We consider for straightforwardness the case of one space dimension.

Consequently our preliminary point is to write the equation $m^2\Box\phi(x)+\Box\Box\phi(x)=m^2$ as:

$$\frac{d^4\phi(x)}{dx^4}-m^2\frac{d^2\phi(x)}{dx^2}=m^2, \quad (9)$$

where the solution is given by:

$$\phi(x)=\frac{c_1}{m^2}e^{mx}+\frac{c_2}{m^2}e^{-mx}-\frac{x^2}{2}+c_3x+c_4, \quad (10)$$

c_i, $i=1, 2, 3, 4, \ldots$ are constants of integration. Assuming the boundary conditions $\phi(0)=0$ and $\phi'(0)=0$, this equation is reduced to:

$$\phi(x)=\frac{c_1}{m^2}(e^{mx}-1-x)+\frac{c_2}{m^2}(e^{-mx}-1+x)-\frac{x^2}{2}. \quad (11)$$

Notice that for a positive value of m, this equation is approximated for a very large distance to $\phi(x)=c_1e^{mx}/m^2$.

The second class of modified Klein–Gordon equation corresponds for $m^2\Box\phi(x)+\Box\Box\phi(x)=m^2-dU(x)/dx$ which in space dimension is written as:

$$\frac{d^4\phi(x)}{dx^4}-m^2\frac{d^2\phi(x)}{dx^2}=m^2-\frac{dU(x)}{dx}. \quad (12)$$

We choose the quadratic potential $U(x)=\frac{1}{2}m^2x^2$. Then the solution of Eq. (12) is given by:

$$\phi(x)=\frac{1}{6}\left(\frac{6(c_5e^{mx}+c_6e^{-mx})}{m^2}+(x-3)x^2\right)+c_7x+c_8. \quad (13)$$

With the boundary conditions $\phi(0)=0$ and $\phi'(0)=0$, this equation is reduced to:

$$\phi(x)=\frac{c_5}{m^2}(e^{mx}-mx-1)+\frac{c_6}{m^2}(e^{-mx}+mx-1)+(x-3)x^2. \quad (14)$$

This equation is approximated for a very large distance and in particular for positive m to $\phi(x)=c_5e^{mx}/m^2$.

The third class is $\Box\Box\phi(x)+(m_1^2+m_2^2)\Box\phi(x)+m_1^2m_2^2\phi(x)=0$ which is written in space dimension as:

$$\frac{d^4\phi(x)}{dx^4}-(m_1^2+m_2^2)\frac{d^2\phi(x)}{dx^2}+m_1^2m_2^2\phi(x)=0. \quad (15)$$

The solution is given by:

$$\phi(x)=c_9e^{m_1x}+c_{10}e^{-m_1x}+c_{11}e^{m_2x}+c_{12}e^{-m_2x}. \quad (16)$$

If, for instance, $m_1^2=-m_2^2$, then $\Box\Box\phi(x)-m_1^4\phi(x)=0$ or $d^4\phi(x)/dx^4-m_1^4\phi(x)=0$ and then the solution is reduced with $\phi(0)=0$ and $\phi'(0)=0$ to:

$$\phi(x)=c_{13}(e^{m_1x}-\sin(m_1x)-\cos(m_1x)) + c_{14}(e^{-m_1x}+\sin(m_1x)-\cos(m_1x)). \quad (17)$$

For large distance, this equation is approximated by $\phi(x)\approx c_{13}e^{m_1x}$ whereas for very short distances, Eq. (17) is approximated by $\phi(x)\approx c_{13}(1-\sin(m_1x)-\cos(m_1x))+c_{14}(1+\sin(m_1x)-\cos(m_1x))$ which resembles the harmonic oscillator solutions.

3. Modified Dirac equation

All the previous arguments may be repeated for the case of a Dirac electrodynamics complex spinor field, i.e. $\phi(x) \leftrightarrow \psi(x)$ with its complex conjugate $\bar{\psi}$, which are invariant under the transformation $\psi \to \psi' = e^{i\Omega}\psi$ and $\bar{\psi} \to \bar{\psi}' = e^{-i\Omega}\bar{\psi}$ where Ω is the infinitesimal arbitrary function of x [1]. Assuming for simplicity the $(0+1)$-dimensional Lagrangian $L = L(t, \psi(t), \dot{\psi}(t))$, it is an easy exercise to check that the corresponding Euler–Lagrange equation is:

$$\frac{\partial L}{\partial \psi(t)} - \frac{d}{dt}\left(\frac{\partial L}{\partial(\dot{\psi}(t))}\right) = \left(\frac{\partial L}{\partial t} + \dot{\psi}(t)\left(\frac{\partial L}{\partial \psi(t)}\right) + \ddot{\psi}(t)\left(\frac{\partial L}{\partial(\dot{\psi}(t))}\right)\right)\frac{\partial L}{\partial(\dot{\psi}(t))}. \tag{18}$$

The one associated to the complex conjugate of the spinor field takes again the form:

$$\frac{\partial L}{\partial \bar{\psi}(t)} - \frac{d}{dt}\left(\frac{\partial L}{\partial(\dot{\bar{\psi}}(t))}\right) = \left(\frac{\partial L}{\partial t} + \dot{\bar{\psi}}(t)\left(\frac{\partial L}{\partial \bar{\psi}(t)}\right) + \ddot{\bar{\psi}}(t)\left(\frac{\partial L}{\partial(\dot{\bar{\psi}}(t))}\right)\right)\frac{\partial L}{\partial(\dot{\bar{\psi}}(t))}. \tag{19}$$

To illustrate, we consider the free standard electron Lagrangian in $(0+1)$-dimensions $L = \bar{\psi}(t)[i\gamma_0\dot{\psi}(t) - m\psi(t)]$ where $\psi(t)$ is a two-components spinor, $\bar{\psi}(t) = \psi^\dagger(t)\gamma_0$ is the adjoint field operator, m is a constant parameter with the dimension of the mass and

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{20}$$

In fact, it was argued in [31] that quantum field theory in $(0+1)$ dimensions is formally equivalent to quantum mechanics and besides it simplifies the mathematical concepts considerably. Then Eqs. (18) and (19) give:

$$-m\bar{\psi}(t) - i\gamma_0\dot{\bar{\psi}}(t) = i\gamma_0\bar{\psi}(t)\bar{\psi}(t)\left(i\gamma_0\ddot{\psi}(t) - m\dot{\psi}(t)\right), \tag{21}$$

and

$$i\gamma_0\dot{\psi}(t) - m\psi(t) = 0. \tag{22}$$

Deriving Eq. (22) with respect to time and replace into the RHS of Eq. (21) gives:

$$i\gamma_0\dot{\bar{\psi}}(t) + m\bar{\psi}(t) = 0. \tag{23}$$

Equations (22) and (23) are the well-known Dirac equations. It is an easy exercise to check that these equations hold as well for a time-dependent mass, i.e. $L = \bar{\psi}(t)[i\gamma_0\dot{\psi}(t) - m(t)\psi(t)]$.

One supplementary illustration concerns the non-standard Lagrangian $L = \ln\left(i\gamma_0\psi(t)\dot{\bar{\psi}}(t)\right) - 2\ln\left(\psi(t)\bar{\psi}(t)\right) + \alpha\dot{\psi}(t) + \beta\dot{\bar{\psi}}(t)$ where α and β are real parameters. Equations (18) and (19) give accordingly:

$$\alpha^2\psi(t)\ddot{\psi}(t) - \alpha\dot{\psi}(t) = -1, \tag{24}$$

and

$$\frac{m}{\bar{\psi}(t)} + \ddot{\bar{\psi}}(t)\frac{1}{\dot{\bar{\psi}}(t)} = \left(\frac{1}{\dot{\bar{\psi}}(t)} + \beta\right)\left(-2\dot{\bar{\psi}}(t)\frac{1}{\bar{\psi}(t)} + \ddot{\bar{\psi}}(t)\frac{1}{\dot{\bar{\psi}}(t)}\right). \tag{25}$$

Equation (25) gives for $\forall\beta$

$$\ddot{\bar{\psi}}(t)\bar{\psi}(t) - 2\dot{\bar{\psi}}^2(t) = 0. \tag{26}$$

We choose $\alpha = \pm 1$ and then the solutions of Eqs. (24) and (26) are respectively:

$$\psi(t) = (d_1 + t)(d_2 \pm \log(d_1 + t)), \tag{27}$$

and

$$\psi(t) = \frac{d_3}{t + d_4}. \tag{28}$$

Here d_i, $i = 1, 2, 3, 4$ are constants of integration.

Up to now, we have derived the equations of motion for some scalar and spinor field systems characterized by both standard and non-standard Lagrangians in an easy way. It is notable that when applying the EAF to those fields holding standard and non-standard Lagrangians, the equations of motion will appear to be absolutely different from what is obtained when we use the standard action principle characterized by the action functional $S = \int L dt$. Of course, further Lagrangians may be examined as well; hitherto more mathematical analysis of the far-reaching equations of motion will be addressed with awareness in a forthcoming work.

4. Conclusions

In conclusion, we have derived in this work the modified Klein–Gordon and the modified Dirac equations starting from the exponential action functional $S = \int e^L dt$ recently introduced by the author. Both the standard and non-standard Lagrangians are considered and discussed. It is noteworthy that classical and quantum dynamics with non-standard Lagrangians are still in their infancies and much work is required. Yet the modified Klein–Gordon equation derived from the standard Klein–Gordon Lagrangian is somewhat complicated and required more numerical analysis. Nevertheless, it is in this work that, starting from an exponential action, the scalar field equation $\Box\Box\phi(x) + (m_1^2 + m_2^2)\Box\phi$

$(x) + m_1^2 m_2^2 \phi(x) = 0$ which appears in many theories dealing with quantum gravitational corrections arise. Besides, when we have applied the EAF to the Dirac electrodynamics spinor field characterized by its usual Lagrangian, it is that the Dirac equations for both the spinor field and its complex conjugate are not affected and acquire their standard form. We argue that these outcomes may have many physical applications, e.g. Higgs field [32], black-holes fields [33] and more advanced field theories [34–36]. The main advantage of the results obtained in this work is that they offer a newfangled view of quantum field theory, yet much work is required for a better understanding of the theory. We speculate that these new arguments can have several consequences in several modified field theories, e.g. higher-derivative theories and quantum field theory on noncommutative spacetime that deserve future inquiries. Associated work that lies ahead could include the formulation of quantum field theories starting from different types of action functional and derivatives operators [37–42].

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