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Superhorizon fluctuations and acoustic oscillations in relativistic heavy ion collisions

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Abstract : We focus on the initial state spatial anisotropies, originating at the thermalization stage, for central collisions in relativistic heavy-ion collisions. We argue that the physics of fluctuations at the early stages of heavy ion collisions has strong similarities with the physics of density fluctuations in the early universe which give rise to remarkable acoustic peaks in the cosmic microwave background radiation (CMBR) power spectrum. Following the method of analysis in CMBR physics, we propose that a plot of root mean square values of the flow coefficients $\sqrt{v_n^2} \equiv v_n^{rms}$, calculated in a laboratory fixed coordinate system, for *a* large range of n from 1 to about 30, can give non-trivial information about the initial stages of the system and its evolution. We also argue that for all wavelengths λ of the anisotropy (at the surface of the plasma region) much larger than the acoustic horizon size H_s^{fr} at the freezeout stage, the resulting values of v_n^{rms} should be suppressed by a factor of order $2H_s^{fr}/\lambda$.

Keywords : Flow, quark gluon plasma, initial conditions

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1. Introduction

In relativistic heavy ion collision experiments (RHICE), the observation of elliptic flow has given strong evidence for very early thermalization and of the collective behavior of the partonic matter produced. It has also been noticed [1] that even in central collisions, due to initial state fluctuations, one can get non-zero anisotropies in particle distribution (and hence in final particle momenta) in a given event, though these will be typically much smaller in comparison to the non-central collisions. These will average out to zero when large number of central events are considered. We argue that there is a deep correspondence between the physics of these fluctuations and that of the density fluctuations in the early universe which give rise to th anisotropies in CMBR (with obvious

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difference of the absence of gravity effects for RHICE). Following the successes of the analysis of the CMBR anisotropy power spectrum, we argue below that, for central events in RHICE, a plot of the root mean square values of the flow coefficients $\sqrt{\overline{v_n^2}} \equiv v_n^{rms}$, calculated in a laboratory fixed coordinate system, for a large range of n from 1 to about 30, can give non-trivial information about the initial stages of the system and its evolution.

2. The model

We focus mainly on central events. Just as for the elliptic flow, here also we consider only transverse fluctuations by assuming Bjorken scaling for the longitudinal expansion. For a given central event, azimuthal distribution of particles and energy density are in general anisotropic due to fluctuations of nucleon coordinates as well as due to localized nature of parton production during initial nucleon collisions. As an example, for central Au-Au collision at 200 GeV/A center of mass energy and rapidity window $\Delta(y) = 1$ centered at y = 0, we show the contour plot of initial transverse energy density at $\tau_{eq} = 1$ fm obtained from HIJING in Fig. 1. We assume that by this time the produced partons thermalize and hydrodynamic description becomes applicable for subsequent times.

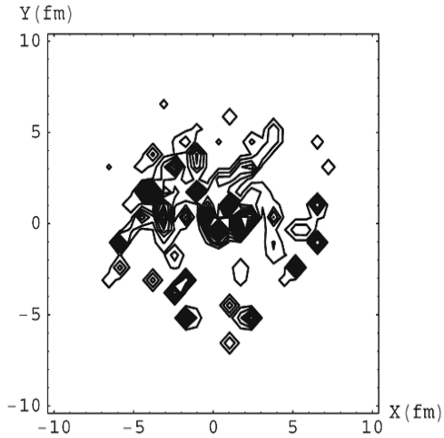

Figure 1. Contour plot of the initial transverse energy density.

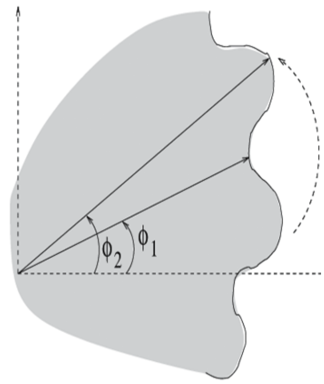

Figure 2. Schematic diagram of a part of the region showing spatial anisotropies of small wavelength.

Azimuthal anisotropy of produced partons is manifest in Fig. 1. It is thus reasonable to expect that the equilibrated matter resulting from this parton distribution will also have azimuthal anisotropies (as well as radial fluctuations) of similar level. According to flow results, thermalization happens quickly, within proper time $\tau_{eq} \leq 1$ fm for RHIC energies. Hence, no homogenization can be expected to occur beyond length scales larger than this at thermalization. Thus we see that anisotropies with wavelengths larger than the causal horizon scale should be necessarily present at the thermalization stage. This brings us to the most important correspondence between the universe and the RHICE. It is the presence of fluctuations with superhorizon wavelengths. In the universe, inflation stretches out quantum fluctuations of subhorizon scale to density fluctuations of superhorizon scales. Here, the transverse fluctuations are superhorizon because of simultaneous collisions of different (transverse) parts of the system with large transverse dimensions compared to the causal horizon at the thermalization stage which is given by $c\tau$. It is more appropriate to use the sound horizon, $H_s \sim c_s \tau$ where c_s is the sound speed, as we are interested in the flow arising from pressure gradients. At the stage of equilibration, $c\tau_{eq}$ is at most 1 fm, with corresponding acoustic horizon H_s^{eq} being even smaller. Thus every fluctuation of wavelength larger than H_s^{eq} is superhorizon. With the nucleon size being about 1.6 fm, the equilibrated matter will necessarily have density inhomogeneities with superhorizon wavelengths (see Fig. 1). As the system evolves beyond τ_{eq}, fluctuations of larger wavelengths enter the horizon.

To estimate spatial anisotropies for the system as in Fig. 1 we will use the following procedure. We take τ_{eq} to be 1 fm. We calculate the anisotropies in the fluctuations in the spatial extent $R(\phi)$ at this stage, where $R(\phi)$ represents the energy density weighted average of the transverse radial coordinate in the angular bin at azimuthal coordinate . We divide the region in 50 - 100 bins of azimuthal angle ϕ, and calculate the Fourier coefficients of the anisotropies in $\delta R/R \equiv \left(R(\phi) - \bar{R}\right)/\bar{R}$ where $\bar{R}$ is the angular average of $R(\phi)$. Here we are representing all fluctuations essentially in terms of fluctuations at the boundary of the initial region. Important thing is that, in contrast to the conventional discussions of the elliptic flow, here one does not try to determine any special reaction plane on event-by-event basis.

We know that these anisotropies are experimentally accessible only if they leave some imprints on the final particle momenta at freezeout just like the density fluctuations at the surface of last scattering are accessible through their imprints on the CMBR. What one is looking for, therefore, is the evolution of spatial anisotropies of different wavelengths, and corresponding buildup of momentum anisotropies (*i.e.* essentially different flow coefficients), existing at the freezeout stage.

The two most crucial aspects of the inflationary density fluctuations leading to the remarkable signatures of acoustic peaks in CMBR are coherence and acoustic oscillations. Let us consider them in turn to see if any such features are expected for RHICE. Coherence of inflationary density fluctuations essentially results from the fact that the fluctuations at superhorizon sizes are frozen out dynamically. Later, at the stage of re-entering the acoustic horizon, when these fluctuations start growing due to gravity, and subsequently start oscillating due to radiation pressure, the fluctuations start with zero velocity. For an oscillating fluctuation, this will mean that only $\cos\omega t$ term survives. This should be reasonably true for RHICE, especially as we are considering transverse fluctuations. Transverse velocity to begin with is expected to be zero. Though, note that with initial state fluctuations, there may be some residual transverse velocities even at the earliest stages, but for wavelengths significantly larger than the nucleon size, it is unlikely that the fluid will develop any significant collective velocity at the thermalization stage. Thus the fluctuations entering the sound horizon should be essentially coherent.

Let us now address the possibility of the oscillatory behavior for the fluctuations. In the universe, attractive forces of gravity and counter balancing forces from radiation pressure lead to acoustic oscillations. For RHICE, there is no gravity, but there is a non-zero pressure present in the system. First let us follow the conventional analysis as in the case of elliptic flow. Due to unequal pressure gradients in different azimuthal directions, the spatial anisotropy decreases in time by the buildup of momentum anisotropy (starting from isotropic momentum distribution), it eventually crosses zero and becomes negative. This forces momentum anisotropy to saturate first (when spatial anisotropy becomes zero), and then start decreasing. In principle, one could imagine momentum anisotropy to decrease to zero, becoming negative eventually. The whole cycle could then be repeated, resulting in an oscillatory behavior for the spatial anisotropy as well as for the momentum anisotropy.

For the elliptic flow, in hydrodynamic simulations one does see saturation of the flow, and possibly turn over part, but due to the build up of strong radial flow, the momentum anisotropy does not become zero, and there is never any indication of an oscillatory behavior. This can be seen from the evolution of transverse velocity in hydrodynamics simulations. But this may not be the case for fluctuations of shorter wavelengths. Shorter wavelength fluctuations enter the sound horizon earlier and can be expected to complete some oscillations by the time the flow freezes out due to radial expansion.

For elliptic flow the flow freezeout is usually accounted for by referring to the flow build up time scale of order R/c_s where R represents the initial average transverse extent of the region. This is the time scale when transverse expansion is expected to become strong. For central collisions, we will take $R = \bar{R}(\tau_{eq}) \equiv \bar{R}$. For $Au - Au$ collision at 200 GeV, the value of $\bar{R}$ is obtained to be about 3 fm from HIJING. This gives us $\tau_{fr} = \bar{R}/c_s + \tau_{eq} \simeq 6$ fm. (We use velocity of sound $c_s = 1/\sqrt{3}$). We will use the freezeout stage as given by $\tau_{fr} - \tau_{eq} = \bar{R}/c_s$. The size of the acoustic horizon at $\tau = \tau_{fr}$ is then simply $H_s^{fr} = c_s(\tau_{fr} - \tau_{eq}) = \bar{R} \simeq 3$ fm for $Au - Au$ collision at 200 GeV.

Now let us consider spatial anisotropy with a wavelength which is much shorter than H_s^{fr} at the freezeout stage, say λ being of order 2 fm, see Fig. 2. One will expect that due to initial unequal pressure gradients in the two directions ϕ_1 and ϕ_2, momentum anisotropy would have built up in these two directions in relatively short time. Thus, we expect that spatial anisotropy should reverse sign in time of order $\lambda/(2c_s) \simeq 2$ fm. This is the time when we say that the mode has "entered the horizon". The momentum anisotropy should then reach saturation, and start decreasing by this time. Due to short time scale of evolution here, radial expansion may still not be most dominant and there may be possibility of momentum anisotropy changing sign, leading to some sort of oscillatory behavior.

An entirely different line of argument supporting oscillatory behavior can be given as follows. If the confinement-deconfinement transition is first order, an interface will be present at the boundary of the QGP region and vacuum with a nonzero surface tension and small perturbations will propagate on the surface at the speed of sound appropriate for the QGP region enclosed. The interface will eventually disappear after the quark-hadron phase transition stage.

We have seen that when the acoustic horizon is less that $\lambda/2$, the flow corresponding to that wavelength will build up slowly and reaches its maximum as the horizon becomes equal to $\lambda/2$. Once the mode has entered the horizon, it will start oscillating. This means that the the momentum anisotropies corresponding to the modes which are superhorizon at the freezeout would not have grown to its maximum, *i.e.*, the flow coefficients corresponding to these modes will be suppressed.

The largest wavelength mode which will be unsuppressed, will have max $\lambda_{\max} \simeq 2c_s\left(\tau_{fr} - \tau_{eq}\right)$. Thus in terms of the corresponding flow coefficients, for modes with $\lambda/2 \ge H_s^{fr}$, we expect,

$$\left(v_n\right)_{observed} = \frac{2H_s^{fr}}{\lambda}\left(v_n\right)_{\max} \tag{1}$$

where $\lambda \sim 2\pi\bar{R}^{fr}/n$, $(n \ge 1)$, is the measure of the wavelength of the anisotropy corresponding to the n^{th} Fourier coefficient. Here $\bar{R}^{fr}$ represents the transverse radius at the stage τ_{fr}.

For the case of elliptic flow several studies have shown that for near central events, the momentum anisotropy is related to the initial spatial anisotropy by a proportionality constant of about 0.2. The relation between the Fourier coefficients of the spatial anisotropy and resulting momentum anisotropy in our model can only be obtained using a full hydrodynamical simulation, and in the absence of such a simulation, we make a strong assumption here that all Fourier coefficients for momentum anisotropy are related to the corresponding coefficients for spatial anisotropy by roughly the same proportionality factor, which we take to be 0.2 for definiteness.

A rough estimate will show that for $Au - Au$ collision at 200 GeV, the fluctuation modes with $n < n_{min} \simeq 4.5 \simeq 5$ the values of v_n^{rms} will be suppressed due to being superhorizon. Note in particular that this implies that the mode with $n = 2$, which corresponds to the elliptic flow, will be expected to be suppressed by a factor of order $f \simeq 1/2$. This suppression is roughly of same order as the suppression factor for the elliptic flow discussed in the literature.

For the CMBR case, as there is only one CMBR sky to observe for each l mode of the spherical harmonic, there are only $2l + 1$ independent measurements available. For small l values this limits the accuracy by the so called cosmic variance. In contrast, the accuracy in heavy ion collisions, for any Fourier mode, is only limited by the number of events one includes in the analysis. Therefore it should be possible to resolve any signal present in these events as discussed in this paper. Note, due to the absence of any special reflection symmetry here (which was present in the elliptic flow case) both even and odd flow coefficients give non-zero contributions to v_n^{rms}, and the sine terms give same values as the cosine terms. In the plots of v_n^{rms} in the next section, we show the sum of these two contributions.

3. Numerical results

We have generated events using HIJING and we present sample results for Au-Au collision at 200 GeV/A center of mass energy. All the curves are obtained using 10000 events. The solid curve in Fig. 3 shows the root mean square values v_n^{rms} of the flow Fourier coefficients which are obtained from spatial $F_n s$ using proportionality factor of 0.2. For comparison, we show here the plot of v_n^{rms} obtained when partons are distributed in a nucleus size region with uniform probability with momentum distribution from HIJING (dotted plot in Fig. 3). We see that the resulting values of v_n^{rms} are significantly smaller than the values shown by the solid plot as well as the plot is flat in contrast to the non-trivial shape of the solid plot. This suggests that statistical fluctuations in parton positions may remain sub dominant and a genuine fluctuation in parton positions (as depicted in Fig. 1) may be visible by the non-trivial shape of the solid plot in Fig. 3.

The solid curve in Fig. 4 shows plot of values of v_n^{rms} (as given by the solid curve in Fig. 3) with the inclusion of suppression factor for superhorizon fluctuations, and the dashed curve in Fig. 4 includes this suppression factor as well as the cos!“oscillatory factor for subhorizon anisotropies (the oscillations are modeled here). The dotted curve in Fig. 4, which is close to zero, shows the plot of average values of v_n corresponding to the solid plot in Fig. 3. This shows that we have enough statistics to control fluctuations due to statistical errors. Note that various plots here represent smooth joining of the points obtained at integer *n*.

For near central collisions (with impact parameter $b \simeq 2$ fm) the elliptic flow has been estimated earlier in simulations. The average value of v_2 was found to be about 0.005 with

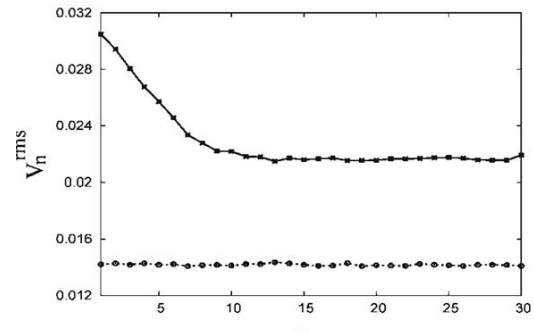

Figure 3. Plots representing v_n^{rms} calculated from the spatial anisotropy from HIJING.

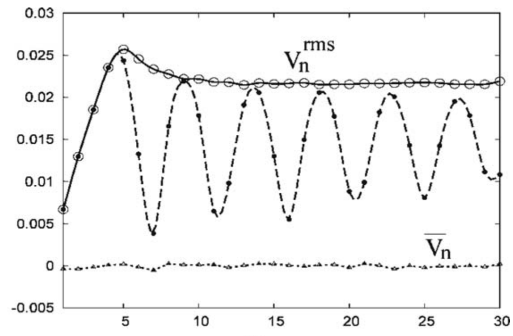

Figure 4. Plots obtained from the values of v_n^{rms} of the solid plot in Fig. 3 by including suppression and oscillations

the standard deviation $\delta \simeq$ 0.013. This is consistent with the experimental results. The plots of v_n^{rms} in Fig. 3 show that the value we obtain for n = 2 is in reasonably good agreement with these values quoted in the literature. The agreement is better for curves in Fig. 4 which include the suppression factor, as compared to the solid curve in Fig. 3.

We have checked the effects of varying various parameters like $\tau_{fr}, \tau_{eq}, c_s$ center of mass energy, rapidity window *etc* as well as changing the nuclei on the shape of these curves. All these changes causes slight shifts in the positions of the peaks as expected but the overall shape of the plots remain unchanged. For non-central collisions the plot of v_n^{rms} (as given by the solid plot in Fig. 3) shows a peak at n = 2 as expected.

4. Conclusions

In this paper, we argue that important information about initial anisotropies of the system and their evolution in relativistic heavy-ion collisions can be obtained by plotting the root mean square values of the Fourier coefficients v_n^{rms} of the anisotropies in the fluctuations of the particle momenta, starting from n = 1 up to large values of $n \simeq$ 30. For central collisions there is the possibility of a peak near n ~ 5 (for Au – Au collision at 200 GeV) and of subsequent peaks for larger n. If any of these non-trivial features are detected in the particle momentum spectra then it can open up a new way of accessing the information about initial stages of the matter produced. Just as for CMBR where the location of the first peak refers to the decoupling stage, here also the location of the first peak will give information about the freezeout stage, including the all important equation of state which could distinguish a QGP phase from a hadronic phase. For CMBR, successive peaks yield important information about baryon content etc. which couple to the radiation and hence contribute to the acoustic oscillations. In the same way for RHICE, if successive peaks are present, then they may give information about the detailed properties of the matter present at that stage, and any dissipative factors, the presence and nature of any phase transition, *etc*.

Reference

[1] A P Mishra, R K Mohapatra, P S Saumia and A M Srivastava, arXiv:0711.1323, and references therein.