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Gauge symmetry and supersymmetry in the Higgsless standard model

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Abstract : The group $SO(6) \otimes SO(5)$ is shown to be the gauge group as well as supersymmetry group of a four dimensional superstring model. Here, we discuss how supersymmetry is realised in 4-dimensions and further, we successfully reproduce the gauge symmetry results. Using the $SO(6) \otimes SO(5)$ group, all the known aspects of the string theory are obtained. The model reduces to the Standard Model which has the capability of containing the ingredients of a successful theory of the present day physics. However, there are no Higgs in the model.

Keywords : 4-Dimensional superstring, Higgs, supersymmetry.

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1. Introduction

In a supersymmetric theory of weak, electromagnetic and strong interactions, spin $1/2$ gluinos and heavy spin-0 s-quarks are associated with ordinary vector gluons and quarks [1]. With a class of s-leptons which include charge one, a photonic neutrino and Higgsinos, there is a large proliferation of elementary particles in SUSY standard models and there has not been any experimental signal for SUSY particles. The standard model [2], without SUSY, has been remarkably successful in explaining almost all the experimental data accurately. However, many physicists suspected that there could be a deeper symmetry in the standard model [3] and in this paper we find that the two such underlying symmetries are the supersymmetry and the gauge symmetry.

One essential requirement for the existence of supersymmetry is that the fermionic and bosonic degrees of freedom be equal. We have three coloured left

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handed doublets (u_L, d_L), two singlets u_R and d_R of quarks, one left handed lepton doublet $\left(\nu_L,\ e_L^-\right)$ and the right handed electron e_R. Thus eleven fermions of one type in one generation exist in nature. In the bosonic sector, there are eight gluons (G_μ), three *W*-bosons and one $U_Y(1)$ gauge boson (B_μ). One, out of twelve bosons, is taken away by reduction in the rank, as will be seen in the end of Section 8. Researches for 4-dimensional string [4] have been in progress for the latter half of eighties. Antoniadis *et al* [5] have constructed a four dimensional superstring supplemented by 18 real fermions in trilinear coupling. The central charge in this theory is 15. Chang and Kumar [6] have discussed the problem with Thiring fermions.

For $D = 26$, bosonic string has been discussed by Casher *et al* [7]. Englert *et al* [8], in extension of this work, have shown brane fusion in bosonic string resulting in a fermionic string. Fairly recently Buchmueller *et al* [9] have discussed a supersymmetric string model from the Heterotic string. However, we are pursuing in a direct way of dealing the problem in four dimensional string model starting from a 26-dimensional model which exist from the early days of string theory.Our purpose is also to show the gauge symmetry as well as supersymmetry of the four dimensional model which will be considered in Section 7.

In the Nambu-Goto [10,11] bosonic string theory in the world-sheet (σ, τ) in 26 dimensions, the string action in 26 dimensions is

$$S_B = -\frac{1}{2\pi}\int d^2\sigma(\partial_\alpha X^\mu \partial^\alpha X_\mu),\ \mu = 0,\ 1,\ 2,\ ...,\ 25\,, \tag{1}$$

where $\partial_\alpha = (\partial_\sigma, \partial_\tau)$. The central charge for bosons is found by using the general expression for the two energy momentum tensors at two world sheet points z and ω

$$2 < T(z)T(\omega) >= \frac{C}{(z-\omega)^4} + . \tag{2}$$

The coefficient C of the most divergent term in eq. (2) is the central charge. Methods and principles of calculation of the central charge and those for a variety of strings have been given in reference [12]. For free bosons, the central charge is $C_B = \delta^\mu_\mu$. The action S_B in eq. (1) is not anomaly free. To make the action (1) anomaly free, one can write the action S_{FP} of conformal ghosts (c^+, b_{++}) and the generator L_m^{FP} with quanta (c, b),

$$S_{FP} = \frac{1}{\pi}\int (c^+\partial_- b_{++} + c^-\partial_+ b_{--})d^2\sigma,$$

$$L_m^{FP} = \sum_n (m-n) b_{m+n} c_{-n}, \tag{3}$$

which has a central charge –26, independent of the dimensionality of the string and cancels the central charge arising from the free bosons $C = \delta^\mu_\mu = D = 26$. So the string is physical only in D = 26 dimensions with the total central charge zero.

Using Mandelstam's [13] proof of equivalence between one boson to two fermionic modes in the infinite volume limit in 1+1 dimensional field theory, one can rewrite the action as the sum of the four bosonic coordinates X^μ of $SO(3,1)$ and forty four fermions having internal symmetry $SO(44)$, thereby losing Lorentz invariance. This is however anomaly free. So this is also true in finite intervals or a circle. The Majoraana fermions can be in bosonic representation of the Lorentz group $SO(3,1)$, the forty four fermions are grouped into eleven Lorentz vectors of $SO(3,1)$ which look as a commuting internal symmetry group when viewed from the other internal quantum number space. The action becomes

$$S_{FB} = -\frac{1}{2\pi}\int d^2\sigma \left[\partial^\alpha X^\mu(\sigma,\tau)\partial_\alpha X_\mu(\sigma,\tau) - i\sum_{j=1}^{11}\bar{\psi}^{\mu,j}\rho^\alpha\partial_\alpha\psi_{\mu,j}\right]. \tag{4}$$

This is anomaly free with S_{FP} of eq. (3). The upper index j refers to a row and the lower to a column, and

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix},\quad \bar{\psi} = \psi^\dagger\rho^0. \tag{5}$$

Here ρ^α's are imaginary, so the Dirac operators $\rho^\alpha\partial_\alpha$ are real. In this representation of Dirac algebra, the components of the world sheet spinor $\psi^{\mu,j}$ are real and they are Majorana spinors.

The action given by eq. (4), however, is not supersymmetric. The eleven $\psi_A^{\mu,j}$ have to be further divided into two species; $\psi^{\mu,j}$, j = 1, 2,..., 6 and $\phi^{\mu,k}$, k = 7, 8, ..., 11. For the group of six, the positive and negative parts of $\psi^{\mu,j} = \psi^{(+)\mu,j} + \psi^{(-)\mu,j}$, whereas for the group of five, the allowed freedom of phase of creation operators for Majorana fermions in $\phi^{\mu,k} = \phi^{(+)\mu,k} - \phi^{(-)\mu,k}$. This is due to the fact that we have Majorana like neutrinos rather than Dirac like ones. In fact 'neutrinos' are 6 x 4 = 24 in number and can be taken as right handed. The left handed ones can be 5 x 4 = 20 in number [14].

The action is now

$$S = -\frac{1}{2\pi}\int d^2\sigma \left[\partial_\alpha X^\mu\partial^\alpha X_\mu - i\bar{\psi}^{\mu,j}\rho^\alpha\partial_\alpha\psi_{\mu,j} + i\bar{\phi}^{\mu,k}\rho^\alpha\partial_\alpha\phi_{\mu,k}\right]. \tag{6}$$

Besides the $SO(3,1)$, the action (6) is invariant under $SO(6) \otimes SO(5)$. It is also invariant under the transformation

$$\delta X^\mu = \in (e^j\psi_j^\mu - e^k\phi_k^\mu), \tag{7}$$

$$\delta\psi^{\mu,j} = -ie^j\rho^\alpha\partial_\alpha X^\mu \in, \tag{8}$$

and

$$\delta\phi^{\mu,k} = ie^k\rho^\alpha\partial_\alpha X^\mu \in. \tag{9}$$

Here $\in$ is a constant anticommuting spinor. e^j and e^k are eleven numbers of a row with $e^j e_j$ = 6 and $e^k e_k$ = 5. In the formulation of the theory, one must have the proof that the commutator of two supersymmetric transformations gives a world sheet translation. Therefore, it is necessary to have an exact framework in which the super Virasoro conditions can emerge as gauge conditions. We can however demand that the spinor is $\Psi^{\mu} = e^j \psi_j'^{\mu} - e^j \phi_k'^{\mu}$. There are four bosons and effectively four fermions in the model.

2. The supersymmetric results

Introducing another supersymmetric pair, the Zweibein $e^{\alpha}(\sigma,\ \tau)$ and the gravitons $\chi_{\alpha} = \partial \nabla_{\alpha} \in$, the local 2-*d* supersymmetric action, first written down by Brink, Di Vecchia, Howe, Deser and Zumino [15,16]

$$S = -\frac{1}{2\pi}\int d^2\sigma e\Big[h^{\alpha\beta}\partial_{\alpha}X^{\mu}\partial_{\beta}X_{\mu} - i\bar{\Psi}^{\mu}\rho^{\alpha}\partial_{\alpha}\bar{\Psi}_{\mu}$$

$$+2\bar{\chi}_{\alpha}\rho^{\beta}\rho^{\alpha}\Psi^{\mu}\partial_{\beta}\chi^{\mu} + \frac{1}{2}\bar{\Psi}^{\mu}\Psi_{\mu}\bar{\chi}_{\beta}\rho^{\beta}\rho^{\alpha}\chi_{\alpha}\Big], \tag{10}$$

where $\Psi^{\mu} = e^j \psi_j^{\mu} - e^k \phi_k^{\mu}$. A detailed derivation is given in reference [17]. The Einstein-Hilbert action, with R is Ricci scalar, $\int eR\, d^2\sigma$ can be added, but this does not change the classical analysis. The action is invariant under local transformations

$$\delta X^{\mu} = \bar{\in}\Psi^{\mu},\ \ \delta\Psi^{\mu} = -i\rho^{\alpha} \in (\partial_{\alpha}X^{\mu} - \bar{\Psi}^{\mu}\chi_{\alpha}),\ \ \delta e_{\alpha}^{a} = -2i\,\bar{\in}\,\rho^{a}\chi_{\alpha},\ \ \delta\chi_{\alpha} = \nabla_{\alpha} \in. \tag{11}$$

There are two other important transformations,
(a) the Weyl transformations

$$\delta X^{\mu} = 0,\quad \delta\Psi^{\mu} = -\frac{1}{2}\Lambda\Psi^{\mu},\quad \delta e_{\alpha}^{a} = \Delta e_{\alpha}^{a},\quad \text{and}\quad \delta\chi_{\alpha} = \frac{1}{2}\Lambda\chi_{\alpha}, \tag{12}$$

(b) the local fermionic symmetry, with η, an arbitrary Majorana spinor

$$\delta\chi_{\alpha} = i\rho_{\alpha}\eta,\quad \delta e_{\alpha}^{a} = \delta\psi^{\mu} = \delta X^{\mu} = 0. \tag{13}$$

The invariance (*b*) requires the identity $\rho^{\alpha}\rho_{\beta}\rho_{\alpha}$ = 0, which is true in two dimensions. All these transformation properties imply that the action (10) is superconformal invariant. Varying the field and Zweibein, the Noether current J^{α} and the energy momentum tensor $T_{\alpha\beta}$ vanish,

$$j_{\alpha} = \frac{\pi}{2e}\frac{\delta S}{\delta\chi^{\alpha}} = \rho^{\beta}\rho_{\alpha}\bar{\Psi}^{\mu}\partial_{\beta}X_{\mu} = 0, \tag{14}$$

and

$$T_{\alpha\beta} = \partial_{\alpha}X^{\mu}\partial_{\beta}X_{\mu} - \frac{i}{2}\bar{\Psi}^{\mu}\rho_{(\alpha}\partial_{\beta)}\Psi_{\mu} = 0. \tag{15}$$

These are the super Virasoro constrain equations as derived from the algebra.

In a light cone basis, the vanishing of the lightcone components are obtained from variation of the action (10) *i.e.* eqs. (14) and (15)

$$J_{\pm} = \partial_{\pm} X_{\mu} \Psi^{\mu}_{\pm} = 0 \tag{16}$$

and

$$T_{\pm\pm} = \partial_{\pm} X^{\mu} \partial_{\pm} X_{\mu} + \frac{i}{2} \psi^{\mu j}_{\pm} \partial_{\pm} \psi_{\pm\mu,j} - \frac{i}{2} \phi^{\mu k}_{\pm} \partial_{\pm} \phi_{\pm\mu,k}, \tag{17}$$

where $\partial_{\pm} = \frac{1}{2}(\partial_{\tau} \pm \partial_{\sigma})$.

The gravitino χ_{α} has been gauged away. So this string is anomaly free, even without them. We shall put it back in the path integral and retrieve the superconformal ghost action. Using η and $\in$,

$$\chi_{\alpha} = i\rho_{\alpha}\eta + \nabla_{\alpha} \in . \tag{18}$$

We have

$$\delta\chi_{3/2} = \nabla_1 \in_{1/2}, \quad \delta\chi_{1/2} = \nabla_1 \in_{-1/2} + \eta_{1/2},$$

$$\delta\chi_{-1/2} = \nabla_{-1} \in_{1/2} + \eta_{-1/2} \text{ and } \delta\chi_{-3/2} = \nabla_{-1} \in_{-1/2} . \tag{19}$$

Two of the above four are important. Changing variables from $\chi_{3/2}$ to $\in_{1/2}$, the Jacobian is [17]

$$J_{3/2} = \det{}^{-1} \nabla_1^{1/2\to 3/2} = \int D\gamma_{1/2} D\beta_{-3/2} \exp\left(-\frac{1}{\pi}\int d^2\sigma \beta_{-3/2} \nabla_1 \gamma_{1/2}\right), \tag{20}$$

and from $\chi_{-3/2}$ to $\in_{-1/2}$, the other Jacobian is,

$$J_{-3/2} = \int D\gamma_{-1/2} D\beta_{3/2} \exp\left(-\frac{1}{\pi}\int d^2\sigma \beta_{3/2} \nabla_{-1} \gamma_{-1/2}\right). \tag{21}$$

The resulting ghost action is [17]

$$S_{SC} = -\frac{1}{2\pi}\int d^2\sigma e h^{\alpha\beta} \bar{\gamma} \nabla_{\alpha} \beta_{\beta}, \tag{22}$$

with the energy momentum tensor is

$$T_{++} = -\frac{1}{4}\gamma\partial_{+}\beta - \frac{3}{4}\beta\partial_{+}\gamma .$$

This constitutes the central charge 11 (or –11). After the usual commutator quantisation, the superconformal ghost generator is

$$L_m^{gh,sc} = \sum \left(\frac{1}{2} m + n \right) : \beta_{m-n} \gamma_n : . \tag{23}$$

With the new ghosts, anomaly cancellation becomes subtler. The covariant formulation for the energy momentum correlation for one loop is

$$2 \left\langle T_+(\sigma) T_+(\sigma') \right\rangle = \frac{1 - 3k^2}{(\sigma - \sigma')^4} , \tag{24}$$

The central charge for the one loop particle is $C = \pm(1 - 3k^2)$. The ± signs depend on its statistics. k is related to the conformal dimension $J = (1 + k)/2$. For each boson $k = 0$ and $C = 1$, for conformal ghost $k = 3$ and $C = -26$, and for the superconformal ghost $k = 2$ and $C = 11$ due to statistics. So the central charge of the four bosons of the action (6) is 4, of the conformal ghost of eq. (3) is –26 and of the superconformal ghost (23) is 11. All these add up to 4 – 26 + 11 = –11. The 44 fermions of the action (6) have been left out. To cancel this anomaly, each fermionic loop is to be characterised by $k = 1/\sqrt{6}$, so that the central charge for each one of a pair is 1/4 and hence the total contribution of the 44 fermionic mode is equal to 11. Thus there is a total cancellation of all anomalies and, the superstring is anomaly free.

Due to the gravitino consideration, the action (6) has central charge 4 + 11 = 15, same as that of the 10-*d* superstring, *i.e.*, 10 + 5 = 15. Interestingly, this string theory falls into $N = 1$ supersymmetry as per the table of the complete list given by Polchinski [18], since $C^{gh} = -15$. It is also appropriate to consider the gravitinos to have been gauged away and their contribution to the loop integral can be included due to the 44 Mandelstam Majorana fermions. As they are normal fermions, $k = 0$ or $J = 1/2$, the 22 pairs contribute 22 to the central charge. Together with 4 bosons, the central charge is 26, the same as that of the Nambu-Goto string. We shall follow this alternative as they should be identical descriptions. The sum of all super Virasoro generators without any anomaly is expressed in the following equation.

$$L_m^{\text{sum}} = L_m^{FP} + \frac{1}{\pi} \int_{-\pi}^{\pi} e^{im\sigma}\, T_{++} d\sigma = L_m^{FP} + L_m^{gh,sc} + \frac{1}{\pi} \int_{-\pi}^{\pi} e^{im\sigma} (\, T_{++} - k \partial_+ T_+^F) d\sigma, \tag{25}$$

where

$$T_+^F = \frac{i}{2} \left(\psi_+^{j,\mu} \psi_{+j,\mu} - \phi_+^{k,\mu} \phi_{+k,\mu} \right). \tag{26}$$

The coefficient k of the total derivative of the last term in eq. (25) is $1/\sqrt{6}$. $J = (1 + 1/\sqrt{6})$. In either case $a = -1$.

3. The super Virasoro algebra

For a brief outline, let L_m, G_r and F_m be the super Virasoro generators of energy, momenta and currents. Let α's denote the quanta of X^{μ} fields, b's and b''s denote quanta of ψ and ϕ fields in *NS* formulation and d, d' in *R* formulation. Then, to be specific,

$$L_m = \frac{1}{\pi}\int_{-\pi}^{\pi} d\sigma e^{im\sigma} T_{++}$$

$$= \frac{1}{2}\sum_{-\infty}^{\infty} :\alpha_{-n}\cdot\alpha_{m+n}: + \frac{1}{2}\sum_{r\in z+\frac{1}{2}}\left(r+\frac{1}{2}m\right):\left(b_{-r}\cdot b_{m+r} - b'_{-r}\cdot b'_{m+r}\right):, \quad NS$$

$$= \frac{1}{2}\sum_{-\infty}^{\infty} :\alpha_{-n}\cdot\alpha_{m+n}: + \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(n+\frac{1}{2}m\right):\left(d_{-n}\cdot d_{m+n} - d'_{-n}\cdot d'_{m+n}\right):, R \quad (27)$$

$$G_r = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} d\sigma e^{ir\sigma} J_{+} = \sum_{n=-\infty}^{\infty} \alpha_{-n}\cdot\left(e^j b_{n+r,j} - e^k b'_{n+r,k}\right), \quad NS \quad (28)$$

and

$$F_m = \sum_{-\infty}^{\infty} \alpha_{-n}\cdot\left(e^j d_{n+m,j} - e^k d'_{n+m,k}\right): \quad R \quad (29)$$

and satisfy the super Virasoro algebra with central charge C = 26 for the action of eq. (6),

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{C}{12}(m^3 - m)\delta_{m,-n}, \quad (30)$$

$$[L_m, G_r] = \left(\frac{1}{2}m - r\right)G_{m+r}, \quad NS \quad (31)$$

$$\{G_r, G_s\} = 2L_{s+r} + \frac{C}{3}\left(r^2 - \frac{1}{4}\right)\delta_{r,-s}, \quad (32)$$

$$[L_m, F_n] = \left(\frac{1}{2}m - n\right)F_{m+n}, \quad R \quad (33)$$

$$\{F_m, F_n\} = 2L_{m+n} + \frac{C}{3}(m^2 - 1)\delta_{m,-n}, \; m \neq 0 \,. \quad (34)$$

Eqs. (32) and (34) can be obtained using Jacobi identity.

This is also known that the normal ordering constant of L_0 is equal to one and we define the physical states, satisfying

$$(L_0 - 1)\left|\phi\right\rangle = 0,\; L_m\left|0\right\rangle = 0,\; G_r\left|\phi\right\rangle = 0 \text{ for } r, m > 0: \quad NS \text{ Bosonic} \tag{35}$$

$$L_m\left|\psi\right\rangle = F_m\left|\psi\right\rangle = 0, \text{ for } m > 0, \quad : \quad R \text{ Fermionic} \tag{36}$$

and $$(L_0 - 1)\left|\psi\right\rangle_\alpha = (F_0^2 - 1)\left|\psi\right\rangle_\alpha = 0. \tag{37}$$

So we have,

$$(F_0 + 1)\left|\psi_+\right\rangle_\alpha = 0 \text{ and } (F_0 - 1)\left|\psi_-\right\rangle_\alpha = 0. \quad : \quad R \tag{38}$$

These conditions make the string ghost free. Simply the L_0 condition makes the state $\alpha^\mu_{-1}\left|0, k\right\rangle$ massless. The L_1 constraint gives the Lorentz condition $k^\mu\left|0, k\right\rangle = 0$, implying a transverse photon with $\alpha^0_{-1}\left|\phi\right\rangle = 0$ as Gupta-Bleuler condition. Applying L_2, L_3 ... constraints, gives $\alpha^0_m\left|\phi\right\rangle = 0$. Further, since $\left[\alpha^0_{-1}, G_{r+1}\right]\left|\phi\right\rangle = 0$, we have $b^0_{r,j}\left|\phi\right\rangle = 0$ and $b'^0_{r,k}\left|\phi\right\rangle = 0$. All the time components are thus eliminated from the Fock space.

4. The BRST charge and unitarity

The conformal dimension of γ is '–1/2' and that of γ is '3/2', which can be deduced from eq. (23). The part of the BRST charge which comes from the usual conformal Lie algebra technique, is

$$(Q_1)^{NS,R} = \sum (L_{-m}c_m)^{NS,R} - \frac{1}{2}\sum (m-n) : c_{-m}c_{-n}b_{m+n} : -ac_0;\; Q_1^2 = 0 \text{ for } a = 1. \tag{39}$$

Using the graded Lie algebra, we get the additional BRST charge, in a straight forward way, in NS and R,

$$Q'_{NS} = \sum G_{-r}\gamma_r - \sum \gamma_{-r}\gamma_{-s}b_{r+s}, \tag{40}$$

$$Q'_R = \sum F_{-m}\gamma_m - \sum \gamma_{-m}\gamma_{-n}b_{n+m}, \tag{41}$$

and

$$Q_{\text{BRST}} = Q_1 + Q', \tag{42}$$

such that $Q^2_{\text{BRST}} = 0$ in both the NS and the R sectors [19,20]. In proving $\left\{Q', Q'\right\} + 2\left\{Q_1, Q'\right\} = 0$, we have used the Fourier transforms, the wave equations and integration by parts such that

$$\sum\sum r^2\gamma_r\gamma_s\delta_{r,-s} = \sum_r\sum_s \gamma_r\gamma_s\delta_{r,-s} = 0. \tag{43}$$

Thus the theory is unitary and ghost free. There are no harmful effective tachyons in the model.

5. The mass spectrum

Ghosts do not couple to physical states, therefore these states must be of the form (up to null states) [21]

$$\left|\{m\}\, p\right\rangle_M \otimes c_1 \left|0\right\rangle_G ,$$

where $\left|\{m\}\, p\right\rangle_M$ denotes the occupation numbers and momentum of physical matter states. The operator c_1 lowers the energy of the states by one unit and is necessary for BSRT invariance. The ghost is responsible for lowering the ground state energy producing shiftable tachyon (F_2 picture)

$$\left(L_0^M - 1\right)\left|\text{phys}\right\rangle = 0$$

Therefore, the mass shell condition is

$$\alpha' M^2 = N^B + N_{NS}^F - 1 : \; NS, \tag{44}$$

$$\alpha' M^2 = N^B + N_R^F - 1 : \; R, \tag{45}$$

where

$$N^B = \sum_{m=1}^{\infty} \alpha_{-m}\alpha_m \tag{46}$$

$$N_{NS}^F = \sum_{r=1/2}^{\infty} r\left(b_r \cdot b_r + b_r' \cdot b_r'\right) : \; NS, \tag{47}$$

and

$$N_R^F = \sum_{m=1}^{\infty} m\left(d_r \cdot d_r + d_r' \cdot d_r'\right) : \; R \tag{48}$$

In general, $\alpha' M^2 = 1, -1/2, 0, 1/2, 1, 3/2, \ldots$ in the *NS* sector. This is in the shifted Hilbert state, $\alpha' M^2 = -1/2, 0, 1/2, 1, 3/2, \ldots$. Due to presence of Ramond and Neveu-Schwartz sectors, with periodic and anti-periodic boundary conditions, we can make a GSO projection [22] on the mass spectrum on NSR model [23]. The projection as desired is

$$G = \frac{1}{2}\left(1 + (-1)^{F+F''}\right), \tag{49}$$

where $F = \sum b_r \cdot b_r$ and $F' = b_r' \cdot b_r'$. This will eliminate the half integral value from the mass spectrum by choosing G = 1 including the tachyon at αM^2 = $-1/2$.

We have, therefore

$$\alpha' M^2 = -1, 0, 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots R. \tag{50}$$

The G.S.O. projection eliminates the half integral values. The tachyonic self energy of bosonic sector $-\left\langle 0\left|\left(L_0+1\right)^{-1}\right|0\right\rangle$ is cancelled by $-\left\langle\left|\left(F_0+1\right)^{-1}\left(F_0-1\right)^{-1}\right|0\right\rangle_R$, the negative sign being due to the fermionic loop. Such tadpole cancellations have been noted also by Chattaraputi *et al* in [24]. One can proceed a step further and write down the world sheet charge

$$Q_\omega = \frac{i}{\pi}\int_0^\pi \rho^0\rho^{\dagger\alpha}\partial_\alpha X^\mu \Psi_\mu d\sigma, \tag{51}$$

and find, as one may expect

$$\sum\left\{Q_\alpha^\dagger, Q_\alpha\right\} = 2H \quad \text{and} \quad \sum\left|Q_\alpha \left|\phi_\circ\right\rangle\right|^2 = 2\left\langle\phi_\circ\left|H\right|\phi_\circ\right\rangle. \tag{52}$$

6. Partition function

The ground state is of zero energy. There are no overall tachyons in this Superstring. To prove the modular invariance, one has to use the G.S.O. condition. In covariant formulation, the number of degrees of freedom of fermions is the number obtained after substraction of constraints from the total number. In our case, the total number is 44 and there are four constraints. So the physical fermionic modes are 40. Let us cast them in $SO(6)$ and $SO(5)$ invariant way. Let $b_{r,\alpha}^k$ be the annihilation quanta, α runs from 1 to 8 and k from 1 to 5.

We have in the NS sector, the Hamiltinian $H^{NS}=\sum_{k=1}^5 H_k^{NS}$, where $H_k^{NS}=\sum_r b_{r,\alpha}^{k\dagger} b_{r,\alpha}^k - \frac{8}{48}$. But $\left[H_k^{NS}, H_k^{\prime NS}\right]=0$. The partition function of the forty member assembly will be the partition function of the eight fermions raised to the power of five.

The path integral function for the eight fermions is given by Seiberg and Witten [25]. In their notation,

$$A_8\left((--),\tau\right)=\left(\frac{\Theta_3(\tau)}{\eta(\tau)}\right)^4$$

$$A_8\left((+-),\tau\right)=A_8\left((--),\frac{\tau}{1+\tau}\right)=-\left(\frac{\Theta_2(\tau)}{\eta(\tau)}\right)^4$$

$$A_8\left((-+),\tau\right)=A_8\left((+-),\frac{-1}{\tau}\right)=-\left(\frac{\Theta_4(\tau)}{\eta(\tau)}\right)^4$$

$$A_8\left((++),\tau\right)=0\,, \tag{53}$$

where $\Theta(\tau)$ is the Jacobi Theta function and $\eta(\tau)$ is the Dedekind eta function. The sum due to all the spin structures is

$$A_8(\tau)=\left(\Theta_3(\tau)/\eta(\tau)\right)^4-\left(\Theta_2(\tau)/\eta(\tau)\right)^4-\left(\Theta_4(\tau)/\eta(\tau)\right)^4, \tag{54}$$

and

$$A_8(1+\tau) = -A(\tau) = A\left(-\frac{1}{\tau}\right). \tag{55}$$

The total partition function is given by $Z = \left|A_8(\tau)\right|^5$. This is not only modular invariant but it also vanishes due to the Jacobi relation. This is also the necessary condition for space time supersymmetry. Thus all the criteria for supersymmetry are satisfied.

7. Gauge symmetry and the standard model

For the gauge symmetry, one has to find massless vector bosons which are generators of a group. We proceed in the line of the work of Li [26].

In *O(n)* there are $\frac{1}{2}n(n-1)$ generators represented by

$$L_{ij} = X_i \frac{\partial}{\partial X_j} - X_j \frac{\partial}{\partial X_i}, \; i, \; j = 1, \dots, \; n. \tag{56}$$

The commutation among the generators, called Lie algebra can be worked out. The rule is to write

$$\left[\frac{\partial}{\partial X_i}, X_j\right] = \delta_{ij}$$

so that the Lie algebra, which is the commutator relation among the generators can be written as

$$\left[L_{ij}, L_{kl}\right] = \delta_{jk}L_{il} + \delta_{il}L_{jk} - \delta_{ik}L_{jl} - \delta_{jl}L_{ik}. \tag{57}$$

Hence one must have $\frac{1}{2}n(n-1)$ vector gauge bosons W_{ij}^{μ} with the transformation law

$$W_{ij}^{\mu} \to W_{ij}^{\mu} + \in_{ik} W_{kj}^{\mu} + \in_{jl} W_{li}^{\mu}, \;\; W_{ij}^{\mu} = -W_{ji}^{\mu}, \tag{58}$$

where $\in_{ij} = -\in_{ji}$ are the infinitesimal parameters which characterise such rotation in $O(n)$. Under gauge transformation of the second kind, we have

$$W_{ij}^{\mu} \to W_{ij}^{\mu} + \in_{ik} W_{kj}^{\mu} + \in_{jl} W_{li}^{\mu} + \frac{1}{g}\partial^{\mu} \in_{ij}. \tag{59}$$

The Yang-Mills Lagrangian is then written as

$$L = -\frac{1}{4}\left|F_{ij}^{\mu\nu}\right|^2, \tag{60}$$

with

$$F_{ij}^{\mu\nu} = \partial^{\mu}W_{ij}^{\nu} - \partial^{\nu}W_{ij}^{\mu} + g\left(W_{ik}^{\mu}W_{kj}^{\nu} - W_{ik}^{\nu}W_{kj}^{\nu}\right), \tag{61}$$

$F\frac{\mu\nu}{ij}$ has the obvious properties, namely,

$$\Box F_{ij}^{\mu\nu} = 0, \;\; \partial_\mu F_{ij}^{\mu\nu} = \partial_\nu F_{ij}^{\mu\nu} = 0, \;\; F_{ij}^{\mu\mu} = 0. \tag{62}$$

There are two sets of field strength tensors which are found in the model, one for $SO(6)$ and the other for $SO(5)$. Since $\Box F_{ij}^{\mu\nu} = 0$, one can take plane wave solution and write

$$F_{ij}^{\mu\nu}(x) = F_{ij}^{\mu\nu}(p)e^{ipx}, \tag{63}$$

so that,

$$p^2 F_{ij}^{\mu\nu}(p) = p_\mu F_{ij}^{\mu\nu}(p) = p_\nu F_{ij}^{\mu\nu}(p) = F_{ij}^{\nu\nu}(p) = 0 \quad and \quad F_{ij}^{\mu\nu}(p) = -F_{ij}^{\mu\nu}(p). \tag{64}$$

with

$$F_{ij}^{\mu\nu}(p) = b_i^{\mu\,\dagger} b_j^{\nu\,\dagger} \left|0, p\right\rangle, \tag{65}$$

and (i, j) = 1, ..., 6 for $O(6)$. In terms of the excitation quanta of the string, the vector generators of $O(6)$

$$W_{ij}^{\mu} = \frac{1}{\sqrt{2ng}} \eta_\kappa \in^{\kappa\mu\nu\sigma} b_{\nu,i} b_{\sigma,j}, \tag{66}$$

where n_κ is the time-like four vector and is taken as (1,0,0,0). One finds that

$$\partial^\mu W_{ij}^\nu - \partial^\nu W_{ij}^\mu = p^\mu W_{ij}^\nu - p^\nu W_{ij}^\mu$$

$$= \frac{1}{\sqrt{2ng}} \left(n_\kappa \in^{\kappa\nu\lambda\sigma} p^u - \eta_\kappa \in^{\kappa\mu\lambda\sigma} p^\nu \right) b_{\lambda,i} b_{\sigma,j}. \tag{67}$$

As μ must be equal to ν, if κ, λ and σ are the same, this term vanishes and

$$g\left(W_{ik}^\mu W_{kj}^\nu - W_{ik}^\nu W_{kj}^\mu\right) = \frac{1}{2n}\left(n_\kappa \in^{\kappa\mu\lambda\sigma} b_{\lambda,i} b_{\sigma,k} \eta_{\kappa'} \in^{\kappa'\nu\lambda'\sigma'} b_{\lambda',k} b_{\sigma,j} - \mu \leftrightarrow \nu\right)$$

$$= b_i^{\dagger\,\mu} b_j^{\dagger\,\nu}, \tag{68}$$

we have used

$$\left\{b_{\lambda' k}, b_{\sigma k}\right\} = \eta_{\lambda'\sigma}\delta_{kk} = n\eta_{\lambda'\sigma}.$$

Since the product of pairs of b and b' commute, the gauge group of the action (6) is the product group $SO(6) \otimes SO(5)$. The indices i, j are taken from 1 to 5 for $SO(5)$, b' is replaced by b. It is important to note that this is same as the symmetry group of the action (6).

8. The standard model with Wilson loop

Symmetry breaking by Wilson loop or flux breaking mechanism or the Hosotani mechanism [27,28] takes place due to nonvanishing of closed loop integrals in a multiple connected integral space. Here let us choose the loop integral as

$$U_\lambda = P\exp\left(\oint_\gamma A_\mu dx^\mu\right). \tag{69}$$

P represents the ordering of each term with respect to one closed path γ. $SO(6) = SU(4)$ descends to $SU_C(3) \otimes U_{B-L}(1)$. This breaking can be accomplished by choosing one element of U_0 of $SU(4)$ such that

$$U_0^2 = 1. \tag{70}$$

The permutation group Z_2 is generated by this element. So we get

$$\frac{SO(6)}{Z_2} = SU_C(3) \otimes U_{B-L}(1), \tag{71}$$

without breaking the supersymmetry. Similarly $SO(5) \to SO(3) \otimes SO(2) = SU(2) \otimes U(1)$. We also have,

$$\frac{SO(5)}{Z_2} = SU(2) \otimes U(1). \tag{72}$$

Here the supersymmetry is not broken. Thus

$$\frac{SO(6)\otimes SO(5)}{Z_2 \otimes Z_2} = SU_C(3) \otimes U_{B-L}(1) \otimes U_R(1) \otimes SU_L(2). \tag{73}$$

This makes us aware of the identification with the usual low energy phenomenology. But this is not the standard model. We have an additional $U(1)$. A Z_6 orbifold compactification of $E_8 \times E_8$ heterotic string model leading to supersymmetric standard model gauge group is given in Ref. [9]. It is also seen that Yukawa couplings are suppressed. In a heterotic string model, Buchmueller *et al* [9] introduced a new heterotic string model after compactification with a Z_2 group and had a single term of Higgs. Following the similar idea [17] as in E_6, we take

$$U_\gamma = (\alpha_\gamma) \otimes \begin{pmatrix} \beta & & \\ & \beta_\gamma & \\ & & \beta_\gamma^{-2} \end{pmatrix} \otimes \begin{pmatrix} \delta_\gamma & \\ & \delta_\gamma^{-1} \end{pmatrix}. \tag{74}$$

$\alpha_\gamma^3 = 1 \cdot \alpha_\gamma$ is the cube root of unity. This structure inevitably lowers the rank by one. We, therefore, get

$$\frac{SO(6)\otimes SO(5)}{Z_3} = SU_C(3) \otimes U_L(2) \otimes U_Y(1), \tag{75}$$

and thus find our supersymmetric standard model.

We now elaborately discuss the Z_3, described by Mishra [28],

$$g(\theta_1, \theta_2, \theta_3) = \left(\frac{2\pi}{3} - 2\theta_1, \frac{2\pi}{3} + \theta_2, \frac{2\pi}{3} + \theta_3,\right). \tag{76}$$

For the first Wilson loop, the angle integral for θ_1 varies from $\theta_1 = \frac{2\pi}{9}$ to $\frac{2\pi}{3} - \frac{4\pi}{9} = \frac{2\pi}{9}$. So the first loop integral vanishes. θ_2 = 0 to $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$ for the second loop is described by a length parameter R, θ_3 = 0 to $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$ for the remaining loop with the same R. We take the polar components of the gauge fields as non-zero constants,

$$gA^{15}_{\theta_2} = \vartheta_{15}$$

for $SO(6) = SU(4)$ for which the diagonal generator t_{15} breaks the symmetry. Further

$$g' A^{10}_{\theta_3} = \vartheta'_{10}$$

for $SO(5)$, the diagonal generator being t'_{10}. The generators of both $SO(6)$ and $SO(5)$ are 4 × 4 matrices. We can write the Z_3 group as

$$T = T_{\theta_1} T_{\theta_2} T_{\theta_3}. \tag{77}$$

We have $T_{\theta_1} = 1$. This leaves the unbroken symmetry $SU(3)$ x $SU(2)$ untouched

$$T_{\theta_2} = \exp\left(it_{15} \int_{\circ}^{\frac{4\pi}{3}} \vartheta_{15} R d\theta_2 \right). \tag{78}$$

$T_{\theta_2} \neq 1$ breaks the $SU(4)$ symmetry. Again,

$$T_{\theta_3} = \exp\left(it'_{10} \int_{\circ}^{\frac{4\pi}{3}} \vartheta'_{10} R d\theta_3 \right). \tag{79}$$

$T_{\theta_3} \neq 1$ breaks the $SO(5)$ symmetry. But the remaining product of Z_3 is

$$T_{\theta_2} T_{\theta_3} = \exp\left(i \int_{\circ}^{\frac{4\pi}{3}} \left(\vartheta'_{10} t'_{10} + \vartheta_{15} t_{15} \right) R d\theta \right). \tag{80}$$

ϑ_{15} and ϑ'_{10} are arbitrary constants. We can choose them in such a way that

$$t_{15}\vartheta_{15} + t'_{10}\vartheta'_{10} = 0, \frac{3}{2R}, \ldots$$

The term in the exponential is zero or multiples of $2\pi i$. Thus $T = U(1)$ and eq. (72) is obtained. The rank is reduced by one. This is what we intended to achieve. This confirms that our group and supersymmetry give exact result.

9. The SUSY particles and their gauges

We now proceed to construct the Hamiltonian which is supersymmetric as well as $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ invariant following the ideas presented by one of us [29,30]. The hyperon vector B_μ, in the NS sector vector field V_l^μ with l = 1, ..., 8 and W_l^μ with l = 9, 10, 11 for the fields are picked up by the product of b^μ and b'^μ's. The particle spectrum and the quantum numbers are given in Table 1 in reference [29]. Let us take the color vector fields denoted by V_μ^l, l = 1, ..., 8, are gluon fields. l = 9 is the $U_Y(1)$ field and l = 10, 11, 12 stand for W-mesons fields. We shall use the temporal gauge [31], where $V_0^l = 0$. For each V_i^l and W_i^l, there are electric field strength E_i^l and magnetic field strength B_i^l. Following Nambu [32], the combination

$$F_i^l = \frac{1}{\sqrt{2}}\left[E_i^l + B_i^l\right], \tag{81}$$

satisfies the only nonvanishing equal time commutation relation

$$\left[F_i^{\dagger\, l}(x), F_j^m(y)\right] = i\delta^{lm} \in_{ijk} \partial^k (x-y). \tag{82}$$

We then construct Wilson's loop line integrals to convert the ordinary derivatives acting on fermion fields to respective gauge covariant derivatives.

The phase function for the colour is

$$U_C(x) = \exp\left(ig\int_0^x \sum_{l=1}^{\infty} \lambda^l V_i^l dy_i\right). \tag{83}$$

We denote $Y(x)$ as

$$Y(x) = g'\int_0^x B_i dy_i\,. \tag{84}$$

The isospin phase functions are then given by,

$$U_Q(x) = \exp\left(\frac{ig}{2}\int_0^x \tau\cdot \boldsymbol{W}_i dy_i - \frac{i}{6}Y(x)\right), \tag{85}$$

$$U(x) = \exp\left(\frac{ig}{2}\int_0^x \tau\cdot \boldsymbol{W}_i dy_i - \frac{i}{2}Y(x)\right), \tag{86}$$

$$U_1(x) = \exp\left(-\frac{2i}{3}Y(x)\right), \tag{87}$$

$$U_2(x) = \exp\left(\frac{i}{3}Y(x)\right), \tag{88}$$

and

$$U_R(x) = \exp\left(iY(x)\right). \tag{89}$$

The (ψ^I, ψ^*) will denote the fermions, I = 1, 2, 3 refer to the coloured quark doublets. The sum of the products

$$\sum_{I=1}^{3}\left(F_i^{*I}, F_i^{*I+3}\right)\begin{pmatrix}\psi^I \\ \psi^{*I+3}\end{pmatrix} \tag{90}$$

$$= \sum_{I=1}^{3}\left(F_i^{*I}, F_i^{*I+3}\right)U_Q U_C\begin{pmatrix}u_L^I \\ d_L^{I+3}\end{pmatrix}. \tag{91}$$

The singlet quarks which are single phased, are

$$\psi^I = U_1 U_c u_R^I, \quad I = 4, 5, 6 \tag{92}$$

and

$$\psi^I = U_2 U_c u_R^I, \quad I = 7, 8, 9. \tag{93}$$

This leaves us with the electrons and the neutrinos. The Q = 1 singlet is given by $\psi_R = U_R e^R = \psi_{10}$ and the lepton doublet

$$\psi_L = U\begin{pmatrix}\nu_e \\ e^-\end{pmatrix} = \begin{pmatrix}\psi_{11} \\ \psi_{12}\end{pmatrix}. \tag{94}$$

Our results in 4-dimensions, are the same as that of Sen [34].

In writing down the supersymmetric charge Q, we shall use 2 × 2 matrices, σ^μ, μ = 0, 1, 2, 3; σ^0 = I and σ's are the supers the three Pauli spin matrices,

$$Q = \int d^3x \sum_{I=1}^{12}\left(\sigma \cdot F^{\dagger\, I}\psi^I(x)\right). \tag{95}$$

We use the usual commutators and anticommutators at equal times and the $\sigma_i \sigma_j = \delta_{ij} + i \in_{ijk} \sigma_k$ and obtain

$$\left\{Q_\alpha^\dagger, Q_\beta\right\} = \left(\sigma^\mu P_\mu\right)_{\alpha\beta}, \tag{96}$$

P_0 is the Hamiltonian H and P is the total momentum. The $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ invariant Hamiltonian is

$$H = \int d^3x \left[\sum_{l=1}^{12} \frac{1}{2} \left(\boldsymbol{E}^{l2} + \boldsymbol{B}^{l2} + i\psi^{l\dagger}(x)\sigma \cdot \nabla \psi^l(x) \right) \right]. \tag{97}$$

Supersymmetric transformations exhibiting the superpartners are

$$\delta\psi^l = \sigma \cdot \boldsymbol{F}^l \in, \tag{98}$$

$$\delta \boldsymbol{F}_i^l = -i \,\bar{\in}\, (\sigma \times \nabla)_i \psi^l . \tag{99}$$

Thus the Susy particles and their partners are related as given above. It is satisfying to note that this relation is actually deducible. There are no Higgs in the model. The Higgs have not yet been observed by any experiment.

10. Family replication

The supersymmetry at electroweak scale becomes $Z_3 \times SU_C(3) \times SU_L(2) \times U_Y(1)$. It is denoted by GZ. The number of generations of the theory n_G is given by the Euler number χ, because

$$n_G = \frac{1}{2} \chi(GZ). \tag{100}$$

The following formula will be used. The space with known Euler number is

$$CP_N = \frac{SU(N+1)}{U(N)} \tag{101}$$

which leads to

$$\chi(CP_N) = (N+1). \tag{102}$$

Since we have $\chi(GZ) = 6$, we get $n_G = 3$. Thus this group structure gives the correct number of generations to be 3. Indeed this number is topological because of Dirac index theorem. The Dirac equation, in general, can have zero modes,

$$\gamma \cdot p\psi = \not{p}\psi = \not{D}\psi = 0 . \tag{103}$$

The index of this operator is equal to the difference between the positive, n_+ and negative, n_- chiralities of the zero modes of Dirac equation. The exact theorem with n_G, is

$$n_G = \frac{1}{2} \chi = \text{index}\,(\not{D}) = n_+ - n_-, \tag{104}$$

which is the difference of the zero modes. It is necessary to find the massless four dimensional Dirac spinors. The creation operators of the Ramond sector is

$$D^{\mu\dagger} = e^j d_{-1\cdot j} - e^k d_{-1\cdot k}. \tag{105}$$

such that a zero mass spinorial state [12] is

$$\phi_{0+} = D^{\mu}_{-1} \left|0\right\rangle u_{\mu} = D^{\mu\dagger} \left|0\right\rangle u_{\mu} \tag{106}$$

u_{μ} is a spinor four vector.

Similarly, there is another state

$$\phi_{0-} = \alpha^{\mu}_{-1} \left|0\right\rangle \upsilon_{\mu} = \alpha^{\mu\dagger} \left|0\right\rangle \upsilon_{\mu}, \tag{107}$$

due to the possible coordinate excitation of the Ramond sector. υ_{μ} is another four vector spinor. Both (u_{μ},υ_{μ}) are in four dimensions. Thus we can write

$$\gamma_5 u_{\mu} = u_{\mu},$$

and $$\gamma_5 \upsilon_{\mu} = -\upsilon_{\mu}. \tag{109}$$

Together, they form a four component spin vector ψ_{μ}. Since F_0 is essentially the Dirac gamma matrix (γ), the condition $F_0\psi_{\mu}$ gives the zeromass spinor equation

$$\gamma \cdot p\psi_{\mu} = 0. \tag{110}$$

The states $\phi_{0,\pm}$ contains not only a spin(–1/2) but also a spin(–3/2) state due to the addition of both the υ_{μ} and u_{μ}. But one can covariantly separate out the Dirac spin-(–1/2) equation as given in [22]. If ψ_{Dirac} is any Dirac spinor in 4-dimensions satisfying $\not{p}\psi = 0$, then the spin (–1/2) component is

$$\psi_{\mu}^{\frac{1}{2}} = \frac{1}{2}\left[\gamma_{\mu} - p_{\mu}\left(\gamma \cdot \bar{p}\right)\right]\psi_{\text{Dirac}}, \tag{111}$$

where the momentum $\bar{p}_{\mu}$ is conjugate to p_{μ} with $p^2 = \bar{p}^2 = 0$ and $p \cdot \bar{p} = 1$. Thus knowing that we have the zero mass spinors we can find the Dirac objects.

The zero mass modes in the Standard Model are grouped into three families (generations). They are

$$\begin{pmatrix} u \\ d \\ \nu_e \\ e \end{pmatrix}, \begin{pmatrix} c \\ s \\ \nu_{\mu} \\ \mu \end{pmatrix} \text{ and } \begin{pmatrix} t \\ b \\ \nu_{\tau} \\ \tau \end{pmatrix}. \tag{112}$$

The left handed ones are doublets and right handed ones are singlets. The quarks are colored. There are several fermionic zero modes, 24 are with +ve helicity and 21 with negative helicity. Thus $n_+ - n_- = 3$ as per topological findings. So there have to be only three fermions (neutrinos with unpaired helicity) in the standard model.

11. Conclusion

We have made an successful attempt in constructing an anomaly free N = 1, D = 4 superstring from bosonic string with the gauge symmetry $SO(6) \otimes SO(5)$ which, with the help of Wilson loops, descends to the SUSY standard model. There are just three generations. The D = 4 theory has both the positive and negative chirality and they occur in pairs. So the gravitational interaction will not lead to anomalies [23]. This is a very important result which can help not only in phenomenology but also in finding the correct dynamics of interacting string following the bosonic equivalence. It has been summarised by Mohapatra [33] that the uniqueness of the three generations model constructed by Tian and Yau, is the triumph of Calabi-Yau compactification procedure and all such models are equivalent.

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