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ORIGINAL PAPER

Application of Hybrid AHP-TOPSIS Technique in Analyzing Material Performance of Silicon Carbide Ceramic Particulate Reinforced AA2024 Alloy Composite

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Abstract

In this research work, the performance determining criteria's (PDC) measures like mechanical, tribological and thermo-mechanical etc. of AA2024-SiC alloy composite (i.e. ASC-0; ASC-2; ASC-4; ASC-6) are analysed using hybrid Analytic Hierarchy Process (AHP) and Technique for Order Preferences by Similarity to Ideal Solution (TOPSIS) technique (a Multi-Criteria-Decision-Making (MCDM) technique; computationally simple and easy to understand) in-order-to rank the composites formulations. The order of different PDCs as per AHP is: Coefficient-of-friction > Specific wear rate > Tensile strength ~ Cost > Hardness > Impact strength > Elongation ~ Flexural strength > Voids content ~ Actual density > Fracture toughness > Storage modulus > Thermal conductivity ~ Thermo-gravimetric analysis > Tan δ. The ranking order as per TOPSIS is: ASC-6 > ASC-4 > ASC-2 > ASC-0. The sensitivity analysis study reveals robust ranking-order or priority-order of PDCs as obtained by AHP analysis, when the weights changes from ±30%. The obtained results are in tune with the ranking of the formulations on subjective ground. This proves that MCDM techniques like Hybrid AHP-TOPSIS aids in taking skillful decision by ranking the formulations based on performance measures.

Keywords Performance determining criteria (PDC) · Analytic hierarchy process (AHP) · Technique for order preferences by similarity to ideal solution (TOPSIS) · Sensitivity analysis · Multi-criteria-decision-making (MCDM) · Ranking order

1 Introduction

The aluminium alloy like AA2024 having superior mechanical, structural and tribological features is used for making and substitution of conventional materials/parts e.g. sliding surfaces like rotor disk or drum etc. [1]. Further the materials scientists are trying to enhance the characteristics (physical, mechanical, thermal and tribological) of the above mentioned alloy by introducing the ceramic reinforcement. With the viability of such material and taking consideration of so many material property criteria, it is difficult for a design engineer to take decision regarding selection of material for a particular application. Hence one can take the help of quantitative decision making skill like Multi-Criteria-Decision-Making (MCDM) techniques [2]. Numerous material scholars in literatures advocate the case study of such techniques that enable engineers/scholars to learn them and apply in their study [3]. Multi-Criteria Decision Making (MCDM) techniques are briefly summarised by Jahan et al. [4]. They reported various material selection quantitative tools for systematic screening (viz. cost per unit property method, chart method, materials in product selection tools, knowledge-based systems, Neutral networks etc.) and ranking of materials (viz. TOPSIS, ELECTRE, AHP, SAW, fuzzy MCDM, Goal Programming, PROMETHEF etc.). Applications of these techniques in evaluating real time industrial problems are enormous, few of them listed here are; Ishizaka et al. [5] make use of Groups Analytic Hierarchy Process Ordering Method in selection of new production facilities. Xuebin [6] have discussed Non-dominated Sorting Genetic Algorithm (NSGA-II) technique to find Pareto sets and TOPSIS method with entropy weights to choose the best compromising solution in economic and environmental power dispatch problems. Satapathy et al. [7]

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reported balancing and ranking method in evaluation of performance ranking of friction material. Maleque et al. [8] discussed unit cost per property method and digital logic techniques for evaluating material performance and ranking. Maniya et al. [9] suggested preference selection index method for material ranking. Delice et al. [10] integrate heuristic evaluation (HE) and AHP approach to evaluate usability problems encountered on websites. Zhu et al. [11] delivered the concepts of hybrid AHP-PROMETHEF in evaluating the optimal material and ranking of different friction composites formulations consisting of aramid and $CaSO_4$ whiskers. Shyur et al. [12] applied hybrid AHP-TOPSIS for Vendor selection process. Implementations of hybrid AHP-TOPSIS technique over a wider range of problem domain are very well reported in literatures viz. Process designing of a product according to customer requirement suggested by Lin et al. [13], performance improvement of cold chain by Joshi et al. [14], ranking evaluation of different fly-ash based friction formulations by Satapathy et al. [15]. Kranthi et al. [16] suggested neural network technique by using a computational model to simulate experiments with parametric design strategy; and found it effective and efficient in predicting the dry sliding wear response of epoxy composites for various test conditions. Mohanty et al. [17] adopted multi-objective genetic algorithms and the resulting Pareto fronts to determine the optimum production quantity and associated quality of iron content in an iron making rotary kiln. Mohanty et al. [18] discussed evolutionary multi-objective artificial neural network and Genetic Algorithms models to determine the mechanical properties of the interstitial free steel sheets and correlations with various compositional and processing parameters. Gopal and Prakash [19] applied GRA (Grey Relational Analysis) and TOPSIS methods to rank the materials under a set of input factors like particle size and weight content, tool diameter, speed, feed and depth of cut. Akbari et al. [20] applied ANN (Artificial Neural Network) and hybrid non-dominated sorting genetic algorithm-II (NSGA-II) along with TOPSIS for optimization of various mechanical characteristics of A356 matrix composite reinforced with B_4C particulates. Patel et al. [21] have discussed the effect of various squeeze casting parameters like pressure duration (20, 35, 50 s), squeeze pressure (40, 80, 120 MPa), pouring temperature (630, 675, 720 °C) and die temperature (150, 225, 300 °C) on the wear characteristics of fabricated casting of LM20 alloy. For designing the experimentations, CCD and BBD (Box-Behnken Design) were used and to perform the optimization, various tools like DFA (Desirability function approach), GA (Genetic algorithm) and PSO (Particle swarm optimization) have been used. More accurate and similar results were obtained from GA and PSO techniques. Lower wear rate was observed by following squeeze casting method instead of gravity casting. Singh et al. [22] used preference selection index (PSI) method for evaluating the Tribological characteristics of brake friction material having nano-clay and multi walled carbon nano-tube. The tribo-performance of fabricated composite was affected by the addition of nano reinforcement which was also assessed by PSI method.

The present investigation critically examined the ranking of the designed formulations using hybrid AHP-TOPSIS approach under a set of conflicting performance defining criteria. The stability of obtained ranking as a response to fluctuation in evaluating attributes i.e. sensitivity analysis has also been worked out.

2 Materials and Methodology

2.1 Material Selection and Fabrication Procedure of AA2024-SiC Alloy Composites

The ingredients for the preparation of AA2024-SiC alloy composites and the detailed fabrication procedure are discussed in Fig. 1 [23].

2.2 AHP-TOPSIS Methodology

Multi-Criteria-Decision-Making (MCDM) method/technique like hybrid AHP-TOPSIS technique is a mathematical approach which is used to find out the solution of problems having finite alternatives and conflicting criteria. The above mentioned technique is successfully applied in many areas like society, economics, military, management etc. and it is gaining more and more attention over the last decades as seen in literatures [24].

AHP is a ground-breaking and adaptable strategy proposed by Thomas L. Saaty [10, 12–15] around 1970s that provides relative appraisal and prioritization of criteria for decision making. It incorporates both quantitative and subjective part of judgment that reinforced its adaptable appropriateness. It helps a leader in judging best choice suiting the objective and better comprehension of the issue. The principle steps it includes are: hierarchy construction, need priority examination and relative weights assurance with consistency confirmation. The basic steps are [25]:

Step 1: The hierarchy construction of the any complex decision making problem enables clear understanding of the problem. The decisive goal should be at the top, evaluating criteria at the middle and alternatives/options lie at the foot of the hierarchy, as shown in Fig. 2.

Step 2: The pair-wise comparison matrix is then constructed by assigning/quantifying scores based on the human judgement and Sattay's 1–9 scale (Table 2). One PDC is pair-wise compared with PDCs next in level

Fig. 1 Ingredients of the alloy composites and fabrication procedure

1. Base matrix AA2024; supplied by Vijay Prakash Gupta & Sons, New Delhi;
2. Silicon Nitride (Si_3N_4); supplied by Triveni Chemicals, Gujarat; particle size of 44 µm; constant 2 wt.-%
3. Solid lubricant Graphite (Gr); supplied by Central Drug House Private Limited, New Delhi; particle size of 99 µm; constant 2 wt.-%
4. Silicon-Carbide (SiC); supplied by ASES Chemical Works, Jodhpur; particle size of 10 µm

↓

Composite nomenclature and proportion: ASC-0 (having 0 wt.-% SiC); ASC-2 (having 2 wt.-% SiC); ASC-4 (having 4 wt.-% SiC); ASC-6 (having 6 wt.-% SiC)

↓

Fabrication procedure:

1. The measured quantity of AA2024 alloy rods was cleaned and its tiny pieces were melted in graphite crucible using induction furnace. The melt was held at ~ 800°C for 20 minutes; thereafter temperature lowered to 660°C (to mushy zone i.e. between solidus and liquidus temperatures of the alloy).
2. The ceramic particulates were preheated separately at 700°C for 3 h.
3. In-order-to enhance wettability of reinforcing phases in the molten melt 2 wt.-% Mg powder was added.
4. To achieve uniform mix of reinforcing phase in the molten melt, automatic stirrer (stainless steel; speed ~ 500 rpm; time = 10 min.) was used.
5. The mixture was poured into permanent cast iron mould with dimensions 150 × 90 × 10 $mm^3$and allowed to solidify to room temperature in air for one hour.
6. Composite specimen's samples were cut using wire EDM machine as per prevalent ASTM standard dimensions followed by polishing with emery paper

e.g. PDC-1 & PDC-2, PDC-1 & PDC-3 likewise. Pair-wise comparison between similar PDC results in score 1. Let the obtained pair-wise comparison matrix be C, then it appears like:

$$C_{n\times n} = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} 1 & C_{12} & \cdots & C_{1n} \\ C_{21} & 1 & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & 1 \end{bmatrix} \end{array}$$

where, C_{ij} can be interpreted as quantified degree of preference of ith criteria (row) over jth criteria (column) (i, j = 1, 2, 3, − -, *n,* for *n* number of PDCs) therefore matrix C is n^{th} order square matrix having scores at diagonal equals to 1. If there are *n*-PDCs then there were $\frac{n(n-1)}{2}$ pair-wise comparisons such that $C_{ij}=\frac{1}{C_{ji}}$. If the pair-wise comparison matrix $C = [C_{ij}]_{n\times n}$ satisfies $C_{ij} = C_{ik} \times C_{kj}$ for any i, j, k ∈ (1. .. *n*) then matrix *C* is said to be perfectly consistent; otherwise it is said to be inconsistent. Thus for a

Fig. 2 The hierarchy structure of investigated problem using AHP

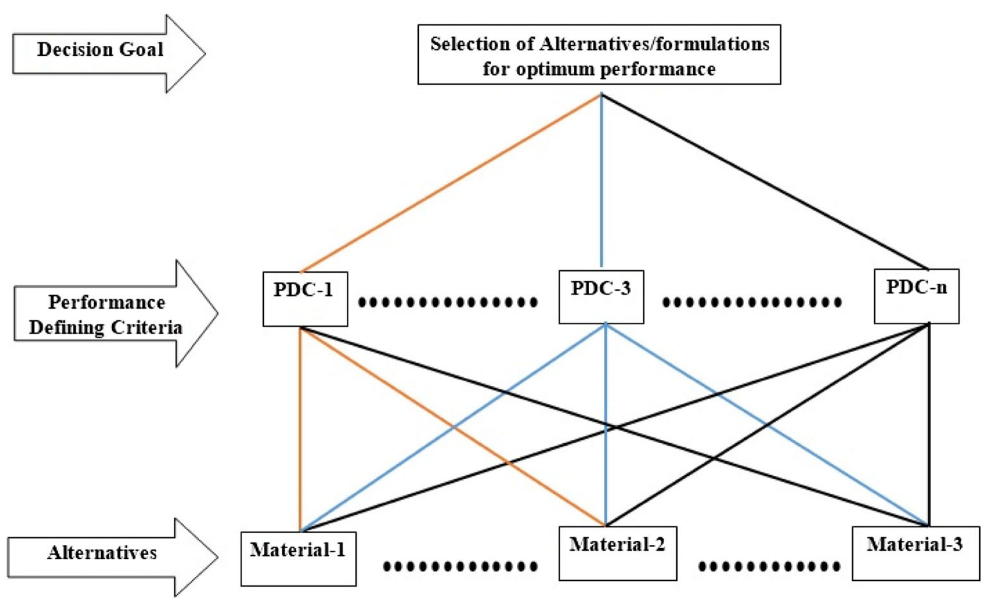

Table 1 Description of PDCs for the evaluation/ranking of alloy composites

PDCs No.	Performance determining criteria (PDC)	Implications of different PDCs on performance	Brief description of PDCs.
PDC-1	Ultimate tensile strength	Higher-the-better	The strength of material that resists any deformation along the axis subjected to externally applied tensile load perpendicular to cross-section.
PDC-2	Elongation %	Higher-the-better	The degree to which a material may be bent, stretched or compressed before it ruptures.
PDC-3	Flexural strength	Higher-the-better	The strength of material that resists any deformation against externally applied lateral load in flexure test.
PDC-4	Impact strength	Higher-the-better	It is the strength measured as energy absorbed by a material during fracture test. Higher impact strength is preferred for the composite to attain higher toughness.
PDC-5	Hardness	Higher-the-better	It is the resistance of material against plastic deformation usually by penetration/scratch. Higher hardness improves wear resistance.
PDC-6	Thermal conductivity	Higher-the-better	It is the material ability to conduct heat to flow through it for its unit thickness in a direction normal to a surface of unit area.
PDC-7	Material stability (TGA)	Higher-the-better	It is defined as the measurement of weight loss with respect to temperature/time in a controlled inert atmosphere.
PDC-8	Fracture toughness	Higher-the-better	It refers to the property of a material which describes the ability of a material containing a crack to resist further fracture.
PDC-9	Storage modulus	Higher-the-better	It is the ability of a material to return or store energy.
PDC-10	Tan δ	Higher-the-better	It is defined as the ability of a material that how efficiently the material loses energy to molecular rearrangements and internal friction.
PDC-11	Voids content	Lower-the-better	It is the presence of voids or pores in a material that may result due to fabrication error. It significantly affects material behaviour in actual situation.
PDC-12	Actual density	Lower-the-better	It is defined as mass per unit volume of a material. Low density with higher strength is needed for higher stiffness of material for light weight applications.
PDC-13	Wear	Lower-the-better	It is defined as the material loss from the surface due to thermo-mechanical-physical phenomenon's as a result of frictional interactions during sliding. The minimum is the wear, the maximum will be the operational life expectancy.
PDC-14	Coefficient-of-Friction	Lower-the-better	It is defined as a measure of amount of resistance that a surface exerts on during the sliding contact with another surface. The lower the coefficient of friction the higher the operational life expectancy.
PDC-15	Cost	Lower-the-better	It is the cost of ingredients (matrix and reinforcement) used for the fabrication of composite specimens.

given pair-wise comparison matrix C, the weight vector w can be determined by solving the characteristic equation: $C.w = \lambda_{max}.w$, where w is the weight vector of the actual absolute weights or eigen vector associated to the eigen value and λ_{max} is the maximal eigen value of matrix C. It has been proved that for perfect consistency $\lambda_{max} = n$ or rank = 1. Notably, inconsistencies in priorities assignment may leads to different values of λ_{max}; in such cases $\lambda_{max} \sim n$ for greater consistency of results. Above mentioned method for calculating the weight vector of a pair-wise comparison

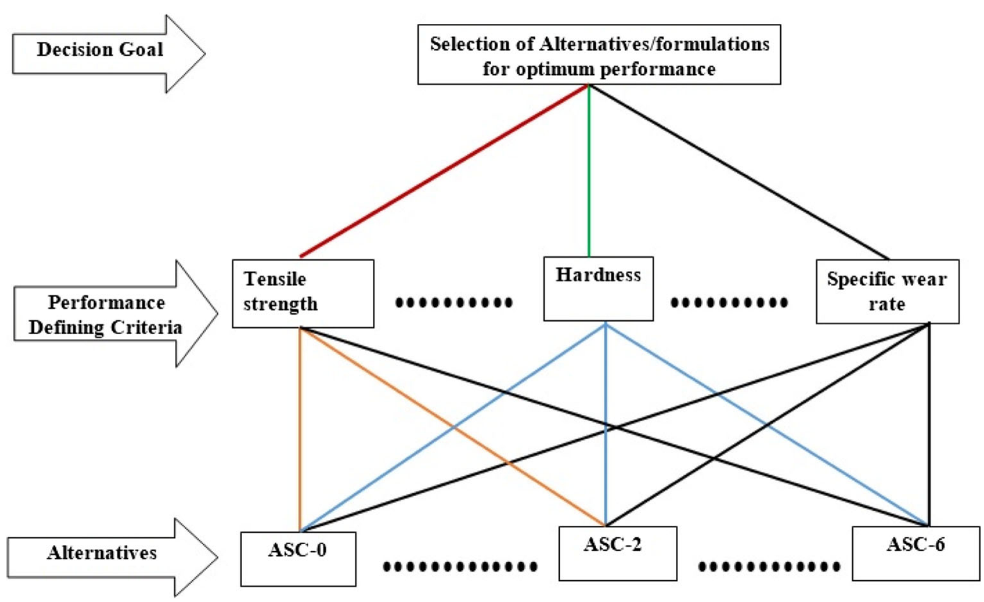

Fig. 3 The hierarchy structure of investigated problem

Table 2 The fundamental relational scale (Saaty's 1–9 scale) for pair-wise comparison [26, 27]

Intensity of importance on an absolute scale	Verbal judgement of preferences
1	'A' is equally preferred to 'B'
2	'A' is equally to moderately preferred over 'B'
3	'A' is moderately preferred over 'B'
4	'A' is moderately to strongly preferred over 'B'
5	'A' is strongly preferred over 'B'
6	'A' is strongly to very strongly preferred over 'B'
7	'A' is very strongly preferred over 'B'
8	'A' is very strongly to extremely preferred over 'B'
9	'A' is extremely preferred over 'B'
Reciprocals	If activity 'A' has one of the above number assigned to it when compared with activity 'B', then 'B' has the reciprocal value when compared with 'A'

matrix is given by Saaty, 1980 [10, 12–15]. It is well reported that consistency of comparison matrix C, greatly affects the results, hence consistency test is performed by calculating Consistency Index (CI) $=\frac{(\lambda_{\max}-n)}{(n-1)}$. For complete consistency $\lambda_{\max} \cong n$ or rank = 1 or CI = 0.

Step 3: Finally, acceptable consistency of matrix C or the extent of consistency or consistency verification is evaluated by computing Consistency Ratio (CR) $=\frac{CI\ (\text{Consistency Index})}{RI\ (\text{Random Index})}$. For consistent pair-wise comparison matrix CR ≤ 0.1 or 10%, otherwise comparison matrix needs to be reconstructed in order to reduce the inconsistency. Thus, the measure CR evaluates the consistency of decision maker as well as the consistency of the hierarchy. The Table 3 shows random index (RI) values for the pair-wise comparison matrices with the order form 1 to 10. The RI is the average of the consistency index of 500 randomly generated matrices.

The concept of TOPSIS is given by Hwang and Yoon [13–15, 18] and its effectiveness in solving real life decisive problems of various domains is very well advocated by literatures. The illustrations of various steps are as below:

Step 1: The multi-alternative (say m-alternatives) and multi-criteria (say n-criteria) of the problem are precisely expressed in the matrix form (say matrix D of $m \times n$ order).

$$D_{m\times n} = \begin{array}{c} \\ \mathrm{A}_1 \\ \mathrm{A}_2 \\ \vdots \\ \mathrm{A}_m \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & \cdots & C_n \end{matrix} \\ \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{bmatrix} \end{array}$$

where $C_1, C_2, ..., C_n$ are the n−criteria and $\mathrm{A}_1, \mathrm{A}_2, ..., \mathrm{A}_m$ are the m−alternatives

The element p_{ij} is the performance value of the ith alternative ($\mathrm{A_i}$) with respect to the jth attribute (C_j) where i = 1, 2, …, m and j = 1,2,…, n.

Step 2: In order to measure all criteria in dimensionless units and to facilitate inter-attribute comparisons; the above matrix is normalized using eq. 1. Thus, obtained normalized matrix is R = $\{r_{ij}\}$ (of the order $m \times n$).

$$r_{ij} = \frac{p_{ij}}{\left[\sum_{i=1}^{m} p_{ij}^{2}\right]^{\frac{1}{2}}}, \text{where } \mathrm{j} = 1, 2, \ldots, n \tag{1}$$

Step 3: The obtained normalized matrix R is then converted into the weighted normalized decision matrix V = $\{\mathrm{V_{ij}}\}$ (using eq. 2).

$$\mathrm{V_{ij}} = w_j \times r_{ij} \quad \text{where, } \mathrm{i} = 1, 2, \ldots, m; \mathrm{j} = 1, 2, \ldots, n; w_j \geq 0; \quad \sum_{j=1}^{n} w_j = 1 \tag{2}$$

Where, w_j is the relative weight or relative importance of jth criteria. It is determined by AHP method as explained above.

Step 4: Now, determine the positive ideal solution (A^+) and the negative ideal solution (A^-). The ideal solution (A^+) is a hypothetical alternative comprising all the

Table 3 Random Index (RI) for pair-wise comparison matrix [10]

n	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.51	1.52	1.54	1.56	1.58	1.59

Table 4 Pair-wise comparison matrix of PDCs and their relative weights

Attributes PDC	Tensile Strength PDC-1	Elongation % PDC-2	Flexural Strength PDC-3	Impact Strength PDC-4	Hardness PDC-5	Thermal Conductivity PDC-6	TGA PDC-7	FT at 0.5 mm PDC-8	Storage Modulus PDC-9	tan δ PDC-10	Void Content PDC-11	Actual Density PDC-12	Specific Wear Rate PDC-13	COF PDC-14	Cost PDC-15	Relative weights
PDC-1	1	1	1	1	1	4	4	2	4	4	2	2	1	1	1	0.096
PDC-2	1	1	1	1	1	3	3	2	3	3	2	2	1/2	1/2	1	0.082
PDC-3	1	1	1	1	1	3	3	2	3	3	2	2	1/2	1/2	1	0.082
PDC-4	1	1	1	1	1	3	3	2	3	3	2	2	1	1/2	1	0.085
PDC-5	1	1	1	1	1	3	3	2	3	4	2	2	1	1	1	0.091
PDC-6	1/4	1/3	1/3	1/3	1/3	1	1	1/2	1	1	1/2	1/2	1/4	1/4	1/4	0.026
PDC-7	1/4	1/3	1/3	1/3	1/3	1	1	1/2	1	1	1/2	1/2	1/4	1/4	1/4	0.026
PDC-8	1/2	1/2	1/2	1/2	1/2	2	2	1	2	2	1	1	1/2	1/4	1/2	0.046
PDC-9	1/4	1/3	1/3	1/3	1/3	1	1	1/2	1	1	1/2	1/2	1/2	1/2	1/4	0.029
PDC-10	1/4	1/3	1/3	1/3	1/4	1	1	1/2	1	1	1/2	1/2	1/5	1/4	1/4	0.025
PDC-11	1/2	1/2	1/2	1/2	1/2	2	2	1	2	2	1	1	1/2	1/2	1/2	0.048
PDC-12	1/2	1/2	1/2	1/2	1/2	2	2	1	2	2	1	1	1/2	1/2	1/2	0.048
PDC-13	1	2	2	1	1	4	4	2	2	5	2	2	1	1	1	0.104
PDC-14	1	2	2	2	1	4	4	4	2	4	2	2	1	1	1	0.114
PDC-15	1	1	1	1	1	4	4	2	4	4	2	2	1	1	1	0.096
																0.998

Table 5 Experimental observations of the investigated alloy composites

PDC / Attribute/ Composite Nomenclature	PDC-1 Tensile Strength	PDC-2 Elongation %	PDC-3 Flexural Strength	PDC-4 Impact Strength	PDC-5 Hardness	PDC-6 Thermal Conductivity	PDC-7 TGA	PDC-8 FT at 0.5 mm	PDC-9 Storage Modulus	PDC-10 tan δ	PDC-11 Void Content	PDC-12 Actual Density	PDC-13 Specific Wear Rate	PDC-14 COF	PDC-15 Cost
ASC-0	161.13	2.93	320.46	32.4	70	170	99.656	30.94	30,740	0.0015	1.07	2.76	0.00175	0.149	434.1
ASC-2	175.24	1.68	314.68	47.3	91	163	99.847	34.73	45,350	0.00166	0.71	2.78	0.00138	0.144	432.5
ASC-4	182.25	1.39	266.59	65.6	104	159	99.839	37.18	55,640	0.00154	0.71	2.79	0.00119	0.137	430.9
ASC-6	202.27	2.33	515.97	81.3	121	148	99.835	40.59	65,830	0.00187	0.35	2.81	0.000997	0.131	429.3

Table 6 Positive Ideal Solution (PIS) A^+ and Negative Ideal Solution (NIS) A^-

A^+ =	0.0537	0.0555	0.0576	0.0580	0.0560	0.0138	0.0130	0.0259	0.0187	0.0142	0.0111	0.0238	0.0382	0.0532	0.0477
A^- =	0.0428	0.0263	0.0298	0.0231	0.0324	0.0120	0.0130	0.0197	0.0087	0.0114	0.0340	0.0242	0.0670	0.0605	0.0483

criteria values corresponds to the best level, while the anti-ideal alternative (A^-) is also a hypothetical alternative comprising of all criteria values corresponds to the worst level. If the ideal solution is exactly one of the feasible solutions, then there is no need for evaluation to make decision, but it happens rarely in real world. The evaluation of each alternative are often found to be higher in some criteria and lower in the other criteria, hence decision makers should consider and calculate cautiously all the criteria in order to select a suitable compromising alternative. The PIS (A^+) and NIS (A^-) are determined (following below criteria) based on the weighted normalized matrix as obtained by eq.2.

$$A^+ = \left(v_1^+, v_2^+, \ldots\ldots\ldots\ldots, v_J^+\right)$$
$$A^- = \left(v_1^-, v_2^-, \ldots\ldots\ldots\ldots, v_J^-\right)$$

Whereas,

$$v_j^+ = \begin{cases} \max \mathrm{V_{ij}}, \text{if j is a benefit criteria or larger–the–better} \\ \min \mathrm{V_{ij}}, \text{if j is a cost criteria or smaller–the–better} \end{cases}$$

Whereas j = 1,2, …, n

$$v_j^- = \begin{cases} \max \mathrm{V_{ij}}, \text{if j is a benefit criteria or larger–the–better} \\ \min \mathrm{V_{ij}}, \text{if j is a cost criteria or smaller–the–better} \end{cases}$$

Step 5: Now compute euclidian distance (D) between each of the alternative and the positive ideal solution and the negative ideal solution using eq. 3,

$$D_i^+ = \sqrt{\sum_{j=1}^{n}\left(v_j^+ - v_{ij}\right)^2} \quad \text{where } i = 1, 2, \ldots., m$$
$$D_i^- = \sqrt{\sum_{j=1}^{n}\left(v_j^- - v_{ij}\right)^2} \tag{3}$$

Step 6: Finally, compute relative closeness or the overall preference or Closeness Coefficient (CC) to the ideal solution for each alternatives using eq. 4. As, D_i^+ and D_i^- both are greater than zero, hence CC∈ (0, 1).

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-} \qquad for\, i = 1, 2, \ldots., m \tag{4}$$

Step 7: At the end, rank the alternatives in descending order of preferences according to the Closeness Coefficient (CC). Larger the CC better the alternatives relative to others.

Further, sensitivity analysis of performance determining criteria's has been performed to investigate the robustness of the above study. This analysis enables the analyst in taking more credible, understandable, compelling or persuasive recommendations. In the present examination, sensitivity analysis is performed by changing (expanding/diminishing) the weights of PDCs by ±30% and altering different PDCs relatively with the end goal that the summation remains unity. This exercise helps in understanding variations in the results i.e. change in the ranking of alternatives with respect to variations in weights of PDCs within the said confidence interval.

3 Results and Discussions

The Table 1 briefly explained various performance determining criteria and their implications while making judgement regarding performance of the said alloy composites. The performance evaluation data as discussed by Bhaskar et al. [23] and values of various characteristics are taken as input to hybrid AHP-TOPSIS technique to understand its algorithm and utility in material selection. Figure 3 presents the hierarchy structure of the investigated problem using AHP. Table 2 highlights the fundamental relational scale (Saaty's 1–9 scale) used for pair-wise comparison while obtaining pair-wise comparison matrix following AHP method. Table 3 presents Random Index values for n (=15)-criteria and used for calculating consistency ratio while Table 4 presents pair-wise comparison matrix and the relative weights thus obtained following AHP method. The experimental values of performance determining criteria as reported by Bhaskar et al. [23] are taken for the analysis and for the validation purpose (shown in Table 5). Table 6 shows computed PIS (A^+) and NIS (A^-)

Table 7 Closeness Coefficient and ranking of composite material

Composite Nomenclature	D+	D-	CC	Ranking
ASC-0	0.0627	0.0298	0.3225	4
ASC-2	0.0482	0.0254	0.3447	3
ASC-4	0.0453	0.0390	0.4626	2
ASC-6	0.0115	0.0674	0.8542	1

while Table 7 presents Closeness Coefficient and ranking of the alloy composites material formulations/alternatives.

The relative weights or priority order of criteria (W_i) as shown in end column of the Table 4 is: PDC-14 (COF) > PDC-13 (Specific wear rate) > PDC-1 (Tensile strength) ~ PDC-15 (Cost) > PDC-5 (Hardness) > PDC-4 (Impact strength) > PDC-2 (Elongation) ~ PDC-3 (Flexural strength) > PDC-11 (Void content) ~ PDC-12 (Actual density) > PDC-8 (Fracture toughness) > PDC-9 (Storage modulus) > PDC-6 (Thermal conductivity) ~ PDC-7 (TGA) > PDC-10 (Tan δ). The consistency verification is also performed and found that maximum Eigen value (λ_{max}) ~ 15; Consistency Index (CI) = 0.0150 and Consistency Ratio (CR) = 0.0094 which is much less than 0.1 (upper bound limit for acceptance of CR), hence the relative weights obtained by means of pair-wise comparison matrix is consistent and could be used as input to TOPSIS algorithm in order to elicit the ranking of the alloy composite material under study.

In this research work, alloy composite formulations (ASC-0, ASC-2, ASC-4, and ASC-6) as discussed by Bhaskar et al. [23] are taken for the analysis. The various performance measures were evaluated experimentally following standards and the same is depicted in the Fig. 3 and used for computation purpose.

Step 1: **Structuring the decision problem:**

Step 2: The experimentally evaluated measures/data of the investigated alloy composites are used for the preparation of the decision matrix (D) and further computation:

The sensitivity analysis study of the above formulations interestingly gives robust/stable observations. The sensitivity analysis study reveals that the ranking order remains quite insensitive/robust/stable as PDCs weights changes from ±30%, within the experimental range.

4 Conclusions

In this research work, the performance determining criteria's (PDC) measures like mechanical, tribological and thermo-mechanical etc. of AA2024-SiC alloy composite are analysed using hybrid AHP-TOPSIS. The significant findings are:

1. The hybrid AHP-TOPSIS MCDM technique is computationally simple and easy to understand in-order-to rank the alloy composite formulation.
2. The relative weights or priority order of criteria (W_i) is: COF > Specific wear rate > Tensile strength ~ Cost > Hardness > Impact strength > Elongation ~ Flexural strength > Void content ~ Actual density > Fracture toughness > Storage modulus > Thermal conductivity ~ TGA > Tan δ.
3. The consistency verification is also performed and found that maximum Eigen value (λ_{max}) ~ 15; Consistency Index (CI) = 0.0150 and Consistency Ratio (CR) = 0.0094 which is much less than 0.1 (upper bound limit for acceptance of CR), hence the relative weights obtained by means of pair-wise comparison matrix is consistent and could be used as input to TOPSIS algorithm in order to elicit the ranking of the alloy composite material. The ranking order as per TOPSIS approach is: ASC-6 > ASC-4 > ASC-2 > ASC-0.
4. The sensitivity analysis study reveals robust ranking-order or priority-order of Performance Determining Criteria (PDCs) as obtained by AHP analysis, when the weights changes from ±30%. The obtained results are in tune with the ranking of the formulations on subjective ground. This proves that MCDM techniques like Hybrid AHP-TOPSIS aids in taking skillful decision by ranking the formulations based on performance measures.

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