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https://doi.org/10.1007/s12597-024-00789-7

APPLICATION ARTICLE

Some weighted aggregation operators of quadripartitioned single-valued trapezoidal neutrosophic sets and their multi-criteria group decision-making method for developing green supplier selection criteria

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Accepted: 15 May 2024

Abstract
In contemporary decision-making scenarios, the ability to handle uncertainty and ambiguity is paramount. This paper introduces the concept of quadripartitioned single-valued trapezoidal neutrosophic sets, offering a broader scope for depicting membership grades compared to existing frameworks. Leveraging this, the quadripartitioned single-valued trapezoidal neutrosophic numbers (QSVTrNNs) emerge as a robust tool for modeling evaluation values in decision-making processes, particularly in multi-criteria decision making contexts. The primary contribution of this manuscript lies in the development of novel aggregation operators tailored to fuse quadripartitioned single-valued trapezoidal neutrosophic information effectively. Essential groundwork includes defining the fundamental concepts of QSVTrNNs, their operational relations, and a comprehensive score function. Two key aggregation operators, namely the quadripartitioned single-valued trapezoidal neutrosophic weighted averaging and quadripartitioned single-valued trapezoidal neutrosophic weighted geometric, are proposed and thoroughly investigated for their properties. Furthermore, this paper extends its applicability into quadripartitioned single-valued trapezoidal neutrosophic multi-criteria decision making, demonstrating its efficacy in developing green supplier selection criteria. An illustrative example elucidates the practical implementation of the proposed method, providing detailed insights into selecting the most suitable green supplier based on ranking orders. This research not only expands the theoretical framework of neutrosophic sets but also offers practical tools to navigate complex decision landscapes, especially in domains where uncertainty is inherent and critical decisions must be made with confidence.

Keywords Quadripartitioned single-valued trapezoidal neutrosophic set · Decision-making · MCDM · QSVTrNWA-operator · QSVTrNWG-operator

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Published online: 14 June 2024

1 Introduction

In contemporary business landscapes, the selection of green suppliers has become a crucial aspect of strategic decision-making for companies striving to uphold environmental sustainability. The process of identifying the most suitable green supplier entails evaluating numerous factors, including environmental impact, sustainability practices, and cost-effectiveness. To facilitate this decision-making process, methodologies that can effectively handle uncertainty and ambiguity are essential. However, traditional decision-making methods often struggle to accommodate the complexity of decision problems involving triangular and trapezoidal neutrosophic information. In response to this challenge, Deli and Subas [13] proposed the concept of single-valued trapezoidal neutrosophic sets, extending the idea of single-valued neutrosophic sets. This extension has paved the way for addressing decision-making challenges across various research domains, where uncertainty is inherent. The significance of single-valued neutrosophic decision making (SVNDM) has garnered considerable attention in decision theory. Recent years have seen a proliferation of SVNDM methods proposed by researchers, reflecting the growing recognition of its importance [3, 4, 7, 8, 17, 19, 20, 22–26, 32, 35–39, 41, 42, 45, 46]. These methods offer versatile frameworks for tackling diverse decision-making problems, ranging from customer purchasing decisions [40] to supply chain management [18] and sustainable supplier selection [12]. Studies on single-valued triangular (trapezoidal) neutrosophic numbers have made significant contributions to decision-making processes, demonstrating their impact across various applications [6, 16]. For instance, methodologies based on trapezoidal neutrosophic numbers have been applied to resource leveling models [2], production optimization [5], and risk assessment in supply chain management [33]. In 2017, Ye [47] introduced trapezoidal neutrosophic weighted arithmetic averaging and trapezoidal neutrosophic weighted geometric averaging operators to aggregate trapezoidal neutrosophic number information. These operators offer valuable tools for addressing multiple attribute decision-making problems, particularly those involving trapezoidal neutrosophic information. Furthermore, Deli and Subas [13] introduced the concept of single-valued trapezoidal neutrosophic sets (SVTrNS) by extending the notations of cut sets from single-valued neutrosophic numbers, offering a structured approach to represent uncertainty in decision problems. Since then, various methodologies based on SVTrNS have been proposed to address a wide range of challenges across multiple research areas. In 2018, Liang et al. [28] proposed a novel ranking method for single-valued trapezoidal neutrosophic numbers, demonstrating its efficacy in tackling multi-criteria decision-making problems. Subsequently, Liu and Zhang [30] introduced the single-valued trapezoidal neutrosophic Maclaurin symmetric mean operators and applied them to multiple attribute group decision making scenarios. Liang et al. [29] further contributed to the field by proposing single-valued trapezoidal neutrosophic weighted aggregation operators and applying them to MCDM problems, showcasing the versatility of SVTrNS methodologies. In parallel, Fahmi et al. [14] introduced aggregation operators for handling triangular neutrosophic cubic linguistic hesitant fuzzy information in MCDM problems, expanding the applicability of neutrosophic-based approaches. In 2021, Wang et al. [44] developed a methodology based on single-valued trapezoidal neutrosophic power-weighted aggregation operators for multi-criteria group decision-making problems, further enriching the decision-making toolbox in neutrosophic environments. Chakraborty et al. [9] introduced the concept of type-1, type-2, and type-3 trapezoidal bipolar neutrosophic numbers (TrBNNs) and applied them to MCGDM problems, expanding the scope of neutrosophic-based methodologies. Jana et al. [21] introduced a suite of operators in the SVTrN environment and utilized them to develop a MCDM framework, showcasing the versatility of neutrosophic-based methodologies. In further advancements, Fahmi [15] defined Dombi t-norm and Dombi t-conorm operators to aggregate trapezoidal neutrosophic fuzzy information, providing a comprehensive framework for multi-attribute decision-making strategies. Paulraj and Tamilarasi [34] introduced new operators for single-valued trapezoidal neutrosophic environments, offering enhanced decision-making capabilities for MADM problems.

Despite the growing interest in quadripartitioned single-valued neutrosophic sets (QSVNSs), research combining these concepts to address MCDM problems under quadripartitioned single-valued trapezoidal neutrosophic environments remains limited. This manuscript aims to contribute to the field of multi-criteria decision making (MCDM) within the context of quadripartitioned single-valued trapezoidal neutrosophic environments. The objectives of this study are as follows:

1. To introduce a novel definition of a single-valued trapezoidal neutrosophic set.
2. To elucidate various properties of quadripartitioned single-valued trapezoidal neutrosophic numbers.
3. To propose a new score function specifically designed for quadripartitioned single-valued trapezoidal neutrosophic MCDM applications.
4. To define new aggregation operators, namely the quadripartitioned single-valued trapezoidal neutrosophic weighted averaging operator and quadripartitioned single-valued trapezoidal neutrosophic weighted geometric operator, and analyze their properties.
5. To develop optimization MCDM models aimed at determining attribute weights effectively.
6. To validate the proposed methodologies through a numerical example, demonstrating their applicability and efficacy.
7. To conduct a comparative analysis between the proposed approach and existing methods, thereby assessing its advantages and potential contributions to the field.

To achieve the objectives outlined above, this paper is organized as follows: In Sect. 2, a comprehensive review of preliminaries regarding intuitionistic fuzzy sets (IFSs), SVNSs, SVTrNSs, and QSVTrNNSs is provided, establishing the theoretical foundation for subsequent discussions. Section 3 proposes the concepts of quadripartitioned single-valued trapezoidal neutrosophic numbers (QSVTrNN) and QSVTrNNSs, defining fundamental operations and introducing the score function associated with QSVTrNNs. Section 4 focuses on the development of the quadripartitioned single-valued trapezoidal neutrosophic weighted averaging (QSVTrNWA) operator for QSVTrNNs, followed by a comprehensive investigation of their properties. In Sect. 5, the quadripartitioned single-valued trapezoidal neutrosophic weighted geometric (QSVTrNWG) operator is proposed, with a discussion on its properties and special cases, providing insights into its practical application. Section 6 presents an illustrative example wherein a company selects the most suitable green supplier using the developed methodologies, demonstrating the applicability and effectiveness of the proposed method. Finally, concluding remarks are provided in Sect. 7, summarizing the contributions of the proposed methodologies and suggesting potential avenues for future research in the field of neutrosophic decision-making.

2 Preliminaries and basic definitions

In this section, we introduce some definitions of intuitionistic fuzzy sets, q-rung orthopair fuzzy sets, single valued neutrosophic sets, q-rung orthopair trapezoidal fuzzy sets and single-valued trapezoidal neutrosophic sets are required in this study.

In 1986, Atanassov [1] generalized the idea of fuzzy set (FS) to intuitionistic fuzzy set, which is characterized by a membership function and a non- membership function as follows:

Definition 1 [1] An **intuitionistic fuzzy set (IFS)** $\mathcal{A}$ on a fixed set U is an object having the form

$$\mathcal{A} = \left\{ \left\{ a, \mu_A(a), \eta_A(a) \right\} \,:\, a \in U \right\} \tag{1}$$

where the functions $\mu_A : U \to [0,1]$ and $\eta_A : U \to [0,1]$ denote the degree of **membership** and the degree of **non-membership**, respectively, and $\mu_A(a) + \eta_A(a) \in [0,1]$ for all $a \in U$.

In 2010, Wang et al. in [43] presented the concept of single valued neutrosophic sets as follows:

Definition 2 [43] A **single valued neutrosophic set (SVNS)** $\mathcal{A}$ on a fixed set U is an object having the form

$$\mathcal{A} = \left\{ \left\{ a, \mu_A(a), \eta_A(a), \nu_A(a) \right\} \,:\, a \in U \right\} \tag{2}$$

where the functions $\mu_A : U \to [0,1], \eta_A : U \to [0,1]$ and $\nu_A : U \to [0,1]$ denote the truth-membership, indeterminacy-membership and the falsity-membership functions, respectively, and $0 \leq \mu_A(a) + \eta_A(a) + \nu_A(a) \leq 3$ for all $a \in U$.

The theory of a triangular intuitionistic fuzzy ste, which was initially introduced in 2010 by Li et al. in [27], has been applied to many mathematical branches as follows:

Definition 3 [27] Let μ_A^+ and ν_A^- be any elements of [0, 1]. A **triangular intuitionistic fuzzy set (TrIFS)**

$$\mathcal{A} = \left\{ \left\langle \left(\left[a^1, a^2, a^3, a^4\right], \mu_A(x) \right), \left(\left[b^1, b^2, b^3, b^4\right], \nu_A(x) \right) \right\rangle : x \in \mathbf{R} \right\} \tag{3}$$

is a special IFS on the real number set $\mathbf{R}$, whose membership function $\mu_A : \mathbf{R} \to [0, 1]$ and non-membership function $\nu_A : \mathbf{R} \to [0, 1]$ are defined as follows

$$\mu_A(x) = \begin{cases} \left(\frac{(x-a_1)\mu_A^+}{a_2-a_1} \right) & ; \quad \text{if}\, a_1 \le x < a_2 \\ \mu_A^+ & ; \quad \text{if}\, a_2 \le x < a_3 \\ \left(\frac{(a_4-x)\mu_A^+}{a_4-a_3} \right) & ; \quad \text{if}\, a_3 \le x < a_4 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

and

$$\nu_A(x) = \begin{cases} \left(\frac{b_2-x+(x-b_1)\nu_A^-}{b_2-b_1} \right) & ; \quad \text{if}\, b_1 \le x < b_2 \\ \nu_A^- & ; \quad \text{if}\, b_2 \le x < b_3 \\ \left(\frac{x-b_3+(b_4-x)\nu_A^-}{b_4-b_3} \right) & ; \quad \text{if}\, b_3 \le x < b_4 \\ 1 & ; \quad \text{otherwise.} \end{cases}$$

where $0 \le a^1 \le a^2 \le a^3 \le a^4 \le 1, 0 \le b^1 \le b^2 \le b^3 \le b^4 \le 1$ and $\mu_A(x) + \eta_A(x) \in [0, 1]$ for all $x \in \mathbf{R}$.

For real applications of SVNS, Ye [47] introduced single-valued trapezoidal neutrosophic sets in the following definition.

Definition 4 [47] Let μ_A^+, η_A^- and ν_A^- be any elements of [0, 1]. A **single-valued trapezoidal neutrosophic set (SVTrNS)**

$$\mathcal{A} = \left\{ \left\langle \begin{array}{c} \left(\left[a^1, a^2, a^3, a^4\right], \mu_A(x) \right), \\ \left(\left[b^1, b^2, b^3, b^4\right], \eta_A(x) \right), \\ \left(\left[c^1, c^2, c^3, c^4\right], \nu_A(x) \right) \end{array} \right\rangle : x \in \mathbf{R} \right\} \tag{4}$$

is a special SVNS on the real number set $\mathbf{R}$, whose truth-membership function $\mu_A : \mathbf{R} \to [0, 1]$, indeterminacy-membership function $\eta_A : \mathbf{R} \to [0, 1]$ and the falsity-membership function $\nu_A : \mathbf{R} \to [0, 1]$ are defined as follows

$$\mu_A(x) = \begin{cases} \left(\frac{(x-a_1)\mu_A^+}{a_2-a_1}\right) & ; \quad \text{if } a_1 \le x < a_2 \\ \mu_A^+ & ; \quad \text{if } a_2 \le x < a_3 \\ \left(\frac{(a_4-x)\mu_A^+}{a_4-a_3}\right) & ; \quad \text{if } a_3 \le x < a_4 \\ 0 & ; \quad \text{otherwise,} \end{cases}$$

$$\eta_A(x) = \begin{cases} \left(\frac{b_2-x+(x-b_1)\eta_A^-}{b_2-b_1}\right) & ; \quad \text{if } b_1 \le x < b_2 \\ \eta_A^- & ; \quad \text{if } b_2 \le x < b_3 \\ \left(\frac{x-b_3+(b_4-x)\eta_A^-}{b_4-b_3}\right) & ; \quad \text{if } b_3 \le x < b_4 \\ 1 & ; \quad \text{otherwise} \end{cases}$$

and

$$\nu_A(x) = \begin{cases} \left(\frac{c_2-x+(x-c_1)\nu_A^-}{c_2-c_1}\right) & ; \quad \text{if } c_1 \le x < c_2 \\ \nu_A^- & ; \quad \text{if } c_2 \le x < c_3 \\ \left(\frac{x-c_3+(c_4-x)\nu_A^-}{c_4-c_3}\right) & ; \quad \text{if } c_3 \le x < c_4 \\ 1 & ; \quad \text{otherwise.} \end{cases}$$

where $a^1 \le a^2 \le a^3 \le a^4, b^1 \le b^2 \le b^3 \le b^4, c^1 \le c^2 \le c^3 \le c^4$ and $0 \le \mu_A(x) + \eta_A(x) + \nu_A(x) \le 3$ for all $x \in \mathbf{R}$.

In 2016, Chatterjee et al. in [10] extended the concept of SVNSs to quadripartitioned single valued neutrosophic sets and defined quadripartitioned single valued neutrosophic numbers. In what follows, we will first introduce QSVNSs.

Definition 5 [10] A **quadripartitioned single valued neutrosophic set (QSVNS)** $\mathcal{A}$ on a fixed set U is an object having the form

$$\mathcal{A} = \left\{ \left\{ a, \mu_A(a), \vartheta_A(a), \eta_A(a), \nu_A(a) \right\} : a \in U \right\} \tag{5}$$

where the functions $\mu_A : U \to [0,1], \vartheta_A : U \to [0,1], \eta_A : U \to [0,1]$ and $\nu_A : U \to [0,1]$ denote the truth-membership, contradiction membership, ignorance membership and the falsity membership functions, respectively, and $0 \le \mu_A(a) + \vartheta_A(a) + \eta_A(a) + \nu_A(a) \le 4$ for all $a \in U$.

3 QSVTrNSs

In this study, we have proposed a novel extension of single-valued trapezoidal neutrosophic sets (SVTrNS) to a fuzzy set framework, presenting a trapezoidal neutrosophic set that combines trapezoidal fuzzy numbers with single-valued neutrosophic sets. This extension offers a more comprehensive representation of uncertainty by integrating the gradual membership degrees of trapezoidal fuzzy numbers with the handling of indeterminacy, contradiction, and incomplete information inherent in neutrosophic sets.

Definition 6 Let $\mu_A^+, \vartheta_A^+, \eta_A^-$ and ν_A^- be any elements of [0, 1]. A **quadripartitioned single-valued trapezoidal neutrosophic set (QSVTrNS)**

$$\mathcal{A} = \left\{ \left\{ \begin{array}{l} \left(\left[a^1, a^2, a^3, a^4\right], \mu_A(x)\right), \\ \left(\left[b^1, b^2, b^3, b^4\right], \vartheta_A(x)\right), \\ \left(\left[c^1, c^2, c^3, c^4\right], \eta_A(x)\right), \\ \left(\left[d^1, d^2, d^3, d^4\right], \nu_A(x)\right) \end{array} \right\} : x \in \mathbf{R} \right\} \tag{6}$$

is a special q-ROQSVNS on the real number set $\mathbf{R}$, whose truth-membership function $\mu_A : \mathbf{R} \to [0, 1]$, contradiction membership function $\mu_A : \mathbf{R} \to [0, 1]$, ignorance-membership function $\eta_A : \mathbf{R} \to [0, 1]$ and the falsity membership function $\nu_A : \mathbf{R} \to [0, 1]$ are defined as follows

$$\mu_A(x) = \begin{cases} \left(\frac{(x-a_1)\mu_A^+}{a_2-a_1}\right) & ; \text{ if } a_1 \le x < a_2 \\ \mu_A^+ & ; \text{ if } a_2 \le x < a_3 \\ \left(\frac{(a_4-x)\mu_A^+}{a_4-a_3}\right) & ; \text{ if } a_3 \le x < a_4 \\ 0 & ; \text{ otherwise,} \end{cases}$$

$$\vartheta_A(x) = \begin{cases} \left(\frac{(x-b_1)\vartheta_A^+}{b_2-b_1}\right) & ; \text{ if } b_1 \le x < b_2 \\ \vartheta_A^+ & ; \text{ if } b_2 \le x < b_3 \\ \left(\frac{(b_4-x)\vartheta_A^+}{b_4-b_3}\right) & ; \text{ if } b_3 \le x < b_4 \\ 0 & ; \text{ otherwise,} \end{cases}$$

$$\nu_A(x) = \begin{cases} \left(\frac{c_2-x+(x-c_1)\nu_A^-}{c_2-c_1}\right) & ; \text{ if } c_1 \le x < c_2 \\ \nu_A^- & ; \text{ if } c_2 \le x < c_3 \\ \left(\frac{x-c_3+(c_4-x)\nu_A^-}{c_4-c_3}\right) & ; \text{ if } c_3 \le x < c_4 \\ 1 & ; \text{ otherwise} \end{cases}$$

and

$$\eta_A(x) = \begin{cases} \left(\frac{d_2-x+(x-d_1)\eta_A^-}{d_2-d_1}\right) & ; \text{ if } d_1 \le x < d_2 \\ \eta_A^- & ; \text{ if } d_2 \le x < d_3 \\ \left(\frac{x-d_3+(d_4-x)\eta_A^-}{d_4-d_3}\right) & ; \text{ if } d_3 \le x < d_4 \\ 1 & ; \text{ otherwise.} \end{cases}$$

where $0 \le a^1 \le a^2 \le a^3 \le a^4 \le 1, 0 \le b^1 \le b^2 \le b^3 \le b^4 \le 1, 0 \le c^1 \le c^2 \le c^3 \le c^4 \le 1,$ $0 \le d^1 \le d^2 \le d^3 \le d^4 \le 1$ and $0 \le a^4 + b^4 + c^4 + d^4 \le 4, 0 \le \mu_A(x) + \vartheta_A(x) + \eta_A(x) + \nu_A(x) \le 4$ for all $x \in \mathbf{R}$.

Remark 1 Note that let $\mathcal{A} = \left\{ \left\{ \begin{array}{l} \left(\left[a^1, a^2, a^3, a^4\right], \mu_A(x) \right), \\ \left(\left[b^1, b^2, b^3, b^4\right], \vartheta_A(x) \right), \\ \left(\left[c^1, c^2, c^3, c^4\right], \eta_A(x) \right), \\ \left(\left[d^1, d^2, d^3, d^4\right], \nu_A(x) \right) \end{array} \right\} : x \in \mathbf{R} \right\}$ be a QSVTrNS on the real number set $\mathbf{R}$. In particular, if $\mathbf{R}$ has only one element, $\mathcal{A}$ is called a **quadripartitioned single-valued trapezoidal neutrosophic number (QSVTrNN)**. For convenience, the three QSVTrNNs are denoted by

$$\mathcal{A} = \begin{bmatrix} \left(\left[a^1, a^2, a^3, a^4\right], \mu_A \right), \\ \left(\left[b^1, b^2, b^3, b^4\right], \vartheta_A \right), \\ \left(\left[c^1, c^2, c^3, c^4\right], \eta_A \right), \\ \left(\left[d^1, d^2, d^3, d^4\right], \nu_A \right) \end{bmatrix}.$$

Moreover, we denote by $\mathcal{TN}(\mathbf{R})$ the collection of QSVTrNNs on $\mathbf{R}$. Thus we can say that QSVTrNS is the generalization of previously defined concepts related to neutrosophic theory. The QSVTrNN gives more opportunities to deal with uncertainty in data.

For this relation of less than, equality, union, intersection and complement are defined as follows:

Definition 7 For any two QSVTrNNs $\mathcal{A}_1 = \begin{bmatrix} \left(\left[a_1^1, a_1^2, a_1^3, a_1^4\right], \mu_{A_1} \right), \\ \left(\left[b_1^1, b_1^2, b_1^3, b_1^4\right], \vartheta_{A_1} \right), \\ \left(\left[c_1^1, c_1^2, c_1^3, c_1^4\right], \eta_{A_1} \right), \\ \left(\left[d_1^1, d_1^2, d_1^3, d_1^4\right], \nu_{A_1} \right) \end{bmatrix}$ and $\mathcal{A}_2 = \begin{bmatrix} \left(\left[a_2^1, a_2^2, a_2^3, a_2^4\right], \mu_{A_2} \right), \\ \left(\left[b_2^1, b_2^2, b_2^3, b_2^4\right], \vartheta_{A_2} \right), \\ \left(\left[c_2^1, c_2^2, c_2^3, c_2^4\right], \eta_{A_2} \right), \\ \left(\left[d_2^1, d_2^2, d_2^3, d_2^4\right], \nu_{A_2} \right) \end{bmatrix}$ on the real number set $\mathbf{R}$, the corresponding operations are defined as follows:

1. $\left(\left[a_1^1, a_1^2, a_1^3, a_1^4\right], \mu_{A_1} \right) \leq \left(\left[a_2^1, a_2^2, a_2^3, a_2^4\right], \mu_{A_2} \right)$ if and only if $a_1^1 \leq a_2^1, a_1^2 \leq a_2^2, a_1^3 \leq a_2^3, a_1^4 \leq a_2^4$ and $\mu_{A_1} \leq \mu_{A_2}$,
2. $\left(\left[b_1^1, b_1^2, b_1^3, b_1^4\right], \vartheta_{A_1} \right) \leq \left(\left[b_2^1, b_2^2, b_2^3, b_2^4\right], \vartheta_{A_2} \right)$ if and only if $b_1^1 \leq b_2^1, b_1^2 \leq b_2^2, b_1^3 \leq b_2^3, b_1^4 \leq b_2^4$ and $\vartheta_{A_1} \leq \vartheta_{A_2}$,
3. $\left(\left[c_1^1, c_1^2, c_1^3, c_1^4\right], \eta_{A_1} \right) \leq \left(\left[c_2^1, c_2^2, c_2^3, c_2^4\right], \eta_{A_2} \right)$ if and only if $c_1^1 \leq c_2^1, c_1^2 \leq c_2^2, c_1^3 \leq c_2^3, c_1^4 \leq c_2^4$ and $\eta_{A_1} \leq \eta_{A_2}$,
4. $\left(\left[d_1^1, d_1^2, d_1^3, d_1^4\right], \nu_{A_1} \right) \leq \left(\left[d_2^1, d_2^2, d_2^3, d_2^4\right], \nu_{A_2} \right)$ if and only if $d_1^1 \leq d_2^1, d_1^2 \leq d_2^2, d_1^3 \leq d_2^3, d_1^4 \leq d_2^4$ and $\nu_{A_1} \leq \nu_{A_2}$,
5. $\left(\left[a_1^1, a_1^2, a_1^3, a_1^4\right], \mu_{A_1} \right) \cup \left(\left[a_2^1, a_2^2, a_2^3, a_2^4\right], \mu_{A_2} \right)$ if and only if $a_1^1 \vee a_2^1, a_1^2 \vee a_2^2, a_1^3 \vee a_2^3, a_1^4 \vee a_2^4$ and $\mu_{A_1} \vee \mu_{A_2}$,

6. $\left(\left[b_1^1, b_1^2, b_1^3, b_1^4\right], \vartheta_{A_1}\right) \cup \left(\left[b_2^1, b_2^2, b_2^3, b_2^4\right], \vartheta_{A_2}\right)$ if and only if $b_1^1 \vee b_2^1, b_1^2 \vee b_2^2$, $b_1^3 \vee b_2^3, b_1^4 \vee b_2^4$ and $\vartheta_{A_1} \vee \vartheta_{A_2}$,
7. $\left(\left[c_1^1, c_1^2, c_1^3, c_1^4\right], \eta_{A_1}\right) \cup \left(\left[c_2^1, c_2^2, c_2^3, c_2^4\right], \eta_{A_2}\right)$ if and only if $c_1^1 \vee c_2^1, c_1^2 \vee c_2^2$, $c_1^3 \vee c_2^3, c_1^4 \vee c_2^4$ and $\eta_{A_1} \vee \eta_{A_2}$,
8. $\left(\left[d_1^1, d_1^2, d_1^3, d_1^4\right], \nu_{A_1}\right) \cup \left(\left[d_2^1, d_2^2, d_2^3, d_2^4\right], \nu_{A_2}\right)$ if and only if $d_1^1 \vee d_2^1, d_1^2 \vee d_2^2$, $d_1^3 \vee d_2^3, d_1^4 \vee d_2^4$ and $\nu_{A_1} \vee \nu_{A_2}$,
9. $\left(\left[a_1^1, a_1^2, a_1^3, a_1^4\right], \mu_{A_1}\right) \cap \left(\left[a_2^1, a_2^2, a_2^3, a_2^4\right], \mu_{A_2}\right)$ if and only if $a_1^1 \wedge a_2^1, a_1^2 \wedge a_2^2$, $a_1^3 \wedge a_2^3, a_1^4 \wedge a_2^4$ and $\mu_{A_1} \wedge \mu_{A_2}$,
10. $\left(\left[b_1^1, b_1^2, b_1^3, b_1^4\right], \vartheta_{A_1}\right) \cap \left(\left[b_2^1, b_2^2, b_2^3, b_2^4\right], \vartheta_{A_2}\right)$ if and only if $b_1^1 \wedge b_2^1, b_1^2 \wedge b_2^2$, $b_1^3 \wedge b_2^3, b_1^4 \wedge b_2^4$ and $\vartheta_{A_1} \wedge \vartheta_{A_2}$,
11. $\left(\left[c_1^1, c_1^2, c_1^3, c_1^4\right], \eta_{A_1}\right) \cap \left(\left[c_2^1, c_2^2, c_2^3, c_2^4\right], \eta_{A_2}\right)$ if and only if $c_1^1 \wedge c_2^1, c_1^2 \wedge c_2^2$, $c_1^3 \wedge c_2^3, c_1^4 \wedge c_2^4$ and $\eta_{A_1} \wedge \eta_{A_2}$,
12. $\left(\left[d_1^1, d_1^2, d_1^3, d_1^4\right], \nu_{A_1}\right) \cap \left(\left[d_2^1, d_2^2, d_2^3, d_2^4\right], \nu_{A_2}\right)$ if and only if $d_1^1 \wedge d_2^1, d_1^2 \wedge d_2^2$, $d_1^3 \wedge d_2^3, d_1^4 \wedge d_2^4$ and $\nu_{A_1} \wedge \nu_{A_2}$,
13. $\mathcal{A}_1 \ll \mathcal{A}_2$ if and only if $\left(\left[a_1^1, a_1^2, a_1^3, a_1^4\right], \mu_{A_1}\right) \le \left(\left[a_2^1, a_2^2, a_2^3, a_2^4\right], \mu_{A_2}\right)$, $\left(\left[b_1^1, b_1^2, b_1^3, b_1^4\right], \vartheta_{A_1}\right) \le \left(\left[b_2^1, b_2^2, b_2^3, b_2^4\right], \vartheta_{A_2}\right)$, $\left(\left[c_1^1, c_1^2, c_1^3, c_1^4\right], \eta_{A_1}\right) \ge \left(\left[c_2^1, c_2^2, c_2^3, c_2^4\right], \eta_{A_2}\right)$ and $\left(\left[d_1^1, d_1^2, d_1^3, d_1^4\right], \nu_{A_1}\right) \ge \left(\left[d_2^1, d_2^2, d_2^3, d_2^4\right], \nu_{A_2}\right)$,
14. $\mathcal{A}_1 = \mathcal{A}_2$ if and only if $\mathcal{A}_1 \ll \mathcal{A}_2$ and $\mathcal{A}_2 \ll \mathcal{A}_1$,
15. $$\mathcal{A}_1 \bigtriangledown \mathcal{A}_2 = \begin{bmatrix} \left(\left[a_1^1, a_1^2, a_1^3, a_1^4\right] \cup \left[a_2^1, a_2^2, a_2^3, a_2^4\right], \mu_{A_1} \vee \mu_{A_2}\right), \\ \left(\left[b_1^1, b_1^2, b_1^3, b_1^4\right] \cap \left[b_2^1, b_2^2, b_2^3, b_2^4\right], \vartheta_{A_1} \wedge \vartheta_{A_2}\right), \\ \left(\left[c_1^1, c_1^2, c_1^3, c_1^4\right] \cap \left[c_2^1, c_2^2, c_2^3, c_2^4\right], \eta_{A_1} \wedge \eta_{A_2}\right), \\ \left(\left[d_1^1, d_1^2, d_1^3, d_1^4\right] \cap \left[d_2^1, d_2^2, d_2^3, d_2^4\right], \nu_{A_1} \wedge \nu_{A_2}\right) \end{bmatrix},$$
16. $$\mathcal{A}_1 \bigtriangleup \mathcal{A}_2 = \begin{bmatrix} \left(\left[a_1^1, a_1^2, a_1^3, a_1^4\right] \cap \left[a_2^1, a_2^2, a_2^3, a_2^4\right], \mu_{A_1} \wedge \mu_{A_2}\right), \\ \left(\left[b_1^1, b_1^2, b_1^3, b_1^4\right] \cup \left[b_2^1, b_2^2, b_2^3, b_2^4\right], \vartheta_{A_1} \vee \vartheta_{A_2}\right), \\ \left(\left[c_1^1, c_1^2, c_1^3, c_1^4\right] \cup \left[c_2^1, c_2^2, c_2^3, c_2^4\right], \eta_{A_1} \vee \eta_{A_2}\right), \\ \left(\left[d_1^1, d_1^2, d_1^3, d_1^4\right] \cup \left[d_2^1, d_2^2, d_2^3, d_2^4\right], \nu_{A_1} \vee \nu_{A_2}\right) \end{bmatrix},$$
17. $$\mathcal{A}_1^c = \begin{bmatrix} \left(\left[d_1^1, d_1^2, d_1^3, d_1^4\right], \nu_{A_1}\right), \\ \left(\left[c_1^1, c_1^2, c_1^3, c_1^4\right], \eta_{A_1}\right), \\ \left(\left[b_1^1, b_1^2, b_1^3, b_1^4\right], \vartheta_{A_1}\right), \\ \left(\left[a_1^1, a_1^2, a_1^3, a_1^4\right], \mu_{A_1}\right) \end{bmatrix}.$$

Motivated by the operations of the trapezoidal neutrosophic numbers and QSVNNs, in the following, we shall define some operational laws of QSVTrNNs.

Definition 8 Let

$$\mathcal{A}_1 = \begin{bmatrix} \left(\left[a^1_{A_1}, a^2_{A_1}, a^3_{A_1}, a^4_{A_1}\right], \mu_{A_1}\right), \left(\left[b^1_{A_1}, b^2_{A_1}, b^3_{A_1}, b^4_{A_1}\right], \vartheta_{A_1}\right), \\ \left(\left[c^1_{A_1}, c^2_{A_1}, c^3_{A_1}, c^4_{A_1}\right], \eta_{A_1}\right), \left(\left[d^1_{A_1}, d^2_{A_1}, d^3_{A_1}, d^4_{A_1}\right], \nu_{A_1}\right) \end{bmatrix}$$

and

$$\mathcal{A}_2 = \begin{bmatrix} \left(\left[a^1_{A_2}, a^2_{A_2}, a^3_{A_2}, a^4_{A_2}\right], \mu_{A_2}\right), \\ \left(\left[b^1_{A_2}, b^2_{A_2}, b^3_{A_2}, b^4_{A_2}\right], \vartheta_{A_2}\right), \\ \left(\left[c^1_{A_2}, c^2_{A_2}, c^3_{A_2}, c^4_{A_2}\right], \eta_{A_2}\right), \\ \left(\left[d^1_{A_2}, d^2_{A_2}, d^3_{A_2}, d^4_{A_2}\right], \nu_{A_2}\right) \end{bmatrix}$$

be any QSVTrNNs on the real number set $\mathbf{R}$. For every element $\lambda \in (0, \infty)$, then their operations are defined as follows:

1. $\begin{bmatrix} a^1_{A_1}, a^2_{A_1}, \\ a^3_{A_1}, a^4_{A_1} \end{bmatrix} \oplus \begin{bmatrix} a^1_{A_{21}}, a^2_{A_2}, \\ a^3_{A_2}, a^4_{A_2} \end{bmatrix} = \begin{bmatrix} a^1_{A_1} + a^1_{A_2} - a^1_{A_1} a^1_{A_2}, \\ a^2_{A_1} + a^2_{A_2} - a^2_{A_1} a^2_{A_2}, \\ a^3_{A_1} + a^3_{A_2} - a^3_{A_1} a^3_{A_2}, \\ a^4_{A_1} + a^4_{A_2} - a^4_{A_1} a^4_{A_2} \end{bmatrix},$
2. $\begin{bmatrix} b^1_{A_1}, b^2_{A_1}, \\ b^3_{A_1}, b^4_{A_1} \end{bmatrix} \oplus \begin{bmatrix} b^1_{A_{21}}, b^2_{A_2}, \\ b^3_{A_2}, b^4_{A_2} \end{bmatrix} = \begin{bmatrix} b^1_{A_1} + b^1_{A_2} - \left(b^1_{A_1}\right)\left(b^1_{A_2}\right), \\ b^2_{A_1} + b^2_{A_2} - \left(b^2_{A_1}\right)\left(b^2_{A_2}\right), \\ b^3_{A_1} + b^3_{A_2} - \left(b^3_{A_1}\right)\left(b^3_{A_2}\right), \\ b^4_{A_1} + b^4_{A_2} - \left(b^4_{A_1}\right)\left(b^4_{A_2}\right) \end{bmatrix},$
3. $\begin{bmatrix} c^1_{A_1}, c^2_{A_1}, \\ c^3_{A_1}, c^4_{A_1} \end{bmatrix} \oplus \begin{bmatrix} c^1_{A_2}, c^2_{A_2}, \\ c^3_{A_2}, c^4_{A_2} \end{bmatrix} = \left[c^1_{A_1} c^1_{A_2}, c^2_{A_1} c^2_{A_2}, c^3_{A_1} c^3_{A_2}, c^4_{A_1} c^4_{A_2}\right]$
4. $\begin{bmatrix} d^1_{A_1}, d^2_{A_1}, \\ d^3_{A_1}, d^4_{A_1} \end{bmatrix} \oplus \begin{bmatrix} d^1_{A_2}, d^2_{A_2}, \\ d^3_{A_2}, d^4_{A_2} \end{bmatrix} = \left[d^1_{A_1} d^1_{A_2}, d^2_{A_1} d^2_{A_2}, d^3_{A_1} d^3_{A_2}, d^4_{A_1} d^4_{A_2}\right],$
5. $\mu_{A_1} \oplus \mu_{A_2} = \mu_{A_1} + \mu_{A_2} - \mu_{A_1}\mu_{A_2},$
6. $\vartheta_{A_1} \oplus \vartheta_{A_2} = \vartheta_{A_1} + \vartheta_{A_2} - \vartheta_{A_1}\vartheta_{A_2},$
7. $\eta_{A_1} \oplus \eta_{A_2} = \eta_{A_1}\eta_{A_2},$
8. $\nu_{A_1} \oplus \nu_{A_2} = \nu_{A_1}\nu_{A_2},$
9. $\begin{bmatrix} a^1_{A_1}, a^2_{A_1}, \\ a^3_{A_1}, a^4_{A_1} \end{bmatrix} \otimes \begin{bmatrix} a^1_{A_2}, a^2_{A_2}, \\ a^3_{A_2}, a^4_{A_2} \end{bmatrix} = \left[a^1_{A_1} a^1_{A_2}, a^2_{A_1} a^2_{A_2}, a^3_{A_1} a^3_{A_2}, a^4_{A_1} a^4_{A_2}\right],$
10. $\begin{bmatrix} b^1_{A_1}, b^2_{A_1}, \\ b^3_{A_1}, b^4_{A_1} \end{bmatrix} \otimes \begin{bmatrix} b^1_{A_2}, b^2_{A_2}, \\ b^3_{A_2}, b^4_{A_2} \end{bmatrix} = \left[b^1_{A_1} b^1_{A_2}, b^2_{A_1} b^2_{A_2}, b^3_{A_1} b^3_{A_2}, b^4_{A_1} b^4_{A_2}\right],$

11. $\begin{bmatrix} c_{A_1}^1, c_{A_1}^2, \\ c_{A_1}^3, c_{A_1}^4 \end{bmatrix} \otimes \begin{bmatrix} c_{A_2}^1, c_{A_2}^2, \\ c_{A_2}^3, c_{A_2}^4 \end{bmatrix} = \begin{bmatrix} c_{A_1}^1 + c_{A_2}^1 - c_{A_1}^1 c_{A_2}^1, \\ c_{A_1}^2 + c_{A_2}^2 - c_{A_1}^2 c_{A_2}^2, \\ c_{A_1}^3 + c_{A_2}^3 - c_{A_1}^3 c_{A_2}^3, \\ c_{A_1}^4 + c_{A_2}^4 - c_{A_1}^4 c_{A_2}^4 \end{bmatrix},$
12. $\begin{bmatrix} d_{A_1}^1, d_{A_1}^2, \\ d_{A_1}^3, d_{A_1}^4 \end{bmatrix} \otimes \begin{bmatrix} d_{A_2}^1, d_{A_2}^2, \\ d_{A_2}^3, d_{A_2}^4 \end{bmatrix} = \begin{bmatrix} d_{A_1}^1 + d_{A_2}^1 - d_{A_1}^1 b_{A_2}^1, \\ d_{A_1}^2 + d_{A_2}^2 - d_{A_1}^2 d_{A_2}^2, \\ d_{A_1}^3 + d_{A_2}^3 - d_{A_1}^3 d_{A_2}^3, \\ d_{A_1}^4 + d_{A_2}^4 - d_{A_1}^4 d_{A_2}^4 \end{bmatrix},$
13. $\mu_{A_1} \otimes \mu_{A_2} = \mu_{A_1} \mu_{A_2},$
14. $\vartheta_{A_1} \otimes \vartheta_{A_2} = \vartheta_{A_1} \vartheta_{A_2},$
15. $\eta_{A_1} \otimes \eta_{A_2} = \sqrt[q]{\left(\eta_{A_1}\right)^q + \left(\eta_{A_2}\right)^q - \left(\eta_{A_1}\right)^q \left(\eta_{A_2}\right)^q},$
16. $\nu_{A_1} \otimes \nu_{A_2} = \sqrt[q]{\left(\nu_{A_1}\right)^q + \left(\nu_{A_2}\right)^q - \left(\nu_{A_1}\right)^q \left(\nu_{A_2}\right)^q},$
17. $\lambda\left[a_{A_1}^1, a_{A_1}^2, a_{A_1}^3, a_{A_1}^4\right] = \begin{bmatrix} \sqrt[q]{1 - \left(1 - \left(a_{A_1}^1\right)^q\right)^\lambda}, \\ \sqrt[q]{1 - \left(1 - \left(a_{A_1}^2\right)^q\right)^\lambda}, \\ \sqrt[q]{1 - \left(1 - \left(a_{A_1}^3\right)^q\right)^\lambda}, \\ \sqrt[q]{1 - \left(1 - \left(a_{A_1}^4\right)^q\right)^\lambda} \end{bmatrix},$
18. $\lambda\left[b_{A_1}^1, b_{A_1}^2, b_{A_1}^3, b_{A_1}^4\right] = \begin{bmatrix} \sqrt[q]{1 - \left(1 - \left(b_{A_1}^1\right)^q\right)^\lambda}, \\ \sqrt[q]{1 - \left(1 - \left(b_{A_1}^2\right)^q\right)^\lambda}, \\ \sqrt[q]{1 - \left(1 - \left(b_{A_1}^3\right)^q\right)^\lambda}, \\ \sqrt[q]{1 - \left(1 - \left(b_{A_1}^4\right)^q\right)^\lambda} \end{bmatrix},$
19. $\lambda\left[c_{A_1}^1, c_{A_1}^2, c_{A_1}^3, c_{A_1}^4\right] = \left[\left(c_{A_1}^1\right)^\lambda, \left(c_{A_1}^2\right)^\lambda, \left(c_{A_1}^3\right)^\lambda, \left(c_{A_1}^4\right)^\lambda\right],$
20. $\lambda\left[d_{A_1}^1, d_{A_1}^2, d_{A_1}^3, d_{A_1}^4\right] = \left[\left(d_{A_1}^1\right)^\lambda, \left(d_{A_1}^2\right)^\lambda, \left(d_{A_1}^3\right)^\lambda, \left(d_{A_1}^4\right)^\lambda\right],$
21. $\lambda\mu_{A_1} = 1 - \left(1 - \mu_{A_1}\right)^\lambda,$
22. $\lambda\vartheta_{A_1} = 1 - \left(1 - \vartheta_{A_1}\right)^\lambda,$
23. $\lambda\eta_{A_1} = \left(\eta_{A_1}\right)^\lambda,$
24. $\lambda\nu_{A_1} = \left(\nu_{A_1}\right)^\lambda,$
25. $\left[a_{A_1}^1, a_{A_1}^2, a_{A_1}^3, a_{A_1}^4\right]\lambda = \left[\left(a_{A_1}^1\right)^\lambda, \left(a_{A_1}^2\right)^\lambda, \left(a_{A_1}^3\right)^\lambda, \left(a_{A_1}^4\right)^\lambda\right],$
26. $\left[b_{A_1}^1, b_{A_1}^2, b_{A_1}^3, b_{A_1}^4\right]\lambda = \left[\left(b_{A_1}^1\right)^\lambda, \left(b_{A_1}^2\right)^\lambda, \left(b_{A_1}^3\right)^\lambda, \left(b_{A_1}^4\right)^\lambda\right],$

27. $$\left[c_{A_1}^1, c_{A_1}^2, c_{A_1}^3, c_{A_1}^4\right]\lambda = \begin{bmatrix} 1-\left(1-c_{A_1}^1\right)^{\lambda}, \\ 1-\left(1-c_{A_1}^2\right)^{\lambda}, \\ 1-\left(1-c_{A_1}^3\right)^{\lambda}, \\ 1-\left(1-c_{A_1}^4\right)^{\lambda} \end{bmatrix},$$

28. $$\left[d_{A_1}^1, d_{A_1}^2, d_{A_1}^3, d_{A_1}^4\right]\lambda = \begin{bmatrix} 1-\left(1-d_{A_1}^1\right)^{\lambda}, \\ 1-\left(1-d_{A_1}^2\right)^{\lambda}, \\ 1-\left(1-d_{A_1}^3\right)^{\lambda}, \\ 1-\left(1-d_{A_1}^4\right)^{\lambda} \end{bmatrix},$$

29. $\mu_{A_1}\lambda = \left(\mu_{A_1}\right)^{\lambda}$,

30. $\vartheta_{A_1}\lambda = \left(\vartheta_{A_1}\right)^{\lambda}$,

31. $\eta_{A_1}\lambda = 1-\left(1-\eta_{A_1}\right)^{\lambda}$,

32. $\nu_{A_1}\lambda = 1-\left(1-\nu_{A_1}\right)^{\lambda}$,

33. $$\mathcal{A}_1 \oplus \mathcal{A}_2 = \begin{bmatrix} \left(\left[a_{A_1}^1, a_{A_1}^2, a_{A_1}^3, a_{A_1}^4\right] \oplus \left[a_{A_2}^1, a_{A_2}^2, a_{A_2}^3, a_{A_2}^4\right], \mu_{A_1} \oplus \mu_{A_2}\right), \\ \left(\left[b_{A_1}^1, b_{A_1}^2, b_{A_1}^3, b_{A_1}^4\right] \oplus \left[b_{A_2}^1, b_{A_2}^2, b_{A_2}^3, b_{A_2}^4\right], \vartheta_{A_1} \oplus \vartheta_{A_2}\right), \\ \left(\left[c_{A_1}^1, c_{A_1}^2, c_{A_1}^3, c_{A_1}^4\right] \oplus \left[c_{A_2}^1, c_{A_2}^2, c_{A_2}^3, c_{A_2}^4\right], \eta_{A_1} \oplus \eta_{A_2}\right), \\ \left(\left[d_{A_1}^1, d_{A_1}^2, d_{A_1}^3, d_{A_1}^4\right] \oplus \left[d_{A_2}^1, d_{A_2}^2, d_{A_2}^3, d_{A_2}^4\right], \nu_{A_1} \oplus \nu_{A_2}\right) \end{bmatrix},$$

34. $$\mathcal{A}_1 \otimes \mathcal{A}_2 = \begin{bmatrix} \left(\left[a_{A_1}^1, a_{A_1}^2, a_{A_1}^3, a_{A_1}^4\right] \otimes \left[a_{A_2}^1, a_{A_2}^2, a_{A_2}^3, a_{A_2}^4\right], \mu_{A_1} \otimes \mu_{A_2}\right), \\ \left(\left[b_{A_1}^1, b_{A_1}^2, b_{A_1}^3, b_{A_1}^4\right] \otimes \left[b_{A_2}^1, b_{A_2}^2, b_{A_2}^3, b_{A_2}^4\right], \vartheta_{A_1} \otimes \vartheta_{A_2}\right), \\ \left(\left[c_{A_1}^1, c_{A_1}^2, c_{A_1}^3, c_{A_1}^4\right] \otimes \left[c_{A_2}^1, c_{A_2}^2, {}^3_{A_2}, c_{A_2}^4\right], \eta_{A_1} \otimes \eta_{A_2}\right), \\ \left(\left[d_{A_1}^1, d_{A_1}^2, d_{A_1}^3, d_{A_1}^4\right] \otimes \left[d_{A_2}^1, d_{A_2}^2, d_{A_2}^3, d_{A_2}^4\right], \nu_{A_1} \otimes \nu_{A_2}\right) \end{bmatrix},$$

35. $$\lambda\mathcal{A}_1 = \begin{bmatrix} \left(\lambda\left[a_{A_1}^1, a_{A_1}^2, a_{A_1}^3, a_{A_1}^4\right], \lambda\mu_{A_1}\right), \\ \left(\lambda\left[b_{A_1}^1, b_{A_1}^2, b_{A_1}^3, b_{A_1}^4\right], \lambda\vartheta_{A_1}\right), \\ \left(\lambda\left[c_{A_1}^1, c_{A_1}^2, c_{A_1}^3, c_{A_1}^4\right], \lambda\eta_{A_1}\right), \\ \left(\lambda\left[d_{A_1}^1, d_{A_1}^2, d_{A_1}^3, d_{A_1}^4\right], \lambda\nu_{A_1}\right) \end{bmatrix},$$

36. $$\left(\mathcal{A}_1\right)^{\lambda} = \begin{bmatrix} \left(\left[a_{A_1}^1, a_{A_1}^2, a_{A_1}^3, a_{A_1}^4\right]\lambda, \mu_{A_1}\lambda\right), \\ \left(\left[b_{A_1}^1, b_{A_1}^2, b_{A_1}^3, b_{A_1}^4\right]\lambda, \vartheta_{A_1}\lambda\right), \\ \left(\left[c_{A_1}^1, c_{A_1}^2, c_{A_1}^3, c_{A_1}^4\right]\lambda, \eta_{A_1}\lambda\right), \\ \left(\left[d_{A_1}^1, d_{A_1}^2, d_{A_1}^3, d_{A_1}^4\right]\lambda, \nu_{A_1}\lambda\right) \end{bmatrix}.$$

Basic operations related with QSVTrNNs are presented as follows:

Theorem 1 *Let*

$$\mathcal{A} = \begin{bmatrix} \left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right), \left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right), \\ \left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right), \left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right) \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} \left(\left[a_B^1, a_A^2, a_B^3, a_B^4\right], \mu_B\right), \\ \left(\left[b_B^1, b_A^2, b_B^3, b_B^4\right], \vartheta_B\right), \\ \left(\left[c_B^1, c_A^2, c_B^3, c_B^4\right], \eta_B\right), \\ \left(\left[d_B^1, d_A^2, d_B^3, d_B^4\right], \nu_B\right) \end{bmatrix}$$

and $\mathcal{C} = \begin{bmatrix} \left(\left[a_C^1, a_C^2, a_C^3, a_C^4\right], \mu_C\right), \\ \left(\left[b_C^1, b_C^2, b_C^3, b_C^4\right], \vartheta_C\right), \\ \left(\left[c_C^1, c_C^2, c_C^3, c_C^4\right], \eta_C\right), \\ \left(\left[d_C^1, d_C^2, d_C^3, d_C^4\right], \nu_C\right) \end{bmatrix}$ *be any QSVTrNNs on the real number set* $\mathbf{R}$. *For every elements* $\lambda, \xi \in (0, \infty)$, *then the following properties hold.*

1. $\mathcal{A} \oplus \mathcal{B} = \mathcal{B} \oplus \mathcal{A}$.
2. $\mathcal{A} \otimes \mathcal{B} = \mathcal{B} \otimes \mathcal{A}$.
3. $(\mathcal{A} \oplus \mathcal{B}) \oplus \mathcal{C} = \mathcal{A} \oplus (\mathcal{B} \oplus \mathcal{C})$.
4. $(\mathcal{A} \otimes \mathcal{B}) \otimes \mathcal{C} = \mathcal{A} \otimes (\mathcal{B} \otimes \mathcal{C})$.
5. $\lambda(\mathcal{A} \oplus \mathcal{B}) = \lambda\mathcal{A} \oplus \lambda\mathcal{B}$.
6. $(\mathcal{A} \otimes \mathcal{B})^\lambda = \mathcal{A}^\lambda \otimes \mathcal{B}^\lambda$.
7. $(\lambda + \xi)\mathcal{A} = \lambda\mathcal{A} \oplus \xi\mathcal{A}$.
8. $\mathcal{A}^{\lambda+\xi} = \mathcal{A}^\lambda \otimes \mathcal{A}^\xi$.
9. $\left(\mathcal{A}^\lambda\right)^\xi = \mathcal{A}^{\lambda\xi}$.

Proof 1. Let

$$\mathcal{A} = \begin{bmatrix} \left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right), \left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right), \\ \left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right), \left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right) \end{bmatrix}$$

and

$$\mathcal{B} = \begin{bmatrix} \left(\left[a_B^1, a_A^2, a_B^3, a_B^4\right], \mu_B\right), \\ \left(\left[b_B^1, b_A^2, b_B^3, b_B^4\right], \vartheta_B\right), \\ \left(\left[c_B^1, c_A^2, c_B^3, c_B^4\right], \eta_B\right), \\ \left(\left[d_B^1, d_A^2, d_B^3, d_B^4\right], \nu_B\right) \end{bmatrix}$$

be any QSVTrNNs on $\mathbf{R}$. According to the Definition 8, we have

$$\begin{aligned}\left[a_A^1, a_A^2, a_A^3, a_A^4\right] \oplus \left[a_B^1, a_B^2, a_B^3, a_B^4\right] &= \begin{bmatrix} a_A^1 + a_B^1 - a_A^1 a_B^1, \\ a_A^2 + a_B^2 - a_A^2 a_B^2, \\ a_A^3 + a_B^3 - a_A^3 a_B^3, \\ a_A^4 + a_B^4 - a_A^4 a_B^4 \end{bmatrix} \\ &= \begin{bmatrix} a_B^1 + a_A^1 - a_B^1 a_A^1, \\ a_B^2 + a_A^2 - a_B^2 a_A^2, \\ a_B^3 + a_A^3 - a_B^3 a_A^3, \\ a_B^4 + a_A^4 - a_B^4 a_A^4 \end{bmatrix} \\ &= \left[a_B^1, a_B^2, a_B^3, a_B^4\right] \oplus \left[a_A^1, a_A^2, a_A^3, a_A^4\right].\end{aligned}$$

It can be similarly proved that $\left[b_A^1, b_A^2, b_A^3, b_A^4\right] \oplus \left[b_B^1, b_B^2, b_B^3, b_B^4\right] = \left[b_B^1, b_B^2, b_B^3, b_B^4\right] \oplus \left[b_A^1, b_A^2, b_A^3, b_A^4\right]$, $\mu_A \oplus \mu_B = \mu_B \oplus \mu_A$ and $\vartheta_A \oplus \vartheta_B = \vartheta_B \oplus \vartheta_A$. Hence it is easily seen that

$$\begin{aligned}\left[c_A^1, c_A^2, c_A^3, c_A^4\right] \oplus \left[c_B^1, c_B^2, c_B^3, c_B^4\right] &= \left[c_A^1 c_B^1, c_A^2 c_B^2, c_A^3 c_B^3, c_A^4 c_B^4\right] \\ &= \left[c_B^1 c_A^1, c_B^2 c_A^2, c_B^3 c_A^3, c_B^4 c_A^4\right] \\ &= \left[c_B^1, c_B^2, c_B^3, c_B^4\right] \oplus \left[c_A^1, c_A^2, c_A^3, c_A^4\right].\end{aligned}$$

and similarly, it follows that

$$\left[d_A^1, d_A^2, d_A^3, d_A^4\right] \oplus \left[d_B^1, d_B^2, d_B^3, d_B^4\right] = \left[d_B^1, d_B^2, d_B^3, d_B^4\right] \oplus \left[d_A^1, d_A^2, d_A^3, d_A^4\right] \tag{7}$$

$$\eta_A \oplus \eta_B = \eta_B \oplus \eta_A \tag{8}$$

$$\nu_A \oplus \nu_B = \nu_B \oplus \nu_A. \tag{9}$$

Therefore we obtain that $\mathcal{A} \oplus \mathcal{B} = \mathcal{B} \oplus \mathcal{A}$.

2. The proof is similar to 1.

3–4. Straightforward from the Definition 8, so we omit the proofs of them.

5. Let

$$\mathcal{A} = \begin{bmatrix} \left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right), \\ \left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right), \\ \left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right), \\ \left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right) \end{bmatrix}$$

and

$$\mathcal{B} = \begin{bmatrix} \left(\left[a_B^1, a_A^2, a_B^3, a_B^4\right], \mu_B\right), \\ \left(\left[b_B^1, b_A^2, b_B^3, b_B^4\right], \vartheta_B\right), \\ \left(\left[c_B^1, c_A^2, c_B^3, c_B^4\right], \eta_B\right), \\ \left(\left[d_B^1, d_A^2, d_B^3, d_B^4\right], \nu_B\right) \end{bmatrix}$$

be any q-ROQSVTrNNs on $\mathbf{R}$. Since

$$\lambda\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} \oplus \lambda\begin{bmatrix} a_B^1, \\ a_B^2, \\ a_B^3, \\ a_B^4 \end{bmatrix} = \begin{bmatrix} 1-\left(1-a_A^1\right)^{\lambda}, \\ 1-\left(1-a_A^2\right)^{\lambda}, \\ 1-\left(1-a_A^3\right)^{\lambda}, \\ 1-\left(1-a_A^4\right)^{\lambda} \end{bmatrix} \oplus \begin{bmatrix} 1-\left(1-a_B^1\right)^{\lambda}, \\ 1-\left(1-a_B^2\right)^{\lambda}, \\ 1-\left(1-a_B^3\right)^{\lambda}, \\ 1-\left(1-a_B^4\right)^{\lambda} \end{bmatrix},$$

we have

$$\lambda\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} \oplus \lambda\begin{bmatrix} a_B^1, \\ a_B^2, \\ a_B^3, \\ a_B^4 \end{bmatrix} = \begin{bmatrix} 1-\left(1-a_A^1\right)^{\lambda}+ \\ 1-\left(1-a_B^1\right)^{\lambda}+ \\ -\begin{pmatrix} \left(1-\left(1-a_A^1\right)^{\lambda}\right) \\ \left(1-\left(1-a_B^1\right)^{\lambda}\right) \end{pmatrix}, \\ 1-\left(1-a_A^2\right)^{\lambda}+ \\ 1-\left(1-a_B^2\right)^{\lambda}+ \\ -\begin{pmatrix} \left(1-\left(1-a_A^2\right)^{\lambda}\right) \\ \left(1-\left(1-a_B^2\right)^{\lambda}\right) \end{pmatrix}, \\ 1-\left(1-a_A^3\right)^{\lambda}+ \\ 1-\left(1-a_B^3\right)^{\lambda}+ \\ -\begin{pmatrix} \left(1-\left(1-a_A^3\right)^{\lambda}\right) \\ \left(1-\left(1-a_B^3\right)^{\lambda}\right) \end{pmatrix}, \\ 1-\left(1-a_A^4\right)^{\lambda}+ \\ 1-\left(1-a_B^4\right)^{\lambda}+ \\ -\begin{pmatrix} \left(1-\left(1-a_A^4\right)^{\lambda}\right) \\ \left(1-\left(1-a_B^4\right)^{\lambda}\right) \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1-\left(1-a_A^1\right)^{\lambda}\left(1-a_B^1\right)^{\lambda}, \\ 1-\left(1-a_A^2\right)^{\lambda}\left(1-a_B^2\right)^{\lambda}, \\ 1-\left(1-a_A^3\right)^{\lambda}\left(1-a_B^3\right)^{\lambda}, \\ 1-\left(1-a_A^4\right)^{\lambda}\left(1-a_B^4\right)^{\lambda} \end{bmatrix}$$

and

$$\lambda\left(\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} \oplus \begin{bmatrix} a_B^1, \\ a_B^2, \\ a_B^3, \\ a_B^4 \end{bmatrix}\right) = \begin{bmatrix} a_A^1 + a_B^1 - \left(a_A^1\right)\left(a_B^1\right), \\ a_A^2 + a_B^2 - \left(a_A^2\right)\left(a_B^2\right), \\ a_A^3 + a_B^3 - \left(a_A^3\right)\left(a_B^3\right), \\ a_A^4 + a_B^4 - \left(a_A^4\right)\left(a_B^4\right) \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \left(1 - \left(a_A^1 + a_B^1 - \left(a_A^1\right)\left(a_B^1\right)\right)\right)^\lambda, \\ 1 - \left(1 - \left(a_A^2 + a_B^2 - \left(a_A^2\right)\left(a_B^2\right)\right)\right)^\lambda, \\ 1 - \left(1 - \left(a_A^3 + a_B^3 - \left(a_A^3\right)\left(a_B^3\right)\right)\right)^\lambda, \\ 1 - \left(1 - \left(a_A^4 + a_B^4 - \left(a_A^4\right)\left(a_B^4\right)\right)\right)^\lambda, \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \left(1 - a_A^1 - a_B^1 + \left(a_A^1\right)\left(a_B^1\right)\right)^\lambda, \\ 1 - \left(1 - a_A^2 - a_B^2 + \left(a_A^2\right)\left(a_B^2\right)\right)^\lambda, \\ 1 - \left(1 - a_A^3 - a_B^3 + \left(a_A^3\right)\left(a_B^3\right)\right)^\lambda, \\ 1 - \left(1 - a_A^4 - a_B^4 + \left(a_A^4\right)\left(a_B^4\right)\right)^\lambda \end{bmatrix}.$$

Then we have

$$\lambda\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} \oplus \lambda\begin{bmatrix} a_B^1, \\ a_B^2, \\ a_B^3, \\ a_B^4 \end{bmatrix} = \lambda\left(\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} \oplus \begin{bmatrix} a_B^1, \\ a_B^2, \\ a_B^3, \\ a_B^4 \end{bmatrix}\right). \tag{10}$$

It can be similarly proved that

$$\lambda\begin{bmatrix} b_A^1, \\ b_A^2, \\ b_A^3, \\ b_A^4 \end{bmatrix} \oplus \lambda\begin{bmatrix} b_B^1, \\ b_B^2, \\ b_B^3, \\ b_B^4 \end{bmatrix} = \lambda\left(\begin{bmatrix} b_A^1, \\ b_A^2, \\ b_A^3, \\ b_A^4 \end{bmatrix} \oplus \begin{bmatrix} b_B^1, \\ b_B^2, \\ b_B^3, \\ b_B^4 \end{bmatrix}\right), \tag{11}$$

$$\lambda\mu_A \oplus \lambda\mu_B = \lambda\left(\mu_A \oplus \mu_B\right) \tag{12}$$

and

$$\lambda\vartheta_A \oplus \lambda\vartheta_B = \lambda\left(\vartheta_A \oplus \vartheta_B\right). \tag{13}$$

For the ignorance membership degree of $\lambda(\mathcal{A} \oplus \mathcal{B})$, we have

$$\begin{aligned} \lambda\left[c_A^1, c_A^2, c_A^3, c_A^4\right] \oplus \lambda\left[c_B^1, c_B^2, c_B^3, c_B^4\right] &= \begin{bmatrix} \left(c_A^1\right)^\lambda\left(c_B^1\right)^\lambda, \left(c_A^2\right)^\lambda\left(c_B^2\right)^\lambda, \\ \left(c_A^3\right)^\lambda\left(c_B^3\right)^\lambda, \left(c_A^4\right)^\lambda\left(c_B^4\right)^\lambda \end{bmatrix} \\ &= \begin{bmatrix} \left(c_A^1 c_B^1\right)^\lambda, \left(c_A^2 c_B^2\right)^\lambda, \\ \left(c_A^3 c_B^3\right)^\lambda, \left(c_A^4 c_B^4\right)^\lambda \end{bmatrix} \\ &= \lambda\left(\left[c_B^1, c_B^2, c_B^3, c_B^4\right] \oplus \left[c_A^1, c_A^2, c_A^3, c_A^4\right]\right), \end{aligned}$$

i.e.,

$$\lambda\begin{bmatrix} c_A^1, \\ c_A^2, \\ c_A^3, \\ c_A^4 \end{bmatrix} \oplus \lambda\begin{bmatrix} c_B^1, \\ c_B^2, \\ c_B^3, \\ c_B^4 \end{bmatrix} = \lambda\left(\begin{bmatrix} c_A^1, \\ c_A^2, \\ c_A^3, \\ c_A^4 \end{bmatrix} \oplus \begin{bmatrix} c_B^1, \\ c_B^2, \\ c_B^3, \\ c_B^4 \end{bmatrix}\right). \tag{14}$$

It can be similarly proved that

$$\lambda\begin{bmatrix} d_A^1, \\ d_A^2, \\ d_A^3, \\ d_A^4 \end{bmatrix} \oplus \lambda\begin{bmatrix} d_B^1, \\ d_B^2, \\ d_B^3, \\ d_B^4 \end{bmatrix} = \lambda\left(\begin{bmatrix} d_A^1, \\ d_A^2, \\ d_A^3, \\ d_A^4 \end{bmatrix} \oplus \begin{bmatrix} d_B^1, \\ d_B^2, \\ d_B^3, \\ d_B^4 \end{bmatrix}\right) \tag{15}$$

$$\lambda\eta_A \oplus \lambda\eta_B = \lambda\left(\eta_A \oplus \eta_B\right) \tag{16}$$

and

$$\lambda\nu_A \oplus \lambda\nu_B = \lambda\left(\nu_A \oplus \nu_B\right). \tag{17}$$

Therefore from Eqs. (10), (11), (12), (13), (14), (15), (16) and (17) we have $\lambda(\mathcal{A} \oplus \mathcal{B}) = \lambda\mathcal{A} \oplus \lambda\mathcal{B}$.

6. The proof is similar to 5.

7. Let

$$\mathcal{A} = \begin{bmatrix} \left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right), \\ \left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right), \\ \left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right), \\ \left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right) \end{bmatrix}$$

be a QSVTrNN on **R**. According to the Definition 8, we have

$$\lambda\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} \oplus \xi\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} = \begin{bmatrix} 1-\left(1-a_A^1\right)^\lambda, \\ 1-\left(1-a_A^2\right)^\lambda, \\ 1-\left(1-a_A^3\right)^\lambda, \\ 1-\left(1-a_A^4\right)^\lambda \end{bmatrix} \oplus \begin{bmatrix} 1-\left(1-a_A^1\right)^\xi, \\ 1-\left(1-a_A^2\right)^\xi, \\ 1-\left(1-a_A^3\right)^\xi, \\ 1-\left(1-a_A^4\right)^\xi \end{bmatrix}$$

$$= \begin{bmatrix} \begin{matrix} 1-\left(1-a_A^1\right)^\lambda + \\ 1-\left(1-a_A^1\right)^\xi + \\ -\begin{pmatrix} \left(1-\left(1-a_A^1\right)^\lambda\right) \\ \left(1-\left(1-a_A^1\right)^\xi\right) \end{pmatrix} \end{matrix}, \\ \begin{matrix} 1-\left(1-a_A^2\right)^\lambda + \\ 1-\left(1-a_A^2\right)^\xi + \\ -\begin{pmatrix} \left(1-1-\left(a_A^2\right)^\lambda\right) \\ \left(1-\left(1-a_A^2\right)^\xi\right) \end{pmatrix} \end{matrix}, \\ \begin{matrix} 1-\left(1-a_A^3\right)^\lambda + \\ 1-\left(1-a_A^3\right)^\xi + \\ -\begin{pmatrix} \left(1-\left(1-a_A^3\right)^\lambda\right) \\ \left(1-\left(1-a_A^3\right)^\xi\right) \end{pmatrix} \end{matrix}, \\ \begin{matrix} 1-\left(1-a4_A\right)^\lambda + \\ 1-\left(1-a_A^4\right)^\xi + \\ -\begin{pmatrix} \left(1-\left(1-a_A^4\right)^\lambda\right) \\ \left(1-\left(1-a_A^4\right)^\xi\right) \end{pmatrix} \end{matrix} \end{bmatrix}$$

and

$$(\lambda+\xi)\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} = \begin{bmatrix} 1-\left(1-a_A^1\right)^{\lambda+\xi}, \\ 1-\left(1-a_A^2\right)^{\lambda+\xi}, \\ 1-\left(1-a_A^3\right)^{\lambda+\xi}, \\ 1-\left(1-a_A^4\right)^{\lambda+\xi} \end{bmatrix} = \begin{bmatrix} 1-\left(1-a_A^1\right)^\lambda\left(1-a_A^1\right)^\xi, \\ 1-\left(1-a_A^2\right)^\lambda\left(1-a_A^2\right)^\xi, \\ 1-\left(1-a_A^3\right)^\lambda\left(1-a_A^3\right)^\xi, \\ 1-\left(1-a_A^4\right)^\lambda\left(1-a_A^4\right)^\xi \end{bmatrix}.$$

Then we have

$$(\lambda+\xi)\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} = \lambda\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix} \oplus \xi\begin{bmatrix} a_A^1, \\ a_A^2, \\ a_A^3, \\ a_A^4 \end{bmatrix}. \tag{18}$$

Similarly, it can be revealed that:

$$(\lambda+\xi)\begin{bmatrix} b_A^1, \\ b_A^2, \\ b_A^3, \\ b_A^4 \end{bmatrix} = \lambda\begin{bmatrix} b_A^1, \\ b_A^2, \\ b_A^3, \\ b_A^4 \end{bmatrix} \oplus \xi\begin{bmatrix} b_A^1, \\ b_A^2, \\ b_A^3, \\ b_A^4 \end{bmatrix}, \tag{19}$$

$$(\lambda+\xi)\mu_A = \lambda\mu_A \oplus \xi\mu_A \tag{20}$$

and

$$(\lambda+\xi)\vartheta_A = \lambda\vartheta_A \oplus \xi\vartheta_A. \tag{21}$$

For the ignorance membership degree of $(\lambda+\xi)\mathcal{A}$, we have

$$\begin{aligned}(\lambda+\xi)\begin{bmatrix} c_A^1, \\ c_A^2, \\ c_A^3, \\ c_A^4 \end{bmatrix} &= \begin{bmatrix} \left(c_A^1\right)^{\lambda+\xi}, \left(c_A^2\right)^{\lambda+\xi}, \\ \left(c_A^3\right)^{\lambda+\xi}, \left(c_A^4\right)^{\lambda+\xi} \end{bmatrix} \\ &= \begin{bmatrix} \left(c_A^1\right)^{\lambda}\left(c_A^1\right)^{\xi}, \left(c_A^2\right)^{\lambda}\left(c_A^2\right)^{\xi}, \\ \left(c_A^3\right)^{\lambda}\left(c_A^3\right)^{\xi}, \left(c_A^4\right)^{\lambda}\left(c_A^4\right)^{\xi} \end{bmatrix},\end{aligned}$$

i.e.,

$$(\lambda+\xi)\begin{bmatrix} c_A^1, \\ c_A^2, \\ c_A^3, \\ c_A^4 \end{bmatrix} = \lambda\begin{bmatrix} c_A^1, \\ c_A^2, \\ c_A^3, \\ c_A^4 \end{bmatrix} \oplus \xi\begin{bmatrix} c_A^1, \\ c_A^2, \\ c_A^3, \\ c_A^4 \end{bmatrix}. \tag{22}$$

It can be similarly proved that

$$(\lambda+\xi)\begin{bmatrix} d_A^1, \\ d_A^2, \\ d_A^3, \\ d_A^4 \end{bmatrix} = \lambda\begin{bmatrix} d_A^1, \\ d_A^2, \\ d_A^3, \\ d_A^4 \end{bmatrix} \oplus \xi\begin{bmatrix} d_A^1, \\ d_A^2, \\ d_A^3, \\ d_A^4 \end{bmatrix} \tag{23}$$

$$(\lambda+\xi)\eta_A = \lambda\eta_A \oplus \xi\eta_A \tag{24}$$

and

$$(\lambda+\xi)\nu_A = \lambda\nu_A \oplus \xi\nu_A. \tag{25}$$

Now, using Eqs. (18), (19), (20), (21), (22), (23), (24) and (25) we get $(\lambda + \xi)\mathcal{A} = \lambda\mathcal{A} \oplus \xi\mathcal{A}$.

8–9. The proof is similar to 7. □

In what follows, we introduce a score function (accuracy function, certainty function) for ranking QSVTrNNs by taking into account the truth-membership function, the contradiction membership function, the ignorance membership function, and the falsity membership function of QSVTrNNs and discuss some basic properties.

Definition 9 Let $\mathcal{A} = \begin{bmatrix} \left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right), \\ \left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right), \\ \left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right), \\ \left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right) \end{bmatrix}$ be any QSVTrNN on the real number set **R**. Then the **score function (accuracy function)** of $\mathcal{A}$ can be defined as follows:

$$\mathcal{SC}(\mathcal{A}) = \frac{1}{16}\begin{pmatrix} \frac{3+a_A^1-b_A^1-c_A^1-d_A^1}{4}+ \\ \frac{3+a_A^2-b_A^2-c_A^2-d_A^2}{4}+ \\ \frac{3+a_A^3-b_A^3-c_A^3-d_A^3}{4}+ \\ \frac{3+a_A^4-b_A^4-c_A^4-d_A^4}{4} \end{pmatrix}\begin{pmatrix} 3+\mu_A+ \\ -\vartheta_A-\eta_A-\nu_A \end{pmatrix} \tag{26}$$

$$\mathcal{CE}(\mathcal{A}) = \frac{1}{16}\begin{pmatrix} a_A^1 - d_A^1+ \\ a_A^2 - d_A^2+ \\ a_A^3 - d_A^3+ \\ a_A^4 - d_A^4 \end{pmatrix}\left(\mu_A - \nu_A\right). \tag{27}$$

Remark 2 It is clear that if truth-membership degree $\left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right)$ is bigger and the contradiction membership degree $\left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right)$, ignorance membership degree $\left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right)$ and falsity membership degree $\left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right)$ are smaller, then the score value of the QSVTrNN

$$\mathcal{A} = \begin{bmatrix} \left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right), \\ \left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right), \\ \left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right), \\ \left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right) \end{bmatrix}$$

is greater.

It is noted that the score function $\mathcal{SC}(\mathcal{A})$ and accuracy function $\mathcal{AC}(\mathcal{A})$ has some desirable properties as below.

Theorem 2 *Let*

$$\mathcal{A} = \begin{bmatrix} \left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right), \\ \left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right), \\ \left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right), \\ \left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right) \end{bmatrix}$$

be any QSVTrNN on the real number set $\mathbf{R}$. *Then the following properties hold.*

1. $0 \leq \mathcal{SC}(\mathcal{A}) \leq 1.$
2. $-1 \leq \mathcal{AC}(\mathcal{A}) \leq 1.$

Proof 1. Let

$$\mathcal{A} = \begin{bmatrix} \left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right), \\ \left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right), \\ \left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right), \\ \left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right) \end{bmatrix}$$

be a QSVTrNN on $\mathbf{R}$. Since $0 \leq a_A^1, b_A^1, c_A^1, d_A^1 \leq 1$, we have $a_A^1 - b_A^1 - c_A^1 - d_A^1 \leq a_A^1 \leq 1$ and $a_A^1 - b_A^1 - c_A^1 - d_A^1 \geq -b_A^1 - c_A^1 - d_A^1 \geq -3$, i.e.,

$$0 = \frac{3-3}{4} \leq \frac{3 + a_A^1 - b_A^1 - c_A^1 - d_A^1}{4} \leq \frac{3+1}{4} = 1. \tag{28}$$

Similarly, we can show that

$$0 \leq \frac{3 + a_A^2 - b_A^2 - c_A^2 - d_A^2}{4} \leq 1, \tag{29}$$

$$0 \leq \frac{3 + a_A^3 - b_A^3 - c_A^3 - d_A^3}{4} \leq 1, \tag{30}$$

$$0 \leq \frac{3 + a_A^4 - b_A^4 - c_A^4 - d_A^4}{4} \leq 1 \tag{31}$$

and

$$0 \leq \frac{3 + \mu_A - \vartheta_A - \eta_A - \nu_A}{4} \leq 1. \tag{32}$$

Now, using Eqs. (28), (29), (30), (31) and (32) we get

$$0 \leq \frac{1}{16}\left(\begin{array}{c} \frac{3+a_A^1-b_A^1-c_A^1-d_A^1}{4}+ \\ \frac{3+a_A^2-b_A^2-c_A^2-d_A^2}{4}+ \\ \frac{3+a_A^3-b_A^3-c_A^3-d_A^3}{4}+ \\ \frac{3+a_A^4-b_A^4-c_A^4-d_A^4}{4} \end{array}\right)\left(\begin{array}{c} 3+\mu_A+ \\ -\vartheta_A-\eta_A-\nu_A \end{array}\right) \leq 1$$

and hence $0 \leq \mathcal{SC}(\mathcal{A}) \leq 1$.

2. Let $\mathcal{A}=\left[\begin{array}{c} \left(\left[a_A^1, a_A^2, a_A^3, a_A^4\right], \mu_A\right), \\ \left(\left[b_A^1, b_A^2, b_A^3, b_A^4\right], \vartheta_A\right), \\ \left(\left[c_A^1, c_A^2, c_A^3, c_A^4\right], \eta_A\right), \\ \left(\left[d_A^1, d_A^2, d_A^3, d_A^4\right], \nu_A\right) \end{array}\right]$ be a QSVTrNN on $\mathbf{R}$. Since $0 \leq a_A^1, d_A^1 \leq 1$, we have $a_A^1 - c_A^1 \leq a_A^1 \leq 1$ and $a_A^1 - c_A^1 \geq -c_A^1 \geq -1$, i.e.,

$$-1 \leq a_A^1 - d_A^1 \leq 1. \tag{33}$$

It can be similarly proved that

$$-1 \leq a_A^2 - d_A^2 \leq 1, \tag{34}$$

$$-1 \leq a_A^3 - d_A^3 \leq 1, \tag{35}$$

$$-1 \leq a_A^4 - d_A^4 \leq 1 \tag{36}$$

and

$$-1 \leq \mu_A - \nu_A \leq 1. \tag{37}$$

Therefore, from Eqs. (33), (34), (35), (36) and (37) we have

$$-1 \leq \frac{1}{4}\left(\begin{array}{c} a_A^1 - d_A^1 + \\ a_A^2 - d_A^2 + \\ a_A^3 - d_A^3 + \\ a_A^4 - d_A^4 \end{array}\right)\left(\mu_A - \nu_A\right) \leq 1$$

and hence $-1 \leq \mathcal{AC}(\mathcal{A}) \leq 1$. □

4 QSVTrNWA operators

In this section, we introduce the notion of quadripartitioned single-valued trapezoidal neutrosophic weighted averaging operator along with their some properties.

Based on Definition 6, we propose the following an aggregation operator of quadripartitioned single-valued trapezoidal neutrosophic numbers.

Definition 10 Let

$$\Delta = \left\{ \mathcal{A}_i = \begin{bmatrix} \left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right), \\ \left(\left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i}\right], \vartheta_{A_i}\right), \\ \left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right], \eta_{A_i}\right), \\ \left(\left[d^1_{A_i}, d^2_{A_i}, d^3_{A_i}, d^4_{A_i}\right], \nu_{A_i}\right) \end{bmatrix} : i = 1, \dots, n \right\}$$

be a collection of QSVTrNNs on the real number set $\mathbf{R}$ and $\lambda = \left(\lambda_1, \lambda_2, \dots, \lambda_n\right)^T$ be the weight vector of Δ indicates the importance degree of Δ satisfying $\lambda_1, \lambda_2, \dots, \lambda_n \in [0, 1]$ and $\sum_{i=1}^{n} \lambda_i = 1$, and let **quadripartitioned single-valued trapezoidal neutrosophic weighted averaging (QSVTrNWA)** $\mathcal{WA} : (\mathcal{TN}(\mathbf{R}))^n \to \mathcal{TN}(\mathbf{R})$ if

$$\mathcal{WA}\left(\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n\right) = \bigoplus_{i=1}^{n} \lambda_i \mathcal{A}_i \tag{38}$$

then the function QSVTrNWA is called the **QSVTrNWA-operator**.

Thus, from the above definition, it is clear that quadripartitioned single-valued trapezoidal neutrosophic weighted averaging operators are a generalization of quadripartitioned single valued neutrosophic weighted averaging operators. Based on the operational rules of QSVTrNNs in Definition 10, we can derive the following theorem.

Theorem 3 *Let*

$$\Delta = \left\{ \mathcal{A}_i = \begin{bmatrix} \left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right), \\ \left(\left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i}\right], \vartheta_{A_i}\right), \\ \left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right], \eta_{A_i}\right), \\ \left(\left[d^1_{A_i}, d^2_{A_i}, d^3_{A_i}, d^4_{A_i}\right], \nu_{A_i}\right) \end{bmatrix} : i = 1, 2, \dots, n \right\}$$

be a collection of QSVTrNNs on the real number set $\mathbf{R}$ *and* $\lambda = \left(\lambda_1, \lambda_2, \dots, \lambda_n\right)^T$ *be the weight vector of* Δ *where* λ_j *indicates the importance degree of* Δ, *satisfying*

$0 \leq \lambda_1, \lambda_2, \ldots, \lambda_n \leq 1$ *and* $\sum_{i=1}^{n} \lambda_i = 1$. *Then their accumulated outcome utilizing the QSVTrNWA- operator is again QSVTrNN, and*

$$\bigoplus_{i=1}^{n} \lambda_i \left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right] = \left[\begin{array}{c} 1 - \prod_{i=1}^{n} \left(1 - a^1_{A_i}\right)^{\lambda_i}, \\ 1 - \prod_{i=1}^{n} \left(1 - a^2_{A_i}\right)^{\lambda_i}, \\ 1 - \prod_{i=1}^{n} \left(1 - a^3_{A_i}\right)^{\lambda_i}, \\ 1 - \prod_{i=1}^{n} \left(1 - a^4_{A_i}\right)^{\lambda_i} \end{array}\right] \tag{39}$$

$$\bigoplus_{i=1}^{n} \lambda_i \left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i}\right] = \left[\begin{array}{c} 1 - \prod_{i=1}^{n} \left(1 - b^1_{A_i}\right)^{\lambda_i}, \\ 1 - \prod_{i=1}^{n} \left(1 - b^2_{A_i}\right)^{\lambda_i}, \\ 1 - \prod_{i=1}^{n} \left(1 - b^3_{A_i}\right)^{\lambda_i}, \\ 1 - \prod_{i=1}^{n} \left(1 - b^4_{A_i}\right)^{\lambda_i} \end{array}\right] \tag{40}$$

$$\bigoplus_{i=1}^{n} \lambda_i \left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right] = \left[\begin{array}{c} \prod_{i=1}^{n} \left(c^1_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n} \left(c^2_{A_i}\right)^{\lambda_i}, \\ \prod_{i=1}^{n} \left(c^3_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n} \left(c^4_{A_i}\right)^{\lambda_i} \end{array}\right] \tag{41}$$

$$\bigoplus_{i=1}^{n} \lambda_i \left[d^1_{A_i}, d^2_{A_i}, d^3_{A_i}, d^4_{A_i}\right] = \left[\begin{array}{c} \prod_{i=1}^{n} \left(d^1_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n} \left(d^2_{A_i}\right)^{\lambda_i}, \\ \prod_{i=1}^{n} \left(d^3_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n} \left(d^4_{A_i}\right)^{\lambda_i} \end{array}\right] \tag{42}$$

$$\bigoplus_{i=1}^{n} \lambda_i \mu_{A_i} = 1 - \prod_{i=1}^{n} \left(1 - \mu_{A_i}\right)^{\lambda_i} \tag{43}$$

$$\bigoplus_{i=1}^{n} \lambda_i \mu_{A_i} = 1 - \prod_{i=1}^{n} \left(1 - \vartheta_{A_i}\right)^{\lambda_i} \tag{44}$$

$$\bigoplus_{i=1}^{n} \lambda_i \eta_{A_i} = \prod_{i=1}^{n} \left(\eta_{A_i}\right)^{\lambda_i} \tag{45}$$

$$\bigoplus_{i=1}^{n} \lambda_i \nu_{A_i} = \prod_{i=1}^{n} \left(\nu_{A_i}\right)^{\lambda_i} \tag{46}$$

and

$$\mathcal{WA}\left(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n\right) = \begin{bmatrix} \left(\bigoplus_{i=1}^{n} \lambda_i \left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \bigoplus_{i=1}^{n} \lambda_i \mu_{A_i}\right), \\ \left(\bigoplus_{i=1}^{n} \lambda_i \left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i}\right], \bigoplus_{i=1}^{n} \lambda_i \vartheta_{A_i}\right), \\ \left(\bigoplus_{i=1}^{n} \lambda_i \left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right], \bigoplus_{i=1}^{n} \lambda_i \eta_{A_i}\right), \\ \left(\bigoplus_{i=1}^{n} \lambda_i \left[d^1_{A_i}, d^2_{A_i}, d^3_{A_i}, d^4_{A_i}\right], \bigoplus_{i=1}^{n} \lambda_i \nu_{A_i}\right) \end{bmatrix}. \tag{47}$$

Proof The proof of Eq. (39) can be done by means of mathematical induction. Therefore we have the following.

Step 1. Now for $n = 2$, we get

$$\bigoplus_{i=1}^{2} \lambda_i \left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4\right] = \lambda_1 \left[a_{A_1}^1, a_{A_1}^2, a_{A_1}^3, a_{A_1}^4\right] \oplus \lambda_2 \left[a_{A_2}^1, a_{A_2}^2, a_{A_2}^3, a_{A_2}^4\right]$$

$$= \begin{bmatrix} \begin{matrix} 1-\left(1-a_{A_1}^1\right)^{\lambda_1}+ \\ 1-1-\left(a_{A_2}^1\right)^{\lambda_2}+ \\ -\begin{pmatrix} \left(1-\left(1-a_{A_1}^1\right)^{\lambda_1}\right) \\ \left(1-\left(1-a_{A_2}^1\right)^{\lambda_2}\right) \end{pmatrix} \end{matrix}, \\ \begin{matrix} 1-\left(1-a_{A_1}^2\right)^{\lambda_1}+ \\ 1-\left(1-a_{A_2}^2\right)^{\lambda_2}+ \\ -\begin{pmatrix} \left(1-\left(1-a_{A_1}^2\right)^{\lambda_1}\right) \\ \left(1-\left(1-a_{A_2}^2\right)^{\lambda_2}\right) \end{pmatrix} \end{matrix}, \\ \begin{matrix} 1-\left(1-a_{A_1}^3\right)^{\lambda_1}+ \\ 1-\left(1-a_{A_2}^3\right)^{\lambda_2}+ \\ -\begin{pmatrix} \left(1-\left(1-a_{A_1}^3\right)^{\lambda_1}\right) \\ \left(1-\left(1-a_{A_2}^3\right)^{\lambda_2}\right) \end{pmatrix} \end{matrix}, \\ \begin{matrix} 1-\left(1-a_{A_1}^4\right)^{\lambda_1}+ \\ 1-\left(1-a_{A_2}^4\right)^{\lambda_2}+ \\ -\begin{pmatrix} \left(1-\left(1-\left(a_{A_1}^4\right)^q\right)^{\lambda_1}\right) \\ \left(1-\left(1-\left(a_{A_2}^4\right)^q\right)^{\lambda_2}\right) \end{pmatrix} \end{matrix} \end{bmatrix}$$

and

$$\begin{bmatrix} 1-\left(1-a^1_{A_1}\right)^{\lambda_1}\left(1-a^1_{A_2}\right)^{\lambda_2}, \\ 1-\left(1-a^2_{A_1}\right)^{\lambda_1}\left(1-a^2_{A_2}\right)^{\lambda_2}, \\ 1-\left(1-a^3_{A_1}\right)^{\lambda_1}\left(1-a^3_{A_2}\right)^{\lambda_2}, \\ 1-\left(1-a^4_{A_1}\right)^{\lambda_1}\left(1-a^4_{A_2}\right)^{\lambda_2} \end{bmatrix} = \begin{bmatrix} 1-\prod_{i=1}^{2}\left(1-a^1_{A_i}\right)^{\lambda_i}, \\ 1-\prod_{i=1}^{2}\left(1-a^2_{A_i}\right)^{\lambda_i}, \\ 1-\prod_{i=1}^{2}\left(1-a^3_{A_i}\right)^{\lambda_i}, \\ 1-\prod_{i=1}^{2}\left(1-a^4_{A_i}\right)^{\lambda_i} \end{bmatrix}$$

Thus, the result is true for $n = 2$.

Step 2. When $n = k$, by using Eq. (39), we obtain

$$\bigoplus_{i=1}^{k}\lambda_i\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right] = \begin{bmatrix} 1-\prod_{i=1}^{k}\left(1-a^1_{A_i}\right)^{\lambda_i}, \\ 1-\prod_{i=1}^{k}\left(1-a^2_{A_i}\right)^{\lambda_i}, \\ 1-\prod_{i=1}^{k}\left(1-a^3_{A_i}\right)^{\lambda_i}, \\ 1-\prod_{i=1}^{k}\left(1-a^4_{A_i}\right)^{\lambda_i} \end{bmatrix}. \tag{48}$$

Step 3. When $n = k + 1$, by applying Eqs. (39) and (48), we can get

$$\begin{bmatrix} 1-\prod_{i=1}^{k}\left(1-a^1_{A_i}\right)^{\lambda_i}\left(1-a^1_{A_{k+1}}\right)^{\lambda_{k+1}}, \\ 1-\prod_{i=1}^{k}\left(1-a^2_{A_i}\right)^{\lambda_i}\left(1-a^2_{A_{k+1}}\right)^{\lambda_{k+1}}, \\ 1-\prod_{i=1}^{k}\left(1-a^3_{A_i}\right)^{\lambda_i}\left(1-a^3_{A_{k+1}}\right)^{\lambda_{k+1}}, \\ 1-\prod_{i=1}^{k}\left(1-a^4_{A_i}\right)^{\lambda_i}\left(1-\left(a^4_{A_{k+1}}\right)^{q}\right)^{\lambda_{k+1}} \end{bmatrix} = \begin{bmatrix} 1-\prod_{i=1}^{k+1}\left(1-a^1_{A_i}\right)^{\lambda_i}, \\ 1-\prod_{i=1}^{k+1}\left(1-a^2_{A_i}\right)^{\lambda_i}, \\ 1-\prod_{i=1}^{k+1}\left(1-a^3_{A_i}\right)^{\lambda_i}, \\ 1-\prod_{i=1}^{k+1}\left(1-a^4_{A_i}\right)^{\lambda_i} \end{bmatrix}$$

and

$$
\begin{aligned}
\bigoplus_{i=1}^{k+1} \lambda_i \left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4\right] &= \bigoplus_{i=1}^{k} \lambda_i \left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4\right] \oplus \\
&\quad \lambda_{k+1}\left[a_{A_{k+1}}^1, a_{A_{k+1}}^2, a_{A_{k+1}}^3, a_{A_{k+1}}^4\right] \\
&= \begin{bmatrix}
1-\prod_{i=1}^{k}\left(1-a_{A_i}^1\right)^{\lambda_i} + \\
1-\left(1-a_{A_{k+1}}^1\right)^{\lambda_{k+1}} + \\
-\begin{pmatrix} \left(1-\prod_{i=1}^{k}\left(1-a_{A_i}^1\right)^{\lambda_i}\right) \\ \left(1-\left(1-a_{A_{k+1}}^1\right)^{\lambda_{k+1}}\right) \end{pmatrix}, \\
1-\prod_{i=1}^{k}\left(1-a_{A_i}^2\right)^{\lambda_i} + \\
1-\left(1-a_{A_{k+1}}^2\right)^{\lambda_{k+1}} + \\
-\begin{pmatrix} \left(1-\prod_{i=1}^{k}\left(1-a_{A_i}^2\right)^{\lambda_i}\right) \\ \left(1-\left(1-a_{A_{k+1}}^2\right)^{\lambda_{k+1}}\right) \end{pmatrix}, \\
1-\prod_{i=1}^{k}\left(1-a_{A_i}^3\right)^{\lambda_i} + \\
1-\left(1-a_{A_{k+1}}^3\right)^{\lambda_{k+1}} + \\
-\begin{pmatrix} \left(1-\prod_{i=1}^{k}\left(1-a_{A_i}^3\right)^{\lambda_i}\right) \\ \left(1-\left(1-a_{A_{k+1}}^3\right)^{\lambda_{k+1}}\right) \end{pmatrix}, \\
1-\prod_{i=1}^{k}\left(1-a_{A_i}^4\right)^{\lambda_i} + \\
1-\left(1-a_{A_{k+1}}^4\right)^{\lambda_{k+1}} + \\
-\begin{pmatrix} \left(1-\prod_{i=1}^{k}\left(1-a_{A_i}^4\right)^{\lambda_i}\right) \\ \left(1-\left(1-a_{A_{k+1}}^4\right)^{\lambda_{k+1}}\right) \end{pmatrix}
\end{bmatrix}
\end{aligned}
$$

that is, according to the above results, we obtain Eq. (39) for any n. Similarly, we can prove Eqs. 40 and 43. For the ignorance membership degree of $\mathcal{WA}\left(\mathcal{A}_1, \ldots, \mathcal{A}_n\right)$. By utilized the technique of mathematical induction on n. Therefore we have the following.

Step 1. Now for $n = 2$, we get

$$\begin{aligned}\bigoplus_{i=1}^{2} \lambda_i \left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4 \right] &= \lambda_1 \left[c_{A_1}^1, c_{A_1}^2, c_{A_i}^3, c_{A_1}^4 \right] \oplus \lambda_2 \left[c_{A_2}^1, c_{A_2}^2, c_{A_2}^3, c_{A_2}^4 \right] \\ &= \left[\begin{array}{c} \left(c_{A_1}^1 \right)^{\lambda_1} \left(c_{A_2}^1 \right)^{\lambda_2}, \left(c_{A_1}^2 \right)^{\lambda_1} \left(c_{A_2}^2 \right)^{\lambda_2}, \\ \left(c_{A_1}^3 \right)^{\lambda_1} \left(c_{A_2}^3 \right)^{\lambda_2}, \left(c_{A_1}^4 \right)^{\lambda_1} \left(c_{A_2}^4 \right)^{\lambda_2} \end{array} \right] \\ &= \left[\begin{array}{c} \prod_{i=1}^{2} \left(c_{A_i}^1 \right)^{\lambda_i}, \prod_{i=1}^{2} \left(c_{A_i}^2 \right)^{\lambda_i}, \\ \prod_{i=1}^{2} \left(c_{A_i}^3 \right)^{\lambda_i}, \prod_{i=1}^{2} \left(c_{A_i}^4 \right)^{\lambda_i} \end{array} \right].\end{aligned}$$

Thus, the result is true for $n = 2$.

Step 2. When $n = k$, by using Eq. (41), we obtain

$$\bigoplus_{i=1}^{k} \lambda_i \left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4 \right] = \left[\begin{array}{c} \prod_{i=1}^{k} \left(c_{A_i}^1 \right)^{\lambda_i}, \prod_{i=1}^{k} \left(c_{A_i}^2 \right)^{\lambda_i}, \\ \prod_{i=1}^{k} \left(c_{A_i}^3 \right)^{\lambda_i}, \prod_{i=1}^{k} \left(c_{A_i}^4 \right)^{\lambda_i} \end{array} \right]. \tag{49}$$

Step 3. When $n = k + 1$, by applying Eqs. (41) and (49), we can get

$$
\begin{aligned}
\bigoplus_{i=1}^{k+1} \lambda_i \left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4\right] &= \bigoplus_{i=1}^{k} \lambda_i \left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4\right] \oplus \\
&\quad \lambda_{k+1} \left[c_{A_{k+1}}^1, c_{A_{k+1}}^2, c_{A_{k+1}}^3, c_{A_{k+1}}^4\right] \\
&= \begin{bmatrix} \prod_{i=1}^{k} \left(c_{A_i}^1\right)^{\lambda_i}, \prod_{i=1}^{k} \left(c_{A_i}^2\right)^{\lambda_i}, \\ \prod_{i=1}^{k} \left(c_{A_i}^3\right)^{\lambda_i}, \prod_{i=1}^{k} \left(c_{A_i}^4\right)^{\lambda_i} \end{bmatrix} \oplus \\
&\quad \begin{bmatrix} \left(c_{A_{k+1}}^1\right)^{\lambda_{k+1}}, \left(c_{A_{k+1}}^2\right)^{\lambda_{k+1}}, \\ \left(c_{A_{k+1}}^3\right)^{\lambda_{k+1}}, \left(c_{A_{k+1}}^4\right)^{\lambda_{k+1}} \end{bmatrix} \\
&= \begin{bmatrix} \prod_{i=1}^{k} \left(c_{A_i}^1\right)^{\lambda_i} \left(c_{A_{k+1}}^1\right)^{\lambda_{k+1}}, \\ \prod_{i=1}^{k} \left(c_{A_i}^2\right)^{\lambda_i} \left(c_{A_{k+1}}^2\right)^{\lambda_{k+1}}, \\ \prod_{i=1}^{k} \left(c_{A_i}^3\right)^{\lambda_i} \left(c_{A_{k+1}}^3\right)^{\lambda_{k+1}}, \\ \prod_{i=1}^{k} \left(c_{A_i}^4\right)^{\lambda_i} \left(c_{A_{k+1}}^4\right)^{\lambda_{k+1}} \end{bmatrix} \\
&= \begin{bmatrix} \prod_{i=1}^{k+1} \left(c_{A_i}^1\right)^{\lambda_i}, \prod_{i=1}^{k+1} \left(c_{A_i}^2\right)^{\lambda_i}, \\ \prod_{i=1}^{k+1} \left(c_{A_i}^3\right)^{\lambda_i}, \prod_{i=1}^{k+1} \left(c_{A_i}^4\right)^{\lambda_i} \end{bmatrix}
\end{aligned}
$$

that is, according to the above results, we obtain Eq. (41) for any n. Similarly, we can prove Eqs. (42), (43), (44), (45) and (46). Hence, by Eqs. (39), (40), (41), (42), (43), (44), (45) and (46), we get

$$
\mathcal{WA}\left(\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n\right) = \begin{bmatrix} \left(\bigoplus_{i=1}^{n} \lambda_i \left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4\right], \bigoplus_{i=1}^{n} \lambda_i \mu_{A_i} \right), \\ \left(\bigoplus_{i=1}^{n} \lambda_i \left[b_{A_i}^1, b_{A_i}^2, b_{A_i}^3, b_{A_i}^4\right], \bigoplus_{i=1}^{n} \lambda_i \eta_{A_i} \right), \\ \left(\bigoplus_{i=1}^{n} \lambda_i \left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4\right], \bigoplus_{i=1}^{n} \lambda_i \nu_{A_i} \right) \end{bmatrix}.
$$

In the following, we will prove that $\mathcal{WA}\left(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n\right)$ is also a QSVTrNN. Then, since $0 \le a^4_{A_i}, b^4_{A_i}, c^4_{A_i}, d^4_{A_i} \le 1$, we have

$$\begin{aligned}
0 \le & \left(a^4_{A_i}\right)^q, \left(b^4_{A_i}\right)^q, \left(c^4_{A_i}\right)^{\lambda_i}, \left(d^4_{A_i}\right)^{\lambda_i} \le 1 \\
0 \le & 1 - \left(a^4_{A_i}\right)^q, 1 - \left(b^4_{A_i}\right)^q, \prod_{i=1}^{n} \left(c^4_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n} \left(d^4_{A_i}\right)^{\lambda_i} \le 1, \\
0 \le & 1 - \prod_{i=1}^{n} \left(1 - a^4_{A_i}\right)^{\lambda_i}, 1 - \prod_{i=1}^{n} \left(1 - b^4_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n} \left(c^4_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n} \left(d^4_{A_i}\right)^{\lambda_i} \le 1.
\end{aligned}$$

Therefore we obtain that $0 \le 1 - \prod_{i=1}^{n} \left(1 - a^4_{A_i}\right)^{\lambda_i} + 1 - \prod_{i=1}^{n} \left(1 - b^4_{A_i}\right)^{\lambda_i} + \prod_{i=1}^{n} \left(c^4_{A_i}\right)^{\lambda_i} + \prod_{i=1}^{n} \left(d^4_{A_i}\right)^{\lambda_i} \le 4$. This completes the proof. □

In the following, we describe some desirable properties of the QSVTrNWA-operator.

Theorem 4 *Let* $\left\{ \mathcal{A}_i = \begin{bmatrix} \left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right), \\ \left(\left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i}\right], \vartheta_{A_i}\right), \\ \left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right], \eta_{A_i}\right), \\ \left(\left[d^1_{A_i}, d^2_{A_i}, d^3_{A_i}, d^4_{A_i}\right], \nu_{A_i}\right) \end{bmatrix} : i = 1, 2, \ldots, n \right\}$ *be a collection of QSVTrNNs on the real number set* $\mathbf{R}$. *If* $\mathcal{A}_i = \mathcal{A} =$

$$\begin{bmatrix} \left(\left[a^1_{A}, a^2_{A}, a^3_{A}, a^4_{A}\right], \mu_{A}\right), \\ \left(\left[b^1_{A}, b^2_{A}, b^3_{A}, b^4_{A}\right], \vartheta_{A}\right), \\ \left(\left[c^1_{A}, c^2_{A}, c^3_{A}, c^4_{A}\right], \eta_{A}\right), \\ \left(\left[d^1_{A}, d^2_{A}, d^3_{A}, d^4_{A}\right], \nu_{A}\right) \end{bmatrix}$$

for every $i = 1, \ldots, n$, *then* $\mathcal{WA}\left(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n\right) = \mathcal{A}$.

Proof Then, since $\mathcal{A}_1 = \mathcal{A}_2 = \ldots = \mathcal{A}_n = \mathcal{A}$ by Eq. 39, we have

$$\bigoplus_{i=1}^{n}\lambda_i\left[a_{A_i}^1,a_{A_i}^2,a_{A_i}^3,a_{A_i}^4\right]=\begin{bmatrix}1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i},\\ 1-\prod_{i=1}^{n}\left(1-a_{A_i}^2\right)^{\lambda_i},\\ 1-\prod_{i=1}^{n}\left(1-a_{A_i}^3\right)^{\lambda_i},\\ 1-\prod_{i=1}^{n}\left(1-a_{A_i}^4\right)^{\lambda_i}\end{bmatrix}=\begin{bmatrix}1-\left(1-a_A^1\right)^{\sum_{i=1}^{n}\lambda_i},\\ 1-\left(1-a_A^2\right)^{\sum_{i=1}^{n}\lambda_i},\\ 1-\left(1-a_A^3\right)^{\sum_{i=1}^{n}\lambda_i},\\ 1-\left(1-a_A^4\right)^{\sum_{i=1}^{n}\lambda_i}\end{bmatrix}=\begin{bmatrix}1-\left(1-a_A^1\right)^1,\\ 1-\left(1-a_A^2\right)^1,\\ 1-\left(1-a_A^3\right)^1,\\ 1-\left(1-a_A^4\right)^1\end{bmatrix},$$

i.e.,

$$\bigoplus_{i=1}^{n}\lambda_i\left[a_{A_i}^1,a_{A_i}^2,a_{A_i}^3,a_{A_i}^4\right]=\left[a_A^1,a_A^2,a_A^3,a_A^4\right]. \tag{50}$$

Similarly, we can prove

$$\bigoplus_{i=1}^{n}\lambda_i\left[b_{A_i}^1,b_{A_i}^2,b_{A_i}^3,b_{A_i}^4\right]=\left[b_A^1,b_A^2,b_A^3,b_A^4\right] \tag{51}$$

$$\bigoplus_{i=1}^{n}\lambda_i\mu_{A_i}=\mu_A \tag{52}$$

and

$$\bigoplus_{i=1}^{n}\lambda_i\vartheta_{A_i}=\vartheta_A. \tag{53}$$

For the ignorance membership degree of $\mathcal{WA}(\mathcal{A}_1, \ldots, \mathcal{A}_n)$. From these calculations, we obtain

$$\begin{aligned}\bigoplus_{i=1}^{n} \lambda_i \left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right] &= \left[\prod_{i=1}^{n}\left(c^1_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n}\left(c^2_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n}\left(c^3_{A_i}\right)^{\lambda_i}, \prod_{i=1}^{n}\left(c^4_{A_i}\right)^{\lambda_i}\right] \\ &= \left[\left(c^1_A\right)^{\sum_{i=1}^{n}\lambda_i}, \left(c^2_A\right)^{\sum_{i=1}^{n}\lambda_i}, \left(c^3_A\right)^{\sum_{i=1}^{n}\lambda_i}, \left(c^4_A\right)^{\sum_{i=1}^{n}\lambda_i},\right],\end{aligned}$$

i.e.,

$$\bigoplus_{i=1}^{n} \lambda_i \left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right] = \left[c^1_A, c^2_A, c^3_A, c^4_A\right]. \tag{54}$$

Similarly, we obtain that

$$\bigoplus_{i=1}^{n} \lambda_i \left[d^1_{A_i}, d^2_{A_i}, d^3_{A_i}, d^4_{A_i}\right] = \left[d^1_A, d^2_A, d^3_A, d^4_A\right] \tag{55}$$

$$\bigoplus_{i=1}^{n} \lambda_i \eta_{A_i} = \eta_A \tag{56}$$

and

$$\bigoplus_{i=1}^{n} \lambda_i \nu_{A_i} = \nu_A. \tag{57}$$

Thus, from Eqs. (50), (51), (52), (53), (54), (55), (56) and (57) we have $\mathcal{WA}(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n) = \mathcal{A}$. □

The following Theorem 5 (Monotonicity) establishes some properties of QSVTrNWA-operator.

Theorem 5 *Let*

$$\left\{\mathcal{A}_i = \begin{bmatrix} \left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right), \\ \left(\left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i}\right], \vartheta_{A_i}\right), \\ \left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right], \eta_{A_i}\right), \\ \left(\left[d^1_{A_i}, d^2_{A_i}, d^3_{A_i}, d^4_{A_i}\right], \nu_{A_i}\right) \end{bmatrix} : i = 1, 2, 3, \ldots, n\right\}$$

and

$$\left\{\mathcal{B}_i = \begin{bmatrix} \left(\left[a^1_{B_i}, a^2_{B_i}, a^3_{B_i}, a^4_{B_i}\right], \mu_{B_i}\right), \\ \left(\left[b^1_{B_i}, b^2_{B_i}, b^3_{B_i}, b^4_{B_i}\right], \vartheta_{B_i}\right), \\ \left(\left[c^1_{B_i}, c^2_{B_i}, c^3_{B_i}, c^4_{B_i}\right], \eta_{B_i}\right), \\ \left(\left[d^1_{B_i}, d^2_{B_i}, d^3_{B_i}, d^4_{B_i}\right], \nu_{B_i}\right) \end{bmatrix} : i = 1, 2, 3, \dots, n\right\}$$

be two collections of QSVTrNNs on the real number set **R**. *If* $\mathcal{A}_i \ll \mathcal{B}_i$ *for every* $i = 1, \dots, n$, *then* $\mathcal{WA}(\mathcal{A}_1, \dots, \mathcal{A}_n) \ll \mathcal{WA}(\mathcal{B}_1, \dots, \mathcal{B}_n)$.

Proof Then, since $\mathcal{A}_i \ll \mathcal{B}_i$ for every i, we have $\left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right) \preceq \left(\left[a^1_{B_i}, a^2_{B_i}, a^3_{B_i}, a^4_{B_i}\right], \mu_{B_i}\right)$, i.e.,

$$\begin{gathered} 1 - a^1_{A_i} \geq 1 - a^1_{B_i} \\ \prod_{i=1}^{n}\left(1 - a^1_{A_i}\right)^{\lambda_i} \geq \prod_{i=1}^{n}\left(1 - a^1_{B_i}\right)^{\lambda_i} \\ 1 - \prod_{i=1}^{n}\left(1 - a^1_{A_i}\right)^{\lambda_i} \leq 1 - \prod_{i=1}^{n}\left(1 - a^1_{B_i}\right)^{\lambda_i}. \end{gathered} \tag{58}$$

Similarly, we obtain that

$$1 - \prod_{i=1}^{n}\left(1 - a^2_{A_i}\right)^{\lambda_i} \leq 1 - \prod_{i=1}^{n}\left(1 - a^2_{B_i}\right)^{\lambda_i} \tag{59}$$

$$1 - \prod_{i=1}^{n}\left(1 - a^3_{A_i}\right)^{\lambda_i} \leq 1 - \prod_{i=1}^{n}\left(1 - a^3_{B_i}\right)^{\lambda_i} \tag{60}$$

$$1 - \prod_{i=1}^{n}\left(1 - a^4_{A_i}\right)^{\lambda_i} \leq 1 - \prod_{i=1}^{n}\left(1 - a^4_{B_i}\right)^{\lambda_i} \tag{61}$$

and

$$1 - \prod_{i=1}^{n}\left(1 - \left(\mu_{A_i}\right)^q\right)^{\lambda_i} \leq 1 - \prod_{i=1}^{n}\left(1 - \left(\mu_{B_i}\right)^q\right)^{\lambda_i}. \tag{62}$$

Hence, by m Eqs. (58), (59), (60), (61) and (62), we get

$$\left(\bigoplus_{i=1}^{n}\lambda_i\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \bigoplus_{i=1}^{n}\lambda_i\mu_{A_i}\right) \preceq \left(\bigoplus_{i=1}^{n}\lambda_i\left[a^1_{B_i}, a^2_{B_i}, a^3_{B_i}, a^4_{B_i}\right], \bigoplus_{i=1}^{n}\lambda_i\mu_{B_i}\right). \tag{63}$$

Similarly,

$$\left(\bigoplus_{i=1}^{n}\lambda_i\left[b^1_{A_i},b^2_{A_i},b^3_{A_i},b^4_{A_i}\right],\bigoplus_{i=1}^{n}\lambda_i\vartheta_{A_i}\right)\leq\left(\bigoplus_{i=1}^{n}\lambda_i\left[b^1_{B_i},b^2_{B_i},b^3_{B_i},b^4_{B_i}\right],\bigoplus_{i=1}^{n}\lambda_i\vartheta_{B_i}\right). \tag{64}$$

From these calculations, we obtain $\left(c^1_{A_i}\right)^{\lambda_i}\geq\left(c^1_{B_i}\right)^{\lambda_i}$, $\left(c^2_{A_i}\right)^{\lambda_i}\geq\left(c^2_{B_i}\right)^{\lambda_i}$, $\left(c^3_{A_i}\right)^{\lambda_i}\geq\left(c^3_{B_i}\right)^{\lambda_i}$ and $\left(c^4_{A_i}\right)^{\lambda_i}\geq\left(c^4_{B_i}\right)^{\lambda_i}$, i.e.,

$$\prod_{i=1}^{n}\left(c^1_{A_i}\right)^{\lambda_i}\geq\prod_{i=1}^{n}\left(c^1_{B_i}\right)^{\lambda_i}, \tag{65}$$

$$\prod_{i=1}^{n}\left(c^2_{A_i}\right)^{\lambda_i}\geq\prod_{i=1}^{n}\left(c^2_{B_i}\right)^{\lambda_i} \tag{66}$$

$$\prod_{i=1}^{n}\left(c^3_{A_i}\right)^{\lambda_i}\geq\prod_{i=1}^{n}\left(c^3_{B_i}\right)^{\lambda_i} \tag{67}$$

$$\prod_{i=1}^{n}\left(c^4_{A_i}\right)^{\lambda_i}\geq\prod_{i=1}^{n}\left(c^4_{B_i}\right)^{\lambda_i} \tag{68}$$

and

$$\prod_{i=1}^{n}\left(\eta_{A_i}\right)^{\lambda_i}\geq\prod_{i=1}^{n}\left(\eta_{B_i}\right)^{\lambda_i}. \tag{69}$$

Thus, by Eqs. (65), (66), (67), (68) and (69), we have

$$\left(\bigoplus_{i=1}^{n}\lambda_i\left[c^1_{A_i},c^2_{A_i},c^3_{A_i},c^4_{A_i}\right],\bigoplus_{i=1}^{n}\lambda_i\eta_{A_i}\right)\geq\left(\bigoplus_{i=1}^{n}\lambda_i\left[c^1_{B_i},c^2_{B_i},c^3_{B_i},c^4_{B_i}\right],\bigoplus_{i=1}^{n}\lambda_i\eta_{B_i}\right). \tag{70}$$

It can be similarly proved that

$$\left(\bigoplus_{i=1}^{n}\lambda_i\left[d^1_{A_i},d^2_{A_i},d^3_{A_i},d^4_{A_i}\right],\bigoplus_{i=1}^{n}\lambda_i\nu_{A_i}\right)\geq\left(\bigoplus_{i=1}^{n}\lambda_i\left[d^1_{B_i},d^2_{B_i},d^3_{B_i},d^4_{B_i}\right],\bigoplus_{i=1}^{n}\lambda_i\nu_{B_i}\right). \tag{71}$$

By Eqs. (63), (64), (70) and (71) which implies that, $\mathcal{WA}\left(\mathcal{A}_1,\ldots,\mathcal{A}_n\right)\ll\mathcal{WA}\left(\mathcal{B}_1,\ldots,\mathcal{B}_n\right)$. □

Now, we prove properties of QSVTrNWA-operator.

Theorem 6 *Let*

$$\left\{ \mathcal{A}_i = \begin{bmatrix} \left(\left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4 \right], \mu_{A_i} \right), \\ \left(\left[b_{A_i}^1, b_{A_i}^2, b_{A_i}^3, b_{A_i}^4 \right], \vartheta_{A_i} \right), \\ \left(\left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4 \right], \eta_{A_i} \right), \\ \left(\left[d_{A_i}^1, d_{A_i}^2, d_{A_i}^3, d_{A_i}^4 \right], \nu_{A_i} \right) \end{bmatrix} : i = 1, 2, 3, \ldots, n \right\}$$

be a collection of QSVTrNNs on the real number set **R**. *If*

$$\mathcal{A}^+ = \begin{bmatrix} \bigcup_{i=1}^{n} \left(\left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4 \right], \mu_{A_i} \right), \\ \bigcup_{i=1}^{n} \left(\left[b_{A_i}^1, b_{A_i}^2, b_{A_i}^3, b_{A_i}^4 \right], \eta_{A_i} \right), \\ \bigcap_{i=1}^{n} \left(\left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4 \right], \eta_{A_i} \right), \\ \bigcap_{i=1}^{n} \left(\left[d_{A_i}^1, d_{A_i}^2, d_{A_i}^3, d_{A_i}^4 \right], \nu_{A_i} \right) \end{bmatrix}$$

and

$$\mathcal{A}^- = \begin{bmatrix} \bigcap_{i=1}^{n} \left(\left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4 \right], \mu_{A_i} \right), \\ \bigcap_{i=1}^{n} \left(\left[b_{A_i}^1, b_{A_i}^2, b_{A_i}^3, b_{A_i}^4 \right], \vartheta_{A_i} \right), \\ \bigcup_{i=1}^{n} \left(\left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4 \right], \eta_{A_i} \right), \\ \bigcup_{i=1}^{n} \left(\left[d_{A_i}^1, d_{A_i}^2, d_{A_i}^3, d_{A_i}^4 \right], \nu_{A_i} \right) \end{bmatrix},$$

then $\mathcal{A}^- \ll \mathcal{WA}\left(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n\right) \ll \mathcal{A}^+$.

Proof For the truth-membership degree of $\mathcal{WA}\left(\mathcal{A}_1, \ldots, \mathcal{A}_n\right)$, we have

$$
\begin{aligned}
&\bigwedge_{i=1}^{n} a_{A_i}^1 \le a_{A_i}^1 \\
&1-\bigwedge_{i=1}^{n} a_{A_i}^1 \ge 1-a_{A_i}^1 \\
&\left(1-\bigwedge_{i=1}^{n} a_{A_i}^1\right)^{\lambda_i} \ge \left(1-a_{A_i}^1\right)^{\lambda_i} \\
&\prod_{i=1}^{n}\left(1-\bigwedge_{i=1}^{n} a_{A_i}^1\right)^{\lambda_i} \ge \prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&1-\prod_{i=1}^{n}\left(1-\bigwedge_{i=1}^{n} a_{A_i}^1\right)^{\lambda_i} \le 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&1-\prod_{i=1}^{n}\left(1-\bigwedge_{i=1}^{n} a_{A_i}^1\right)^{\lambda_i} \le 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&1-\left(1-\bigwedge_{i=1}^{n} a_{A_i}^1\right)^{\sum_{i=1}^{n}\lambda_i} \le 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&1-\left(1-\bigwedge_{i=1}^{n} a_{A_i}^1\right)^{1} \le 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&\bigwedge_{i=1}^{n} a_{A_i}^1 \le 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&\bigwedge_{i=1}^{n} a_{A_i}^1 \le 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i}
\end{aligned}
\tag{72}
$$

and

$$\begin{aligned}
&\bigvee_{i=1}^{n} a_{A_i}^1 \geq a_{A_i}^1 \\
&1-\bigvee_{i=1}^{n} a_{A_i}^1 \leq 1-a_{A_i}^1 \\
&\left(1-\bigvee_{i=1}^{n} a_{A_i}^1\right)^{\lambda_i} \leq \left(1-a_{A_i}^1\right)^{\lambda_i} \\
&\prod_{i=1}^{n}\left(1-\bigvee_{i=1}^{n} a_{A_i}^1\right)^{\lambda_i} \leq \prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&1-\prod_{i=1}^{n}\left(1-\bigvee_{i=1}^{n} a_{A_i}^1\right)^{\lambda_i} \geq 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&1-\prod_{i=1}^{n}\left(1-\bigvee_{i=1}^{n} a_{A_i}^1\right)^{\lambda_i} \geq 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&1-\left(1-\bigvee_{i=1}^{n} a_{A_i}^1\right)^{\sum_{i=1}^{n}\lambda_i} \geq 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&1-\left(1-\bigvee_{i=1}^{n} a_{A_i}^1\right)^{1} \geq 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&\bigvee_{i=1}^{n} a_{A_i}^1 \geq 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \\
&\bigvee_{i=1}^{n} a_{A_i}^1 \geq 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i}.
\end{aligned} \tag{73}$$

Thus, from Eqs. (72) and (73) we have

$$\bigwedge_{i=1}^{n} a_{A_i}^1 \leq 1-\prod_{i=1}^{n}\left(1-a_{A_i}^1\right)^{\lambda_i} \leq \bigvee_{i=1}^{n} a_{A_i}^1. \tag{74}$$

It can be similarly proved that

$$\bigwedge_{i=1}^{n} a_{A_i}^2 \leq \sqrt[q]{1-\prod_{i=1}^{n}\left(1-\left(a_{A_i}^2\right)^q\right)^{\lambda_i}} \leq \bigvee_{i=1}^{n} a_{A_i}^2 \tag{75}$$

$$\bigwedge_{i=1}^{n} a_{A_i}^3 \leq \sqrt[q]{1-\prod_{i=1}^{n}\left(1-\left(a_{A_i}^3\right)^q\right)^{\lambda_i}} \leq \bigvee_{i=1}^{n} a_{A_i}^3 \tag{76}$$

$$\bigwedge_{i=1}^{n} a_{A_i}^4 \le \sqrt[q]{1-\prod_{i=1}^{n}\left(1-\left(a_{A_i}^4\right)^q\right)^{\lambda_i}} \le \bigvee_{i=1}^{n} a_{A_i}^4 \tag{77}$$

and

$$\bigwedge_{i=1}^{n} \mu_{A_i} \le \sqrt[q]{1-\prod_{i=1}^{n}\left(1-\left(\mu_{A_i}\right)^q\right)^{\lambda_i}} \le \bigvee_{i=1}^{n} \mu_{A_i}. \tag{78}$$

Hence, by Eqs. (74), (75), (76), (77) and (78), we get

$$\begin{aligned}\bigcap_{i=1}^{n}\left(\left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4\right], \mu_{A_i}\right) \le &\left(\bigoplus_{i=1}^{n}\lambda_i\left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4\right], \bigoplus_{i=1}^{n}\lambda_i\mu_{A_i}\right)\\ &\le \bigcup_{i=1}^{n}\left(\left[a_{A_i}^1, a_{A_i}^2, a_{A_i}^3, a_{A_i}^4\right], \mu_{A_i}\right).\end{aligned} \tag{79}$$

It can be similarly proved that

$$\begin{aligned}\bigcap_{i=1}^{n}\left(\left[b_{A_i}^1, b_{A_i}^2, b_{A_i}^3, b_{A_i}^4\right], \vartheta_{A_i}\right) \le &\left(\bigoplus_{i=1}^{n}\lambda_i\left[b_{A_i}^1, b_{A_i}^2, b_{A_i}^3, b_{A_i}^4\right], \bigoplus_{i=1}^{n}\lambda_i\vartheta_{A_i}\right)\\ &\le \bigcup_{i=1}^{n}\left(\left[b_{A_i}^1, b_{A_i}^2, b_{A_i}^3, b_{A_i}^4\right], \vartheta_{A_i}\right).\end{aligned} \tag{80}$$

For the ignorance membership degree of $\mathcal{WA}(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n)$, we have

$$\begin{aligned}&\bigwedge_{i=1}^{n} c_{A_i}^1 \le c_{A_i}^1 \le \bigvee_{i=1}^{n} c_{A_i}^1\\ &\left(\bigwedge_{i=1}^{n} c_{A_i}^1\right)^{\lambda_i} \le \left(c_{A_i}^1\right)^{\lambda_i} \le \left(\bigvee_{i=1}^{n} c_{A_i}^1\right)^{\lambda_i}\\ &\prod_{i=1}^{n}\left(\bigwedge_{i=1}^{n} c_{A_i}^1\right)^{\lambda_i} \le \prod_{i=1}^{n}\left(c_{A_i}^1\right)^{\lambda_i} \le \prod_{i=1}^{n}\left(\bigvee_{i=1}^{n} c_{A_i}^1\right)^{\lambda_i}\\ &\left(\bigwedge_{i=1}^{n} c_{A_i}^1\right)^{\sum_{i=1}^{n}\lambda_i} \le \prod_{i=1}^{n}\left(c_{A_i}^1\right)^{\lambda_i} \le \left(\bigvee_{i=1}^{n} c_{A_i}^1\right)^{\sum_{i=1}^{n}\lambda_i}\\ &\bigwedge_{i=1}^{n} c_{A_i}^1 \le \prod_{i=1}^{n}\left(c_{A_i}^1\right)^{\lambda_i} \le \bigvee_{i=1}^{n} c_{A_i}^1.\end{aligned} \tag{81}$$

Similarly, we obtain that

$$\bigwedge_{i=1}^{n} c_{A_i}^2 \le \prod_{i=1}^{n}\left(c_{A_i}^2\right)^{\lambda_i} \le \bigvee_{i=1}^{n} c_{A_i}^2 \tag{82}$$

$$\bigwedge_{i=1}^{n} c_{A_i}^3 \le \prod_{i=1}^{n} \left(c_{A_i}^3\right)^{\lambda_i} \le \bigvee_{i=1}^{n} c_{A_i}^3 \tag{83}$$

$$\bigwedge_{i=1}^{n} c_{A_i}^4 \le \prod_{i=1}^{n} \left(c_{A_i}^4\right)^{\lambda_i} \le \bigvee_{i=1}^{n} c_{A_i}^4 \tag{84}$$

and

$$\bigwedge_{i=1}^{n} \eta_{A_i} \le \prod_{i=1}^{n} \left(\eta_{A_i}\right)^{\lambda_i} \le \bigvee_{i=1}^{n} \eta_{A_i}. \tag{85}$$

Thus, by Eqs. (81), (82), (83), (84) and (85), we get

$$\begin{aligned}\bigcap_{i=1}^{n} \left(\left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4\right], \eta_{A_i}\right) &\le \left(\bigoplus_{i=1}^{n} \lambda_i \left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4\right], \bigoplus_{i=1}^{n} \lambda_i \eta_{A_i}\right) \\ &\le \bigcup_{i=1}^{n} \left(\left[c_{A_i}^1, c_{A_i}^2, c_{A_i}^3, c_{A_i}^4\right], \eta_{A_i}\right).\end{aligned} \tag{86}$$

It can be similarly proved that

$$\begin{aligned}\bigcap_{i=1}^{n} \left(\left[d_{A_i}^1, d_{A_i}^2, d_{A_i}^3, d_{A_i}^4\right], \nu_{A_i}\right) &\le \left(\bigoplus_{i=1}^{n} \lambda_i \left[d_{A_i}^1, d_{A_i}^2, d_{A_i}^3, d_{A_i}^4\right], \bigoplus_{i=1}^{n} \lambda_i \nu_{A_i}\right) \\ &\le \bigcup_{i=1}^{n} \left(\left[d_{A_i}^1, d_{A_i}^2, d_{A_i}^3, d_{A_i}^4\right], \nu_{A_i}\right).\end{aligned} \tag{87}$$

By Eqs. (79), (80), (86) and (87), we have $\mathcal{A}^- \ll \mathcal{WA}\left(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n\right) \ll \mathcal{A}^+$. □

5 QSVTrNWG operators

In this section, we introduce the notion of quadripartitioned single-valued trapezoidal neutrosophic weighted geometric operator along with their some properties.

Based on Definition 6, we propose the following an aggregation operator of quadripartitioned single-valued trapezoidal neutrosophic numbers.

Definition 11 Let

$$\Delta = \left\{ \mathcal{A}_i = \begin{bmatrix} \left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i} \right], \mu_{A_i} \right), \\ \left(\left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i} \right], \eta_{A_i} \right), \\ \left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i} \right], \nu_{A_i} \right), \\ \left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i} \right], \nu_{A_i} \right) \end{bmatrix} : i = 1, \dots, n \right\}$$

be a collection of QSVTrNNs on the real number set **R** and $\lambda = \left(\lambda_1, \dots, \lambda_n\right)^T$ be the weight vector of Δ, where λ_j indicates the importance degree of Δ, satisfying $\lambda_1, \dots, \lambda_n \in [0, 1]$ and $\sum_{i=1}^{n} \lambda_i = 1$, and let **quadripartitioned single-valued trapezoidal neutrosophic weighted geometric (QSVTrNWG)** $\mathcal{WG} : (\mathcal{TN}(\mathbf{R}))^n \to \mathcal{TN}(\mathbf{R})$ if

$$\mathcal{WG}\left(\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n\right) = \bigotimes_{i=1}^{n} \mathcal{A}_i^{\lambda_i}, \tag{88}$$

then the function QSVTrNWG is called the **QSVTrNWG-operator**.

Remark 3 When we need to weight the ordered positions of the quadripartitioned single-valued trapezoidal neutrosophic arguments instead of weighting the arguments themselves, quadripartitioned single valued neutrosophic weighted geometric can be generalized to single valued neutrosophic weighted geometric.

The general expression of the QSVTrNWG-operator is constructed in the following theorem:

Theorem 7 *Let*

$$\Delta = \left\{ \mathcal{A}_i = \begin{bmatrix} \left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i} \right], \mu_{A_i} \right), \\ \left(\left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i} \right], \vartheta_{A_i} \right), \\ \left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i} \right], \eta_{A_i} \right), \\ \left(\left[d^1_{A_i}, d^2_{A_i}, d^3_{A_i}, d^4_{A_i} \right], \nu_{A_i} \right) \end{bmatrix} : i = 1, \dots, n \right\}$$

be a collection of QSVTrNNs on the real number set **R** *and* $\lambda = \left(\lambda_1, \lambda_2, \dots, \lambda_n\right)^T$ *be the weight vector of* Δ, *where* λ_j *indicates the importance degree of* Δ, *satisfying* $\lambda_1, \lambda_2, \dots, \lambda_n \in [0, 1]$ *and* $\sum\limits_{i=1}^{n} \lambda_i = 1$. *Then their accumulated outcome utilizing the QSVTrNWG-operator is again QSVTrNN, and*

$$\bigotimes_{i=1}^{n}\left[a^1_{A_i},a^2_{A_i},a^3_{A_i},a^4_{A_i}\right]\lambda_i=\begin{bmatrix}\prod_{i=1}^{n}\left(a^1_{A_i}\right)^{\lambda_i},\prod_{i=1}^{n}\left(a^2_{A_i}\right)^{\lambda_i},\\ \prod_{i=1}^{n}\left(a^3_{A_i}\right)^{\lambda_i},\prod_{i=1}^{n}\left(a^4_{A_i}\right)^{\lambda_i}\end{bmatrix} \tag{89}$$

$$\bigotimes_{i=1}^{n}\left[b^1_{A_i},b^2_{A_i},b^3_{A_i},b^4_{A_i}\right]\lambda_i=\begin{bmatrix}\prod_{i=1}^{n}\left(b^1_{A_i}\right)^{\lambda_i},\prod_{i=1}^{n}\left(b^2_{A_i}\right)^{\lambda_i},\\ \prod_{i=1}^{n}\left(b^3_{A_i}\right)^{\lambda_i},\prod_{i=1}^{n}\left(b^4_{A_i}\right)^{\lambda_i}\end{bmatrix} \tag{90}$$

$$\bigotimes_{i=1}^{n}\left[c^1_{A_i},c^2_{A_i},c^3_{A_i},c^4_{A_i}\right]\lambda_i=\begin{bmatrix}1-\prod_{i=1}^{n}\left(1-c^1_{A_i}\right)^{\lambda_i},\\ 1-\prod_{i=1}^{n}\left(1-c^2_{A_i}\right)^{\lambda_i},\\ 1-\prod_{i=1}^{n}\left(1-c^3_{A_i}\right)^{\lambda_i},\\ 1-\prod_{i=1}^{n}\left(1-c^4_{A_i}\right)^{\lambda_i}\end{bmatrix} \tag{91}$$

$$\bigotimes_{i=1}^{n}\left[d^1_{A_i},d^2_{A_i},d^3_{A_i},d^4_{A_i}\right]\lambda_i=\begin{bmatrix}1-\prod_{i=1}^{n}\left(1-d^1_{A_i}\right)^{\lambda_i},\\ 1-\prod_{i=1}^{n}\left(1-d^2_{A_i}\right)^{\lambda_i},\\ 1-\prod_{i=1}^{n}\left(1-d^3_{A_i}\right)^{\lambda_i},\\ 1-\prod_{i=1}^{n}\left(1-d^4_{A_i}\right)^{\lambda_i}\end{bmatrix} \tag{92}$$

$$\bigotimes_{i=1}^{n}\left(\mu_{A_i}\right)\lambda_i=\prod_{i=1}^{n}\left(\mu_{A_i}\right)^{\lambda_i} \tag{93}$$

$$\bigotimes_{i=1}^{n}\left(\vartheta_{A_i}\right)\lambda_i=\prod_{i=1}^{n}\left(\vartheta_{A_i}\right)^{\lambda_i} \tag{94}$$

$$\bigotimes_{i=1}^{n}\left(\eta_{A_i}\right)\lambda_i = 1-\prod_{i=1}^{n}\left(1-\eta_{A_i}\right)^{\lambda_i} \tag{95}$$

$$\bigotimes_{i=1}^{n}\left(\nu_{A_i}\right)\lambda_i = 1-\prod_{i=1}^{n}\left(1-\nu_{A_i}\right)^{\lambda_i} \tag{96}$$

$$\mathcal{WG}\left(\mathcal{A}_1,\mathcal{A}_2,\ldots,\mathcal{A}_n\right)=\begin{bmatrix}\left(\bigotimes_{i=1}^{n}\left[a_{A_i}^1,a_{A_i}^2,a_{A_i}^3,a_{A_i}^4\right]\lambda_i,\bigotimes_{i=1}^{n}\left(\mu_{A_i}\right)\lambda_i\right),\\ \left(\bigotimes_{i=1}^{n}\left[b_{A_i}^1,b_{A_i}^2,b_{A_i}^3,b_{A_i}^4\right]\lambda_i,\bigotimes_{i=1}^{n}\left(\vartheta_{A_i}\right)\lambda_i\right),\\ \left(\bigotimes_{i=1}^{n}\left[c_{A_i}^1,c_{A_i}^2,c_{A_i}^3,c_{A_i}^4\right]\lambda_i,\bigotimes_{i=1}^{n}\left(\eta_{A_i}\right)\lambda_i\right),\\ \left(\bigotimes_{i=1}^{n}\left[d_{A_i}^1,d_{A_i}^2,d_{A_i}^3,d_{A_i}^4\right]\lambda_i,\bigotimes_{i=1}^{n}\left(\nu_{A_i}\right)\lambda_i\right)\end{bmatrix}. \tag{97}$$

In the following, we describe some desirable properties of the QSVTrNWG-operator.

Theorem 8 *Let*

$$\left\{\mathcal{A}_i=\begin{bmatrix}\left(\left[a_{A_i}^1,a_{A_i}^2,a_{A_i}^3,a_{A_i}^4\right],\mu_{A_i}\right),\\ \left(\left[b_{A_i}^1,b_{A_i}^2,b_{A_i}^3,b_{A_i}^4\right],\vartheta_{A_i}\right),\\ \left(\left[c_{A_i}^1,c_{A_i}^2,c_{A_i}^3,c_{A_i}^4\right],\eta_{A_i}\right),\\ \left(\left[d_{A_i}^1,d_{A_i}^2,d_{A_i}^3,d_{A_i}^4\right],\nu_{A_i}\right)\end{bmatrix}: i=1,2,3,\ldots,n\right\}$$

be a collection of QSVTrNNs on the real number set **R**. *If*

$$\mathcal{A}_i=\mathcal{A}=\begin{bmatrix}\left(\left[a_A^1,a_A^2,a_A^3,a_A^4\right],\mu_A\right),\\ \left(\left[b_A^1,b_A^2,b_A^3,b_A^4\right],\vartheta_A\right),\\ \left(\left[c_A^1,c_A^2,c_A^3,c_A^4\right],\eta_A\right),\\ \left(\left[d_A^1,d_A^2,d_A^3,d_A^4\right],\nu_A\right)\end{bmatrix}$$

for every $i=1,2,\ldots,n$, *then* $\mathcal{WG}\left(\mathcal{A}_1,\mathcal{A}_2,\ldots,\mathcal{A}_n\right)=\mathcal{A}$.

The following Theorem 9 (Boundedness) establishes some properties of QSVTrNWG-operator.

Theorem 9 *Let*

$$\left\{\mathcal{A}_i = \left[\begin{array}{l} \left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right), \\ \left(\left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i}\right], \vartheta_{A_i}\right), \\ \left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right], \eta_{A_i}\right), \\ \left(\left[d^1_{A_i}, d^2_{A_i}, d^3_{A_i}, d^4_{A_i}\right], \nu_{A_i}\right) \end{array}\right] : i = 1, 2, 3, \dots, n\right\}$$

be a collection of QSVTrNNs on the real number set $\mathbf{R}$. *If*

$$\mathcal{A}^+ = \left[\begin{array}{l} \bigcup_{i=1}^{n}\left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right), \\ \bigcup_{i=1}^{n}\left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right), \\ \bigcap_{i=1}^{n}\left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right], \nu_{A_i}\right), \\ \bigcap_{i=1}^{n}\left(\left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i}\right], \eta_{A_i}\right) \end{array}\right]$$

and

$$\mathcal{A}^- = \left[\begin{array}{l} \bigcap_{i=1}^{n}\left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right), \\ \bigcap_{i=1}^{n}\left(\left[a^1_{A_i}, a^2_{A_i}, a^3_{A_i}, a^4_{A_i}\right], \mu_{A_i}\right), \\ \bigcup_{i=1}^{n}\left(\left[c^1_{A_i}, c^2_{A_i}, c^3_{A_i}, c^4_{A_i}\right], \nu_{A_i}\right), \\ \bigcup_{i=1}^{n}\left(\left[b^1_{A_i}, b^2_{A_i}, b^3_{A_i}, b^4_{A_i}\right], \eta_{A_i}\right) \end{array}\right],$$

then $\mathcal{A}^- \preceq \mathcal{WG}\left(\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n\right) \preceq \mathcal{A}^+$.

6 Multi-criteria decision making method under quadripartitioned single-valued trapezoidal neutrosophic information

In this section, we define a multi-criteria decision-making method termed quadripartitioned single-valued trapezoidal neutrosophic multi-criteria decision-making (QSVTrNMCDM) method. This approach is adopted from a body of existing literature [8, 17, 22, 23, 26, 35, 36, 39, 41].

The proposed QSVTrNMCDM approach employs quadripartitioned single-valued trapezoidal neutrosophic aggregation operators to determine the weights of real numbers within a quadripartitioned single-valued trapezoidal neutrosophic environment. Here, the QSVTrNMCDM method is applied to identify the most favorable alternative for a company selecting the most suitable supplier.

Let A represent a discrete set of alternatives, such that $A = \{A_1, \ldots, A_m\}$, for all $i = 1, 2, \ldots, m$ and $G = \{G_1, G_2, \ldots, G_n\}$ is a set of attributes with weight vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ of the attributes $G = \{G_1, G_2, \ldots, G_n\}$, where $0 \leq \lambda_1, \lambda_2, \ldots, \lambda_n \leq 1$ and $\sum_{i=1}^{n} \lambda_i = 1$. The expert's decisions are stated as QSVTrNNs, i.e., $\mathcal{A}_{ij} = \left(\mu_{A_{ij}}, \eta_{A_{ij}}, \nu_{A_{ij}}\right)$, such that $0 \leq \mu_{A_{ij}}, \eta_{A_{ij}}, \nu_{A_{ij}} \leq 1$ and $0 \leq \left(\mu_{A_{ij}}\right)^p + \left(\eta_{A_{ij}}\right)^r + \left(\nu_{A_{ij}}\right)^q \leq 1$. Hence, the quadripartitioned single-valued trapezoidal neutrosophic decision matrix is given as

$$D = \left[\mathcal{A}_{ij} = \left[\begin{matrix} \left(\left[a^1_{A_{ij}}, a^2_{A_{ij}}, a^3_{A_{ij}}, a^4_{A_{ij}}\right], \mu_{A_{ij}}\right), \\ \left(\left[b^1_{A_{ij}}, b^2_{A_{ij}}, b^3_{A_{ij}}, b^4_{A_{ij}}\right], \vartheta_{A_{ij}}\right), \\ \left(\left[c^1_{A_{ij}}, c^2_{A_{ij}}, c^3_{A_{ij}}, c^4_{A_{ij}}\right], \eta_{A_{ij}}\right), \\ \left(\left[d^1_{A_{ij}}, d^2_{A_{ij}}, d^3_{A_{ij}}, d^4_{A_{ij}}\right], \nu_{A_{ij}}\right) \end{matrix} \right] \right]_{m \times n}$$

and presented as follows:

$$D = \begin{bmatrix} \mathcal{A}_{ij} & \mathcal{A}_{ij} & \ldots & \mathcal{A}_{ij} \\ \mathcal{A}_{ij} & \mathcal{A}_{ij} & \ldots & \mathcal{A}_{ij} \\ \vdots & & & \\ \mathcal{A}_{ij} & \mathcal{A}_{ij} & \ldots & \mathcal{A}_{ij} \end{bmatrix}. \tag{98}$$

The algorithm follows a QSVTrNMCDM method to interpret a QSVTrNMCDM problem under quadripartitioned single-valued trapezoidal neutrosophic information using QSVTrNWA and QSVTrNWG-operators. Through this methodology, we aim to provide a systematic and effective approach for decision-making in complex and uncertain environments, particularly in scenarios where neutrosophic information is prevalent.

Algorithm

Input: quadripartitioned single-valued trapezoidal neutrosophic information

Output: Best alternative

Step 1. Calculate the quadripartitioned single-valued trapezoidal neutrosophic decision matrix.

Step 2. In order to eliminate the influence of attribute type, we consider the following technique and obtain the standardize matrix $R = \left[\mathcal{R}_{ij}\right]_{m \times n}$, where $\mathcal{R}_{ij} = \begin{bmatrix} \left(\left[a^1_{A_{ij}}, a^2_{A_{ij}}, a^3_{A_{ij}}, a^4_{A_{ij}}\right], \mu_{A_{ij}}\right), \\ \left(\left[b^1_{A_{ij}}, b^2_{A_{ij}}, b^3_{A_{ij}}, b^4_{A_{ij}}\right], \vartheta_{A_{ij}}\right), \\ \left(\left[c^1_{A_{ij}}, c^2_{A_{ij}}, c^3_{A_{ij}}, c^4_{A_{ij}}\right], \eta_{A_{ij}}\right), \\ \left(\left[d^1_{A_{ij}}, d^2_{A_{ij}}, d^3_{A_{ij}}, d^4_{A_{ij}}\right], \nu_{A_{ij}}\right) \end{bmatrix}$ is QSVTrNN. Then we have

$$\mathcal{R}_{ij} = \begin{cases} \mathcal{A}_{ij} \ ; & \text{for benefit type attribute} \\ \mathcal{A}^c_{ij} \ ; & \text{for cost type attribute.} \end{cases} \tag{99}$$

Step 3. Based on the standardize matrix, as obtained from step 2, the overall aggregated value of the alternative A_i for all $i = 1, 2, \dots, m$, under the different criteria G_j for all $j = 1, 2, \dots, n$ is obtained by using either

$$r_i = \mathcal{WA}\left(\mathcal{R}_{i1}, \mathcal{R}_{i2}, \dots, \mathcal{R}_{in}\right) \tag{100}$$

or

$$r_i = \mathcal{WG}\left(\mathcal{R}_{i1}, \mathcal{R}_{i2}, \dots, \mathcal{R}_{in}\right) \tag{101}$$

operator and hence get the collective value r_i for each alternative A_i for all $i = 1, 2, \dots, m$.

Step 4. Next, the score values of $\mathcal{SC}\left(r_i\right)$ $(i = 1, 2, 3, \dots, m)$ of the overall QSVTrNNs of $r_i (i = 1, 2, 3, \dots, m)$ is obtained to rank the alternatives $A_i (i = 1, 2, \dots, m)$. If the score values of $\mathcal{SC}\left(r_i\right)$ and $\mathcal{SC}\left(r_j\right)$ are equal for two alternatives A_i and A_j, then it is required to calculate accuracy degrees of $\mathcal{AC}\left(r_i\right)$ and $\mathcal{AC}\left(r_j\right)$ with respect to the overall collective QSVTrNNs to rank the alternatives A_i and A_j, respectively, based on the aforementioned accuracy degrees $\mathcal{AC}\left(r_i\right)$ and $\mathcal{AC}\left(r_j\right)$.

Step 5. We select the best alternative from the rankings of all alternatives A_i $(i = 1, 2, \dots, m)$ according to $\mathcal{SC}\left(r_i\right) \left(\mathcal{SC}\left(r_i\right)\right) (i = 1, 2, \dots, m)$.

Step 6.: Stop

6.1 An illustrative example

We begin by considering a multi-criteria decision-making (MCDM) problem focusing on the selection of green suppliers for a critical part in the product assembly process of a product design company. The QSVTrNMCDM strategy proposed in this study is illustrated through the resolution of a specific MCDM problem adapted from [44].

The MCDM problem at hand involves selecting the most appropriate green suppliers from a pool of four final alternatives, namely:

A_1: Shantui Construction Machinery Company Limited

A_2: Taikai Electric Group Company Limited,

Table 1 Evaluations of decision makers

	G_1	G_2	G_3
A_1	$[[0.6, 0.7, 0.8, 0.9], 0.6]$ $[[0.3, 0.4, 0.5, 0.6], 0.3]$ $[[0.3, 0.4, 0.5, 0.6], 0.2]$ $[[0.1, 0.2, 0.4, 0.5], 0.2]$	$[[0.5, 0.6, 0.7, 0.9], 0.3]$ $[[0.4, 0.5, 0.6, 0.8], 0.3]$ $[[0.2, 0.4, 0.5, 0.6], 0.4]$ $[[0.5, 0.6, 0.7, 0.8], 0.2]$	$[[0.6, 0.7, 0.8, 0.9], 0.7]$ $[[0.3, 0.4, 0.5, 0.6], 0.6]$ $[[0.4, 0.5, 0.6, 0.7], 0.2]$ $[[0.3, 0.4, 0.5, 0.7], 0.4]$
A_2	$[[0.3, 0.4, 0.5, 0.6], 0.6]$ $[[0.4, 0.5, 0.6, 0.7], 0.2]$ $[[0.3, 0.4, 0.5, 0.7], 0.2]$ $[[0.3, 0.4, 0.5, 0.7], 0.3]$	$[[0.6, 0.7, 0.8, 0.9], 0.2]$ $[[0.3, 0.4, 0.5, 0.6], 0.3]$ $[[0.3, 0.4, 0.6, 0.7], 0.2]$ $[[0.1, 0.2, 0.4, 0.5], 0.2]$	$[[0.6, 0.7, 0.8, 0.9], 0.6]$ $[[0.3, 0.4, 0.5, 0.6], 0.3]$ $[[0.3, 0.4, 0.5, 0.6], 0.2]$ $[[0.3, 0.4, 0.6, 0.7], 0.1]$
A_3	$[[0.5, 0.7, 0.8, 0.9], 0.6]$ $[[0.6, 0.7, 0.8, 0.9], 0.1]$ $[[0.7, 0.8, 0.9, 1], 0.3]$ $[[0.4, 0.5, 0.7, 0.9], 0.2]$	$[[0.4, 0.5, 0.6, 0.8], 0.7]$ $[[0.1, 0.3, 0.5, 0.6], 0.2]$ $[[0.3, 0.4, 0.6, 0.7], 0.3]$ $[[0.1, 0.2, 0.3, 0.4], 0.4]$	$[[0.4, 0.5, 0.7, 0.8], 0.3]$ $[[0.3, 0.5, 0.6, 0.7], 0.7]$ $[[0.1, 0.2, 0.6, 0.8], 0.1]$ $[[0.3, 0.5, 0.6, 0.8], 0.3]$

Table 1 (continued)

	G_1	G_2	G_3
A_4	$\begin{bmatrix} \begin{bmatrix} 0.4, \\ 0.5, \\ 0.7, \\ 0.8 \end{bmatrix}, 0.4 \\ \begin{bmatrix} 0.5, \\ 0.6, \\ 0.7, \\ 0.9 \end{bmatrix}, 0.2 \\ \begin{bmatrix} 0.3, \\ 0.5, \\ 0.6, \\ 0.8 \end{bmatrix}, 0.5 \\ \begin{bmatrix} 0.1, \\ 0.2, \\ 0.4, \\ 0.6 \end{bmatrix}, 0.5 \end{bmatrix}$	$\begin{bmatrix} \begin{bmatrix} 0.4, \\ 0.5, \\ 0.6, \\ 0.7 \end{bmatrix}, 0.6 \\ \begin{bmatrix} 0.5, \\ 0.6, \\ 0.7, \\ 0.8 \end{bmatrix}, 0.1 \\ \begin{bmatrix} 0.2, \\ 0.4, \\ 0.5, \\ 0.8 \end{bmatrix}, 0.4 \\ \begin{bmatrix} 0.3, \\ 0.5, \\ 0.6, \\ 0.7 \end{bmatrix}, 0.3 \end{bmatrix}$	$\begin{bmatrix} \begin{bmatrix} 0.5, \\ 0.6, \\ 0.7, \\ 0.9 \end{bmatrix}, 0.3 \\ \begin{bmatrix} 0.4, \\ 0.6, \\ 0.7, \\ 0.8 \end{bmatrix}, 0.4 \\ \begin{bmatrix} 0.2, \\ 0.4, \\ 0.5, \\ 0.6 \end{bmatrix}, 0.4 \\ \begin{bmatrix} 0.5, \\ 0.6, \\ 0.7, \\ 0.8 \end{bmatrix}, 0.3 \end{bmatrix}$

A_3: Sino Trunk,

A_4: Howden Hua Engineering Company.

To evaluate the alternatives, a decision-making group comprising three experts from the engineering, production, and quality inspection departments is formed. The experts identify three independent criteria to serve as evaluation principles:

G_1: environment management,

G_2: product quality,

G_3: pollution control.

Suppose that the four experts'risk attitudes are $\lambda_1 = 0.4, \lambda_2 = 0.2$ and $\lambda_3 = 0.4$. The evaluation information is described by QSVTrNNs, as shown in Table 1.

6.2 The QSVTrNMCDM steps based on QSVTrNWA and QSVTrNWG-operators:

Step 1. Calculate the quadripartitioned single-valued trapezoidal neutrosophic decision matrix. These five alternatives A_1, A_2, A_3 and A_4 are to be evaluated by an expert under the three aspects G_1, G_2 and G_3 by using quadripartitioned single-valued trapezoidal neutrosophic decision matrix $D = \left[\mathcal{A}_{ij}\right]_{4\times3}$ and their corresponding rating values are shown in Eq. (102).

$$D = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{A}_{13} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{A}_{13} \\ \mathcal{A}_{31} & \mathcal{A}_{32} & \mathcal{A}_{33} \\ \mathcal{A}_{41} & \mathcal{A}_{42} & \mathcal{A}_{43} \end{bmatrix}. \tag{102}$$

Step 2. The three criteria $G_j (j = 1, 2, 3)$ are regarded as the benefit-type criterion, so the quadripartitioned single-valued trapezoidal neutrosophic decision matrices change nothing.

Table 2 Aggregated values of the alternatives using QSVTrNWA-operator

r_1	$\begin{bmatrix} ([0.58, 0.68, 0.78, 0.90], 0.60), \\ ([0.32, 0.42, 0.52, 0.65], 0.44), \\ ([0.31, 0.44, 0.54, 0.64], 0.23), \\ ([0.21, 0.33, 0.49, 0.63], 0.26) \end{bmatrix}$
r_2	$\begin{bmatrix} ([0.50, 0.60, 0.71, 0.83], 0.54), \\ ([0.34, 0.44, 0.54, 0.64], 0.26), \\ ([0.30, 0.40, 0.52, 0.66], 0.20), \\ ([0.24, 0.35, 0.51, 0.65], 0.19) \end{bmatrix}$
r_3	$\begin{bmatrix} ([0.44, 0.59, 0.73, 0.85], 0.53), \\ ([0.41, 0.56, 0.68, 0.8], 0.43), \\ ([0.27, 0.40, 0.71, 0.85], 0.19), \\ ([0.27, 0.42, 0.56, 0.73], 0.27) \end{bmatrix}$
r_4	$\begin{bmatrix} ([0.44, 0.54, 0.68, 0.84], 0.41), \\ ([0.46, 0.60, 0.70, 0.85], 0.27), \\ ([0.24, 0.44, 0.54, 0.71], 0.44), \\ ([0.24, 0.37, 0.54, 0.69], 0.37) \end{bmatrix}$

Table 3 Aggregated values of the alternatives using QSVTrNWG-operator

r_1	$\begin{bmatrix} ([0.54, 0.67, 0.79, 0.90], 0.56), \\ ([0.32, 0.42, 0.52, 0.64], 0.40), \\ ([0.32, 0.44, 0.54, 0.64], 0.24), \\ ([0.28, 0.38, 0.51, 0.66], 0.29) \end{bmatrix}$
r_2	$\begin{bmatrix} ([0.45, 0.56, 0.66, 0.77], 0.48), \\ ([0.34, 0.44, 0.54, 0.64], 0.26), \\ ([0.30, 0.40, 0.52, 0.66], 0.20), \\ ([0.26, 0.36, 0.53, 0.67], 0.21) \end{bmatrix}$
r_3	$\begin{bmatrix} ([0.44, 0.57, 0.72, 0.84], 0.47), \\ ([0.32, 0.52, 0.65, 0.75], 0.25), \\ ([0.45, 0.57, 0.77, 1.0], 0.23), \\ ([0.31, 0.45, 0.60, 0.81], 0.28) \end{bmatrix}$
r_4	$\begin{bmatrix} ([0.44, 0.54, 0.68, 0.82], 0.39), \\ ([0.46, 0.60, 0.70, 0.84], 0.23), \\ ([0.24, 0.44, 0.54, 0.74], 0.44), \\ ([0.32, 0.45, 0.58, 0.71], 0.39) \end{bmatrix}$

Table 4 Score values of alternatives using QSVTrNWA and QSVTrNWG-operators

	QSVTrNWA-operator	QSVTrNWG-operator
$\mathcal{SC}(r_1)$	0.27	0.26
$\mathcal{SC}(r_2)$	0.25	0.29
$\mathcal{SC}(r_3)$	0.23	0.22
$\mathcal{SC}(r_4)$	0.21	0.21

Table 5 Ranking order of the alternatives

Aggregation operator	Ranking ordered
QSVTrNWA-operator	$A_1 \gg A_2 \gg A_3 \gg A_4$
QSVTrNWG-operator	$A_2 \gg A_1 \gg A_3 \gg A_4$

Step 3. By following the QSVTrNWA and QSVTrNWG-operators given in Eqs. (100) and (101), we obtain the overall rating values of each alternative A_i (Tables 2, 3).

Step 4. Calculate the scores function of r_i for all $i = 1, 2, 3, 4$ (Table 4).

Step 5.

The results of ranking order of the alternatives based on QSVTrNWA and QSVTrNWG-operators are presented in Table 5. When QSVTrNWA-operator, we obtained a rank of alternatives as $A_1 \gg A_2 \gg A_3 \gg A_4$, here, A_1 is the best choice, A_2 is the best second choice and A_3 is the best third choice, but, when QSVTrNWG-operator, we obtained a rank of alternatives as $A_2 \gg A_1 \gg A_3 \gg A_4$, here, A_2 is the best choice, A_1 is the best second choice and A_3 is the best third choice. Thus, the overall best rank is A_2.

Table 6 Comparison between existing work with the proposed work

Aggregation operator	The score function	Ranking
QSVNPDOWAA operator [11]	Cannot be calculated	No
QSVNPDOWGA operator [11]	Cannot be calculated	No
TNCLHFWAA operator [14]	Cannot be calculated	No
TNCLHFWGA operator [14]	Cannot be calculated	No
TrNDFWA operator [15]	Cannot be calculated	No
SVNTrDWA operator [21]	Cannot be calculated	No
SVTrNDWGA operator [21]	Cannot be calculated	No
SVTNNWBM operator [28]	Cannot be calculated	No
SVTNWAA operator [29]	Cannot be calculated	No
SVTNWGA operator [29]	Cannot be calculated	No
SVTNWMSM operator [30]	Cannot be calculated	No
QSVNDWAA operator [31]	Cannot be calculated	No
QSVNDWGA operator [31]	Cannot be calculated	No
SVTNWHA operator [34]	Cannot be calculated	No
SVTNOWHA operator [34]	Cannot be calculated	No
SVTNGOWHA operator [34]	Cannot be calculated	No
SVTNPA operator [44]	Cannot be calculated	No
SVTNPG operator [44]	Cannot be calculated	No
TNWAA operator [47]	Cannot be calculated	No
TNWGA operator [47]	Cannot be calculated	No
The propounded methodology		
QSVTrNWA-operator	$A_1 \gg A_2 \gg A_3 \gg A_4$	Yes
QSVTrNWG-operator	$A_2 \gg A_1 \gg A_3 \gg A_4$	Yes

6.3 Comparative analysis

In this manuscript, we undertake a comprehensive comparison analysis to evaluate the performance of QSVTrNWA and QSVTrNWG operators in relation to several benchmark operators. Specifically, we compare the results obtained using QSVNPDOWAA and QSVNPDOWGA operators [11], TNCLHFWAA and TNCLHFWGA operators [14], TrNDFWA operator [15], SVNTrDWA and SVTrNDWGA operators [21], SVTNNWBM operator [28], SVTNWAA and SVTNWGA operators [29], SVTNWMSM operator [30], QSVNDWAA and QSVNDWGA operators [31], SVTNWHA, SVTNOWHA and SVTNGOWHA operators [34], SVTNPA and SVTNPG operators [44] and TNWAA and TNWGA operators [21], as summarized in Table 6.

From the comparative analysis presented in Table 6, several key merits of the proposed operators, namely QSVTrNWA and QSVTrNWG, emerge:

1. Distinct decision outcomes: Our results indicate that both the QSVTrNWA and QSVTrNWG operators yield comparable outcomes. However, a nuanced difference arises in the selection of the best third choice, denoted as A_1 for QSVTrNWA and A_2 for QSVTrNWG. This distinction underscores the authenticity and practical applicability of our method in real-world MCDM scenarios, particularly in supplier selection processes.
2. Flexibility and generality: The QSVTrNMCDM model, encapsulating the proposed operators, demonstrates remarkable flexibility, versatility, and generality. Its ability to accommodate diverse decision-making problems, utilizing varying values, underscores its suitability for addressing a wide spectrum of MCDM challenges
3. Handling of quadripartitioned single-valued trapezoidal neutrosophic information: Existing operators, as documented in Debnath [11], Fahmi et al. [14, 15], Jana et al. [21], Liang et al. [28, 29], Liu and Zhang [30], Mohanasundari and Mohana [31], Paulraj and Tamilarasi [34], Wang et al. [44], and Ye [47], do not adequately address scenarios wherein data are presented in quadripartitioned single-valued trapezoidal neutrosophic information form. In contrast, our proposed approach demonstrates enhanced accuracy and precision in handling such data representations, thus offering a more robust solution for MCDM problems.

7 Conclusion

In this study, we have explored the application of QSVTrNS and QSVTrNN as effective tools for modeling evaluation values in decision-making processes. By leveraging the expansive range of membership grades offered by QSVTrNS, we have demonstrated their utility in handling more complex and uncertain circumstances compared to conventional SVNS, SVTrNS, and QSVNS. Our primary contribution lies in the development of new aggregation operators, namely the QSVTrNWA operator and QSVTrNWG operator. These operators enable the fusion of quadripartitioned single-valued trapezoidal neutrosophic information, offering enhanced precision and accuracy in decision-making processes. Additionally, we have introduced the concept of quadripartitioned single-valued trapezoidal neutrosophic multi-criteria decision making (QSVTrNMCDM) method, exemplified through the development of green supplier selection criteria. By utilizing operators such as QSVNDWAA or QSVNDWGA, we have provided a systematic approach for companies to select the most suitable green suppliers, thereby contributing to sustainable and environmentally conscious decision-making practices. As future directions, we suggest exploring weighted average and weighted power geometric operators over QSVTrNNs, with the aim of further enhancing the efficacy and applicability of MCDM methods. Additionally, continued research into innovative MCDM methodologies based on these operators holds promise for addressing increasingly complex decision-making challenges across various domains.

The quadripartitioned single-valued trapezoidal neutrosophic set (QSVTrNS) can supply with more doubtful circumstances than single valued neutrosophic set (SVNS), single-valued trapezoidal neutrosophic set (SVTrNS) and quadripartitioned single valued neutrosophic set (QSVNS) because of their larger range of depicting the membership grades. The quadripartitioned single-valued trapezoidal neutrosophic number (QSVTrNN) is a useful tool to model evaluation values in decision-making (DM) process. To solve multi-criteria decision making (MCDM) problem with quadripartitioned single-valued trapezoidal neutrosophic evaluation values. Thus, the main work of this paper is to develop some new aggregation operators (AOs) to fuse quadripartitioned single-valued trapezoidal neutrosophicinformation. Also, we define the new concepts of a quadripartitioned single-valued trapezoidal neutrosophic number (QSVTrNN), the basic operational relations of QSVTrNNs, and the score function of QSVTrNNs. Then, we develop a quadripartitioned single-valued trapezoidal neutrosophic weighted averaging (QSVTrNWA) operator and quadripartitioned single-valued trapezoidal neutrosophic weighted geometric (QSVTrNWG) operator to aggregate quadripartitioned single-valued trapezoidal neutrosophic information and investigate their properties. Furthermore, we discussed about quadripartitioned single-valued trapezoidal neutrosophic multi-criteria decision making (QSVTrNMCDM) method for developing green supplier selection criteria using QSVNDWAA or QSVNDWGA operator and also an illustrative example of a company selecting the most suitable green supplier is given for the proposed method which gives a detailed results to select the best alternative based upon the ranking orders. In future works, the weighted average operator, weighted power geometric operator over QSVTrNNs may be investigated and study a MCDM methods depending on these operators.

Acknowledgements This work (Grant No. N42A670085) was supported by Office of the Permanent Secretary, Ministry of Higher Education, Science, Research and Innovation (OPS MHESI), Thailand Science Research and Innovation (TSRI).

Author contributions Pairote Yiarayong: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Funding This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data availibility Not applicable.

Declarations

Conflict of interest The author declares that there is no Conflict of interest regarding the publication of this paper.

Consent for publication I hereby agreed that my submitted manuscript "Some weighted aggregation operators of quadripartitioned single-valued trapezoidal neutrosophic sets and their multi-criteria group decision-making method for developing green supplier selection criteria" be published by your firm.

Consent to participate Not applicable.

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