



An improved solution for fully neutrosophic multi-level linear programming problem based on Laplace transform

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Abstract

The main objective of this paper is to present an improved solution approach for a fully neutrosophic generalized multi-level linear programming (MLP) problem in which de-neutrosophication of coefficients and parameters are carried out with the proportional probability density functions of each N Number and the use of Laplace transform. The present paper describes a unique solution methodology for generalized multi-level linear programming involving coefficient parameters in objectives at each level as well as constraints as interval-valued trapezoidal neutrosophic numbers (IVTrpN numbers), based on Laplace Transform. In this approach, we first propose to associate the probability density function to each membership function of each IVTrpN number and obtain an equivalent output value of each N Number using Laplace transform. The proposed algorithm is novel and unique for solving the generalized MLLP problem under N Numbers environment, which converts the neutrosophic problem into an equivalent crisp problem. After that, the multi-level structure of the crisp problem is tackled by formulating separate membership functions for each objective function at each level and decision variables up to the (T-1) level with their best values. A simple solution model is formulated to obtain a satisfactory solution to MLLP problem under the neutrosophic environment with the help of usual goal programming. Further, a comparative study is also carried out between the use of Laplace transform and Melin transform (as suggested by Tamilarasi and Paulraj (SC 26:8497–8507, 2022)) for de-neutrosophication of N numbers in the context of the present problem. Numerical example and complex real problem are illustrated to show the functionality and applicability of the proposed improved approach.

Keywords Multi-level linear programming (MLLP) problem · Interval valued trapezoidal neutrosophic number (IVTrpNN) · Probability density function · Laplace transform · Satisfactory solution

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1 Background of the problem and review recap

Multi-level programming (MLP) problem is an important class of mathematical programming problems (MPPs) that has an in-built hierarchical structure (level-wise) within the problem with more than two levels. Such problems are more suitable for modeling real-world problems as most complex real problems have a hierarchical structure within them. Bi-level programming problem (BLPP) is a reduced case of MLP problem with only two hierarchical levels. MLP problems have the essential characteristic of having interacting decision-making units in the hierarchy, and each decision-maker (DM) is interested in optimizing their objectives and satisfying a standard set of constraints. A comprehensive literature review and analysis on MLP problems and BLPPs have already been detailed by Lachhwani and Dwivedi [1]. In mathematical format, a multi-level linear programming problem (MLLP problem) can be described as:

$$\begin{aligned} \underset{\bar{X}_1}{Max} \left\{ Z_1(X) = C_{11}\bar{X}_1 + C_{12}\bar{X}_2 + \dots + C_{1T}\bar{X}_T \right\} \quad (1st \text{ level}) \\ \underset{\bar{X}_2}{Max} \left\{ Z_2(X) = C_{21}\bar{X}_1 + C_{22}\bar{X}_2 + \dots + C_{2T}\bar{X}_T \right\} \quad (2nd \text{ level}) \\ \cdot \\ \cdot \\ \underset{\bar{X}_T}{Max} \left\{ Z_T(X) = C_{T1}\bar{X}_1 + C_{T2}\bar{X}_2 + \dots + C_{TT}\bar{X}_T \right\} \quad (T - th \text{ level}) \end{aligned}$$

$$\text{Subject to, } \bar{A}_{1l}\bar{X}_1 + \bar{A}_{2l}\bar{X}_2 + \dots + \bar{A}_{Tl}\bar{X}_T (\leq, =, \geq) b_l; 1 \leq l \leq p$$

$$\text{and } \bar{X}_1 \geq 0, \bar{X}_2 \geq 0, \dots, \bar{X}_T \geq 0. \quad (1)$$

where $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_T$ are respective set of controlling decision variables at each level. $Z_t(X) = C_{t1}\bar{X}_1 + C_{t2}\bar{X}_2 + \dots + C_{tT}\bar{X}_T, 1 \leq t \leq T$ is the linear objective function at t -th level DM and other notations have the usual meanings.

Over the last fifteen years, a tremendous amount of research has been carried out in developing new solution methodologies for MLP problems and their other extension problems i.e. MLMOLP problems (multi-level multiobjective linear programming problems), MLMONLP problem (multi-level multi-objective non-linear programming problem), MLMOLFP/MLMOL+FP problem (multi-level multiobjective linear/linear plus fractional programming problem), etc. Some notable recent research contributions are: Pramanik et al. [2] developed and demonstrated a solution technique for solving multi-level multiobjective linear plus fractional programming problems (MLMOL+FP problems). Lachhwani [3] modified the existing FGP methodology for obtaining solutions to MLMOLFP problems comparatively with very less computations. Osman et al. [4] proposed a solution technique for solving MLMOP problem with parameters involved as fuzzy demands. Liu and Yang [5] developed a new technique for solving MLMOLP problem.

In the present era, fuzzy and intuitionistic data are used in the decision-making and problem formulation process of MLP problems due to the availability of rough information (vague and imprecise) to decision-makers. Many scholars have suggested important solution methodologies for fuzzy mathematical programming problems (FMPPs) i.e. Fuzzy LPPs, Fuzzy MOPPs, Fuzzy BLPPs, Fuzzy MLP problems, etc. Some of the research contributions which involve the use of integral transform technique in solving FMPP are: Peraei et al. [6] used the concept of Melin transform for solving FLPP. NA Alauden and MY Sanar [7] used Melin transform for de-fuzzification of fuzzy numbers in solving FLPPs. But the fuzzy theory has a major drawback of ignoring two other aspects of information i.e. indeterminacy and falsity in formulating membership functions. To eliminate the shortcomings of the fuzzy set (Zadeh [8]) and intuitionistic fuzzy theory (Atanassov [9]), Smarandache [10] introduced neutrosophic set theory—theory with three parts of information i.e. truthiness, falsity, and indeterminacy. This theory plays a vital role in describing industrial problems having vague and indeterminate information. Nowadays, MPPs are extended towards new and active research areas ‘under neutrosophic environment’. Many researchers have suggested new research ideas on MPPs in the neutrosophic environment: Ye [11] suggested a novel technique for LPPs with N numbers. Mohamed et al. [12] suggested a method for solving integer programming problems with trapezoidal N numbers. Abdel-Basset et al. [13] suggested a new type of ranking function for N numbers for conversion of N numbers into equivalent crisp values and proposed to apply this new ranking function to solve the fully neutrosophic (IVTr N numbers) linear programming problems (LPPs). Pramanik and Banerjee [14] developed a solution technique for MOLP problem under N number environment. Pramanik and Dey [15] suggested another solution technique for solving two-level programming problems (BLPPs) under N number environments. Bera and Mahapatra [16] developed a new simplex method (big-M method for N numbers) to solve real-life problems formulated as neutrosophic LPPs. Darehmiraki [17] proposed a new parametric ranking function to solve neutrosophic linear programming problems. Basumatary and Broumi [18] proposed a new type of neutrosophic MPP with parameters as interval-valued triangular N numbers. Khatter [19] proposed a solution approach to solve neutrosophic LP problem with a possibilistic mean. Maiti et al. [20] proposed a technique based on goal programming for MLMOP problem under N numbers. Lachhwani [21] suggested an algorithm for solving fully neutrosophic MLMOLP problems with all coefficients and parameters as trapezoidal N numbers. Ahmad [22] suggested a solution technique for MOPP under neutrosophic environment with application in the pharmaceutical supply chain planning problems. Recently, Tamilarasi and Paulraj [23] used Melin transform for de-neutrosophication of triangular type N numbers for developing the technique for linear programming problems.

The entire literature on MPPs under neutrosophic environment reveals that most of the researchers used ranking method, accuracy function value, score function values, etc. for the de-neutrosophication of N numbers. Except for Tamilarasi and Paulraj [23] who used Melin transform on triangular type N numbers), the literature does not provide any use of integral transform for de-neutrosophication of N numbers till now. This motivates us to extend the use of important integral

transform i.e. Laplace transform for de-neutrosophication of fully N numbers i.e. interval values trapezoidal neutrosophic numbers (IVTrpN numbers) in solving more complex multi-level programming problems. The present study of fully neutrosophic MLLP problem with Laplace transform has not been studied in the literature. This makes the proposed study unique and novel in this research area. The main objectives of the paper:

1. To propose a novel method of defining proportional probability density function for IVTrpN numbers and de-neutrosophication of IVTrpN numbers with the use of Laplace transform.
2. To propose an improved solution algorithm for MLLP problem with IVTrpN numbers as coefficients/ parameters in objective functions and constrained functions.

In abstract form, the present paper describes a unique solution methodology for generalized multi-level linear programming involving coefficient parameters in an objective at each level as well as constraints as interval-valued trapezoidal neutrosophic numbers (IVTrp N numbers), based on Laplace transform. In this approach, we first propose to associate the probability density function to each membership function of each IVTrpN numbers and obtain an equivalent output value of each N numbers using Laplace transform. After that, the multi-level structure of the crisp problem is tackled by formulating separate membership functions for each objective function at each level and decision variables up to the (T-1) level with their best values. A simple solution model is formulated to obtain a satisfactory solution of MLLP problem under neutrosophic environment with the help of usual goal programming. Further, a comparative study is also carried out between the use of the Laplace transform and Melin transform (as suggested by Tamilarasi and Paulraj [23]) for de-neutrosophication of SVTrN numbers. Numerical example and solution of complex real problem are illustrated to show the functionality and applicability of the proposed improved approach.

This paper is organized as: in the first section of the paper, a background of the problem along with a review recap on specific MPPs and MPPs under neutrosophic environment are suggested. In Sect. 2, literature review analysis is suggested in tabular form on different solution techniques for MLP problems and extension problems under N number environment. Based on this review analysis, the research gap and significance of the proposed technique are also discussed. Some preliminaries on neutrosophic numbers (N numbers) are described in the next section. The proposed algorithm for MLLP problem with N numbers is presented in detail in Sect. 4. The introduction of proportional probability density functions to each N number and the use of the Laplace transform for finding equivalent crisp values of N numbers are discussed in Sects. 4.1 and 4.2 respectively. The step-by-step process and a flow chart of the proposed algorithm for solving MLP problems with IVTrpN numbers are also suggested in the same section. Further, a comparative study is also carried out between the use of the Laplace transform and Melin transform (as suggested by Tamilarasi and Paulraj

[23]) for de-neutrosophication of N numbers in Sect. 5. In Sect. 6, a numerical example is illustrated with the proposed technique to show its computational simplicity and uniqueness in solving MLP problems with IVTrpN numbers. The application of the proposed technique in solving the real problem is described in the same section. Concluding remarks and future research directions are discussed in the last section.

2 Review analysis and identification of research gap

As detailed in the previous section it can be seen that ‘MPP under N number environment’ is a new area of research and limited number of research scholars have contributed in this area. In continuation of the literature review on solving different types of MPPs with N numbers can be described in tabular form (Table 1). From Table 1, it is evident that researchers preferably used score functions or accuracy functions as ranking methods to convert N numbers. But recently Tamilarasi and Paulraj [23] used Melin transform for de-neutrosophication of triangular type N numbers for developing the technique for linear programming problems. This motivates us to extend the use of important integral transform i.e. Laplace transform for de-neutrosophication of interval values trapezoidal neutrosophic numbers (IVTrpN numbers) in solving more complex multi-level linear programming problems. Further, in the above literature review and analysis, it can be observed that no researcher has proposed a solution method for MLP problems with IVTrpN numbers using Laplace transform. In all, the proposed algorithm is improved and unique for solving MLP problems under N number environment.

3 Neutrosophic set theory—preliminaries

Neutrosophic set theory is a paradigm shift from traditional fuzzy and intuitionistic fuzzy set theory which includes three major parts of information i.e. truthiness, indeterminacy, and falsity information. This N set theory is used to analyze imprecise and inaccurate information. In literature, this theory was extended with properties of N set and neutrosophic statistics by Smarandache ([24, 25]) respectively. Some preliminaries on N numbers and related terms are described as:

Definition 1 (Smarandache [10]): a neutrosophic set A in X is characterized as $A = \{x : \mu_A^T(x), \sigma_A^T(x), v_A^T(x), x \in X\}$ where $\mu_A^T(x), \sigma_A^T(x), v_A^T(x) \in (0, 1)$ represents degree of membership for truthiness, indeterminacy and falsity parts of information respectively along with condition.

$$0^+ \leq \text{Sup } \mu_A^T(x) + \text{Sup } \sigma_A^T(x) + \text{Sup } v_A^T(x) \leq 3^-$$

Definition 2 (Interval valued trapezoidal neutrosophic number IVTrpN number) (Abdel-Basset et al. [13]) Let $\mu_{\tilde{a}}, \sigma_{\tilde{a}}, v_{\tilde{a}} \in [0, 1]$ and $a_i \in R, R$ the real numbers and $a_1 \leq a_2 \leq a_3 \leq a_4$. Then an interval-valued trapezoidal neutrosophic number is

Table 1 Comparison of research contributions on MP problems with fuzzy and/ or N numbers

S. No.	Authors information	MP problem studied	Coefficients/parameters in MP problems		Types of numbers		Research contributions	Integral transform used in developing solution technique ? Name of integral transform used
			fuzzy numbers		N numbers			
			N numbers	N numbers	Fuzzy Numbers	N numbers		
1	Pramanik and Dey [15]	BLPP	×	✓	×	N number Coefficients in linear format $c + dI$ where I indicates indeterminacy	Problem was converted into crisp problem using interval technique	×
2	Maiti et al. [20]	MLMOLP problems	×	✓	×	N number coefficients considered in linear form $c + dI$ where I indicates indeterminacy	N numbers were changed into intervals and accordingly problem was converted into interval problem Goal programming was used to obtain satisfactory solution to the problem	×
3	Abdel-Basset et al. [13]	LP problem	×	✓	×	fully N numbers in the form IVTrpN numbers (interval valued trapezoidal N numbers)	Problem was converted into crisp problem with the help of ranking function	×
4	Ahmad [22]	General MOP problems	×	✓	×	Triangular N numbers	New score function was defined Original MOPP was converted into three different solution models with the use of score function	×

Table 1 (continued)

S. No.	Authors information	MP problem studied	Coefficients/parameters in MP problems		Types of numbers		Research contributions	Integral transform used in developing solution technique ? Name of integral transform used
			fuzzy Numbers		Fuzzy Numbers			
			N numbers	N numbers	N numbers	N numbers		
6	Lachhwani [21]	MLMOP problem	×	✓	×	Fully N number in the form of IV/Trp N numbers	New ranking function was defined Problem was converted into crisp problem with the use of new ranking function	×
7	Khatter [19]	LP problem	×	✓	×	single-valued triangular neutrosophic numbers (SV Tr N numbers)	Possibility mean of truthiness, indeterminacy and falsity membership function was defined N numbers were converted into to weighted value using possibilistic mean and problem converted into crisp problem	×

Table 1 (continued)

S. No.	Authors information	MP problem studied	Coefficients/parameters in MP problems		Types of numbers		Research contributions	Integral transform used in developing solution technique ?
			Fuzzy Numbers	N numbers	Fuzzy Numbers	N numbers		
8	Tamilarasi and Paulraj [23]	LP problem	×	✓	×	single-valued triangular neutrosophic numbers (SVTrN numbers)	Melin transform was used for de-neutrosophication of triangular type N numbers Original problem was converted into crisp LPP	Yes Melin transform
9	Peraei et al. [6]	LP problem	✓	×	×	Fuzzy Numbers	Used concept of Melin transform for solving FLPP	Yes Melin transform
10	Atauden and Sanar [7]	LP problem	✓	×	×	Fuzzy Numbers	De-fuzzification of fuzzy numbers of FLPPs was carried out using Melin transform	Yes Melin transform
11	Darehmiraki [17]	LP problem	×	✓	×	Triangular N numbers	A new parametric ranking function was introduced to solve the problem	×
13	Basumatary and Broumi [18]	LP problem	×	✓	×	interval-valued triangular N numbers	Proposed a new LP problem with parameters as interval-valued triangular N numbers	×

Table 1 (continued)

S. No.	Authors information	MP problem studied	Coefficients/parameters in MP problems		Types of numbers		Research contributions	Integral transform used in developing solution technique ? Name of integral transform used
			fuzzy Numbers	N numbers	Fuzzy Numbers	N numbers		
14	Bera and Mahapatra [16]	LP problem	×	√	×	Single-valued trap-ezoidal N numbers	Developed the big-M simplex method for solving LPP and applied it to real-life problems	×

given as: $\tilde{a} = \langle (a_1, a_2, a_3, a_4); [\mu_a^L, \mu_a^U], [\sigma_a^L, \sigma_a^U], [v_a^L, v_a^U] \rangle$ whose membership function for truthiness, indeterminacy and falsity are

$$\mu_{\tilde{a}}(x) = \begin{cases} \mu_a^U \left(\frac{x-a_1}{a_2-a_1} \right), & a_1 \leq x \leq a_2, \\ \mu_a^U, & a_2 \leq x \leq a_3, \\ \mu_a^U \left(\frac{a_4-x}{a_4-a_3} \right), & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

$$\sigma_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x+\sigma_a^L(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \sigma_a^L, & a_2 \leq x \leq a_3, \\ \frac{x-a_3+\sigma_a^L(a_4-x)}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 1, & \text{otherwise,} \end{cases} \tag{3}$$

$$v_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x+v_a^L(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ v_a^L, & a_2 \leq x \leq a_3, \\ \frac{x-a_3+v_a^L(a_4-x)}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 1, & \text{otherwise,} \end{cases} \tag{4}$$

In view of this definition, there is not much significance of μ_a^L, σ_a^U and v_a^U in achieving membership values. Therefore for simplicity, we here assume $\mu_a^U = \mu_{\tilde{a}}, \sigma_a^L = \sigma_{\tilde{a}}$ and $v_a^L = v_{\tilde{a}}$. Accordingly, IVTrp N numbers are rewritten as:

$$\tilde{a} = \langle (a_1, a_2, a_3, a_4); \mu_{\tilde{a}}, \sigma_{\tilde{a}}, v_{\tilde{a}} \rangle. \tag{5}$$

Definition 3 (Single valued triangular neutrosophic number SVTrN number) (Tamilarasi and Paulraj [23]) Let $\mu_{\tilde{a}}, \sigma_{\tilde{a}}, v_{\tilde{a}} \in [0, 1]$ and $a_i \in R, R$ the real numbers and $a_1 \leq a_2 \leq a_3$. Then a single valued triangular neutrosophic number is given as: $\tilde{a} = \langle (a_1, a_2, a_3); \mu_{\tilde{a}}, \sigma_{\tilde{a}}, v_{\tilde{a}} \rangle$ whose membership function for truthiness, indeterminacy and falsity are

$$\mu_{\tilde{a}}(x) = \begin{cases} \mu_{\tilde{a}} \left(\frac{x-a_1}{a_2-a_1} \right), & a_1 \leq x \leq a_2, \\ \mu_{\tilde{a}}, & x = a_2, \\ \mu_{\tilde{a}} \left(\frac{a_4-x}{a_4-a_3} \right), & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

$$\sigma_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x+\sigma_{\tilde{a}}(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \sigma_{\tilde{a}}, & x = a_2, \\ \frac{x-a_3+\sigma_{\tilde{a}}(a_4-x)}{a_4-a_3}, & a_2 \leq x \leq a_3, \\ 1, & \text{otherwise.} \end{cases} \tag{7}$$

$$v_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x+v_{\tilde{a}}(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ v_{\tilde{a}}, & x = a_2, \\ \frac{x-a_2+v_{\tilde{a}}(a_3-x)}{a_3-a_2}, & a_2 \leq x \leq a_3, \\ 1, & \text{otherwise.} \end{cases} \tag{8}$$

Definition 4 (Score function) (Khatter [19]): the score function for the interval valued neutrosophic number $\tilde{a} = \langle (a_1, a_2, a_3, a_4); [\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U], [\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U], [v_{\tilde{a}}^L, v_{\tilde{a}}^U] \rangle$ is defined as:

$$S(\tilde{a}) = \frac{1}{16}(a_1 + a_2 + a_3 + a_4)(1 + \mu_{\tilde{a}}^L + \mu_{\tilde{a}}^U - \sigma_{\tilde{a}}^L - \sigma_{\tilde{a}}^U - v_{\tilde{a}}^L - v_{\tilde{a}}^U + 1). \tag{9}$$

Definition 5 (Accuracy function) (Khatter [19]): accuracy function for IVTrp N number $\tilde{a} = \langle (a_1, a_2, a_3, a_4); [\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U], [\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U], [v_{\tilde{a}}^L, v_{\tilde{a}}^U] \rangle$ is defined as:

$$A(\tilde{a}) = \frac{1}{8}(a_1 - a_2 - a_3 + a_4)(1 + \mu_{\tilde{a}}^L + \mu_{\tilde{a}}^U - v_{\tilde{a}}^L - v_{\tilde{a}}^U + 1). \tag{10}$$

Definition 6 (Ranking function for interval valued trapezoidal N number) (Abdel-Basset et al. [13]): ranking function for IVTrpN number Let $\tilde{a} = (a_1, a_2, a_3, a_4; \mu_{\tilde{a}}, \sigma_{\tilde{a}}, v_{\tilde{a}})$, then corresponding ranking function for maximization purpose is defined as:

$$R(\tilde{a}) = \left(\frac{a_1 + a_4 + 2(a_3 + a_4)}{2} \right) + (\mu_{\tilde{a}} - \sigma_{\tilde{a}} - v_{\tilde{a}}). \tag{11}$$

It is important to mention that score function, accuracy function and ranking function are used to convert N numbers into equivalent crisp values.

4 Proposed improved solution technique for MLLP problem with IVTrpN numbers

In this part of the paper, we describe the proposed solution methodology for MLLP problem with IVTrpN numbers in three subsections ahead. Firstly, we propose to associate proportional probability density function (*p.d.f.*) and introduce the use of Laplace transform on *p.d.f.* to obtain the equivalent crisp value of the corresponding IVTrpN numbers in the following subsections:

4.1 Proportional probability density function to membership functions

Here, we propose to associate proportional *p.d.f.* corresponding to its truthiness, indeterminacy and falsity membership functions of IVTrpN number $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \mu_{\tilde{a}}, \sigma_{\tilde{a}}, v_{\tilde{a}} \rangle$. For this, let $f_1(x), f_2(x), f_3(x)$ be the *p.d.f.* corresponding to membership functions $\mu_{\tilde{a}}, \sigma_{\tilde{a}}, v_{\tilde{a}}$ respectively. Then $f_1(x) = k_1\mu_{\tilde{a}}(x)$, $f_2(x) = k_2\sigma_{\tilde{a}}(x)$ and $f_3(x) = k_3v_{\tilde{a}}(x)$ where k_1, k_2 and k_3 are constants to be obtained using properties of *p.d.f.*

$$(i) \int_{-\infty}^{\infty} f_1(x)dx = 1 \Rightarrow k_1 \left[\int_{a_1}^{a_2} \mu_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right)dx + \int_{a_2}^{a_3} \mu_{\tilde{a}}dx + \int_{a_3}^{a_4} \mu_{\tilde{a}}\left(\frac{a_4-x}{a_4-a_3}\right)dx \right] = 1$$

$$\Rightarrow k_1 \left[-a_1\mu_{\tilde{a}} - \frac{a_1^2\mu_{\tilde{a}}}{2(-a_1+a_2)} + \frac{a_2^2\mu_{\tilde{a}}}{2(-a_1+a_2)} + (-a_2+a_3)\mu_{\tilde{a}} + a_4\mu_{\tilde{a}} + \frac{a_3^2\mu_{\tilde{a}}}{2(-a_3+a_4)} - \frac{a_4^2\mu_{\tilde{a}}}{2(-a_3+a_4)} \right] = 1$$

$$\text{This will give us } k_1 = -\frac{2}{(a_1 + a_2 - a_3 - a_4)\mu_{\tilde{a}}}. \tag{12}$$

$$(ii) f_2(x) = k_2\sigma_{\tilde{a}}(x), \text{ then } \int_{-\infty}^{\infty} f_2(x)dx = 1$$

$$\Rightarrow k_2 \left[\int_{a_1}^{a_2} \left(\frac{a_2 - x + \sigma_{\tilde{a}}(x - a_1)}{a_2 - a_1} \right) dx + \int_{a_2}^{a_3} \sigma_{\tilde{a}} dx + \int_{a_3}^{a_4} \left(\frac{x - a_3 + \sigma_{\tilde{a}}(a_4 - x)}{a_4 - a_3} \right) dx \right] = 1$$

$$\Rightarrow k_2 \left[(-a_2 + a_3)\sigma_{\tilde{a}} - \frac{1}{2}(a_1 - a_2)(1 + \sigma_{\tilde{a}}) - \frac{1}{2}(a_3 - a_4)(1 + \sigma_{\tilde{a}}) \right] = 1.$$

$$\text{It give us } k_2 = -\frac{2}{(a_1 - a_2 + a_3 - a_4) + \sigma_{\tilde{a}}(a_1 + a_2 - a_3 - a_4)}. \tag{13}$$

$$(iii) \text{ Similarly for } f_3(x) = k_3v_{\tilde{a}}(x) \text{ and } \int_{-\infty}^{\infty} f_3(x)dx = 1 \text{ which yield}$$

$$k_3 = -\frac{2}{(a_1 - a_2 + a_3 - a_4) + v_{\tilde{a}}(a_1 + a_2 - a_3 - a_4)}. \tag{14}$$

Now let $f(x)$ be the complete probability density function of IVTrpN number [defined in (5)] in consideration of all membership functions, then $f(x)$ can be defined as:

$$f(x) = \lambda_1 f_1(x) + \lambda_2 f_2(x) + \lambda_3 f_3(x)$$

$$\text{i.e. } f(x) = \lambda_1 k_1 \mu_{\tilde{a}}(x) + \lambda_2 k_2 \sigma_{\tilde{a}}(x) + \lambda_3 k_3 v_{\tilde{a}}(x) \tag{15}$$

where $\lambda_1, \lambda_2, \lambda_3 \in [0, 1]$ are relative weights to different functions selected by decision maker in accordance with preference information available to DM with condition $\lambda_1 + \lambda_2 + \lambda_3 = 1$.

4.2 Use of Laplace transform

Integral transform is normally used to convert a hard function into a simple one according to functional specifications. The integral transform of a specifically defined linear function is associated with the unique equivalent value of that function. Here we propose to use Laplace Transform on proportional complete probability density function to obtain an equivalent value of N number.

Definition 7 Laplace transform of function $f(x)$ ($0 < x < \infty$) of class ‘A’ is defined as

$$L_X(s) = \int_0^\infty e^{-sx}f(x)dx,$$

Whenever it exists. Here $f(x)$ is complete probability density function associated with IVTrpN number and X denotes the random variable corresponding to the IVTrpN numbers. Laplace Transform has the unique property of one-to-one correspondence in two domains (x, s) i.e. $f(x) \leftrightarrow L_X(s)$. This property is taken as a tool for finding equivalent value for each of IVTrpN number with the assumption of least value of s, i.e. $s = 1$. Laplace transform of *p.d.f.* of IVTrpN number (12) can be described as:

$$\begin{aligned} L_X(s) &= \int_0^\infty e^{-sx}[\lambda_1f_1(x) + \lambda_2f_2(x) + \lambda_3f_3(x)]dx \\ \Rightarrow L_X(s) &= \frac{2\lambda_1}{(a_3 + a_4 - a_1 - a_2)\mu_a^2} \left[\int_{a_1}^{a_2} e^{-sx} \mu_a \left(\frac{x - a_1}{a_2 - a_1} \right) dx + \int_{a_2}^{a_3} e^{-sx} \mu_a dx + \int_{a_3}^{a_4} e^{-sx} \mu_a \left(\frac{a_4 - x}{a_4 - a_3} \right) dx \right] \\ &- \frac{2\lambda_2}{(a_1 - a_2 + a_3 - a_4 + \sigma_a a_1 + \sigma_a a_2 - \sigma_a a_3 - \sigma_a a_4)} \left[\int_{a_1}^{a_2} e^{-sx} \left(\frac{a_2 - x + \sigma_a(x - a_1)}{a_2 - a_1} \right) dx + \int_{a_2}^{a_3} e^{-sx} \sigma_a dx \right. \\ &+ \left. \int_{a_3}^{a_4} e^{-sx} \left(\frac{x - a_3 + \sigma_a(a_4 - x)}{a_4 - a_3} \right) dx \right] \\ &- \frac{2\lambda_3}{(a_1 - a_2 + a_3 - a_4 + v_a a_1 + v_a a_2 - v_a a_3 - v_a a_4)} \left[\int_{a_1}^{a_2} e^{-sx} v_a \left(\frac{a_2 - x + v_a(x - a_1)}{a_2 - a_1} \right) dx + \int_{a_2}^{a_3} e^{-sx} v_a dx \right. \\ &+ \left. \int_{a_3}^{a_4} e^{-sx} \left(\frac{x - a_3 + v_a(a_4 - x)}{a_4 - a_3} \right) dx \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow L_X(s) = & \frac{2\lambda_1}{(a_3 + a_4 - a_1 - a_2)\mu_{\tilde{a}}} \left[\mu_{\tilde{a}} \left(\frac{e^{-a_2s} - e^{-a_3s}}{s} \right) - \mu_{\tilde{a}} \frac{e^{-(a_1+a_2)s}(e^{a_2s} + e^{a_1s}(1 + a_1s - a_2s))}{(a_1 - a_2)s^2} \right. \\ & \left. + \mu_{\tilde{a}} \frac{e^{-(a_3+a_4)s}(-e^{a_3s} + e^{a_4s}(1 + a_3s - a_4s))}{(a_3 - a_4)s^2} \right] \\ & - \frac{2\lambda_2}{(a_1 - a_2 + a_3 - a_4 + \sigma_{\tilde{a}}a_1 + \sigma_{\tilde{a}}a_2 - \sigma_{\tilde{a}}a_3 - \sigma_{\tilde{a}}a_4)} \left[\frac{e^{-(a_1+a_2)s}(-e^{-a_2s}(-1 - a_1s + a_2s + \sigma_{\tilde{a}}))}{(a_1 - a_2)s^2} \right. \\ & + \frac{e^{-(a_1+a_2)s}(e^{a_1s}(-1 + \sigma_{\tilde{a}} - a_1s\sigma_{\tilde{a}} + a_2s\sigma_{\tilde{a}}))}{(a_1 - a_2)s^2} + \frac{e^{-(a_3+a_4)s}(-e^{-a_3s}(-1 + a_3s - a_4s + \sigma_{\tilde{a}}))}{(a_3 - a_4)s^2} \\ & \left. + \frac{e^{-(a_3+a_4)s}(e^{a_4s}(-1 + \sigma_{\tilde{a}} + a_3s\sigma_{\tilde{a}} - a_4s\sigma_{\tilde{a}}))}{(a_3 - a_4)s^2} + \frac{(e^{-a_2s} - e^{-a_3s})\sigma_{\tilde{a}}}{s} \right] \\ & - \frac{2\lambda_3}{(a_1 - a_2 + a_3 - a_4 + v_{\tilde{a}}a_1 + v_{\tilde{a}}a_2 - v_{\tilde{a}}a_3 - v_{\tilde{a}}a_4)} \left[\frac{e^{-(a_1+a_2)s}(e^{-a_2s}(-1 - a_1s + a_2s + v_{\tilde{a}}))}{(a_1 - a_2)s^2} \right. \\ & + \frac{e^{-(a_1+a_2)s}(e^{a_1s}(-1 + v_{\tilde{a}} - a_1sv_{\tilde{a}} + a_2sv_{\tilde{a}}))}{(a_1 - a_2)s^2} + \frac{e^{-(a_3+a_4)s}(-e^{-a_3s}(-1 + a_3s - a_4s + v_{\tilde{a}}))}{(a_3 - a_4)s^2} \\ & \left. + \frac{e^{-(a_3+a_4)s}(e^{a_4s}(-1 + v_{\tilde{a}} + a_3sv_{\tilde{a}} - a_4sv_{\tilde{a}}))}{(a_3 - a_4)s^2} + \frac{(e^{-a_2s} - e^{-a_3s})v_{\tilde{a}}}{s} \right] \end{aligned} \tag{16}$$

In order to find equivalent crisp values $E(\tilde{a})$ of each of neutrosophic coefficient/parameter of the problem with minimum mathematical computation, here we have considered the least value of s i.e. $s = l$. It is to be noted that $E(\tilde{a})$ only represents the corresponding equivalent crisp value of N number.

For $s = l$, the corresponding equivalent value will be as:

$$\begin{aligned} E(\tilde{a}) = & \frac{2\lambda_1}{(a_3 + a_4 - a_1 - a_2)} \left[\frac{e^{-a_1-a_2-a_3-a_4}((-a_1 + a_2)e^{a_1+a_2+a_3} + (a_1 - a_2)e^{a_1+a_2+a_4}}{(a_1 - a_2)(a_3 - a_4)} \right. \\ & \left. + \frac{e^{-a_1-a_2-a_3-a_4}((a_3 - a_4)e^{a_1+a_3+a_4} + (-a_3 + a_4)e^{a_2+a_3+a_4})}{(a_1 - a_2)(a_3 - a_4)} \right] \\ & + \frac{2\lambda_2}{(a_2 - a_3 + a_4 - \sigma_{\tilde{a}}a_2 + \sigma_{\tilde{a}}a_3 + \sigma_{\tilde{a}}a_4 - a_1(1 + \sigma_{\tilde{a}}))} \left[\frac{e^{-(a_1+a_2)}(-e^{-a_2}(-1 - a_1 + a_2 + \sigma_{\tilde{a}}))}{(a_1 - a_2)} \right. \\ & + \frac{e^{-(a_1+a_2)}(e^{a_1}(-1 + \sigma_{\tilde{a}} - a_1\sigma_{\tilde{a}} + a_2\sigma_{\tilde{a}}))}{(a_1 - a_2)} + \frac{e^{-(a_3+a_4)}(-e^{-a_3}(-1 + a_3 - a_4 + \sigma_{\tilde{a}}))}{(a_3 - a_4)} \\ & \left. + \frac{e^{-(a_3+a_4)}(e^{a_4}(-1 + \sigma_{\tilde{a}} + a_3\sigma_{\tilde{a}} - a_4\sigma_{\tilde{a}}))}{(a_3 - a_4)} + \frac{(e^{-a_2} - e^{-a_3})\sigma_{\tilde{a}}}{1} \right] \\ & + \frac{2\lambda_3}{(a_2 - a_3 + a_4 - v_{\tilde{a}}a_2 + v_{\tilde{a}}a_3 + v_{\tilde{a}}a_4 - a_1(1 + v_{\tilde{a}}))} \left[\frac{e^{-(a_1+a_2)}(-e^{-a_2}(-1 - a_1 + a_2 + v_{\tilde{a}}))}{(a_1 - a_2)} \right. \\ & + \frac{e^{-(a_1+a_2)}(e^{a_1}(-1 + v_{\tilde{a}} - a_1v_{\tilde{a}} + a_2v_{\tilde{a}}))}{(a_1 - a_2)} + \frac{e^{-(a_3+a_4)}(-e^{-a_3}(-1 + a_3 - a_4 + v_{\tilde{a}}))}{(a_3 - a_4)} \\ & \left. + \frac{e^{-(a_3+a_4)}(e^{a_4}(-1 + v_{\tilde{a}} + a_3v_{\tilde{a}} - a_4v_{\tilde{a}}))}{(a_3 - a_4)} + \frac{(e^{-a_2} - e^{-a_3})v_{\tilde{a}}}{1} \right] \end{aligned} \tag{17}$$

This gives the unique equivalent value corresponding to each of IVTrpN number, therefore it is proposed to use this equivalent value for the purpose of converting the problem into crisp problem.

4.3 Improved solution of MLLP problems with interval valued TrpN numbers: Laplace transform approach

In this section, we apply the proportional *p.d.f.* to each N number and Laplace transform on complete *p.d.f.* to obtain equivalent crisp values in solving MLLP problem under N number environment in following manner:

Mathematically MLLP problem with interval valued trapezoidal N numbers is described in matrix form as:

$$\begin{aligned}
 & \underset{\bar{X}_1}{Max/Min} \left\{ \widetilde{Z}_1(\bar{X}) = \widetilde{C}_{11}\bar{X}_1 + \widetilde{C}_{12}\bar{X}_2 + \dots + \widetilde{C}_{1T}\bar{X}_T \right\} \\
 & \underset{\bar{X}_2}{Max/Min} \left\{ \widetilde{Z}_2(\bar{X}) = \widetilde{C}_{21}\bar{X}_1 + \widetilde{C}_{22}\bar{X}_2 + \dots + \widetilde{C}_{2T}\bar{X}_T \right\} \\
 & \cdot \\
 & \cdot \\
 & \underset{\bar{X}_T}{Max/Min} \left\{ \widetilde{Z}_T(\bar{X}) = \widetilde{C}_{T1}\bar{X}_1 + \widetilde{C}_{T2}\bar{X}_2 + \dots + \widetilde{C}_{TT}\bar{X}_T \right\} \\
 & \text{Subject to, } \widetilde{A}_1\bar{X}_1 + \widetilde{A}_2\bar{X}_2 + \dots + \widetilde{A}_T\bar{X}_T (\leq, =, \geq) \widetilde{b}
 \end{aligned}$$

$$\text{And } \bar{X}_1, \bar{X}_2, \dots, \bar{X}_T \geq 0. \tag{18}$$

where $\widetilde{A}_j = [\widetilde{a}_{ij}]$, $\widetilde{b} = [\widetilde{b}_{ij}]$, $1 \leq i \leq T; 1 \leq j \leq T$ and superscript \sim indicates all coefficients/parameters are IVTrp N number in format $\widetilde{a} = \langle (a_1, a_2, a_3, a_4); \mu_{\widetilde{a}}, \sigma_{\widetilde{a}}, v_{\widetilde{a}} \rangle$. With the techniques discussed in Sects. 4.1 and 4.2, these interval valued trapezoidal neutrosophic coefficients/parameters are translated into equivalent crisp values as given in Eq. (14), then accordingly MLLP problem with IVTrp N numbers reduces into equivalent crisp problem as:

$$\begin{aligned}
 & \underset{\bar{X}_1}{Max/Min} \left\{ [E(\widetilde{C}_{11})]\bar{X}_1 + [E(\widetilde{C}_{12})]\bar{X}_2 + [E(\widetilde{C}_{13})]\bar{X}_3 + \dots + [E(\widetilde{C}_{1T})]\bar{X}_T \right\} \\
 & \underset{\bar{X}_2}{Max/Min} \left\{ [E(\widetilde{C}_{21})]\bar{X}_1 + [E(\widetilde{C}_{22})]\bar{X}_2 + [E(\widetilde{C}_{23})]\bar{X}_3 + \dots + [E(\widetilde{C}_{2T})]\bar{X}_T \right\} \\
 & \cdot \\
 & \cdot \\
 & \underset{\bar{X}_T}{Max/Min} \left\{ [E(\widetilde{C}_{T1})]\bar{X}_1 + [E(\widetilde{C}_{T2})]\bar{X}_2 + [E(\widetilde{C}_{T3})]\bar{X}_3 + \dots + [E(\widetilde{C}_{TT})]\bar{X}_T \right\} \\
 & \text{Subject to, } \widetilde{A}_1\bar{X}_1 + \widetilde{A}_2\bar{X}_2 + \dots + \widetilde{A}_T\bar{X}_T (\leq, =, \geq) \widetilde{b}
 \end{aligned}$$

$$\text{And } \bar{X}_1, \bar{X}_2, \dots, \bar{X}_T \geq 0. \tag{19}$$

where $E(\bar{C}_{11}), E(\bar{C}_{12}), \dots, E(\bar{C}_{1T}); \dots E(\bar{C}_{T1}), E(\bar{C}_{T2}), \dots, E(\bar{C}_{TT}); A(\bar{a}_{1ij}); A(\bar{a}_{2ij}), \dots, A(\bar{a}_{Tij})$ and b_j are corresponding equivalent values of parameters and coefficients obtained by technique discussed in Sect. 4. Now, to tackle the multi-level structure of crisp problem (16), we extend the modified approach suggested by Lachhwani [3] for a multi-level single objective linear programming problem. For this, we estimate the ideal values of each objective function at each level (\bar{Z}_t and \underline{Z}_t) of the crisp problem (19). Then, we construct the linear membership functions for each objective function with individual maximum and minimum values as:

$$M_t(Z_t(\bar{X})) = \begin{cases} 1 & \text{if } Z_t(\bar{X}) \geq \bar{Z}_t \\ \frac{Z_t(\bar{X}) - \underline{Z}_t}{\bar{Z}_t - \underline{Z}_t} & \text{if } \underline{Z}_t \leq Z_t(\bar{X}) \leq \bar{Z}_t \\ 0 & \text{if } Z_t(\bar{X}) \leq \underline{Z}_t \end{cases} \quad \forall t = 1, 2, \dots, T \tag{20}$$

Further, in order to avoid decision deadlock in between of different levels due to multi-level structure we construct the linear membership function for the decision vector X_1 (up to T-1 levels) as:

$$N_{X_t}(X_t) = \begin{cases} 1 & \text{for } X_t \geq \bar{X}_t \\ \frac{X_t - X_t}{\bar{X}_t - X_t} & \text{for } \underline{X}_t \leq X_t \leq \bar{X}_t \\ 0 & \text{for } X_t \leq \underline{X}_t \end{cases} \quad \forall t = 1, 2, \dots, T - 1 \tag{21}$$

where \bar{X}_t and \underline{X}_t are considered as respective values of decision vectors at t -th level ($\forall t = 1, 2, \dots, T - 1$) which yield the maximum and minimum values of crisp objective functions ($\bar{Z}_t(\bar{X})$ and $\underline{Z}_t(\underline{X})$) of problem (19). The flexible membership goals [(22) and (23)] having aspired level unity and negative deviational variables only can be described as:

$$M_t(Z_t(\bar{X})) + d_{Z_t}^- \geq 1 \quad 1 \leq t \leq T \tag{22}$$

$$N_{X_t}(X_t) + d_t^- \geq 1 \quad 1 \leq t \leq T - 1 \tag{23}$$

Now using the simplest version of goal programming, the solution model based on proposed algorithm for MLLP problem with IVTrpN numbers can be given as:

4.3.1 Solution model

$$\min \lambda = \sum_{t=1}^T d_{Z_t}^- + \sum_{t=1}^{T-1} d_t^-$$

$$\text{Subject to, } -\bar{Z}_t + Z_t + d_{Z_t}^-(\bar{Z}_t - \underline{Z}_t) \geq 0 \quad 1 \leq t \leq T$$

$$-\bar{X}_t + X_t + d_{X_t}^-(\bar{X}_t - \underline{X}_t) \geq 0$$

$$\widetilde{A}_1\bar{X}_1 + \widetilde{A}_2\bar{X}_2 + \dots + \widetilde{A}_T\bar{X}_T (\leq, =, \geq) \widetilde{b}$$

$$\text{And } \bar{X}_1, \bar{X}_2, \dots, \bar{X}_T \geq 0. \tag{24}$$

With the above discussion, the proposed improved solution methodology for MLLP problem with IVTrpN numbers can be summarised in the following stepwise algorithm:

1. Obtain the corresponding probability density function for each membership function and complete probability density function of IVTrpN numbers as described in sub-Sect. 4.1.
2. Set the value $\lambda_1, \lambda_2, \lambda_3 \in [0, 1]$ as per the decision maker’s information on the relative importance of truthiness, intuitionistic and falsity of information and write a complete *p.d.f.* of each N numbers as given in (15).
3. Obtain the equivalent value (17) of each *p.d.f.* using Laplace transform as given in sub-Sect. 4.2.
4. Convert each interval-valued trapezoidal neutrosophic coefficient and parameters of MLLP problem into corresponding crisp values using equivalent function values and convert the problem into an equivalent crisp problem (19).
5. Estimate ideal values for each objective function at each level along with ideal values of decision variables (up to T-1 levels) irrespective of multi-level structure.
6. Construct the fuzzy linear membership function for each objective function (for T levels) and the decision variables (up to T-1 levels) as defined in Eqs. (20) and (21), respectively.
7. Formulate the solution model (24) for this reduced problem.
8. Solve solution model (24) with programming technique or software tool to obtain a satisfactory solution of MLLP problem with IVTrpN numbers.

The flow chart of the proposed algorithm for MLP problems under N number environment can be presented (in Fig. 1) as:

5 Comparison between laplace and melin transform on SVTrN numbers

Tamilarasi and Paulraj [23] used Melin transform for de-neutrosophication (converting neutrosophic numbers into equivalent crisp values) of single valued triangular neutrosophic (SVTrN) numbers for developing the technique for linear programming problems. In this section, we explore the use of the Laplace Transform on SVTrN numbers and compare the problem’s coefficients/parameters as SVTrN

numbers with that obtained through the Melin Transform as given by Tamarasi and Paulraj [23].

Let $\tilde{a} = \langle (a_1, a_2, a_3); \mu_{\tilde{a}}, \sigma_{\tilde{a}}, \nu_{\tilde{a}} \rangle$ be SVTrN numbers (as defined in Definition 3) given as coefficients/parameters of the problem under consideration. Let $f_1(x), f_2(x), f_3(x)$ be the *p.d.f.* corresponding to membership functions $\mu_{\tilde{a}}, \sigma_{\tilde{a}}, \nu_{\tilde{a}}$ respectively then these *p.d.f.* can be described (Tamarasi and Paulraj [23]) as:

$$f_1(x) = \begin{cases} \left(\frac{2(x-a_1)}{(a_2-a_1)(a_3-a_1)} \right), & a_1 \leq x \leq a_2, \\ \frac{(a_3-a_1)}{2}, & x = a_2, \\ \left(\frac{2(a_3-x)}{(a_3-a_2)(a_3-a_1)} \right), & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise.} \end{cases} \tag{25}$$

$$f_2(x) = \begin{cases} \frac{a_2-x+\sigma_{\tilde{a}}(x-a_1)}{a_2-a_1} \frac{2}{(a_3-a_1)(1+\sigma_{\tilde{a}})}, & a_1 \leq x \leq a_2, \\ \frac{\sigma_{\tilde{a}}}{(a_3-a_1)(1+\sigma_{\tilde{a}})}, & x = a_2, \\ \frac{x-a_3+\sigma_{\tilde{a}}(a_4-x)}{a_4-a_3} \frac{2}{(a_3-a_1)(1+\sigma_{\tilde{a}})}, & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

$$f_3(x) = \begin{cases} \frac{a_2-x+\nu_{\tilde{a}}(x-a_1)}{a_2-a_1} \frac{2}{(a_3-a_1)(1+\nu_{\tilde{a}})}, & a_1 \leq x \leq a_2, \\ \frac{\nu_{\tilde{a}}}{(a_3-a_1)(1+\nu_{\tilde{a}})}, & x = a_2, \\ \frac{x-a_2+\nu_{\tilde{a}}(a_3-x)}{a_3-a_2} \frac{2}{(a_3-a_1)(1+\nu_{\tilde{a}})}, & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

Accordingly, probability density function for single valued triangular neutrosophic (SVTrN) number $\tilde{a} = \langle (a_1, a_2, a_3); \mu_{\tilde{a}}, \sigma_{\tilde{a}}, \nu_{\tilde{a}} \rangle$ can be described as:

$$f(x) = \lambda f_1(x) + (1 - \lambda) f_2(x) + (1 - \lambda) f_3(x) \tag{28}$$

Now, we apply Laplace transform on *p.d.f.* (28) of SVTrN number of the problem in following manner as:

$$\Rightarrow L_X(s) = \frac{2\lambda}{(a_3 - a_1)} \left[\frac{e^{-(a_1+a_2)s}(-e^{a_2s} + e^{a_1s}(1 - a_1s + a_2s))}{(a_1 - a_2)s^2} + \frac{e^{-(a_2+a_3)s}(-e^{a_2s} + e^{a_3s}(1 + a_2s - a_3s))}{(a_2 - a_3)s^2} \right] \\ - \frac{2(1 - \lambda)}{(a_3 - a_1)(1 + \nu_{\tilde{a}})} \left[\frac{e^{-(a_2+a_3)s}(-e^{-a_2s} + e^{a_3s}(1 + a_2s - a_3s))(1 - \nu_{\tilde{a}})}{(a_2 - a_3)s^2} \right. \\ \left. + \frac{e^{-(a_1+a_2)s}(-e^{a_2s}(-1 - a_1s + a_2s + \nu_{\tilde{a}}) + e^{a_1s}(-1 - a_1s\nu_{\tilde{a}} + a_2s\nu_{\tilde{a}} + \nu_{\tilde{a}}))}{(a_1 - a_2)s^2} \right] \\ - \frac{2(1 - \lambda)}{(a_3 - a_1)(1 + \sigma_{\tilde{a}})} \left[\frac{e^{-(a_2+a_3)s}(-e^{-a_2s} + e^{a_3s}(1 + a_2s - a_3s))(1 - \sigma_{\tilde{a}})}{(a_2 - a_3)s^2} \right. \\ \left. + \frac{e^{-(a_1+a_2)s}(-e^{a_2s}(-1 - a_1s + a_2s + \sigma_{\tilde{a}}) + e^{a_1s}(-1 - a_1s\sigma_{\tilde{a}} + a_2s\sigma_{\tilde{a}} + \nu_{\tilde{a}}))}{(a_1 - a_2)s^2} \right] \tag{29}$$

For $s = I$, the corresponding equivalent value will be as:

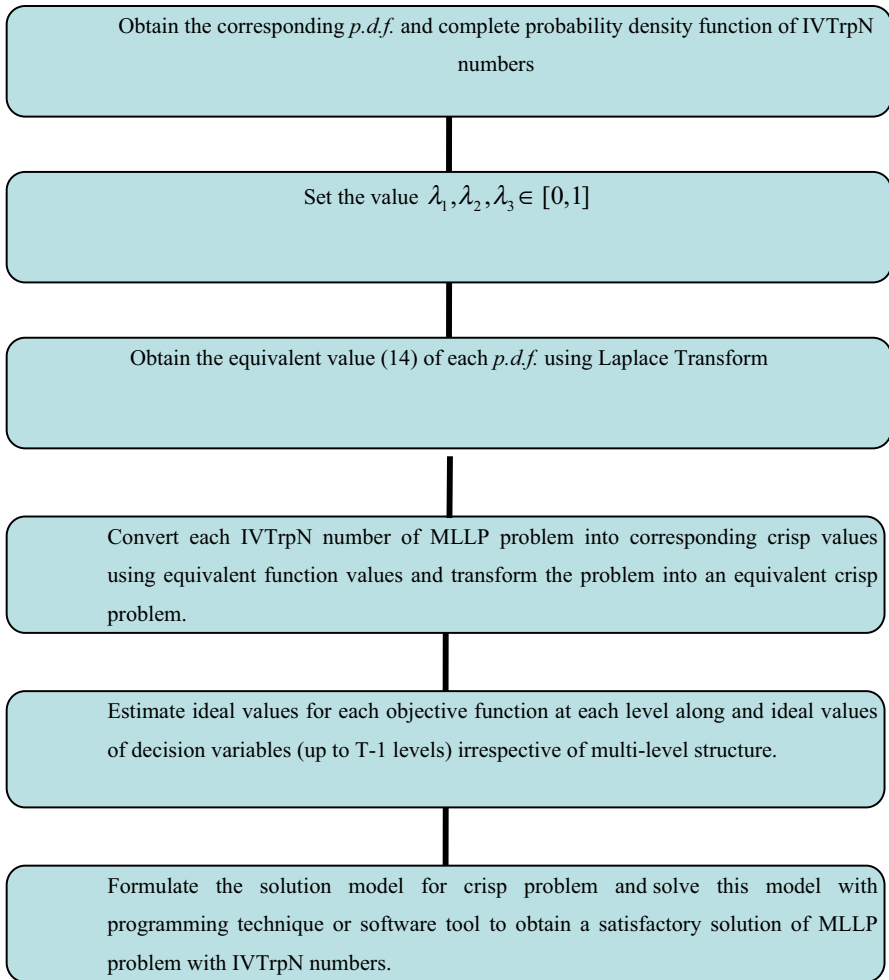


Fig. 1 Flow chart of proposed algorithm

$$\begin{aligned}
 E(\tilde{a}) = & \frac{2\lambda}{(a_3 - a_1)} \left[\frac{e^{-(a_1+a_2)}(-e^{a_2} + e^{a_1}(1 - a_1 + a_2))}{(a_1 - a_2)} + \frac{e^{-(a_2+a_3)}(-e^{a_2} + e^{a_3}(1 + a_2 - a_3))}{(a_2 - a_3)} \right] \\
 & - \frac{2(1 - \lambda)}{(a_3 - a_1)(1 + v_{\tilde{a}})} \left[\frac{e^{-(a_2+a_3)}(-e^{-a_2} + e^{a_3}(1 + a_2 - a_3))(1 - v_{\tilde{a}})}{(a_2 - a_3)} \right. \\
 & \left. + \frac{e^{-(a_1+a_2)}(-e^{a_2}(-1 - a_1 + a_2 + v_{\tilde{a}}) + e^{a_1}(-1 - a_1 v_{\tilde{a}} + a_2 v_{\tilde{a}} + v_{\tilde{a}}))}{(a_1 - a_2)} \right] \\
 & - \frac{2(1 - \lambda)}{(a_3 - a_1)(1 + \sigma_{\tilde{a}})} \left[\frac{e^{-(a_2+a_3)}(-e^{-a_2} + e^{a_3}(1 + a_2 - a_3))(1 - \sigma_{\tilde{a}})}{(a_2 - a_3)} \right. \\
 & \left. + \frac{e^{-(a_1+a_2)}(-e^{a_2}(-1 - a_1 + a_2 + \sigma_{\tilde{a}}) + e^{a_1}(-1 - a_1 \sigma_{\tilde{a}} + a_2 \sigma_{\tilde{a}} + v_{\tilde{a}}))}{(a_1 - a_2)} \right]
 \end{aligned} \tag{30}$$

This quickly gives equivalent crisp values corresponding to SVTrN numbers $\tilde{a} = \langle (a_1, a_2, a_3); \mu_{\tilde{a}}, \sigma_{\tilde{a}}, v_{\tilde{a}} \rangle$ by the Laplace transform which can be used in converting coefficients/parameters (SVTrN numbers) of the concern problem into corresponding equivalent crisp values. On the other hand, Tamilarasi and Paulraj [23] used Melin transform (with $s=2$) on SVTrN numbers which yield the following equivalent crisp values as:

$$E(\tilde{a}) = \lambda \left\{ \frac{(a_1 + a_2 + a_3)}{3} \right\} + (1 - \lambda) \left\{ \frac{\sigma_{\tilde{a}}(a_1 + a_2 + a_3) + 2(a_1 + a_3) - a_2}{3(1 + \sigma_{\tilde{a}})} \right\} \\ + (1 - \lambda) \left\{ \frac{v_{\tilde{a}}(a_1 + a_2 + a_3) + 2(a_1 + a_3) - a_2}{3(1 + v_{\tilde{a}})} \right\} \quad (31)$$

If we compare these results [(30) and (31)], it can be easily observed that both the transform gives equivalent crisp values corresponding to N numbers which are required to convert such problems (like MLLP problem with IVTrN numbers, LPP with SVTrN numbers, etc.) into corresponding crisp problems. This comparison shows that the proposed methodology based on the Laplace transform for MLLP problem with IVTrN numbers is improved in terms of computational difficulties than Melin Transform on single valued triangular neutrosophic numbers (SVTrNNs). Further, it shows that the Laplace transform can also be applied on SVTrN numbers for converting into their crisp values.

6 Numerical illustration and application of proposed solution technique in real world problem

6.1 Numerical illustrations

In this section, we illustrate one numerical example in following stepwise manner to show the functionality of algorithm for MLLP problems with IVTrpN numbers:

Example 1 A maximization type tri-level linear programming problem under N number environment is considered as:

$$\text{Max}_{x_1, x_2} \{ Z_1(X) = \widetilde{c}_{11}x_1 + \widetilde{c}_{12}x_2 + \widetilde{c}_{13}x_3 + \widetilde{c}_{14}x_4 \} \quad (1st \text{ level})$$

$$\text{Max}_{x_3} \{ Z_2(X) = \widetilde{c}_{21}x_1 + \widetilde{c}_{22}x_2 + \widetilde{c}_{23}x_3 + \widetilde{c}_{24}x_4 \} \quad (2nd \text{ level})$$

$$\text{Max}_{x_4} \{ Z_3(X) = \widetilde{c}_{31}x_1 + \widetilde{c}_{32}x_2 + \widetilde{c}_{33}x_3 + \widetilde{c}_{34}x_4 \} \quad (3rd \text{ level})$$

$$\text{Subject to, } \widetilde{a}_{11}x_1 + \widetilde{a}_{12}x_2 + \widetilde{a}_{13}x_3 + \widetilde{a}_{14}x_4 \leq 50$$

$$\widetilde{a}_{21}x_1 + \widetilde{a}_{22}x_2 - \widetilde{a}_{23}x_3 + \widetilde{a}_{24}x_4 \leq 100$$

$$\widetilde{a}_{31}x_1 + \widetilde{a}_{32}x_2 + \widetilde{a}_{33}x_3 + \widetilde{a}_{34}x_4 \leq 200$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

where coefficients in the objective function and constraint are IVTrpN numbers for-

mat respectively as: $\widetilde{Z}_i = \sum_{j=1}^4 \left\langle (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4); \mu_{\widetilde{c}_{ij}}, \sigma_{\widetilde{c}_{ij}}, \nu_{\widetilde{c}_{ij}} \right\rangle x_j$

$$\text{and } \widetilde{a}_{ij} = \left\langle (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4), \mu_{\widetilde{a}_{ij}}, \sigma_{\widetilde{a}_{ij}}, \nu_{\widetilde{a}_{ij}} \right\rangle \quad \forall j = 1, 2, 3, 4; i = 1, 2;$$

$$\widetilde{Z}_1 = \langle (1, 1.5, 2.5, 4), 0.25, 0.75, 0.25 \rangle x_1 + \langle (2, 2.5, 3, 5), 0.25, 0.75, 0.25 \rangle x_2 + \langle (2.5, 3.0, 3.5, 6); 0.20, 0.80, 0.20 \rangle x_3 + \langle (2.5, 3, 3.5, 6); 0.20, 0.80, 0.20 \rangle x_4$$

$$\widetilde{Z}_2 = \langle (0.5, 1.5, 2.5, 4.5); 0.20, 0.80, 0.20 \rangle x_1 + \langle (1.5, 2.5, 3, 5.5); 0.20, 0.80, 0.20 \rangle x_2 + \langle (2, 3, 3.5, 6.5); 0.20, 0.80, 0.20 \rangle x_3 + \langle (2.5, 3.5, 4, 5.5); 0.20, 0.80, 0.20 \rangle x_4$$

$$\widetilde{Z}_3 = \langle (0.5, 1.5, 2.5, 4.5); 0.25, 0.75, 0.25 \rangle x_1 + \langle (1.5, 2.5, 3, 5.5); 0.25, 0.75, 0.25 \rangle x_2 + \langle (2, 3, 3.5, 4.5); 0.20, 0.80, 0.20 \rangle x_3 + \langle (2.5, 3.5, 4, 6); 0.20, 0.80, 0.20 \rangle x_4$$

Subject to,

$$\langle (1.5, 1.75, 2.0, 6.25); 0.20, 0.80, 0.50 \rangle x_1 + \langle (1.5, 1.75, 2.0, 6.50); 0.40, 0.50, 0.30 \rangle x_2 + \langle (1.0, 1.75, 2.0, 6.75); 0.20, 0.80, 0.30 \rangle x_3 + \langle (1.5, 1.75, 2.0, 6.25); 0.30, 0.60, 0.20 \rangle x_4 \leq 50$$

$$\langle (2.0, 2.5, 3.0, 11.5); 0.30, 0.80, 0.50 \rangle x_1 + \langle (1.5, 1.75, 2.0, 6.50); 0.30, 0.50, 0.40 \rangle x_2 + \langle (1.0, 1.75, 2.0, 6.75); 0.30, 0.80, 0.20 \rangle x_3 + \langle (1.5, 1.75, 2.0, 6.25); 0.20, 0.50, 0.20 \rangle x_4 \leq 100$$

$$\langle (1.5, 1.75, 2.0, 6.25); 0.40, 0.70, 0.20 \rangle x_1 + \langle (1.5, 1.75, 2.0, 6.50); 0.40, 0.50, 0.30 \rangle x_2 + \langle (1.0, 1.75, 2.0, 6.75); 0.30, 0.70, 0.20 \rangle x_3 + \langle (1.5, 1.75, 2.0, 6.25); 0.40, 0.60, 0.30 \rangle x_4 \leq 200$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

With the proposed approach in a stepwise manner as:

Step 1 we obtain the corresponding proportional *p.d.f.* of each N number coefficient in each objective function and constraint as discussed in Sect. 4.1. For this, we use MATLAB software and the values of k_1, k_2 and k_3 for each N number coefficient of objectives and constraints are tabulated (Tables 2 and 3) as:

Step 2 setting the values of λ_1, λ_2 and λ_3 for each of IVTrpN numbers present as coefficient in objective and constraint as: $\lambda_1 = 0.20, \lambda_2 = 0.50, \lambda_3 = 0.30$ for each N numbers coefficients in each objective and $\lambda_1 = 0.30, \lambda_2 = 0.40, \lambda_3 = 0.30$ for each

N numbers coefficients in each of constraint respectively. With these values, the complete *p.d.f.* of each N number coefficient can be given as:

$$f_{Z_1}(X) = [0.40\mu_a(x_1) + 0.1429\sigma_a(x_1) + 0.15v_a(x_1)]x_1 + [0.4444\mu_a(x_2) + 0.1408\sigma_a(x_2) + 0.1364v_a(x_2)]x_2 \\ + [0.40\mu_a(x_3) + 0.1250\sigma_a(x_3) + 0.1200v_a(x_3)]x_3 + [0.5714\mu_a(x_4) + 0.1887\sigma_a(x_4) + 0.1875v_a(x_4)]x_4$$

$$f_{Z_2}(X) = [0.40\mu_a(x_1) + 0.20\sigma_a(x_1) + 0.20v_a(x_1)]x_1 + [0.4571\mu_a(x_2) + 0.1951\sigma_a(x_2) + 0.1778v_a(x_2)]x_2 \\ + [0.50\mu_a(x_3) + 0.1613\sigma_a(x_3) + 0.1579v_a(x_3)]x_3 + [0.50\mu_a(x_4) + 0.1613\sigma_a(x_4) + 0.1579v_a(x_4)]x_4$$

$$f_{Z_3}(X) = [0.32\mu_a(x_1) + 0.1481\sigma_a(x_1) + 0.1412v_a(x_1)]x_1 + [0.3556\mu_a(x_2) + 0.1455\sigma_a(x_2) + 0.1297v_a(x_2)]x_2 \\ + [0.6667\mu_a(x_3) + 0.2273\sigma_a(x_3) + 0.2308v_a(x_3)]x_3 + [0.50\mu_a(x_4) + 0.1613\sigma_a(x_4) + 0.1579v_a(x_4)]x_4$$

Subject to,

$$x_1 + [0.2857\mu_{a_{12}}(x_2) + 0.1085\sigma_{a_{12}}(x_2) + 0.0949k_3v_{a_{12}}(x_2)]x_2 \\ + [0.50\mu_{a_{13}}(x_3) + 0.0777\sigma_{a_{13}}(x_3) + 0.0822v_{a_{13}}(x_3)]x_3 + [0.40\mu_{a_{14}}(x_4) + 0.1067\sigma_{a_{14}}(x_4) + 0.1091v_{a_{14}}(x_4)]x_4 \leq 50$$

$$x_1 + [0.3810\mu_{a_{12}}(x_2) + 0.1085\sigma_{a_{12}}(x_2) + 0.0876v_{a_{12}}(x_2)]x_2 \\ + [0.3333\mu_{a_{13}}(x_3) + 0.0777\sigma_{a_{13}}(x_3) + 0.0896v_{a_{13}}(x_3)]x_3 + [0.60\mu_{a_{14}}(x_4) + 0.1143\sigma_{a_{14}}(x_4) + 0.1091v_{a_{14}}(x_4)]x_4 \leq 100$$

$$x_1 + [0.2857\mu_{a_{12}}(x_2) + 0.1085\sigma_{a_{12}}(x_2) + 0.0949v_{a_{12}}(x_2)]x_2 \\ + [0.3333\mu_{a_{13}}(x_3) + 0.0825\sigma_{a_{13}}(x_3) + 0.0896v_{a_{13}}(x_3)]x_3 + [0.30\mu_{a_{14}}(x_4) + 0.1067\sigma_{a_{14}}(x_4) + 0.10v_{a_{14}}(x_4)]x_4 \leq 200$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

Step 4 in this step. We obtain the equivalent value of each *p.d.f.* using Laplace Transform as given in (14). For this, we use MATHEMATICA© software and equivalent crisp problem as:

$$\text{Max}_{x_1, x_2} \{Z_1(X) = 0.0627x_1 + 0.0213x_2 + 0.0102x_3 + 0.0137x_4\} \quad (1st \text{ level})$$

$$\text{Max}_{x_3} \{Z_2(X) = 0.0041x_1 + 0.0052x_2 + 0.0103x_3 + 0.0103x_4\} \quad (2nd \text{ level})$$

$$\text{Max}_{x_4} \{Z_3(X) = 0.0556x_1 + 0.0297x_2 + 0.0321x_3 + 0.0102x_4\} \quad (3rd \text{ level})$$

$$\text{Subject to, } 0.0156x_1 + 0.0169x_2 + 0.0155x_3 + 0.0208x_4 \leq 50$$

$$0.0013x_1 + 0.0064x_2 + 0.0204x_3 + 0.0164x_4 \leq 100$$

Table 2 Values of k_1, k_2 and k_3 for each N number coefficient of objective functions

i	$\widetilde{Z}_i = \widetilde{c}_{i1}x_1$			$\widetilde{c}_{i2}x_2$			$\widetilde{c}_{i3}x_3$			$\widetilde{c}_{i4}x_4$		
	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3
$i=1$	2	0.2857	0.5	2.2222	0.2816	0.4545	2	0.25	0.4	2.8571	0.3773	0.625
$i=2$	2	0.4	0.6666	2.2857	0.3902	0.5925	2.5	0.3225	0.5263	2.5000	0.3225	0.5263
$i=3$	1.6	0.2962	0.4705	1.7777	0.2909	0.4324	3.3333	0.4545	0.7692	2.5	0.3225	0.5263

Table 3 Values of k_1, k_2 and k_3 for each N number coefficient of constraint coefficients

i	$\widetilde{a}_{i1}x_1$			$\widetilde{a}_{i2}x_2$			$\widetilde{a}_{i3}x_3$			$\widetilde{a}_{i4}x_4$		
	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3
2		0.2352	0.2857	0.9523	0.2711	0.3162	1.6666	0.1941	0.2739	1.3333	0.2667	0.3636
	0.6666	0.1176	0.1428	1.2698	0.2711	0.2919	1.1111	0.1941	0.2985	2	0.2857	0.3636
1		0.25	0.3636	0.9523	0.2711	0.3162	1.1111	0.2061	0.2985	1	0.2667	0.3333

$$0.0165x_1 + 0.0147x_2 + 0.0181x_3 + 0.0163x_4 \leq 200$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

Step 5 calculating the ideal values (maximum and minimum values) for each objective function at each level along with ideal values of decision variables (up to T-1 levels) (as shown in Table 4).

Step 6 construct membership functions for each objective functions at each level and decision variables (i.e. x_1, x_2, x_3) of first two levels as defined in Eqs. (17) and (18) respectively as shown below:

$$F_1(Z_1(\bar{X})) = \frac{0.0627x_1 + 0.0213x_2 + 0.0102x_3 + 0.0137x_4 - 0}{200.9615 - 0}$$

$$F_2(Z_2(\bar{X})) = \frac{0.0041x_1 + 0.0052x_2 + 0.0103x_3 + 0.0103x_4 - 0}{33.22581 - 0}$$

$$F_3(Z_3(\bar{X})) = \frac{0.0556x_1 + 0.0297x_2 + 0.0321x_3 + 0.0102x_4 - 0}{178.2051 - 0};$$

$$F_{x_1}(x_1) = \frac{x_1 - 0}{3205.128 - 0}; F_{x_3}(x_3) = \frac{x_3 - 0}{3.225806 - 0}$$

Step 7 with these values, solution model (21) for MLLP problem with IVTrpN numbers can be given as:

$$\min \lambda = d_1^- + d_2^- + d_3^- + d_{x_1}^- + d_{x_3}^-$$

$$\text{Subject to, } 0.0627x_1 + 0.0213x_2 + 0.0102x_3 + 0.0137x_4 + 200.9615d_1^- \geq 200.9615$$

$$0.0041x_1 + 0.0052x_2 + 0.0103x_3 + 0.0103x_4 + 33.22581d_2^- \geq 33.22581$$

$$0.0556x_1 + 0.0297x_2 + 0.0321x_3 + 0.0102x_4 + 178.2051d_3^- \geq 178.2015$$

$$x_1 + 3205.128d_{x_1}^- \geq 3205.128$$

$$x_3 + 33.22581d_{x_1}^- \geq 33.22581$$

$$0.0156x_1 + 0.0169x_2 + 0.0155x_3 + 0.0208x_4 \leq 50$$

$$0.0013x_1 + 0.0064x_2 + 0.0204x_3 + 0.0164x_4 \leq 100$$

$$0.0165x_1 + 0.0147x_2 + 0.0181x_3 + 0.0163x_4 \leq 200$$

and $x_1, x_2, x_3, x_4, d_1^-, d_2^-, d_3^-, d_{x_1}^-, d_{x_3}^- \geq 0$.

Solving this programming problem model with LINGO© tool, we obtain the satisfactory solution to the original problem: $\lambda = 0.6214752$, $x_1 = 3172.115$, $x_2 = 0$, $x_3 = 33.22581$, $x_4 = 0$ with the objective function values as: $\widetilde{Z}_{11} = 199.2305$, $\widetilde{Z}_2 = 13.34790$, and $\widetilde{Z}_3 = 177.4362$.

6.2 Application of proposed solution technique in real world problem

Multi-level programming approaches under neutrosophic environment have can be used in solving real world problem e.g. Luo et al. [26] applied different approach with N number in solving a pricing decision-making case of satellite image data products. Proposed solution algorithm can be applied in solving complex real problems arising in day-to-day life. Here, formulation of production problem as MLLP problem with N numbers and its solution with proposed technique is described as:

Let us consider a real problem of ‘XYZ’ production firm producing mainly three types of products P, Q and R. This firm has three hierarchical levels of functioning namely manufacturing unit (1st level), distributor (2nd level) and retailer (3rd unit). At each level, the firm has different objective functions (say $Z_1(X)$, $Z_2(X)$, $Z_3(X)$) as: at manufacturing unit, maximization of market share of product of firm; at distributor level, maximization of sales revenue of product and at 3rd level, maximization of profit on net product cost. There are different decision variables involved in decision making such as x_1 - profit on product P, x_2 - profit on product Q and x_3 - profit on product R. Besides these, coefficients in each objective function and each constraint are in the form of interval valued trapezoidal N numbers. This real production problem can be formulated in mathematical format as:

$$Max_{x_1} \left\{ \begin{aligned} &Z_1(X) = \langle(1.0, 1.75, 2.0, 6.75);0.20, 0.80, 0.30\rangle x_1 + \langle(1.5, 1.75, 2.0, 6.50);0.30, 0.50, 0.40\rangle x_2 \\ &+ \langle(1.0, 1.75, 2.0, 6.75);0.30, 0.70, 0.20\rangle x_3 \end{aligned} \right\} \quad (1st \text{ level})$$

$$Max_{x_2} \left\{ \begin{aligned} &Z_2(X) = \langle(1.5, 1.75, 2.0, 6.25);0.40, 0.70, 0.20\rangle x_1 + \langle(1.5, 1.75, 2.0, 6.25);0.20, 0.80, 0.50\rangle x_2 \\ &+ \langle(1.5, 1.75, 2.0, 6.50);0.40, 0.50, 0.30\rangle x_3 \end{aligned} \right\} \quad (2nd \text{ level})$$

$$Max_{x_3} \left\{ \begin{aligned} &Z_3(X) = \langle(1.5, 1.75, 2.0, 6.25);0.40, 0.60, 0.30\rangle x_1 + \langle(1.0, 1.75, 2.0, 6.75);0.30, 0.70, 0.20\rangle x_2 \\ &+ \langle(1.5, 1.75, 2.0, 6.25);0.20, 0.80, 0.50\rangle x_3 \end{aligned} \right\} \quad (3rd \text{ level})$$

Subject to, $\langle(0.5, 1.5, 2.5, 4.5);0.20, 0.80, 0.20\rangle x_1 + \langle(1.5, 2.5, 3, 5.5);0.20, 0.80, 0.20\rangle x_2$
 $+ \langle(1.0, 1.75, 2.0, 6.75);0.20, 0.80, 0.30\rangle x_3 + \langle(1.5, 1.75, 2.0, 6.25);0.30, 0.60, 0.20\rangle x_4 \leq 850$

$\langle(2, 3, 3.5, 6.5);0.20, 0.80, 0.20\rangle x_1 + \langle(2.0, 2.5, 3.0, 11.5);0.30, 0.80, 0.50\rangle x_2$
 $+ \langle(1.0, 1.75, 2.0, 6.75);0.30, 0.80, 0.20\rangle x_3 + \langle(1.5, 1.75, 2.0, 6.25);0.20, 0.50, 0.20\rangle x_4 \leq 900$

$\langle(1.5, 1.75, 2.0, 6.50);0.30, 0.50, 0.40\rangle x_1 + \langle(1.0, 1.75, 2.0, 6.75);0.30, 0.80, 0.20\rangle x_2$
 $+ \langle(1.5, 1.75, 2.0, 6.25);0.40, 0.70, 0.20\rangle x_3 + \langle(1.5, 1.75, 2.0, 6.25);0.40, 0.60, 0.30\rangle x_4 \leq 1100$

and $x_1, x_2, x_3, x_4 \geq 0$

Using the step 1 of proposed solution technique, the values of k_1, k_2 and k_3 for each N number coefficient of objectives and constraints are obtained (Tables 5 and 6) as:

Thereafter, with the values $\lambda_1 = 0.20, \lambda_2 = 0.50, \lambda_3 = 0.30$ and $\lambda_1 = 0.30, \lambda_2 = 0.40, \lambda_3 = 0.30$ for N numbers coefficient in objective function and constraint respectively, the complete *p.d.f.* of each coefficient as mentioned in step 2 of algorithm can be obtained and accordingly their equivalent value can be obtained using Laplace Transform as given in (14) and mentioned in step 4. With these equivalent values of each N numbers, we convert the problem into equivalent crisp problem as:

$$Max_{x_1} \{ Z_1(X) = 0.0155x_1 + 0.0064x_2 + 0.0204x_3 \} \quad (1st \text{ level})$$

$$Max_{x_2} \{ Z_2(X) = 0.0165x_1 + 0.0156x_2 + 0.0147x_3 \} \quad (2nd \text{ level})$$

$$Max_{x_3} \{ Z_3(X) = 0.0163x_1 + 0.0181x_2 + 0.0156x_3 \} \quad (3rd \text{ level})$$

Subject to, $0.0627x_1 + 0.0213x_2 + 0.0155x_3 \leq 850$

$0.0102x_1 + 0.0041x_2 + 0.0204x_3 \leq 900$

$0.0052x_1 + 0.0103x_2 + 0.0556x_3 \leq 1100$

Table 4 Maximum and minimum values of objective functions and corresponding decision variables

S. No	Ist level		IInd level		IIIrd level	
	Maximization	Minimization	Maximization	Minimization	Maximization	Minimization
1	$\bar{Z}_1 = 200.9615$ at $(3205.128, 0, 0, 0)$	$\underline{Z}_1 = 0$ at $(0, 0, 0, 0)$	$\bar{Z}_1 = 33.22581$ at $(0, 0, 3.225806, 0)$	$\underline{Z}_1 = 0$ at $(0, 0, 0, 0)$	$\bar{Z}_3 = 178.2051$ at $(3205.128, 0, 0, 0)$	$\underline{Z}_3 = 0$ at $(0, 0, 0, 0)$
\bar{X}_1 and \underline{X}_1	$\bar{x}_1 = 3205.128, \bar{x}_2 = 0$ and	$x_1 = 0, x_2 = 0$	$\bar{x}_3 = 3.225806, x_3 = 0$			

Table 5 Values of k_1, k_2 and k_3 for each N number coefficient of objective functions

i	$\widetilde{Z}_i = \widetilde{c}_{i1}x_1$			$\widetilde{c}_{i2}x_2$			$\widetilde{c}_{i3}x_3$		
	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3
$i=1$	1.6666	0.1941	0.2739	1.2698	0.2711	0.2919	0.1111	0.1941	0.2985
$i=2$	1	0.25	0.3636	2	0.2352	0.2857	0.9523	0.2711	0.3162
$i=3$	1	0.2667	0.3333	0.3162	1.1111	0.2061	2	0.2352	0.2857

Table 6 Values of k_1, k_2 and k_3 for each N number coefficient of constraint

i	$\widetilde{a}_{i1}x_1$			$\widetilde{a}_{i2}x_2$			$\widetilde{a}_{i3}x_3$			$\widetilde{a}_{i4}x_4$		
	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3
2	0.2857	0.5	2.2222	0.2816	0.4545	1.6666	0.1941	0.2739	1.3333	0.2667	0.3636	
2	0.25	0.4	2	0.4	0.6666	1.1111	0.1941	0.2985	2	0.2857	0.3636	
	2.2857	0.3902	0.5925	2.5	0.3225	0.5263	1.6	0.2962	0.4705	1	0.2667	0.3333

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

Solving this crisp problem irrespective of level structure, we obtain the ideal values of decision variables and objective function and formulate the linear membership function for each objective function and variables (up to T-1 level i.e. x_1, x_2). Finally, we have the solution model for MLLP problem with IVTrp N numbers as:

$$\min \lambda = d_1^- + d_2^- + d_3^- + d_{x_1}^- + d_{x_3}^-$$

$$\text{Subject to, } 0.0155x_1 + 0.0064x_2 + 0.0204x_3 + 524.1711d_1^- \geq 524.1711$$

$$0.0165x_1 + 0.0156x_2 + 0.0147x_3 + 670.4845d_2^- \geq 670.4845$$

$$0.0163x_1 + 0.0181x_2 + 0.0156x_3 + 757.0840d_3^- \geq 757.0840$$

$$x_1 + 8870.892d_{x_1}^- \geq 8870.892$$

$$x_2 + 29483.81d_{x_2}^- \geq 29483.81$$

$$0.0627x_1 + 0.0213x_2 + 0.0155x_3 \leq 850$$

$$0.0102x_1 + 0.0041x_2 + 0.0204x_3 \leq 900$$

$$0.0052x_1 + 0.0103x_2 + 0.0556x_3 \leq 1100$$

$$\text{and } x_1, x_2, x_3, d_1^-, d_2^-, d_3^-, d_{x_1}^-, d_{x_2}^- \geq 0$$

Solving this programming problem model with simplex method or LINGO© tool, we obtain the satisfactory solution to the original problem: $\lambda = 1.082608$ $x_1 = 0.001104$, $x_2 = 29483.81$, $x_3 = 14322.24$ with the objective function values as: $\tilde{Z}_{11} = 480.8702$, $\tilde{Z}_2 = 670.4844$, and $\tilde{Z}_3 = 757.0840$.

7 Concluding remarks and future research directions

This paper presents an improved and unique solution technique for MLLP problem with interval-valued trapezoidal neutrosophic numbers (IVTrpN numbers). The unique feature of the proposed technique is the use of proportional *p.d.f.* to each N number and use of Laplace transform to convert problem into crisp problem. Besides this, proposed technique is comparatively simpler with reduced number of computational steps as there is no use of ranking function, score function, accuracy function etc. values of N numbers in the de-neutrosophication process of N numbers. However, proposed technique has limitation of computation difficulties of Laplace transform of associated *p.d.f.* but this limitation can be dealt with the use of computing software like MATHEMATICA, MATLAB and LINGO. The proposed technique can be extended for the solution of more complex problems like MOP problems, BLP problems, MLMOP problems etc. by the future researchers.

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