APPLICATION ARTICLE

Nonlinear fuzzy chance constrained approach for multi-objective mixed fuzzy-stochastic optimization problem

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Abstract
Fuzzy set theory currently has a wide range of applications to model real-world issues with ambiguous or incomplete information, which to some extent captures reality. On the other hand stochastic environment also deals with uncertainties with different approach (probability distribution). In order to deal with decision problems involving more than one objective, where the parameters and the objectives both are uncertain, the mixed fuzzy stochastic programming approach have been introduced. In this paper, a new solution named as fuzzy stochastic pareto optimal solution is defined. Here we have developed an iterative method for the decision making of a multi-objective optimization problem in the fuzzy stochastic environment. Further a numerical illustration of the developed methodology has been given and the superiority of the proposed method has been established by comparing the obtained results with some well known existing methods.

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1 Introduction

Many real-world decision-making problems involve uncertain data because the information is not precisely known. Lack of knowledge about the data or inaccurate information or the dependency upon the nature of the market is the sources of uncertainty. Two approaches to dealing with uncertainty that make it simple to integrate with fuzzy and probabilistic preferences are theory of possibility and theory of probability.

In any decision making system the experts need to consider multiple objectives with different natures (maximization/minimization) at the same time with restricted resources. It is very difficult or may not be possible to obtain an optimal solution which optimizes each objective simultaneously. So there are various approaches available to obtain compromise solution or still the scope of new methodology is always there to obtain better solutions. Here a better approach considering the mixed fuzzy stochastic environment which deals the situation better than the existing has been given.

The study comprises the following novelty to this approach as given below:

- The mixed fuzzy stochastic environment handles the real situation effectively instead of treating a fuzzy or stochastic environment as separate entities.
- This approach is very effective and can be used in various domains where the experts find the decision making very crucial.
- The paper uses a non linear membership function (Gaussian) to define fuzzy mean and fuzzy variance of a fuzzy random variable.
- For the computational purpose crisp conversion of the problem has been defined by considering the α-cut of Gaussian membership function.
- A new solution methodology has been developed to deal with both (fuzzy and stochastic) types of uncertainties simultaneously.
- The solution obtained by proposed method is better than some well known existing methods.
- The proposed technique is the generalization of both the fuzzy and stochastic optimization techniques.
- The mixed fuzzy stochastic approach is better tool to handle any real life decision making problem with uncertainty and vagueness.

The paper is organized as follows: Sect. 2 comprises with the literature review related to the work. In Sect. 3, some basic definitions are given which has been used to develop the methodology. In Sect. 4, problem formulation and methodology for solution has been developed and in Sect. 5, stepwise computational algorithm is given. Numerical illustration has been presented in Sect. 6 and the obtained results are discussed in Sect. 7. Conclusion of the work done and the future scope has been summarised in Sects. 8 and 9 respectively.

2 Literature review

Many researchers have worked in the area of decision making problem in different environments. To build up the proposed theory, number of literature survey has been done; some of the important works are mentioned in this section. In the theory of possibility, some or all the parameters of the problem are regarded as fuzzy variables Parra et al. [22], whereas in the theory of probability, some or all the parameters of the problem are seen as random variables [26]. The probability distributions in probability theory plays the same roles as the possibility distributions, the foundation of which was first introduced by Zadeh [29] and developed by Dubois and Prade [10]. To solve a linear programming problem with imprecise constraint coefficients, Lai and Hwang [14] suggested an auxiliary multi-objective linear programming model. A linear programme with unknown parameters that are either treated as random variables or as fuzzy variables is provided by Buckley [6]. Inuiguchi and Sakawa [12] proposed an equivalent condition between a stochastic linear programming problem with a multivariate normal distribution and a probabilistic linear programming problem with a quadratic membership function. When we formulate the real-world decision-making problem as a multi-objective programming problem the nature of parameters may be taken as crisp, fuzzy, random or fuzzy random (Bellman & Zadeh [3]; Guang & Zhong [11]; Wang & Watada 28). Nanda et al. [19] proposed a new solution method for solving fuzzy chance constrained programming problem where fuzzy random variables involve in chance constraints. Barik et al. [2] introduced a method for solving stochastic programming problem involving pareto distribution. Panda et al. [20] presented a method to solve nonlinear fuzzy chance constraint programming problem. Pradhan et al. [23] have also presented a methodology based on chance constraints programming method for solving a multi-choice probabilistic linear programming problem. Dash et al. [9] introduced optimal solution for a single period inventory model with fuzzy cost and demand as a fuzzy random variable. Mohanty et al. [17] developed a methodology for solving chance constraint linear programming problem in which parameters of the constraint follows some different continuous distributions. Several authors have tried to deal with fuzziness and randomness in optimization problems. Recently, Nabavi et al. [18] introduced a new method for solving the fuzzy stochastic multi-objective programming problems involve fuzzy random variable coefficient in constraint, in which they used the concept of chance constrained technique and fuzzy inequality relation to transform the original problem into the conventional deterministic problem. Arjmandzadeh et al. [1] proposed a neural network model for solving the multi-objective linear programming problem involve random variable parameters. Solid transportation problem (STP) is an important real life problem of stochastic programming, due to random demands, supplies and conveyance capacities decision makers want to design a transportation schedule that minimize both the time and cost of the transportation. Singh et al. [27] construct a formulation of multi-objective STP involve some random parameters with gamma distribution and their methodology for solving this problem is based on chance constraint programming method.

Sharma et al. [25] also developed a concept for solving multi-objective bi-level chance constraint optimization problem in fuzzy intuitionistic environment. Pandey et al. [21] characterized a probabilistic and fuzzy approach for uncertainty consideration in water distribution network. Bharati [4, 5] has modelled certain type of uncertainty using an interval valued intutionistic hesitant fuzzy methodology and hesitant intutionistic fuzzy algorithm for multi-objective optimization problem. In this paper, we have solved a multi-objective mixed fuzzy stochastic programming problem with chance constrained and having coefficients as fuzzy random variable (FRV), the characteristic of FRV are taken as fuzzy numbers with Gaussian membership function and compare the results obtained with [4, 5].

3 Preliminaries

Definition 1 (*Gaussian membership function*) The Gaussian membership function of a set B of a non-empty universal set X is defined as,

$$\mu_B(x) = e^{\frac{-(x-n)^2}{2\sigma^2}}, \tag{1}$$

For all $x \in X$(universal set), n represents the modal value of the fuzzy element and $\sigma > 0$ represents the standard deviation of the element. The Gaussian function will take a bell-shaped curve and the smaller σ the narrower the bell, which is shown in Fig. 1 [15].

Definition 2 (*α-cut of Gaussian membership function*) α-cut of the fuzzy element with Gaussian membership function has been defined as,

$$B_\alpha = \left\{ x \in X : \mu_B(x) = e^{\frac{-(x-n)^2}{2\sigma^2}} \geq \alpha \right\} \quad \forall \alpha \in (0, 1] \tag{2}$$

For a particular value of α, the corresponding element of fuzzy set can be.

obtained as, $e^{\frac{-(x-n)^2}{2\sigma^2}} = \alpha$.
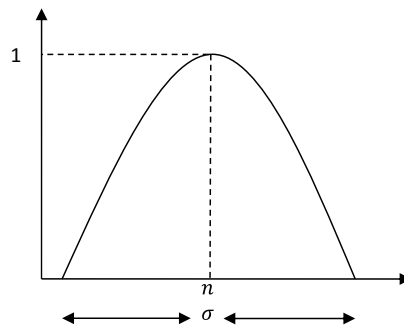
Apply logarithm throughout we have,

Fig. 1 Gaussian membership function

$$\frac{-(x-n)^2}{2\sigma^2} = \log\alpha,$$

$$x = n \pm \sigma\sqrt{-2\log\alpha}. \tag{3}$$

Definition 3 (*Mixed fuzzy stochastic programming problem*) A mathematical programming problem that considers optimization under some random uncertainty and imprecision, for which the probabilistic uncertainty and fuzzy uncertainty are to be considered simultaneously. First kind of uncertainty is due to the randomness in some parameters and the second kind is due to vagueness in some decision parameters.

Definition 4 (*Fuzzy chance constraint programming method*) A chance constrained programming method is suggested by Charnes and Cooper in [8]. Further to deal with both kinds of uncertainties (discussed previously) simultaneously a fuzzy chance constrained programming method is developed. This is a method for solving stochastic programming problem involves some chance constraints that involve both kinds of uncertainties with constraints having violation up to a pre-specified probability levels. Mathematically fuzzy stochastic programming problem with fuzzy chance constrained can be formulated as,

$$\text{Minimize: } z = \sum_{j=1}^{n} c_j x_j$$

s.t.

$$P\left(\sum_{j=1}^{n} \widetilde{a}_{ij} x_j \le b_i\right) \ge \widetilde{\beta}_i$$

$$x_j \ge 0, \quad i = 1, 2, 3, 4 \ldots \ldots m, \quad 0 \le \beta_i \le 1. \tag{4}$$

In the above formulation the coefficients $\widetilde{a}_{ij}$ in chance constrained are fuzzy random variables and $\widetilde{\beta}_i$ is the pre-specified fuzzy probability level for chance constrained.

Definition 5 (*Fuzzy random variable* (FRV)) Kwakernaak [13] has introduced the concept of FRV after work of Zadeh on Fuzzy sets as a basis for a theory of possibility (1978). Many researchers Puri and Ralescu [24], Liu and Liu [16] etc.) have developed this notion, based on various measurability criteria. Some of the parameters in the density function of a continuous probability distribution might be unknown. These uncertainties were expressed as fuzzy numbers by Buckely and Eslami [7] to create a fuzzy probability density function for a continuous random variable. In this study, we focused at a chance constrained programming problem with continuous random variables, which often have normal distributions.

Definition 6 (*Mean and variance of a continuous* FRV) Let X be a continuous random variable with probability density function $f(x, \gamma)$, where γ is a scale parameter for the density function. If we denote $\widetilde{X}$ as continuous random variable with fuzzy

parameter $\widetilde{\gamma}$, then $\widetilde{X}$ is called continuous FRV with density $f(x, \widetilde{\gamma})$. The probability of an event $A = [a, b]$ of the continuous FRV $\widetilde{X}$ is a fuzzy number whose α-cut is.

$$P\big(a \leq \tilde{X} \leq b\big)[\alpha] = \left\{ \int_a^b f(x, \gamma) dx | \gamma \in \tilde{\gamma}[\alpha] \right\} = \left[S_*(\alpha), S^*(\alpha) \right]. \tag{5}$$

$$\text{Where } S_*(\alpha) = \min \left\{ \int_a^b f(x, \gamma) dx \right\}, \quad S^*(\alpha) = \max \left\{ \int_a^b f(x, \gamma) dx \right\}. \tag{6}$$

Mean and variance of a continuous FRV $\widetilde{X}$ with the probability density function $f(x, \widetilde{\gamma})$ are fuzzy number whose α-cuts are defined as,

$$m_x(\tilde{\gamma})[\alpha] = \left\{ \int_{-\infty}^{\infty} x f(x, \gamma) dx | \gamma \in \tilde{\gamma}[\alpha] \right\}, \tag{7}$$

$$\sigma_x^2(\tilde{\gamma})[\alpha] = \left\{ \int_{-\infty}^{\infty} \big(x - m_x(\gamma)\big)^2 f(x, \gamma) dx | \gamma \in \tilde{\gamma}[\alpha], m_x(\gamma) \in m_x(\tilde{\gamma})[\alpha] \right\}. \tag{8}$$

4 Problem formulation and methodology

A crisp chance constraints programming problem with FRV coefficients in the constraints is defined as,

$$Z = \min \sum_{j=1}^n c_J x_j$$

subject to constraints (s.t.)

$$P\left(\sum_{j=1}^n \widetilde{a}_{ij} x_j \leq b_i \right) \geq \widetilde{\beta}_i \tag{9}$$

$$\sum_{j=1}^n \widetilde{b_{kj}} x_j \leq \widetilde{h}_k$$

$$x_j \geq 0.$$

$$i = 1, 2, \ldots \ldots m, \quad k = m + 1, m + 2, \ldots, m + n, \quad 0 \leq \beta_i \leq 1, \quad x_j \in R.$$

In this problem, we considered a presence of FRV only in coefficient of chance constraint. i.e. $\tilde{a}_{ij}$ is normally distributed FRV whose mean and variance are fuzzy number having Gaussian membership function and additionally the remaining constraints involve coefficients expressed as fuzzy number and the probability level $\tilde{\beta}_i$ is also taken as a fuzzy number.

Let $v_i = \sum \tilde{a}_{ij} x_j$ is the linear combinations of n number of FRVs, then v_i is also a FRV. Fuzzy mean and Fuzzy variance is denoted by m_{v_i} and $\sigma_{v_i}^2$ respectively. α-cut of these fuzzy numbers are defined as,

$$m_{v_i}[\alpha] = \left[m_{v_{i_*}}(\alpha), m_{v_{i*}}(\alpha) \right].$$

$$\sigma_{v_i}^2[\alpha] = \left[\sigma_{v_{i_*}}^2(\alpha), \sigma_{v_{i*}}^2(\alpha) \right].$$

$$\beta_i[\alpha] = \left[\beta_{i_*}(\alpha), \beta_{i*}(\alpha) \right]$$

Theorem *If the parameters $\tilde{a}_{ij}$ of the chance constrained of a stochastic programming problem are independent normally distributed FRV, then chance constraint is equivalent to,*

$$F\left(\frac{b_i - m_{v_{i*}}(\alpha)}{\sigma_{v_{i*}}(\alpha)} \right) \geq \beta_{i*}(\alpha), \tag{10}$$

For all $\alpha \in [0,1]$ and F is the cumulative distribution function, here we have taken $N(0,1)$ distribution. The proof of the Theorem has been given by Nanda et al. [19].

Using above Theorem, the crisp conversion of the fuzzy chance constraint programming problem (9) becomes,

$$Z = \min \sum_{j=1}^{n} c_J x_j$$

$$\text{s.t.} \quad F\left(\frac{b_i - m_{v_{i*}}(\alpha)}{\sigma_{v_{i*}}(\alpha)} \right) \geq \beta_{i*}(\alpha)$$

$$\sum_{j=1}^{n} (b_{kj})_\alpha^l x_j \leq (h_k)_\alpha^l \tag{11}$$

$$\sum_{j=1}^{n} (b_{kj})_\alpha^u x_j \leq (h_k)_\alpha^u$$

$$x_j \geq 0$$

$$i = 1, 2, \ldots m, \quad k = m+1, m+2, \ldots, m+n, \quad 0 \leq \beta_i \leq 1, \quad x_j \in R.$$

5 The fuzzy mixed-stochastic programming algorithm

Step 1 Calculate α-cut of each fuzzy mean and fuzzy variance of parameters of the chance constraints and write it in the form of linear combination of each constraint.

Step 2 Convert these chance constraints into nonlinear crisp constraint by using relation (10) and for the crisp conversion of fuzzy constraints we have used α-cut method, as given by (2) and (3).

Step 3 Consider single objective with the constraints as obtained in step 2, to find the optimal solution and the optimal value of objective function.

Step 4 Compute values of remaining objective functions at optimal solution obtained in Step 3.

Step 5 Repeat step 3 and step 4 for each remaining objective functions and constructs a table for these positive ideal solutions.

Step 6 From these ideal solutions, obtain lower and upper bound for each objective function.

Step 7 Construct linear membership function for each objective by using the bounds obtained in step 6.

Step 8 Find pareto optimal solution for the multi objective stochastic programming problem by using fuzzy programming technique.

Step 9 Repeat step 1 to step 8 for distinct values of α.

Step 10 Repeat step 1 to step 9 for different values of β**.**

6 Numerical illustration (application of proposed algorithm)

For the comparative study and to validate the proposed method we have considered the same numerical of production planning problem as taken by Bharati [4, 5]. As the domain of the problem involves human observation, market risks, quality of the raw materials, therefore to handle such types of uncertainties we are here formulate the problem as fuzzy stochastic programming problem and further by the developed algorithm we have solved the problem and compare the obtained results in different environment.

Mathematical formulation of the problem in stochastic environment is given as,

$$\text{Max } z^{(1)} = 50x + 100y + 17.5z$$

$$\text{Max } z^{(2)} = 92x + 75y + 50z$$

$$\text{Max } z^{(2)} = 92x + 75y + 50z$$

$$\text{s.t.}$$

$$P\left[\widetilde{12}x + \widetilde{17}y \leq 1400\right] \geq \widetilde{0.6}$$

$$P\left[\tilde{3}x + \tilde{9}y + \tilde{8}z \leq 1000\right] \geq \widetilde{0.6}$$

$$P\left[\widetilde{10}x + \widetilde{13}y + \widetilde{15}z \leq 1750\right] \geq \widetilde{0.6} \tag{12}$$

$$P\left[\tilde{6}x + \widetilde{16}z \leq 1325\right] \geq \widetilde{0.6}$$

$$P\left[\widetilde{12}x + \tilde{7}y \leq 900\right] \geq \widetilde{0.6}$$

$$\widetilde{9.5}x + \widetilde{9.5}y + \tilde{4}z \leq \widetilde{1075}$$

$$x, y, z \geq 0.$$

In the above problem, x, y and z are the units of three different products produced. The coefficients of the chance constraints are FRV whose mean and variance are taken as fuzzy number with Gaussian membership function, and the fuzzy constraint which involves coefficients as fuzzy numbers with Gaussian membership function. Fuzzy mean and fuzzy variance of the constraints are given in Table 1.

Step 1 α-cut of each fuzzy mean and fuzzy variance as given in Table 1 are calculated from (7) and (8), and for $\alpha = 0.5$ we obtain,

$$\widetilde{m_1}[\alpha] = x\widetilde{m}\left(\widetilde{12}\right)[\alpha] + y\widetilde{m}\left(\widetilde{17}\right)[\alpha] = (7.77x + 11y, 16.22x + 22.99y).$$

$$\widetilde{m_2}[\alpha] = x\widetilde{m}\left(\widetilde{3}\right)[\alpha] + y\widetilde{m}\left(\widetilde{9}\right)[\alpha] + z\widetilde{m}\left(\widetilde{8}\right)[\alpha]$$
$$= (1.94x + 5.82y + 5.18z, 4.05x + 12.17y + 10.81z)$$

$$\widetilde{m_3}[\alpha] = x\widetilde{m}\left(\widetilde{10}\right)[\alpha] + y\widetilde{m}\left(\widetilde{13}\right)[\alpha] + z\widetilde{m}\left(\widetilde{15}\right)[\alpha]$$
$$= (6.47x + 8.41y + 9.71z, 13.52x + 17.58y + 20.28z).$$

$$\widetilde{m_4}[\alpha] = x\widetilde{m}\left(\widetilde{6}\right)[\alpha] + z\widetilde{m}\left(\widetilde{16}\right)[\alpha] = (3.88x + 10.36z, 8.11x + 21.63z).$$

Table 1 Fuzzy Mean and Fuzzy variance of coefficients of the constraints of problem (12)

Fuzzy mean (defined as Gaussian membership function with modal value and spread)		Fuzzy variance (defined as Gaussian membership function with modal value and spread)	
$\widetilde{m}(\widetilde{12})$	$(12, 3.6)$	$\widetilde{\sigma^2}(\widetilde{12})$	$(6, 1.8)$
$\widetilde{m}(\widetilde{17})$	$(17, 5.1)$	$\widetilde{\sigma^2}(\widetilde{17})$	$(8.5, 2.55)$
$\widetilde{m}(\widetilde{3})$	$(3, 0.9)$	$\widetilde{\sigma^2}(\widetilde{3})$	$(1.5, 0.45)$
$\widetilde{m}(\widetilde{9})$	$(9, 2.7)$	$\widetilde{\sigma^2}(\widetilde{9})$	$(4.5, 1.35)$
$\widetilde{m}(\widetilde{8})$	$(8, 2.4)$	$\widetilde{\sigma^2}(\widetilde{8})$	$(4, 1.2)$
$\widetilde{m}(\widetilde{10})$	$(10, 3)$	$\widetilde{\sigma^2}(\widetilde{10})$	$(5, 1.5)$
$\widetilde{m}(\widetilde{13})$	$(13, 3.9)$	$\widetilde{\sigma^2}(\widetilde{13})$	$(6.5, 1.95)$
$\widetilde{m}(\widetilde{15})$	$(15, 4.5)$	$\widetilde{\sigma^2}(\widetilde{15})$	$(7.5, 2.25)$
$\widetilde{m}(\widetilde{6})$	$(6, 1.8)$	$\widetilde{\sigma^2}(\widetilde{6})$	$(3, 0.9)$
$\widetilde{m}(\widetilde{16})$	$(16, 4.8)$	$\widetilde{\sigma^2}(\widetilde{16})$	$(8, 2.4)$
$\widetilde{m}(\widetilde{12})$	$(12, 3.6)$	$\widetilde{\sigma^2}(\widetilde{12})$	$(6, 1.8)$
$\widetilde{m}(\widetilde{7})$	$(7, 2.1)$	$\widetilde{\sigma^2}(\widetilde{7})$	$(3.5, 1.05)$

$$\widetilde{m_5}[\alpha] = y\widetilde{m}\left(\widetilde{12}\right)[\alpha] + z\widetilde{m}(\tilde{7})[\alpha] = (7.77y + 4.53z, 16.22y + 9.46z).$$

$$\widetilde{\sigma_1^2}[\alpha] = x^2\widetilde{\sigma^2}\left(\widetilde{12}\right)[\alpha] + y^2\widetilde{\sigma^2}\left(\widetilde{17}\right)[\alpha] = \left(3.88x^2 + 5.5y^2, 8.11x^2 + 11.49y^2\right).$$

$$\widetilde{\sigma_2^2}[\alpha] = x^2\widetilde{\sigma^2}(\tilde{3})[\alpha] + y^2\widetilde{\sigma^2}(\tilde{9})[\alpha] + z^2\widetilde{\sigma^2}(\tilde{8})[\alpha]$$
$$= \left(0.97x^2 + 2.91y^2 + 2.59z^2, 2.02x^2 + 6.08y^2 + 5.4z^2\right).$$

$$\widetilde{\sigma_3^2}[\alpha] = x^2\widetilde{\sigma^2}\left(\widetilde{10}\right)[\alpha] + y^2\widetilde{\sigma^2}\left(\widetilde{13}\right)[\alpha] + z^2\widetilde{\sigma^2}\left(\widetilde{15}\right)[\alpha]$$
$$= \left(3.23x^2 + 4.2y^2 + 4.85z^2, 6.76x^2 + 8.79y^2 + 10.14z^2\right).$$

$$\widetilde{\sigma_4^2}[\alpha] = x^2\widetilde{\sigma^2}(\tilde{6})[\alpha] + z^2\widetilde{\sigma^2}\left(\widetilde{16}\right)[\alpha] = \left(1.94x^2 + 5.18z^2, 4.05x^2 + 10.81z^2\right).$$

$$\widetilde{\sigma_5^2}[\alpha] = y^2\widetilde{\sigma^2}\left(\widetilde{12}\right)[\alpha] + z^2\widetilde{\sigma^2}(\tilde{7})[\alpha] = \left(3.88y^2 + 2.26z^2, 8.11y^2 + 4.73z^2\right).$$

Since the probability level of the chance constraints and the coefficients of a constraint are taken as Gaussian fuzzy number with modal value and spread is given as,

$$\widetilde{0.6} = (0.6, 0.18), \widetilde{9.5} = (9.5, 2.85), \widetilde{4} = (4, 1.2), \widetilde{1075} = (1075, 322.5).$$

For $\alpha = 0.5$, The α-cut of these fuzzy numbers are, $\widetilde{0.6}[\alpha] = [0.38, 0.81]$, $\widetilde{9.5}[\alpha] = [6.15, 12.84]$, $\widetilde{4}[\alpha] = [2.59, 5.40]$, $\widetilde{1075}[\alpha] = [696.14, 1453]$.

Step 2 crisp conversion of each chance constraints of the problem (12), using (10) are equivalent to,

First chance constraint,

$$F\left(\frac{1400 - 16.22x - 22.99y}{\sqrt{8.11x^2 + 11.49y^2}}\right) \geq 0.81$$
$$\left(\frac{1400 - 16.22x - 22.99y}{\sqrt{8.11x^2 + 11.49y^2}}\right) \geq F^{-1}(0.81) = 0.87$$
$$\Rightarrow 6.13x^2 + 8.69y^2 - (1400 - 16.22x - 22.99y)^2 \leq 0.$$

Similarly, the second constraint,

$$F\left(\frac{1000 - 4.05x - 12.17y - 10.81z}{\sqrt{2.02x^2 + 6.08y^2 + 5.4z^2}}\right) \geq 0.81$$

$$\left(\frac{1000 - 4.05x - 12.17y - 10.81z}{\sqrt{2.02x^2 + 6.08y^2 + 5.4z^2}}\right) \geq F^{-1}(0.81) = 0.87$$

$$\Rightarrow 1.52x^2 + 4.6y^2 + 4.08z^2 - (1000 - 4.05x - 12.17y - 10.81z)^2 \leq 0.$$

The third constraint,

$$F\left(\frac{1750 - 13.52x - 17.58y - 20.28z}{\sqrt{6.76x^2 + 8.79y^2 + 10.14z^2}}\right) \geq 0.81$$

$$\left(\frac{1750 - 13.52x - 17.58y - 20.28z}{\sqrt{6.76x^2 + 8.79y^2 + 10.14z^2}}\right) \geq F^{-1}(0.81) = 0.87$$

$$\Rightarrow 5.11x^2 + 6.65y^2 + 7.67z^2 - (1750 - 13.52x - 17.58y - 20.28z)^2 \leq 0.$$

The fourth constraint,

$$F\left(\frac{1325 - 8.11x - 21.63z}{\sqrt{4.05x^2 + 10.81z^2}}\right) \geq 0.81$$

$$\left(\frac{1325 - 8.11x - 21.63z}{\sqrt{4.05x^2 + 10.81z^2}}\right) \geq F^{-1}(0.81) = 0.87$$

$$\Rightarrow 3.06x^2 + 8.18z^2 - (1325 - 8.11x - 21.63z)^2 \leq 0.$$

The fifth constraint,

$$F\left(\frac{900 - 16.22y - 9.46z}{\sqrt{8.11y^2 + 4.73z^2}}\right) \geq 0.81$$

$$\left(\frac{900 - 16.22y - 9.46z}{\sqrt{8.11y^2 + 4.73z^2}}\right) \geq F^{-1}(0.81) = 0.87$$

$$\Rightarrow 6.13y^2 + 3.58z^2 - (900 - 16.22y - 9.46z)^2 \leq 0.$$

The sixth constraint,

$$6.15x + 6.15y + 2.59z \leq 696.14$$

And the seventh constraint

$$12.84x + 12.84y + 5.40z \leq 1453$$

By using the above crisp nonlinear constraints equivalent to chance constrains, the problem (12) becomes a nonlinear multi-objective programming problem as,

Max $z^{(1)} = 50x + 100y + 17.5z$

Max $z^{(2)} = 92x + 75y + 50z$

Max $z^{(3)} = 25x + 100y + 75z$

s.t.

$$6.13x^2 + 8.69y^2 - (1400 - 16.22x - 22.99y)^2 \leq 0$$

$$1.52x^2 + 4.6y^2 + 4.08z^2 - (1000 - 4.05x - 12.17y - 10.81z)^2 \leq 0$$

$$5.11x^2 + 6.65y^2 + 7.67z^2 - (1750 - 13.52x - 17.58y - 20.28z)^2 \leq 0 \qquad (13)$$

$$3.06x^2 + 8.18z^2 - (1325 - 8.11x - 21.63z)^2 \leq 0$$

$$6.13y^2 + 3.58z^2 - (900 - 16.22y - 9.46z)^2 \leq 0$$

$$6.15x + 6.15y + 2.59z \leq 696.14$$

$$12.84x + 12.84y + 5.40z \leq 1453$$

$$x, y, z \geq 0$$

Step 3, 4 and 5 Solving each objective of (13) as single objective nonlinear programming problem using MATLAB (online free version) software, the solutions obtained for each of the objective and the computed values of other remaining objectives functions with each solution sets are placed in Table 2.

Step 6 lower and upper bounds of each objective are obtained from Table 2 as,

$$\max z^{(1)} = 9581.8, \quad \min z^{(1)} = 6728.02$$

$$\max z^{(2)} = 10706, \quad \min z^{(2)} = 9789$$

$$\max z^{(3)} = 14704, \quad \min z^{(3)} = 8986.5$$

Step 7 construct linear membership function for each objective functions as,

$$\mu_{z^{(1)}} = \begin{cases} 0 & \text{if } z^{(1)} \leq \min z^{(1)} \\ \frac{z^{(1)} - \min z^{(1)}}{\max z^{(1)} - \min z^{(1)}} & \text{if } \min z^{(1)} \leq z^{(1)} \leq \max z^{(1)} \\ 1 & \text{if } z^{(1)} \geq \max z^{(1)} \end{cases} . \qquad (14)$$

Step 8 To find the pareto optimal solution of (13), using the linear membership functions for each objective as defined in (14) and applying the fuzzy programming technique, we formulate the problem as,

Table 2 Solution of NLPP (13), taking one objective at a time and the values of other two objectives for each solution set

$x = 0$	$x = 59.69$	$x = 0$
$y = 83.46$	$y = 26.02$	$y = 69.85$
$z = 70.59$	$z = 65.23$	$z = 102.91$
$\max z^{(1)} = 9581.8$	$z^{(1)} = 6728.02$	$z^{(1)} = 8785.92$
$z^{(2)} = 9789$	$\max z^{(2)} = 10706$	$z^{(2)} = 10384.25$
$z^{(3)} = 13640.25$	$z^{(3)} = 8986.5$	$\max z^{(3)} = 14704$

		$z^{(1)}$	$z^{(2)}$	$z^{(3)}$
Table 3 Solution of (12) for distinct values of α				
$\beta = 0.6$	$\alpha = 0.1$	6602.2	9294.74	8676
	$\alpha = 0.3$	8696.2	8680.69	11,147
	$\alpha = 0.5$	8731.22	9296.11	12,759.5
	$\alpha = 0.7$	9156.27	8825.93	11,601.25
	$\alpha = 1$	8470.17	10,002.09	11,931.5

Maximize λ

s.t.

$$50x + 100y + 17.5z - 2853.78\lambda \geq 6728.02$$

$$92x + 75y + 50z - 917\lambda \geq 9789$$

$$25x + 100y + 75z - 5717.5\lambda \geq 8986.5$$

$$6.13x^2 + 8.69y^2 - (1400 - 16.22x - 22.99y)^2 \leq 0$$

$$1.52x^2 + 4.6y^2 + 4.08z^2 - (1000 - 4.05x - 12.17y - 10.81z)^2 \leq 0 \qquad (15)$$

$$5.11x^2 + 6.65y^2 + 7.67z^2 - (1750 - 13.52x - 17.58y - 20.28z)^2 \leq 0$$

$$3.06x^2 + 8.18z^2 - (1325 - 8.11x - 21.63z)^2 \leq 0$$

$$6.13y^2 + 3.58z^2 - (900 - 16.22y - 9.46z)^2 \leq 0$$

$$6.15x + 6.15y + 2.59z \leq 696.14$$

$$12.84x + 12.84y + 5.40z \leq 1453$$

$$x, y, z \geq 0$$

By using MATLAB (online free version) software, the above NLPP (15) can be solved and the solutions obtained are as follows,

$$x = 2.33, \quad y = 73.71, \quad z = 71.07, \quad \lambda = 0.5$$

$$z^{(1)} = 8731.22, \quad z^{(2)} = 9296.11, \quad z^{(3)} = 12759.5$$

Step 9 In this step the problem (step 1 to step 8) for distinct values of α, has been solved and the results obtained are placed in Table 3.

Step 10 Solution of the chance constrained problem for different values of β with distinct $\alpha's$ are placed in Table 4.

7 Results and discussion

As the objective function of the original problem is of maximization type, and we can easily see that for $\alpha = 0.5$, we have solution which gives the better values for each objective**,** therefore we compare the result obtained for different values of β with $\alpha = 0.5$.

We have also seen that for $0.8 \leq \beta \leq 1$, there is no feasible solution obtained with any $\alpha - cut$.

Table 4 Solution for different values of β and α-cut

		$z^{(1)}$	$z^{(2)}$	$z^{(3)}$
$\beta = 0.4$	$\alpha = 0.1$	7299	9049.08	10,124.75
	$\alpha = 0.3$	7307.22	9482.77	9197.25
	$\alpha = 0.5$	7449.67	9734.5	9462.25
	$\alpha = 0.7$	7535.47	8882.29	9013
	$\alpha = 1$	7874.5	9929.08	9749.75
$\beta = 0.7$	$\alpha = 0.1$	7093.1	8999.56	9209.25
	$\alpha = 0.3$	6946.7	9902.5	9844.75
	$\alpha = 0.5$	7116.95	9931.77	9187.2
	$\alpha = 0.7$	7939.2	9248.95	9573.5
	$\alpha = 1$	8282.82	9123.85	12,185.25

8 Conclusion

Here, we have developed a methodology for solving mixed fuzzy chance constrained Multi-Objective Programming Problem (MOPP), which involves FRV coefficient in chance constrained and parameter of the random variable is taken as fuzzy number with Gaussian membership function. Fuzzy chance constraint has been converted to crisp constraints by using method given by Nanda et al. [19], where they have taken triangular membership function. Here, the Gaussian membership function has been considered to generalize the existing method. For numerical illustration we have taken the same problem as Bharati [4, 5] and do the comparison with the solution obtained at $\alpha = 0.5$ and for different values of β by proposed method as given in the Table 5.

The results placed in Table 5 depict that the solution obtained by proposed method is better than that of Bharati [4, 5]. Thus the method clearly shows its superiority over existing as also depicted from Fig. 2.

9 Future scopes

In future, one can consider this approach for solving mixed fuzzy stochastic programming problem involves FRV with different fuzzy parameters of the random variable with different probability distributions and can also apply this study in transportation problem, crop production problem, portfolio optimization problem and in many more areas where such type of vagueness and uncertainties are involved. Here we have taken crisp objectives, the methodology can be generalised for objectives with uncertainties.

Table 5 Compare the solution of proposed method with Bharati [4, 5]

	Bharati [4]	Bharati [5]	Proposed method		
			$\beta_1 = 0.4$	$\beta_2 = 0.6$	$\beta_3 = 0.7$
$z^{(1)}$	7246.41	7906.48	7449.67	8731.22	7116.95
$z^{(2)}$	10,348.50	10,087.87	9734.5	9296.11	9931.77
$z^{(3)}$	8530.42	9269.28	9462.25	12,759.5	9187.2
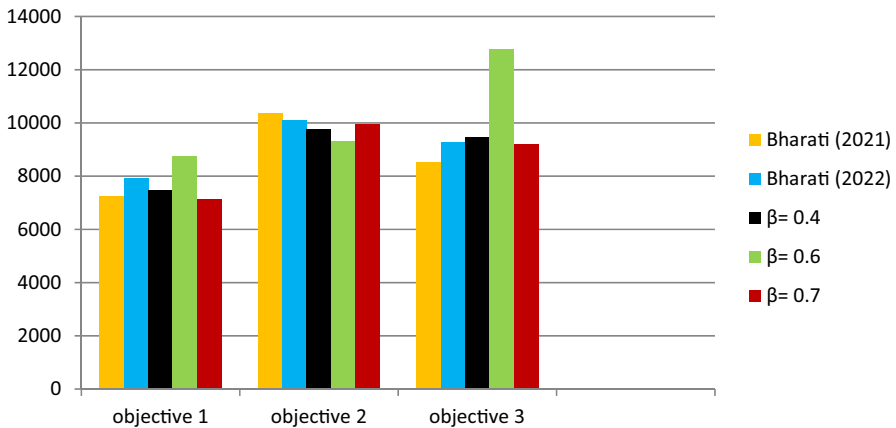
Fig. 2 Comparison bar chart for the proposed method with two existing methods

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Availability of data and materials Not applicable.

Declarations

Conflict of interest We have no competing interests.

Ethical approval and consent to participate Not applicable.

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