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THEORETICAL ARTICLE

Solving zero-sum two-person game with triangular fuzzy number payoffs using new fully fuzzy linear programming models

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Abstract

Many situations involve uncertainty, which we can handle with the help of triangular fuzzy numbers (TFNs). Many scenarios arise in which players in a matrix game cannot reliably estimate their payoffs using crisp numbers, as in real-world scenarios. In these circumstances, TFNs are helpful in game theory. Solving a zero-sum two-player game when all the decision variables and parameters are fuzzy is a worldwide topic of interest to scholars. This article presents a novel solution methodology to solve the zero-sum two-person fully fuzzy matrix game. The payoff matrix, decision variables, and strategies are all taken as TFNs. Two subsidiaries' fully fuzzy linear programming problem (FFLPP) models for both players have been developed to achieve the objective. These two FFLPP models are converted into crisp linear programming problems (LPPs). This procedure uses a ranking approach to the objective function and introduces fuzzy surplus and fuzzy slack variables in constraints. These crisp LPPs are then solved using TORA software (2.0 version) to get optimal strategies and results. The proposed solution methodology in the paper is followed by a real-world example, 'Plastic Ban Problem', and two other examples to prove its applicability and validity.

Keywords Fully fuzzy matrix games · Triangular fuzzy number payoffs · Zero-sum two-person game · Fully fuzzy linear programming problem

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1 Introduction

Game theory is a mathematical approach for investigating situations with competing interests involving decision makers (DMs). American economists Neumann and Morgenstern [49] conceptualized game theory for the first time in their work "Theory of Games, and Economic Behavior". The growth of the work in game theory got pace due to the publication of works by Nash, Shapley [44], Aumann and Peleg [6], Gillies [22, 23], Simon [45] and Raiffa [36], Raiffa and Luce [35] during the decade of 1950. Classical game theory assumes appropriate information and reliable data; nevertheless, it is hard to analyze information accurately in real games due to a lack of knowledge of some elements and a player's vague understanding of events. In such cases, fuzzy set theory has shown to be highly effective in game theory. Zadeh [51] was the first mathematician to conceptualize fuzzy sets. This was effectively used in the problems of decision-making by Bellman and Zadeh [9]. This idea was further explored by Tanaka et al. [47] for solving fuzzy mathematical programming problems. Zimmermann [52] conceptualized the idea of Fuzzy linear programming problem (FLPP) models for the first time, in which decision variables were taken as fuzzy numbers. When all the variables, parameters and coefficients used in an LPP are taken as fuzzy numbers, such an LPP model is known as a fully fuzzy linear programming problem (FFLPP)

Campos [16] firstly developed a method to find the solution to fuzzy matrix games using a ranking function to put fuzzy numbers in order. Later, Buckley and Feuring [15] introduced a solution for an FLPP by converting the objective function of FLPP to a multiple objective LPP model. Liu [32] introduced a method to solve FLPP based on the quantum of satisfaction of constraints involved in the FLPP. Bector et al. [8] explained duality in FLP Models and used it to solve fuzzy matrix games. Vijay et al. [48] used the fuzzy relation approach to solve FMG with a predefined fuzzy goal. Li [30] solved FMGs with TFN payoff matrix with the help of α-cuts. Seikh et al. [40] used a novel approach to game solving based on the concept of the weighted average operator and the score function. Kumar and Kaur [27] gave a solution method to solve FFLPP using a ranking function. Ammar et al. [3] and Brikaa et al. [13] studied continuous differential games under a fuzzy rough environment and gave a saddle point solution. Brikaa et al. [14] applied the Mehar approach to solve games with payoff matrix filled with dual hesitant triangular fuzzy numbers. Brikaa et al. [11, 12] studied zero-sum multi-criteria matrix games and constrained matrix games in a rough fuzzy environment and provided a program algorithm for their solution. Das et al. [19] used TFNs to solve FFLPP by introducing fuzzy slack and surplus variables to convert the fuzzy constraint inequalities into equalities. Mahmoodirad et al. [34] converted the fuzzy problem into a crisp multi-objective non-linear problem and used the principles of fuzzy theory and componentwise optimization.Triangular Pythagorean fuzzy numbers were extended by Akram et al. [1] to the concept of crisp linear programming problems in a Pythagorean fuzzy environment.

Intuitionistic Fuzzy sets (IFS) were presented by Atanassov [4, 5] to add more accuracy to the results by adding a second component of the degree of non-belongingness to fuzzy sets. The Minkowski distance of Type-2 Intuitionistic fuzzy sets (T2IFSs) based on the Hausdorff metric was proposed by Karmakar et al. [24] to construct matrix games in the T2IF environment and then used the proposed distance measure to solve the matrix games by producing a similarity measure of T2IFS. They tested the applicability and validity of the methodology by applying it to the bio-gas plant installation challenge. Using the TOPSIS Technique and dominance property of game theory, Bhaumik et al. [10] studied the fast-rising problem of human trafficking and viewed it through the famous game of "Prisoners dilemma" using hesitant interval-valued IFS. Seikh and Dutta [37] proposed a method to convert the framed game situation into a deterministic model by breaking the single interval type objective function into multi-objective functions. Li [28, 29] proposed a two-tier LPP model for solving FMGs with payoff matrix as TFNs, widely known as 'Li's Model'. From Li's model, it's clear that in an FMG with TFN payoffs, a player's loss ceiling and gain floor may not be identical; however they may be TFNs. Ammar and Birkaa [2] cracked the puzzle of constrained matrix games under a fuzzy rough environment by giving an effective method to break it into four crisp LPPs.

Later Smarandache [46] provided a better generalization of IFS to introduce Neutrosophic fuzzy sets by adding a third component of the degree of hesitancy to IFS. Trapezoidal neutrosophic linear programming (TrNLP) problems with uncertainties were solved using a new computing approach that simplifies the presentation of the complex problem by Das and Chakraborty [21]. A new methodology for solving neutrosophic linear programming problems (NLPPs) known as the pentagonal neutrosophic (PN) approach has been described by Das et al. [18], in which objectives and constraints were represented by pentagonal neutrosophic numbers (PNN). Seikh et al. [38], Seikh and Dutta [41] addressed two aspects of neutrosophic mathematical programming using two different approaches and to demonstrate the validity and effectiveness of the two methodologies, they provided a market share case study in telecom sector and another numerical example. Das et al. [20] solved fractional FLPP and illustrated its applications in the industry sector. Seikh et al. [42] gave a solution technique of FMG with payoff matrix of dense triangular fuzzy lock sets (DTFLS) by defining a new ranking function, further Seikh and Karmakar [39] solved FMGs with DTFLS payoff matrix by defining credibility equilibrium strategy for the decision makers.

Most of the work discussed above involves fuzziness, either in the payoff matrix or in the objective function, with a few exceptions dealing with fuzzy payoffs and fuzzy goals. Very little work deals with fully fuzzified matrix games, where even the players' strategies are taken as fuzzy numbers. In this paper, we will look at the case of a fully fuzzified matrix game (FFMG) in which the payoffs and strategies of the players are treated as arbitrary TFNs. A few researchers who have worked with FFMGs have solved the problem using the traditional Yager's [50] resolution method. This has motivated us to work in the direction of solving Fully Fuzzy Matrix Games (FFMG) using a novel technique, hence this article. This paper proposes a new, effective, and simple linear programming (LP) model technique for solving fully fuzzified matrix games with TFNs as payoffs. We defuzzified the objective function using a ranking function and introduced fuzzy surplus and slack variables in constraint inequalities, which are fuzzy too. As a result, our method is a novel approach to solving such challenges.

According to the following outline, this paper is written: Some preliminary definitions of the Fuzzy set theory are introduced in Sect. 2. In Sect. 3, the focus is on FFMG concepts, and its solution methodology is explained. In Sect. 4, three numerical are solved to show the practical applicability of the proposed solution methodology. The results obtained have been discussed in Sect. 5. Section 6 compares the results obtained by some other renowned researchers. Finally, the conclusion arrives in Sect. 7.

2 Preliminaries

This section is devoted to some fundamental notions and definitions of fuzzy set theory and its algebra (Fig. 1).

Definition 1 [25] A Triplet $\tilde{E} = (e_L, e, e_R)$ is known as **Triangular Fuzzy Number** (TFN) if its membership function $\tilde{E}(z) : \mathbb{R} \to [0, 1]$ is given by

$$\tilde{E}(z) := \begin{cases} \frac{z-e_L}{e-e_L} & \text{if } e_L \le z \le e \\ \frac{e_R-z}{e_R-e} & \text{if } e \le z \le e_R \\ 0 & \text{otherwise} \end{cases}$$

Definition 2 [25] The α **-cut** $\tilde{E}_\alpha$ of TFN $\tilde{E} = \left(e_L, e, e_R\right)$ is written as the crisp set $\tilde{E}_\alpha = \left[\left(e - e_L\right)\alpha + e_L, \left(e - e_R\right)\alpha + e_R\right] = \left[{}^{L}E_\alpha, {}^{R}E_\alpha\right]$ (say) for some $0 \le \alpha \le 1$.

Definition 3 [25] A TFN $\tilde{E} = \left(e_L, e, e_R\right)$ is known as

(a) a **non-negative** fuzzy number iff $e_L \ge 0$.
(b) a **non-positive** fuzzy number iff $e_R \le 0$.
(c) an **unrestricted** fuzzy number iff $e_L < 0$. and $e_R > 0$
(d) a **zero** fuzzy number iff $e_L = e = e_R = 0$ and we write it as $\tilde{0} = (0, 0, 0)$
(e) a **unit** fuzzy number iff $e_L = e = e_R = 1$ and we write it as $\tilde{1} = (1, 1, 1)$

Remark Any real number λ can be written as a TFN $\tilde{\lambda} = (\lambda, \lambda, \lambda)$

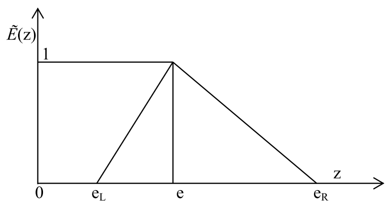

Fig. 1 membership function of TFN

Definition 4 [25] Two TFNs $\tilde{E} = \left(e_L, e, e_R\right)$ and $\tilde{G} = \left(g_L, g, g_R\right)$ are taken to be equal iff $e_L = g_L$; $e = g$; $e_R = g_R$.

Definition 5 [31] A **ranking or defuzzifying function** F is a real function defined with domain as the set of all fuzzy numbers, which maps each fuzzy numbers into the real line, where a natural order exists, i.e $F : N(\mathbb{R}) \to \mathbb{R}$, where $N(\mathbb{R})$ is a set of all fuzzy numbers. If $E = (e_L, e, e_R) \in N(\mathbb{R})$ is a TFN then we take $F(E) = \frac{1}{4}\left[e_L + 2e + e_R\right]$.

Definition 6 [25] **Arithmetic operations** on two TFNs $\tilde{E} = \left(e_L, e, e_R\right)$ and $\tilde{G} = \left(g_L, g, g_R\right)$ are outlined as follows:-

(i) **Addition**: $\tilde{E} \oplus \tilde{G} = \left(e_L + g_L, e + g, e_R + g_R\right)$.
(ii) **Subtraction**: $\tilde{E} \ominus \tilde{G} = \left(e_L - g_R, e - g, e_R - g_L\right)$.
(iii) **Multiplication**: Let $\tilde{E} = \left(e_L, e, e_R\right)$ be any TFN and $\tilde{G} = \left(g_L, g, g_R\right)$ be a non-neg TFN (i.e. $g_L \geq 0$) Then

$$\tilde{E} \otimes \tilde{G} = \begin{cases} \left(e_L g_L, eg, e_R g_R\right) & \text{if } e_L \geq 0 \\ \left(e_L g_R, eg, e_R g_L\right) & \text{if } e_R < 0 \\ \left(e_L g_R, eg, e_R g_R\right) & \text{if } e_L < 0; e_R \geq 0. \end{cases}$$

(iv) **Scalar Multiplication**:

$$\lambda \tilde{E} = \begin{cases} \left(\lambda e_L, \lambda e, \lambda e_R\right) & \text{if } \lambda \geq 0 \\ \left(\lambda e_R, \lambda e, \lambda e_L\right) & \text{if } \lambda < 0 \end{cases}$$

(v) **Division**: Let $\tilde{E} = \left(e_L, e, e_R\right)$ be any TFN and $\tilde{G} = \left(g_L, g, g_R\right)$ be a non-neg TFN, then
$$\tilde{E} \oslash \tilde{G} = \left(\frac{e_L}{g_R}, \frac{e}{g}, \frac{e_R}{g_L}\right).$$

Definition 7 [25] The **ordering** of two TFNs $\tilde{E} = \left(e_L, e, e_R\right)$ and $\tilde{G} = \left(g_L, g, g_R\right)$ is conceptualised as follows

(i) $\tilde{G} \preceq \tilde{E}$ if and only if $F(\tilde{G}) \leq F(\tilde{E})$.
(ii) $\tilde{G} \prec \tilde{E}$ if and only if $F(\tilde{G}) < F(\tilde{E})$.

where F is the defuzzification function.

Note:- $F(\tilde{G} + \tilde{E}) = F(\tilde{G}) + F(\tilde{E})$ and the extension of this property for n-fuzzy numbers $\tilde{E}_1, \tilde{E}_2, \ldots, \tilde{E}_n$, may be given as $F\left(\sum_{k=1}^{n} \tilde{E}_k\right) = \sum_{k=1}^{n} F\left(\tilde{E}_k\right)$.

3 Fuzzy matrix games (FMG)

3.1 Fully fuzzified matrix games (FFMG)

Let $\tilde{E} = \left(\tilde{a}_{jk}\right)_{m\times n} \in (\mathrm{TFN}(\mathbb{R}))^{m\times n}$ be fuzzy matrix with TFN entries where $\tilde{a}_{jk} = \left((a_{jk})_L, a_{jk}, (a_{jk})_R\right) \in (\mathrm{TFN}(\mathbb{R}))^{m\times n}$, TFN($\mathbb{R}$) being the set of all TFNs. The matrix $\tilde{E}$ is called the fuzzy payoff matrix for player-I and the fuzzy entity $(\tilde{z}^T\tilde{E}\,\tilde{t})$ for $\tilde{z} \in \tilde{S}^I$ and $\tilde{t} \in \tilde{S}^{II}$ is called fuzzy expected payoff value set to player-I. Then by a fully fuzzified matrix game we mean a triplet $\left(\tilde{S}^I, \tilde{S}^{II}, \tilde{E}\right)$=FFMG (say), where $\tilde{S}^I$ and $\tilde{S}^{II}$ are sets of all fuzzy mixed TFN strategies for player-I and player-II respectively, i.e.

$$\tilde{S}^I = \left\{ \tilde{z} = \left(\tilde{z}_1, \tilde{z}_2, \dots ., \tilde{z}_m\right) : \sum_{j=1}^{m} \tilde{z}_j \simeq \tilde{1}, \tilde{z}_j \in \mathrm{TFN}(\mathbb{R}) \right\}$$

$$\tilde{S}^{II} = \left\{ \tilde{t} = \left(\tilde{t}_1, \tilde{t}_2, \dots ., \tilde{t}_n\right) : \sum_{k=1}^{n} \tilde{t}_k \simeq \tilde{1}, \tilde{t}_k \in \mathrm{TFN}(\mathbb{R}) \right\}$$

Then a couplet $(\tilde{u}, \tilde{v})$ where $\tilde{u}, \tilde{v} \in \mathrm{TFN}(\mathbb{R})$, is known as a **reasonable solution** of FFMG if $\exists\, \tilde{z}^* \in \tilde{S}^I$ and $\tilde{t}^* \in \tilde{S}^{II}$ satisfying

$$(\tilde{z}^*)^T \tilde{E}\,\tilde{t} \geq \tilde{u} \;\; \forall \;\; \tilde{t} \in \tilde{S}^{II}$$
$$(\tilde{z})^T \tilde{E}\,\tilde{t}^* \leq \tilde{v} \;\; \forall \;\; \tilde{z} \in \tilde{S}^{I}$$

and $\tilde{u}$ and $\tilde{v}$ are known as **reasonable values** of FFMG for player-I and player-II respectively.

let's take $\tilde{\Lambda}_1$, $\tilde{\Lambda}_2$ as reasonable value sets for player-I and player-II respectively.

Then if $\exists\, \tilde{u}^* \in \tilde{\Lambda}_1$ and $\tilde{v}^* \in \tilde{\Lambda}_2$ such that

$$F(\tilde{u}^*) \geq F(\tilde{u}) \quad \forall\, \tilde{u} \in \tilde{\Lambda}_1 \quad \text{and} \quad F(\tilde{v}^*) \leq F(\tilde{v}) \quad \forall\, \tilde{v} \in \tilde{\Lambda}_2.$$

Then the quadruple $(\tilde{z}^*, \tilde{t}^*, \tilde{u}^*, \tilde{v}^*)$ is called the solution of FFMG=$\left(\tilde{S}^I, \tilde{S}^{II}, \tilde{E}\right)$ and $\tilde{u}^*, \tilde{v}^*$ are called the fuzzy optimum TFN values of FFMG for player-I and player-II respectively and

$\tilde{z}^*$, $\tilde{t}^*$ are called the fuzzy optimal TFN strategies of FFMG for player-I and player-II respectively.

3.2 Proposed solution methodology

In this section, we now present a new method to overcome the shortcomings of existing methods discussed in Sect. 6.1 in solving a given FFMG $\left(\tilde{S}^I, \tilde{S}^{II}, \tilde{E}\right)$. The steps are as follows:-

Step-1: To write FFLPPs for both the players.

Using the concepts discussed in Sect. 3, we can obtain the FFLPP for the two players-

For player-1 $(\text{FFLPP})^{\text{I}}$.

$$\begin{array}{ll} \text{maximize} & \tilde{u} \text{ where } \tilde{u} = (u_L, u, u_R) \\ \text{subject to} & \tilde{z}^T \tilde{E} \tilde{t} \succeq \tilde{u} \ \ \forall \ \tilde{t} \in \tilde{S}^{II}; \\ & \tilde{z} \in \tilde{S}^I; \tilde{z}, \tilde{t} \ \text{ are TFNs.} \end{array} \tag{1}$$

For player-2 $(\text{FFLPP})^{\text{II}}$.

$$\begin{array}{ll} \text{minimize} & \tilde{v} \text{ where } \tilde{v} = (v_L, v, v_R) \\ \text{subject to} & \tilde{z}^T \tilde{E} \tilde{t} \preceq \tilde{v} \ \ \forall \ \tilde{z} \in \tilde{S}^I \\ & \tilde{t} \in \tilde{S}^{II}. \end{array} \tag{2}$$

Now, it is customary to take the stationary points (i.e. pure strategy) of the convex fuzzy polytope sets $\tilde{S}^I$ and $\tilde{S}^{II}$ in the constraints, we get the following FFLPP's.

$(\text{FFLPP})^{\text{I}}$

$$\begin{array}{ll} \text{maximize} & \tilde{u} \\ \text{subject to} & \tilde{z}^T \tilde{E}_k \succeq \tilde{u}. \\ & e^T \tilde{z} \approx \tilde{1} \\ & \tilde{z} \succeq \tilde{0} \end{array} \tag{3}$$

where E_k $(k = 1, 2, \dots., n)$ is kth column of pay off matrix $\tilde{E}$ and $e^T = (1, 1, \dots, 1)_{1\times m}$.

$(\text{FFLPP})^{\text{II}}$

$$\begin{array}{ll} \text{minimize} & \tilde{v} \\ \text{subject to} & \tilde{E}_j \tilde{t} \preceq \tilde{v}. \\ & e^T \tilde{t} \approx \tilde{1} \\ & \tilde{t} \succeq \tilde{0} \end{array} \tag{4}$$

where E_j $(j = 1, 2, \dots., m)$ is jth row of pay off matrix $\tilde{E}$. Here $e^T = (1, 1, \dots, 1)_{1\times n}$.

This further yields

$(\text{FFLPP})^{\text{I}}$

$$\begin{aligned} &\text{maximize} && \tilde{u} \\ &\text{subject to} && \textstyle\sum_{j=1}^{m} \tilde{a}_{jk} \otimes \tilde{z}_j \succeq \tilde{u} \quad (k = 1, 2, \ldots., n) \\ & && \textstyle\sum_{j=1}^{m} \tilde{z}_j \simeq (1, 1, 1) \\ & && \tilde{z}_j \succeq \tilde{0} \quad (j = 1, 2, \ldots., m). \end{aligned} \tag{5}$$

$(\text{FFLPP})^{\text{II}}$

$$\begin{aligned} &\text{minimize} && \tilde{v} \\ &\text{subject to} && \textstyle\sum_{k=1}^{n} \tilde{a}_{jk} \otimes \tilde{t}_k \preceq \tilde{v} \quad (j = 1, 2, \ldots., m) \\ & && \textstyle\sum_{k=1}^{n} \tilde{t}_k \simeq (1, 1, 1) \\ & && \tilde{t}_k \succeq \tilde{0} \quad (k = 1, 2, \ldots., n). \end{aligned} \tag{6}$$

Step-2: To get FFLPPs with equality constraints and non negative conditions.

Now using fuzzy surplus and slack TFN variables $\tilde{S}_k$ $(k = 1, 2, ..., n)$ and $\tilde{s}_j$ $(j = 1, 2, ..., m)$. we get

$(\text{FFLPP})^{\text{I}}$

$$\begin{aligned} &\text{maximize} && \tilde{u} \\ &\text{subject to} && \textstyle\sum_{j=1}^{m} \tilde{a}_{jk} \otimes \tilde{z}_j \ominus \tilde{S}_k \simeq \tilde{u} \quad (k = 1, 2, \ldots., n) \\ & && \sum_{j=1}^{m} \tilde{z}_j \simeq (1, 1, 1) \\ & && \tilde{S}_k \succeq \tilde{0}, \tilde{z}_j \succeq \tilde{0} \qquad (j = 1, 2, \ldots., m). \end{aligned} \tag{7}$$

$(\text{FFLPP})^{\text{II}}$

$$\begin{aligned} &\text{minimize} && \tilde{v} \\ &\text{subject to} && \sum_{k=1}^{n} \tilde{a}_{jk} \otimes \tilde{t}_k \oplus \tilde{s}_j \simeq \tilde{v} \quad (j = 1, 2, \ldots., m) \\ & && \sum_{k=1}^{n} \tilde{t}_k \simeq (1, 1, 1) \\ & && \tilde{s}_j \succeq \tilde{0}, \tilde{t}_k \succeq \tilde{0} \qquad (k = 1, 2, \ldots., n). \end{aligned} \tag{8}$$

Since $\tilde{S}_k, \tilde{s}_j, \tilde{a}_{jk}, \tilde{z}_j, \tilde{t}_k$ are all TFNs, lets us take $\tilde{S}_k = (S_k, R_k, Q_k), \tilde{s}_j = (s_j, r_j, q_j), \tilde{a}_{jk} = (a_{jk}, b_{jk}, c_{jk}), \tilde{z}_j = (z_1^j, z_2^j, z_3^j), \tilde{t}_k = (t_1^k, t_2^k, t_3^k)$ in Eq. (8) we get.

$(\text{FFLPP})^{\text{I}}$

$$\begin{aligned}
&\text{maximize} && (u_L, u, u_R) \\
&\text{subject to} && \sum_{j=1}^{m} (a_{jk}, b_{jk}, c_{jk}) \otimes (z_1^j, z_2^j, z_3^j) \ominus (S_k, R_k, Q_k) \simeq (u_L, u, u_R) \\
& && \sum_{j=1}^{m} (z_1^j, z_2^j, z_3^j) \simeq (1, 1, 1) \\
& && (z_1^j, z_2^j, z_3^j) \succeq 0, (j = 1, 2, \dots, m) \\
& && (S_k, R_k, Q_k) \succeq \tilde{0} \quad (k = 1, 2, \dots, n).
\end{aligned} \tag{9}$$

$(\text{FFLPP})^{\text{II}}$

$$\begin{aligned}
&\text{minimize} && (v_L, v, v_R) \\
&\text{subject to} && \sum_{k=1}^{n} (a_{jk}, b_{jk}, c_{jk}) \otimes (t_1^k, t_2^k, t_3^k) \oplus (s_j, r_j, q_j) \simeq (v_L, v, v_R) \\
& && \sum_{k=1}^{n} (t_1^k, t_2^k, t_3^k) \simeq (1, 1, 1) \\
& && (t_1^k, t_2^k, t_3^k) \succeq \tilde{0}, (k = 1, 2, \dots, n) \\
& && (s_j, r_j, q_j) \succeq \tilde{0} \quad (j = 1, 2, \dots, m).
\end{aligned} \tag{10}$$

If we take $(a_{jk}, b_{jk}, c_{jk}) \otimes (z_1^j, z_2^j, z_3^j) = (D_{jk}, E_{jk}, F_{jk})$ and

$(a_{jk}, b_{jk}, c_{jk}) \otimes (t_1^k, t_2^k, t_3^k) = (d_{jk}, e_{jk}, f_{jk})$ in Eqs. (9) and (10) respectively, Then we get.

$(\text{FFLPP})^{\text{I}}$

$$
\begin{aligned}
&\text{maximize} && (u_L, u, u_R) \\
&\text{subject to} && \sum_{j=1}^{m}(D_{jk}, E_{jk}, F_{jk}) \ominus (S_k, R_k, Q_k) \simeq (u_L, u, u_R) \;\; (k = 1, 2, \ldots., n) \\
& && \left(\sum_{j=1}^{m} z_1^j, \sum_{j=1}^{m} z_2^j, \sum_{j=1}^{m} z_3^j\right) \simeq (1, 1, 1) \\
& && z_2^j - z_1^j \geq 0 \\
& && z_3^j - z_2^j \geq 0 \\
& && R_k - S_k \geq 0 \\
& && Q_k - R_k \geq 0 \\
& && S_k \geq 0, \quad z_1^j \geq 0 \quad (j = 1, 2, \ldots., m).
\end{aligned} \tag{11}
$$

$(\mathrm{FFLPP})^{\mathrm{II}}$

$$
\begin{aligned}
&\text{minimize} && (v_L, v, v_R) \\
&\text{subject to} && \sum_{k=1}^{n}(d_{jk}, e_{jk}, f_{jk}) \oplus (s_j, r_j, q_j) \simeq (v_L, v, v_R) \;\; (j = 1, 2, \ldots., m) \\
& && \left(\sum_{k=1}^{n} t_1^k, \sum_{k=1}^{n} t_2^k, \sum_{k=1}^{n} t_3^k\right) \simeq (1, 1, 1) \\
& && t_2^k - t_1^k \geq 0 \\
& && t_3^k - t_2^k \geq 0 \\
& && r_k - s_k \geq 0 \\
& && q_k - r_k \geq 0 \\
& && t_1^k \geq 0; \;\; s_k \geq 0. \quad (k = 1, 2, \ldots., n).
\end{aligned} \tag{12}
$$

Step-3: To get crisp $(\mathrm{LPP})^{\mathrm{I}}$ & $(\mathrm{LPP})^{\mathrm{II}}$ for player-I & II respectively.

Now using the defuzzification function in objective functions and algebra of fuzzy numbers ‘Definition 6’ for constraints in Eqs. (11) and (12) gives us the following crisp LPPs for Player-I and player-II respectively.

$(\mathrm{LPP})^{\mathrm{I}}$

$$
\begin{aligned}
\text{maximize} \quad & \tfrac{1}{4}(u_L + 2u + u_R) \\
\text{subject to} \quad & \sum_{j=1}^{m} D_{jk} - Q_k = u_L \\
& \sum_{j=1}^{m} E_{jk} - R_k = u \\
& \sum_{j=1}^{m} F_{jk} - S_k = u_R \quad (k = 1, 2, \dots, n) \\
& z_2^j - z_1^j \geq 0 \\
& z_3^j - z_2^j \geq 0 \\
& R_k - S_k \geq 0 \\
& Q_k - R_k \geq 0 \\
& S_k \geq 0, \quad z_1^j \geq 0 \quad (j = 1, 2, \dots, m).
\end{aligned} \tag{13}
$$

$(\mathrm{LPP})^{\mathrm{II}}$

$$
\begin{aligned}
\text{minimize} \quad & \tfrac{1}{4}(v_L + 2v + v_R) \\
\text{subject to} \quad & \sum_{k=1}^{n} d_{jk} + s_j = v_L \\
& \sum_{k=1}^{n} e_{jk} + r_j = v \quad (j = 1, 2, \dots, m) \\
& \sum_{k=1}^{n} f_{jk} + q_j = v_R \\
& t_2^k - t_1^k \geq 0 \\
& t_3^k - t_2^k \geq 0 \\
& r_j - s_j \geq 0 \\
& q_j - r_j \geq 0 \\
& t_1^k \geq 0; \;\; s_j \geq 0, (k = 1, 2, \dots, n).
\end{aligned} \tag{14}
$$

Step-4: To get the optimum solution of FFMG for the players.

Now, solving crisp LPP's (13) and (14) will give the complete fuzzy optimum solution of the FFMG. $\tilde{z}_j, (j = 1, 2, \dots, m)$ gives the optimal fuzzy strategies for Player-I and $\tilde{t}_k, (k = 1, 2, \dots, n)$ gives the optimal fuzzy strategies for Player-II, $\tilde{u} = (u_L, u, u_R)$ and $\tilde{v} = (v_L, v, v_R)$ are the optimum fuzzy values of the FFMG for Player-I and Player-II respectively.

3.3 Flow chart of the solution methodology

A visual representation of suggested solution methodology is given in the following flow chart (Fig. 2).

The next section contains three examples that demonstrates the application and computing procedure of the proposed solution methodology.

4 Numerical examples

Example 1 (Campos [16]) Consider the FFMG defined by the following payoff matrix of TFNs

$$\tilde{E} = \begin{bmatrix} (175, 180, 190) & (150, 156, 158) \\ (80, 90, 100) & (175, 180, 190) \end{bmatrix}$$

Let us write $\tilde{180} = (175, 180, 190), \tilde{156} = (150, 156, 158), \tilde{90} = (80, 90, 100)$ and assuming that $\tilde{z}_1, \tilde{z}_2$ are the fuzzy optimal strategies and $\tilde{u}$ is the fuzzy optimal value of game for player-I, we have the following FFLPP-I for *I*st player

$$\begin{aligned} &\text{maximize} && F(\tilde{u}) \\ &\text{subject to} && \tilde{180} \otimes \tilde{z}_1 \oplus \tilde{90} \otimes \tilde{z}_2 \ominus \tilde{S}_1 \simeq \tilde{u} \\ &&& \tilde{156} \otimes \tilde{z}_1 \oplus \tilde{180} \otimes \tilde{z}_2 \ominus \tilde{S}_2 \simeq \tilde{u} \\ &&& \tilde{z}_1 \oplus \tilde{z}_2 \simeq \tilde{1} \\ &&& \tilde{z}_1, \tilde{z}_2, \tilde{S}_1, \tilde{S}_2 \succeq \tilde{0}, \quad where\ \tilde{0} = (0, 0, 0) \end{aligned} \tag{15}$$

Taking $\tilde{z}_1 = (z_1^1, z_2^1, z_3^1); \tilde{z}_2 = (z_1^2, z_2^2, z_3^2); \tilde{u} = (u_L, u, u_R); \tilde{S}_1 = (S_1, R_1, Q_1); \tilde{S}_2 = (S_2, R_2, Q_2)$ in Eq. (15) we get

$$
\begin{aligned}
\text{maximize} \quad & \tfrac{1}{4}(u_L + 2u + u_R) \\
\text{subject to} \quad & (175, 180, 190) \otimes (z_1^1, z_2^1, z_3^1) \oplus (80, 90, 100) \\
& \quad \otimes (z_1^2, z_2^2, z_3^2) \ominus (S_1, R_1, Q_1) \simeq (u_L, u, u_R) \\
& (150, 156, 158) \otimes (z_1^1, z_2^1, z_3^1) \oplus (175, 180, 190) \\
& \quad \otimes (z_1^2, z_2^2, z_3^2) \ominus (S_2, R_2, Q_2) \simeq (u_L, u, u_R) \\
& (z_1^1, z_2^1, z_3^1) \oplus (z_1^2, z_2^2, z_3^2) \simeq (1, 1, 1) \qquad (16) \\
& (z_1^1, z_2^1, z_3^1) \geq \tilde{0} \\
& (z_1^2, z_2^2, z_3^2) \geq \tilde{0} \\
& (S_1, R_1, Q_1) \geq \tilde{0} \\
& (S_2, R_2, Q_2) \geq \tilde{0}. \qquad \textit{where } \tilde{0} = (0, 0, 0)
\end{aligned}
$$

On solving the above fuzzy equations using algebra of fuzzy numbers and using the defuzzification function, we have to solve the following crisp LPP for Player-I

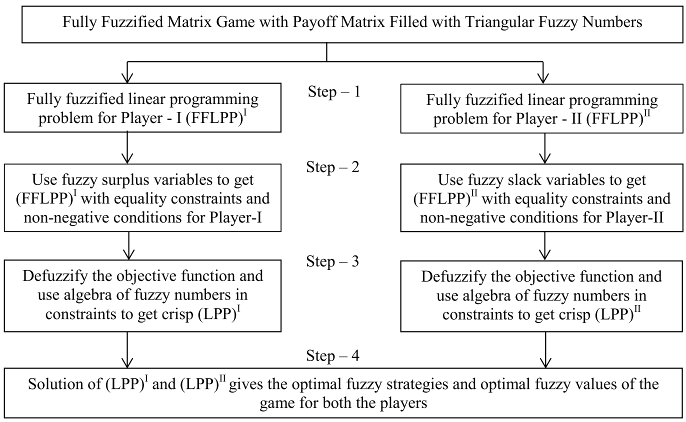

Fig. 2 Flowchart of the solution methodology

$$
\begin{aligned}
\text{maximize} \quad & \tfrac{1}{4}(u_L + 2u + u_R) \\
\text{subject to} \quad & 175z_1^1 + 80z_1^2 - Q_1 = u_L \\
& 180z_2^1 + 90z_2^2 - R_1 = u \\
& 190z_3^1 + 100z_3^2 - S_1 = u_R \\
& 150z_1^1 + 175z_1^2 - Q_2 = u_L \\
& 156z_2^1 + 180z_3^2 - R_2 = u \\
& 158z_3^1 + 190z_3^2 - S_2 = u_R \\
& z_1^1 + z_1^2 = 1 \\
& z_2^1 + z_2^2 = 1 \\
& z_3^1 + z_3^2 = 1 \\
& z_2^1 - z_1^1 \geq 0 \\
& z_3^1 - z_2^1 \geq 0 \\
& z_2^2 - z_1^2 \geq 0 \\
& z_3^2 - z_2^2 \geq 0 \\
& R_1 - S_1 \geq 0 \\
& Q_1 - R_1 \geq 0 \\
& R_2 - S_2 \geq 0 \\
& Q_2 - R_2 \geq 0 \\
& z_1^1, z_1^2, S_1, S_2 \geq 0.
\end{aligned}
\tag{17}
$$

Solving the above crisp LPP in 15 variables and 18 constraints, we get

$$
\begin{aligned}
&\tilde{z}_1 = (z_1^1, z_2^1, z_3^1) = (0.74, 0.74, 0.74) = \tilde{0.74} \\
&\tilde{z}_2 = (z_1^2, z_2^2, z_3^2) = (0.26, 0.26, 0.26) = \tilde{0.26} \\
&\tilde{u} = (u_L, u, u_R) = (150.08, 156.39, 166.39). \\
&\text{Maximum objective function value} = 157.32
\end{aligned}
$$

Now assuming that $\tilde{t}_1, \tilde{t}_2$ are the fuzzy optimal strategies and $\tilde{v}$ is the fuzzy optimal value of the game for Player-II we have the following FFLPP-II

$$\begin{aligned} &\text{minimize} && F(\tilde{v}) \\ &\text{subject to} && \tilde{180} \otimes \tilde{t}_1 \oplus \tilde{156} \otimes \tilde{t}_2 \oplus \tilde{s}_1 \simeq \tilde{v} \\ & && \tilde{90} \otimes \tilde{t}_1 \oplus \tilde{180} \otimes \tilde{t}_2 \oplus \tilde{s}_2 \simeq \tilde{v} \\ & && \tilde{t}_1 \oplus \tilde{t}_2 \simeq \tilde{1} \\ & && \tilde{t}_1, \tilde{t}_2, \tilde{s}_1, \tilde{s}_2 \succeq \tilde{0} \quad \textit{where } \tilde{0} = (0,0,0) \end{aligned} \tag{18}$$

Taking $\tilde{t}_1 = (t_1^1, t_2^1, t_3^1)$;$\tilde{t}_2 = (t_1^2, t_2^2, t_3^2)$;$\tilde{v} = (v_L, v, v_R)$;$\tilde{s}_1 = (s_1, r_1, q_1)$;$\tilde{s}_2 = (s_2, r_2, q_2)$ in Eq. (18) we get

$$\begin{aligned} &\text{minimize} && F(\tilde{v}) \\ &\text{subject to} && (175, 180, 190) \otimes (t_1^1, t_2^1, t_3^1) \oplus (150, 156, 156) \\ & && \quad \otimes (t_1^2, t_2^2, t_3^2) \oplus (s_1, r_1, q_1) \simeq (v_L, v, v_R) \\ & && (80, 90, 100) \otimes (t_1^1, t_2^1, t_3^1) \oplus (175, 180, 190) \\ & && \quad \otimes (t_1^2, t_2^2, t_3^2) \oplus (s_2, r_2, q_2) \simeq (v_L, v, v_R) \\ & && (t_1^1, t_2^1, t_3^1) \oplus (t_1^2, t_2^2, t_3^2) \simeq (1,1,1) \\ & && (t_1^1, t_2^1, t_3^1) \succeq \tilde{0} \\ & && (t_1^2, t_2^2, t_3^2) \succeq \tilde{0} \\ & && (s_1, r_1, q_1) \succeq \tilde{0} \\ & && (s_2, r_2, q_2) \succeq \tilde{0}, \qquad \textit{where } \tilde{0} = (0,0,0). \end{aligned} \tag{19}$$

On solving the above fuzzy equations using algebra of fuzzy numbers and using the defuzzification function, we have to solve the following crisp LPP to get optimum values for Player-II

$$
\begin{aligned}
\text{minimize} \quad & \tfrac{1}{4}(v_L + 2v + v_R) \\
\text{subject to} \quad & 175t_1^1 + 150t_1^2 + s_1 = v_L \\
& 180t_2^1 + 156t_2^2 + r_1 = v \\
& 190t_3^1 + 158t_3^2 + q_1 = v_R \\
& 80t_1^1 + 175t_1^2 + s_2 = v_L \\
& 90t_2^1 + 180t_2^2 + r_2 = v \\
& 100t_3^1 + 190t_3^2 + q_2 = v_R \\
& t_1^1 + t_1^2 = 1 \\
& t_2^1 + t_2^2 = 1 \\
& t_3^1 + t_3^2 = 1 \\
& t_2^1 - t_1^1 \geq 0 \\
& t_3^1 - t_2^1 \geq 0 \\
& t_2^2 - t_1^2 \geq 0 \\
& t_3^2 - t_2^2 \geq 0 \\
& r_1 - s_1 \geq 0 \\
& q_1 - r_1 \geq 0 \\
& r_2 - s_2 \geq 0 \\
& q_2 - r_2 \geq 0 \\
& t_1^1 \geq 0; t_1^2 \geq 0; s_1 \geq 0; s_2 \geq 0.
\end{aligned}
\tag{20}
$$

Now solving the above crisp LPP in 15 variables and 18 constraints, we get

$\tilde{t}_1 = (t_1^1, t_2^1, t_3^1) = (0.21, 0.21, 0.21) = \tilde{0.21}$
$\tilde{t}_2 = (t_1^2, t_2^2, t_3^2) = (0.79, 0.79, 0.79) = \tilde{0.79}$
$\tilde{v} = (v_L, v, v_R) = (155.21, 161.25, 171.25)$.
Minimum objective function value = 162.24

Let us now discuss one more example of FFMG where the payoff matrix consists of some non-positive TFNs.

Example 2 (Li [28]) Consider the FFMG defined by the following payoff matrix of TFNs

$$\tilde{E} = \begin{bmatrix} (18, 20, 23) & (-21, -18, -16) \\ (-33, -32, -27) & (38, 40, 43) \end{bmatrix}$$

Let us write $\tilde{20} = (18, 20, 23), \tilde{40} = (38, 40, 43), \widetilde{(-18)} = (-21, -18, -16)$ and $\widetilde{(-32)} = (-33, -32, -27)$ and assuming that $\tilde{z}_1, \tilde{z}_2$ are the fuzzy optimal strategies and $\tilde{u}$ is the fuzzy optimal value of game for player-I, we have the following FFLPP-I for *I*st player

$$\begin{aligned} \text{maximize} \quad & F(\tilde{u}) \\ \text{subject to} \quad & \tilde{20} \otimes \tilde{z}_1 \oplus \widetilde{(-32)} \otimes \tilde{z}_2 \ominus \tilde{S}_1 \simeq \tilde{u} \\ & \widetilde{(-18)} \otimes \tilde{z}_1 \oplus \tilde{40} \otimes \tilde{z}_2 \ominus \tilde{S}_2 \simeq \tilde{u} \\ & \tilde{z}_1 \oplus \tilde{z}_2 \simeq \tilde{1} \\ & \tilde{z}_1, \tilde{z}_2, \tilde{S}_1, \tilde{S}_2 \succeq \tilde{0} \quad \textit{where } \tilde{0} = (0, 0, 0) \end{aligned} \tag{21}$$

Taking $\tilde{z}_1 = (z_1^1, z_2^1, z_3^1); \tilde{z}_2 = (z_1^2, z_2^2, z_3^2); \tilde{u} = (u_L, u, u_R); \tilde{S}_1 = (S_1, R_1, Q_1); \tilde{S}_2 = (S_2, R_2, Q_2)$ in Eq. (21) we get

$$\begin{aligned} \text{maximize} \quad & \tfrac{1}{4}(u_L + 2u + u_R) \\ \text{subject to} \quad & (18, 20, 23) \otimes (z_1^1, z_2^1, z_3^1) \oplus (-33, -32, -27) \\ & \quad \otimes (z_1^2, z_2^2, z_3^2) \ominus (S_1, R_1, Q_1) \simeq (u_L, u, u_R) \\ & (-21, -18, -16) \otimes (z_1^1, z_2^1, z_3^1) \oplus (38, 40, 43) \\ & \quad \otimes (z_1^2, z_2^2, z_3^2) \ominus (S_2, R_2, Q_2) \simeq (u_L, u, u_R) \\ & (z_1^1, z_2^1, z_3^1) \oplus (z_1^2, z_2^2, z_3^2) \simeq (1, 1, 1) \\ & (z_1^1, z_2^1, z_3^1) \succeq \tilde{0} \\ & (z_1^2, z_2^2, z_3^2) \succeq \tilde{0} \\ & (S_1, R_1, Q_1) \succeq \tilde{0} \\ & (S_2, R_2, Q_2) \succeq \tilde{0}. \quad \textit{where } \tilde{0} = (0, 0, 0) \end{aligned} \tag{22}$$

On solving the above fuzzy equations using algebra of fuzzy number and using the defuzzification function, we have to solve the following crisp LPP to get optimal values for Player-I

$$
\begin{aligned}
\text{maximize} \quad & \tfrac{1}{4}(u_L + 2u + u_R) \\
\text{subject to} \quad & 18z_1^1 - 33z_3^2 - Q_1 = u_L \\
& 20z_2^1 - 32z_2^2 - R_1 = u \\
& 23z_3^1 - 27z_1^2 - S_1 = u_R \\
& -21z_3^1 + 38z_1^2 - Q_2 = u_L \\
& -18z_2^1 + 40z_2^2 - R_2 = u \\
& -16z_1^1 + 43z_3^2 - S_2 = u_R \\
& z_1^1 + z_1^2 = 1 \\
& z_2^1 + z_2^2 = 1 \\
& z_3^1 + z_3^2 = 1 \\
& z_2^1 - z_1^1 \geq 0 \\
& z_3^1 - z_2^1 \geq 0 \\
& z_2^2 - z_1^2 \geq 0 \\
& z_3^2 - z_2^2 \geq 0 \\
& R_1 - S_1 \geq 0 \\
& Q_1 - R_1 \geq 0 \\
& R_2 - S_2 \geq 0 \\
& Q_2 - R_2 \geq 0 \\
& z_1^1, z_1^2, S_1, S_2 \geq 0.
\end{aligned}
\tag{23}
$$

Solving the above crisp LPP in 15 variables and 18 constraints, we get

$$\tilde{z}_1 = (z_1^1, z_2^1, z_3^1) = (0.64, 0.64, 0.64) = \tilde{0.64}$$
$$\tilde{z}_2 = (z_1^2, z_2^2, z_3^2) = (0.36, 0.36, 0.36) = \tilde{0.36}$$
$$\tilde{u} = (u_L, u, u_R) = (-1.25, 1.39, 5.11).$$
Maximum objective function value = 1.66

Now assuming that $\tilde{t}_1, \tilde{t}_2$ are the fuzzy optimal strategies and $\tilde{v}$ is the fuzzy optimal value of the game for Player-II we have the following FFLPP-II

$$\begin{aligned}
&\text{minimize} && F(\tilde{v}) \\
&\text{subject to} && \tilde{20} \otimes \tilde{t}_1 \oplus \widetilde{(-18)} \otimes \tilde{t}_2 \oplus \tilde{s}_1 \simeq \tilde{v} \\
& && \widetilde{(-32)} \otimes \tilde{t}_1 \oplus \tilde{40} \otimes \tilde{t}_2 \oplus \tilde{s}_2 \simeq \tilde{v} \\
& && \tilde{t}_1 \oplus \tilde{t}_2 \simeq \tilde{1} \\
& && \tilde{t}_1, \tilde{t}_2, \tilde{s}_1, \tilde{s}_2 \succeq \tilde{0} \quad \textit{where } \tilde{0} = (0,0,0)
\end{aligned} \tag{24}$$

Taking $\tilde{t}_1 = (t_1^1, t_2^1, t_3^1)$;$\tilde{t}_2 = (t_1^2, t_2^2, t_3^2)$;$\tilde{v} = (v_L, v, v_R)$;$\tilde{s}_1 = (s_1, r_1, q_1)$;$\tilde{s}_2 = (s_2, r_2, q_2)$ in equation (24) we get

$$\begin{aligned}
&\text{minimize} && F(\tilde{v}) \\
&\text{subject to} && (18, 20, 23) \otimes (t_1^1, t_2^1, t_3^1) \oplus (-21, -18, -16) \\
& && \quad \otimes (t_1^2, t_2^2, t_3^2) \oplus (s_1, r_1, q_1) \simeq (v_L, v, v_R) \\
& && (-33, -32, -27) \otimes (t_1^1, t_2^1, t_3^1) \oplus (38, 40, 43) \\
& && \quad \otimes (t_1^2, t_2^2, t_3^2) \oplus (s_2, r_2, q_2) \simeq (v_L, v, v_R) \\
& && (t_1^1, t_2^1, t_3^1) \oplus (t_1^2, t_2^2, t_3^2) \simeq (1, 1, 1) \\
& && (t_1^1, t_2^1, t_3^1) \succeq \tilde{0} \\
& && (t_1^2, t_2^2, t_3^2) \succeq \tilde{0} \\
& && (s_1, r_1, q_1) \succeq \tilde{0} \\
& && (s_2, r_2, q_2) \succeq \tilde{0} \quad \textit{where } \tilde{0} = (0,0,0)
\end{aligned} \tag{25}$$

On solving the above fuzzy equations using algebra of fuzzy numbers and using the defuzzification function, we have to solve the following crisp LPP to get optimum values for Player-II

$$
\begin{aligned}
\text{minimize} \quad & \tfrac{1}{4}(v_L + 2v + v_R) \\
\text{subject to} \quad & 18t_1^1 - 21t_3^2 + s_1 = v_L \\
& 20t_2^1 - 18t_2^2 + r_1 = v \\
& 23t_3^1 - 16t_1^2 + q_1 = v_R \\
& -33t_3^1 + 38t_1^2 + s_2 = v_L \\
& -32t_2^1 + 40t_2^2 + r_2 = v \\
& -27t_1^1 + 43t_3^2 + q_2 = v_R \\
& t_1^1 + t_1^2 = 1 \\
& t_2^1 + t_2^2 = 1 \\
& t_3^1 + t_3^2 = 1 \\
& t_2^1 - t_1^1 \geq 0 \\
& t_3^1 - t_2^1 \geq 0 \\
& t_2^2 - t_1^2 \geq 0 \\
& t_3^2 - t_2^2 \geq 0 \\
& r_1 - s_1 \geq 0 \\
& q_1 - r_1 \geq 0 \\
& r_2 - s_2 \geq 0 \\
& q_2 - r_2 \geq 0 \\
& t_1^1 \geq 0; t_1^2 \geq 0; s_1 \geq 0; s_2 \geq 0.
\end{aligned}
\tag{26}
$$

Solving the above crisp LPP in 15 variables and 18 constraints, we get

$$
\begin{aligned}
&\tilde{t}_1 = (t_1^1, t_2^1, t_3^1) = (0.54, 0.54, 0.54) = 0.\tilde{5}4 \\
&\tilde{t}_2 = (t_1^2, t_2^2, t_3^2) = (0.46, 0.46, 0.46) = 0.\tilde{4}6 \\
&\tilde{v} = (v_L, v, v_R) = (-.08, 2.38, 6.45). \\
&\text{Minimum objective function value} = 2.78
\end{aligned}
$$

We now take a practical problem known as plastic ban problem in Seikh et al. [43] to show how our methodology can be used to tackle a real world problem.

Example 3 (Plastic Ban Problem: Seikh et al. [43]) Consider the FFMG defined by the following payoff matrix of TFNs

$$\tilde{E} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} (115,120,125) & (108,110,112) & (73,75,77) \\ (175,180,185) & (115,120,125) & (70,72,75) \\ (140,143,146) & (140,143,146) & (145,150,155) \end{bmatrix} \end{array}$$

where A_1, A_2, A_3 and B_1, B_2, B_3 are the policies initiated by player-I and player-II respectively to reduce the consumption of plastic (Seikh et al. [43]). Let us write $1\tilde{2}0 = (115, 120, 125), 1\tilde{8}0 = (175, 180, 185), 1\tilde{4}3 = (140, 143, 146), 1\tilde{1}0 = (108, 110, 112), \tilde{7}5 = (73, 75, 77), \tilde{7}2 = (70, 72, 75), 1\tilde{5}0 = (145, 150, 155)$ **and assuming that $\tilde{z}_1, \tilde{z}_2, \tilde{z}_3$ are the fuzzy optimal strategies and $\tilde{u}$ is the fuzzy optimal value of game for player-I, we have the following FFLPP-I for *I*st-player**

$$\begin{aligned} \text{maximize} \quad & F(\tilde{u}) \\ \text{subject to} \quad & 1\tilde{2}0 \otimes \tilde{z}_1 \oplus 1\tilde{8}0 \otimes \tilde{z}_2 \oplus 1\tilde{4}3 \otimes \tilde{z}_3 \ominus \tilde{S}_1 \simeq \tilde{u} \\ & 1\tilde{1}0 \otimes \tilde{z}_1 \oplus 1\tilde{2}0 \otimes \tilde{z}_2 \oplus 1\tilde{4}3 \otimes \tilde{z}_3 \ominus \tilde{S}_2 \simeq \tilde{u} \\ & \tilde{7}5 \otimes \tilde{z}_1 \oplus \tilde{7}2 \otimes \tilde{z}_2 \oplus 1\tilde{5}0 \otimes \tilde{z}_3 \ominus \tilde{S}_3 \simeq \tilde{u} \\ & \tilde{z}_1 \oplus \tilde{z}_2 \oplus \tilde{z}_3 \simeq \tilde{1} \\ & \tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \tilde{S}_1, \tilde{S}_2, \tilde{S}_3 \geq \tilde{0}, \quad \textit{where } \tilde{0} = (0,0,0) \end{aligned}$$

Taking $\tilde{z}_1 = (z_1^1, z_2^1, z_3^1), \tilde{z}_2 = (z_1^2, z_2^2, z_3^2), \tilde{z}_3 = (z_1^3, z_2^3, z_3^3), \tilde{u} = (u_L, u, u_R), \tilde{S}_1 = (S_1, R_1, Q_1), \tilde{S}_2 = (S_2, R_2, Q_2), \tilde{S}_3 = (S_3, R_3, Q_3)$ in above FFLPP-I, Using the algebra of TFNs [6] and solution methodology [3.2] we have the following crisp LPP-I for *I*st-player

$$
\begin{aligned}
\text{maximize} \quad & \tfrac{1}{4}(u_L + 2u + u_R) \\
\text{subject to} \quad & 115z_1^1 + 175z_1^2 + 140z_1^3 - Q_1 = u_L \\
& 120z_2^1 + 180z_2^2 + 143z_2^3 - R_1 = u \\
& 125z_3^1 + 185z_3^2 + 147z_3^3 - S_1 = u_R \\
& 108z_1^1 + 115z_1^2 + 140z_1^3 - Q_2 = u_L \\
& 110z_2^1 + 120z_2^2 + 143z_2^3 - R_2 = u \\
& 112z_3^1 + 125z_3^2 + 146z_3^3 - S_2 = u_R \\
& 73z_1^1 + 70z_1^2 + 145z_1^3 - Q_3 = u_L \\
& 75z_2^1 + 72z_2^2 + 150z_2^3 - R_3 = u \\
& 77z_3^1 + 75z_3^2 + 155z_3^3 - S_3 = u_R \\
& z_1^1 + z_1^2 + z_1^3 = 1 z_2^1 + z_2^2 + z_2^3 = 1 z_3^1 + z_3^2 + z_3^3 = 1 \\
& z_2^1 - z_1^1 \geq 0 z_3^1 - z_2^1 \geq 0 \\
& z_2^2 - z_1^2 \geq 0 \\
& z_3^2 - z_2^2 \geq 0 z_2^3 - z_1^3 \geq 0 z_3^3 - z_2^3 \geq 0 \; R_1 - S_1 \geq 0 \\
& Q_1 - R_1 \geq 0 \\
& R_2 - S_2 \geq 0 \\
& Q_2 - R_2 \geq 0 \\
& R_3 - S_3 \geq 0 \\
& Q_3 - R_3 \geq 0 z_1^1, z_1^2, z_1^3, S_1, S_2, S_3 \geq 0.
\end{aligned} \tag{27}
$$

Solving the above crisp LPP involving 21 variables and 24 constraints, we get

$$\tilde{z}_1 = (z_1^1, z_2^1, z_3^1) = (0, 0, 0) = \tilde{0}$$
$$\tilde{z}_2 = (z_1^2, z_2^2, z_3^2) = (0, 0, 0) = \tilde{0}$$
$$\tilde{z}_3 = (z_1^3, z_2^3, z_3^3) = (1, 1, 1) = \tilde{1}$$
$$\tilde{u} = (u_L, u, u_R) = (136, 141, 146).$$
Maximum objective function value = 141

Now assuming that $\tilde{t}_1, \tilde{t}_2, \tilde{t}_3$ are the fuzzy optimal strategies and $\tilde{v}$ is the fuzzy optimal value of the game for Player-II, we have the following FFLPP-II for *II*nd-player

$$\begin{array}{ll}
\text{minimize} & F(\tilde{v}) \\
\text{subject to} & \tilde{120} \otimes \tilde{t}_1 \oplus \tilde{110} \otimes \tilde{t}_2 \oplus \tilde{75} \otimes \tilde{t}_3 \oplus \tilde{s}_1 \simeq \tilde{v} \\
& \tilde{180} \otimes \tilde{t}_1 \oplus \tilde{120} \otimes \tilde{t}_2 \oplus \tilde{72} \otimes \tilde{t}_3 \oplus \tilde{s}_2 \simeq \tilde{v} \\
& \tilde{143} \otimes \tilde{t}_1 \oplus \tilde{143} \otimes \tilde{t}_2 \oplus \tilde{150} \otimes \tilde{t}_3 \oplus \tilde{s}_3 \simeq \tilde{v} \\
& \tilde{t}_1 \oplus \tilde{t}_2 \oplus \tilde{t}_3 \simeq \tilde{1} \\
& \tilde{t}_1, \tilde{t}_2, \tilde{t}_3, \tilde{s}_1, \tilde{s}_2, \tilde{s}_3 \succeq \tilde{0}, \quad \textit{where } \tilde{0} = (0, 0, 0)
\end{array}$$

Taking $\tilde{t}_1 = (t_1^1, t_2^1, t_3^1), \tilde{t}_2 = (t_1^2, t_2^2, t_3^2), \tilde{t}_3 = (t_1^3, t_2^3, t_3^3), \tilde{v} = (v_L, v, v_R), \tilde{s}_1 = (s_1, r_1, q_1)$, $\tilde{s}_2 = (s_2, r_2, q_2), \tilde{s}_3 = (s_3, r_3, q_3)$ in above FFLPP-II, Using the algebra of TFNs [6] and solution methodology [3.2] we have the following crisp LPP-II for *II*nd-player

$$
\begin{aligned}
\text{minimize} \quad & \tfrac{1}{4}(v_L + 2v + v_R) \\
\text{subject to} \quad & 115t_1^1 + 108t_1^2 + 73t_1^3 + s_1 = v_L \\
& 120t_2^1 + 110t_2^2 + 75t_2^3 + r_1 = v \\
& 125t_3^1 + 112t_3^2 + 77t_3^3 + q_1 = v_R \\
& 175t_1^1 + 115t_1^2 + 70t_1^3 + s_2 = v_L \\
& 180t_2^1 + 120t_2^2 + 72t_2^3 + r_2 = v \\
& 185t_3^1 + 125t_3^2 + 75t_3^3 + q_2 = v_R \\
& 140t_1^1 + 140t_1^2 + 145t_1^3 + s_3 = v_L \\
& 143t_2^1 + 143t_2^2 + 150t_2^3 + r_3 = v \\
& 146t_3^1 + 146t_3^2 + 155t_3^3 + q_3 = v_R \\
& t_1^1 + t_1^2 + t_1^3 = 1 \\
& t_2^1 + t_2^2 + t_2^3 = 1 \\
& t_3^1 + t_3^2 + t_3^3 = 1 \\
& t_2^1 - t_1^1 \geq 0 \\
& t_3^1 - t_2^1 \geq 0 \\
& t_2^2 - t_1^2 \geq 0\, t_3^2 - t_2^2 \geq 0 \\
& t_2^3 - t_1^3 \geq 0\, t_3^3 - t_2^3 \geq 0 \\
& r_1 - s_1 \geq 0 q_1 - r_1 \geq 0 r_2 - s_2 \geq 0 \\
& q_2 - r_2 \geq 0 r_3 - s_3 \geq 0\, q_3 - r_3 \geq 0 \\
& t_1^1, t_1^2, t_1^3, S_1, S_2, S_3 \geq 0.
\end{aligned} \tag{28}
$$

Solving the above crisp LPP involving 21 variables and 24 constraints, we get

$$\tilde{t}_1 = (t_1^1, t_2^1, t_3^1) = (0.42, 0.42, 0.42) = 0.\tilde{42}$$
$$\tilde{t}_2 = (t_1^2, t_2^2, t_3^2) = (0.58, 0.58, 0.58) = 0.\tilde{58}$$
$$\tilde{t}_3 = (t_1^3, t_2^3, t_3^3) = (0, 0, 0) = \tilde{0}$$
$$\tilde{v} = (v_L, v, v_R) = (140, 145, 150).$$

Minimum objective function value = 145

5 Results and discussion

1. In Example-1, solution of Eqs. (17) and (20) using TORA software (2.0 version) gives TFN optimal strategies $\tilde{z}^* = (\tilde{z}_1, \tilde{z}_2) = (.\tilde{74}, .\tilde{26})$ and $\tilde{t}^* = (\tilde{t}_1, \tilde{t}_2) = (.\tilde{21}, .\tilde{79})$ for player-I and player-II respectively. Expected payoff of the game for player-I $\tilde{z}^{*T}\tilde{E}\tilde{t}^* = .\tilde{74} \otimes \tilde{180} \otimes .\tilde{21} + .\tilde{74} \otimes \tilde{156} \otimes .\tilde{79} + .\tilde{26} \otimes \tilde{90} \otimes .\tilde{21} + .\tilde{26} \otimes \tilde{180} \otimes .\tilde{79} = (155.198, 161.0556, 166.3788)$, which is a TFN. Defuzzifying it we get the valve of the game as $\frac{1}{4}(155.198 + 2 \times 161.0556 + 166.3788) = 160.922$. It means that Player-I will receive around 160.922 units if he opts his strategies I and II with probabilities $0.\tilde{74}$ and $0.\tilde{26}$ and player-II opts his strategies I and II with probabilities $0.\tilde{21}$ and $.\tilde{79}$ respectively.
2. In Example-2, solution of Eqs. (23) and (26) using TORA software (2.0 version), gives TFN optimal strategies $\tilde{z}^* = (\tilde{z}_1, \tilde{z}_2) = (.\tilde{64}, .\tilde{36})$ and $\tilde{t}^* = (\tilde{t}_1, \tilde{t}_2) = (.\tilde{54}, .\tilde{46})$ for player-I and player-II respectively. Expected payoff of the game for player-I $\tilde{z}^{*T}\tilde{E}\tilde{t}^* = .\tilde{64} \otimes \tilde{20} \otimes .\tilde{54} + .\tilde{64} \otimes -\tilde{18} \otimes .\tilde{46} + .\tilde{36} \otimes -\tilde{32} \otimes .\tilde{54} + .\tilde{36} \otimes \tilde{40} \otimes .\tilde{46} = (-0.084, 2.016, 5.1104)$, which is a TFN. Defuzzifying it we get the valve of the game as $\frac{1}{4}(-.084 + 2 \times 2.016 + 5.1104) = 2.2646$. It means that Player-I will receive around 2.2646 units if he opts his strategies I and II with probabilities $0.\tilde{64}$ and $0.\tilde{36}$ and player-II opts his strategies I and II with probabilities $0.\tilde{54}$ and $.\tilde{46}$ respectively.
3. In the plastic ban problem in Seikh et al. [43] (Example-3) solving the Eqs. (27) and (28)using TORA software (2.0 version) we get TFN optimal strategies $\tilde{z}^* = (\tilde{z}_1, \tilde{z}_2, \tilde{z}_3) = (\tilde{0}, \tilde{0}, \tilde{1})$ and $\tilde{t}^* = (\tilde{t}_1, \tilde{t}_2, \tilde{t}_3) = (.\tilde{42}, .\tilde{58}, \tilde{0})$ for player-I and player-II respectively. Expected payoff of the game for player-I $\tilde{z}^{*T}\tilde{E}\tilde{t}^*$ is calculated as

$$\begin{aligned}\tilde{z}^{*T}\tilde{E}\tilde{t}^* &= (\tilde{1}) \otimes (140, 143, 146) \otimes (.\tilde{42}) + (\tilde{1}) \otimes (140, 143, 146) \otimes (.\tilde{58}) \\ &= (58.8, 60.06, 61.32) + (81.2, 82.94, 84.68) = (140, 143, 146),\end{aligned}$$

which is a TFN. Defuzzifying it we get the value of the game as $\frac{1}{4}(140 + 2 \times 143 + 146) = 143$. It means that if the NGOs in alliance with the government uses strategy A_3 completely and AIPMA uses 42% of strategy B_1, 58% of strategy B_2 then the consumption of plastic will possibly reduce by 143 units with a maximum upto 146 and a minimum to 140 units. Clearly NGO and government alliance will never use their strategies A_1 and A_2 and AIPMA will never use strategy B_3 to achieve desired goal.

6 Comparative analysis

6.1 Existing methods

Various researchers like Campos [16], Bector et al. [8], Li [28–30], Cevikel et al. [17], Kumar and Jangid, Seikh et al. [43] have solved the examples that we have taken in this article by their respective methodologies to obtain the fuzzy optimum solution of FMG with payoff as TFNs. Most of the researchers have used predetermined fuzzy goals, adequacies, aspirations, α-cut set, degree of satisfaction or any other parameter to solve FMG with TFNs as payoffs. Their solutions can be influenced by changing these predetermined fuzzy goals, adequacies, aspirations, etc. So their results cannot be best relied on. Rest of the researchers have used the classical Yager's [50] resolution method to solve FMG, which also involves a parameters. Their results may vary by changing the value of these parameters as well.

6.2 Advantage of our methodology

Our solution methodology has an advantage over the other methods proposed by earlier mentioned researchers because of the following reasons:-

1. We have not used the classical Yager's [50] resolution method to solve FMG, so our is a new solution methodology.
2. We have not used any predetermined adequacies, aspirations or parameters etc., in our solution methodology, so our results can't be influenced by changing them. That's why our results are more realistic and reliable.
3. We have used fuzzy slack and surplus variables like we do in solving the crisp LPP models, so our solution methodology is easy to understand and less on calculations.
4. The results obtained by our solution methodology are very close to the results obtained by researchers for all three examples in this article, which shows that our proposed solution methodology is valid. A representation of comparison with several researchers has been depicted with the help of three comparison tables, table-[1], [2], [3] for examples-1, 2, 3 respectively.

7 Conclusion

In this study, we have suggested a new solution methodology for finding the optimal solution to a fully fuzzified two-player zero-sum matrix game. TFNs indicate the FFMG's fuzzy payoff matrix and fuzzy strategies for both players. Since we deal with a fully fuzzified game model, we have chosen strategies as TFNs for both the

Table 1 Comparison of our work with the results of other researchers for example-1

References	Adequacies, margin aspiration level or any parameter used	Player-Is optimal strategy	Player-IIs optimal strategy	Player-Is value of game	Player-IIs value of game
Campos [16]	Yes	(.77,.23)	(.23,.77)	Around(162.58)	Around(162.58)
		(.79,.21)	(.21,.79)	Around(158.13)	Around(158.13)
Bector et al. [7]	Yes	(.77,.23)	(.23,.77)	Around(160.9)	Around(160.6)
		(.79,.21)	(.21,.79)	(155,161.05,164.73)	(155.26,161.25,171.25)
Cevikel and Ahlatioglu [17]	Yes (α = .8)	(.78,.22)	(.22,.78)	Not computed	Not computed
Vijay et al. [48]	Yes (α = .7)	(.77,.23)	(.23,.77)	Around(162.59)	Around(162.59)
Kumar and Jangid	Yes				
	(k=.75, η=.5)	(.74,.26)	(.21,.79)	Around(166.44)	Around(155.13)
	(k=.25, η=1)	(.74,.26)	(.21,.79)	Around(166.39)	Around(155.2)
Li [30]	No	(.789,.211)	(.211,.789)	(154.95,161.06,164.98)	(155.28,161.06,171.01)
Our work	No	(.74,.26)	(.21,.79)	(150.08,156.39,166.39)	(155.21,161.25,171.25)

Table 2 Comparison of our work with the results of other researchers for example-2

References	Adequacies, margin, aspiration level or any parameter used	Player-Is optimal strategy	Player-IIs optimal strategy	Player-Is value of game	Player-IIs value of game
Li-[28]	Yes	(.648,.352)	(.534,.466)	(−0.254, 1.715, 4.746)	(0.241, 2.303, 5.601)
Our work	No	(0.64, 0.36)	(0.54, 0.46)	(−1.25, 1.39, 5.11)	(−0.08, 2.38, 6.45)

Table 3 Comparison of our work with the results of other researchers for example-3

References	Adequacies, margin, aspiration level or any parameter used	Player-Is optimal strategy	Player-IIs optimal strategy	Expected pay-off	Value of the game
Seikh et al. [43]	yes	(0, 0, 1)	(.336,.664, 0)	(140,143,146;.5,.17)	142.904
Our work	No	(0, 0, 1)	(.42,.58, 0)	(140, 143, 146)	143

players, though they may eventually come as reals belonging to interval [0, 1]. The proposed solution methodology in this study guarantees that a matrix game with TFN payoffs has a TFN type of optimal value, which can be defuzzified with the help of the ranking function utilized in this work to produce a crisp/accurate game value for each player. Furthermore, unlike previous researchers, we do not need any pre-existing fuzzy goals, adequacies, aspirations, α-cut set, degree of satisfaction, or any other parameter to solve the fully fuzzified matrix game with TFNs as the payoff in our proposed methodology. Nonetheless, our method is simple to explain, highly easy, and convenient to calculate. We next showed our work with three cases, and the results have been compared to those achieved by some of the notable researchers mentioned above. Scholars may use the proposed approach in the "Telecom market share problem," described by Seikh et al. [41] as well as in constrained fuzzy matrix games, bimatrix games, and other games.

Data Availability Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors do not have any conflicts of interest to declare.

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