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THEORETICAL ARTICLE

Periodic inventory management when demand stochastically depends on shelf-stock

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Abstract

Marketing researches reveal that displayed inventory and allocated shelf space affect the demand of some retail items. We study periodic inventory policy when demand is uncertain and its probability distribution function depends on inventory level and period length. Profit function including sale revenue, holding, shortage, and ordering costs is presented. To capture real world situation we propose natural properties for demand probability distribution. Considering those properties well fitted demand distributions are presented. Finally optimal policy is explored via numerical examples. Numerical study reveals that optimal value of decision variables is not monotonic with respect to demand sensitivity to the inventory.

Keywords Inventory · Retailing · Inventory-dependent-demand · Stochastic demand · *(R, T)* policy

1 Introduction

In traditional Inventory models it is usually assumed that demand is an exogenous variable and is not influenced by inventory level. However, as many markets observed, demand is related to the displayed inventory or shelf space (see Saha and Nielsen [1] for more details). Inventory models with stock or inventory dependent demand have found much attention in recent decades [2]. Inventory-dependent demand is experienced when a large amount of inventory motivates customers to buy, such as fashion or novelty items. Empirical evidences of the effect of inventory on demand are provided by Wolfe [3], Achabal, McIntyre and Smith [4], Koschat [5] and Pak et al. [6]. Balakrishnan et al. [7] mentioned a variety of reasons that inventory might stimulate demand.

In some cases, high inventory in stock or displayed, send the message that it is popular. Shelf space could have a similar effect on demand. More displayed inventory and shelf

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space could also increase visibility. Negative correlation between demand and inventory has been observed for some special cases. Such a phenomenon occurs when it signals scarcity and popularity. Obviously in this case shelf space still has a positive effect.

Stimulating effect will result in the Retailer's tendency to stock and display more inventories. It should maintain an optimal balance between holding costs and profit from additional demand and sales.

We will study a periodic review inventory system with stochastic and inventory-dependent demand. As an example that fits a real world condition, consider a retailer selling cosmetic or fashion products. For such products displayed inventory not only assure availability but also stimulate demand. Facing uncertain demand, retailer should decide how often and how much to replenish. Therefore inventory policy is determined by the period length and initial inventory position. Demand probability distribution function in each period depends on both decision variables: period length and initial inventory. To our knowledge this paper is the first to consider those issues together. In pervious periodic review models with stochastic inventory dependent demand [8, 9], period length is fixed and not a decision variable. We specify two natural properties of inventory-dependent demand distributions. Stimulation and saturation conditions imply that the expected demand during a period increases with inventory level at a decreasing rate. Well fitted time-dependent probability distributions are proposed and demonstrated to hold stimulation and saturation conditions.

In the next section we review the related literature. Section 3 presents assumption, notations and mathematical formulation. In Sect. 4 we analyze demand distribution function properties and some well fitted distributions are suggested. Section 5 provides numerical studies. Finally conclusions and summary are presented in Sect. 6.

2 Literature review

2.1 Deterministic models

Many authors have attended the stimulating effect of inventory on sale. Most of them examined a deterministic inventory model. Probably earliest contribution to inventory-dependent demand is Whitin [10]. Wolfe [3], made observations indicated dependency between inventory or shelf space with sale of fashion products. Literature of deterministic models can be categorized into two main streams. In first category demand is a function of initial inventory level and in second, demand is a function of instantaneous inventory level. The first models are consistent with shelf space demand dependency. Gupta and Vart [11], Mandal and Phaujdar [12], Baker and Urban [13], Goh [14], and Liao et al. [15] are some of the models in which demand rate depends on initial stock level.

Among deterministic models instantaneous inventory-dependent demand models are more frequent. The common approach is determining the inventory level over time by solving the differential equations generated from demand function. Depending on the demand function solving those differential equations could be complicated. 16, 17 studied deterministic models with demand dependence to the instantaneous inventory. Balakrishnan et al. [7] developed an EOQ model in which demand rate increases with inventory.

Baron et al. [18] solved the joint inventory optimization and shelf space allocation when demand depends on both instantaneous inventory and shelf space. Smith and Agrawal [19] suggested a multi store inventory model when demand is price and inventory dependent. They determined optimal pricing policy and inventory allocation and showed that store consolidation can be beneficial. Boada-Collado and Martínez-de-Albéniz [20] studied the impact of inventory on demand in a fashion store. They considered a log form for demand function and characterized optimal inventory policy for their model. Zhang et al. [21] proposed a price and inventory dependent demand model and determined the optimal inventory and pricing strategy for non-instantaneous deteriorating items. In a very recent paper Pando et al. [22] considered similar property for the demand function. They obtained the optimal price and inventory policy.

2.2 Stochastic models

However deterministic models are much more common in the area, recently some researches focused on models deal with stochastic demand. An early contribution is Gerchak and Wang [23]. They presented a multiplicative random demand function containing a deterministic part that is increasing in initial inventory and an independent random variable. A price/inventory-dependent demand newsvendor model was considered by Dana and Petruzzi [24]. Balakrishnan et al. [25] proposed a general stochastic demand modeling framework to capture real world situation. They used that framework to study the joint inventory and pricing optimal policy for the classical newsvendor problem. Stavrulaki [26] used the same framework to capture the demand's dependency on inventory. She studied properties of the optimal policy for two substitutable products. Yang and Zhang [9] and more recently Xue et al. [8] proposed negative relationship between stochastic demand and displayed inventory for periodic review inventory models. Akkas [27] formulated a shelf space selection problem as a Markov chain model and approximately computed the optimum shelf space. A queueing system with inventory was considered by Hanukov et al. [28] when demand depends on stock level. They derived closed-form expression for steady state probabilities. As a recent paper, Hübner et al. [29] optimized allocated shelf space when shelf space (initial inventory level) attracts customers.

3 Model description and analysis

Consider a retailer facing uncertain demand and system follows well known *(R, T)* inventory policy. It should decide about the length of replenishment periods *(T)* and maximum inventory position *(R)* at the beginning of the periods. The total arrived demand during each period is stochastic and obviously its distribution depends on the length of period. It also depends on initial stock level. We assume that maximum inventory is hold in the beginning of the period (zero lead time) and there is no backroom inventory. Therefore throughout the paper the initial stock is equivalent to shelf space.

3.1 Notations and assumptions

The following notations are used throughout the paper:

C	Unit purchase cost
H	Per unit holding cost per unit time
A	Fixed ordering cost
P	Unit selling price
S	Per unit shortage cost
R	Initial inventory level (decision variable)
T	Period length (decision variable)
i	Discount rate of sale price
$f(x, R, T)$	Probability density function of demand at given R during a period with length of T
$F(x, R, T)$	Cumulative distribution function of demand at given R during a period with length of T
$\mu(R, T)$	Expected value of demand during a period with length of T and initial inventory R

Balakrishnan et al. [25] proposed natural properties of the demand distribution when inventory increases demand. We propose assumptions with similar concept to capture the effect of initial inventory on demand distribution function and provide a modeling framework for demand probability distribution.

Assumption 1 (promotional effect) As inventory stimulates demand, the expected demand $\mu(R, T)$ increases with the initial inventory R or $\partial\mu(R, T)/\partial R > 0$.

Assumption 2 (saturation condition) Promotional effect of inventory gradually decreases as the inventory increases $\partial^2\mu(R, T)\big/\partial R^2 < 0$.

Regardless of the total demand in a period and its probability distribution, we assume that demand rate during the cycle is constant (see Fig. 1). Therefore, if demand exceeds the inventory in a period *($x > R$)*, the average inventory will be $\frac{R^2}{2xT}$ and if demand is less than R in a period, the average inventory will be $\frac{2R-x}{2}$.

3.2 Model 1: Constant sale price

Assume sale price of goods does not change over time. This assumption is relevant to the cases wherein the inventory has no expiration date and does not lose its value. Without loss of generality it can be assumed that the purchase cost of unsold products at the end of period is refunded. Hence the expected purchasing cost of a period is:

$$CR - C\int_{0}^{R}(R-x)f(x,R,T)dx$$

We next represent the expected profit function per unit time with inventory dependent demand.

$$\begin{aligned}E(Z) = \frac{1}{T}\Bigg[&P\left(R\int_{R}^{\infty}f(x,R,T)dx + \int_{0}^{R}xf(x,R,T)dx\right) - S\int_{R}^{\infty}(x-R)f(x,R,T)dx\\ &- H\left(T\int_{0}^{R}\left(\frac{2R-x}{2}\right)f(x,R,T)dx + \int_{R}^{\infty}\left(\frac{R^2}{2x}\right)f(x,R,T)dx\right)\\ &-C\left(R - \int_{0}^{R}(R-x)f(x,R,T)dx\right) - A\Bigg]\end{aligned} \quad (1)$$

The first term is the expected sales revenue, the second term is the expected shortage cost and third term is the expected holding cost, per unit time.

Differentiating the profit function applying Leibnitz rule, the first order optimality conditions respect to R yield:

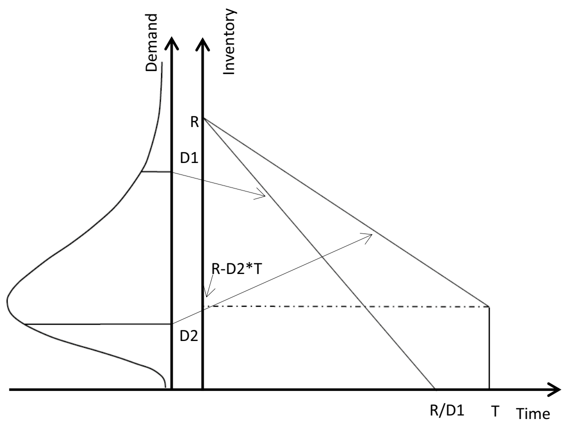

Fig. 1 Computing the average on-hand inventory

$$F(R)(C-HT-S-P)=C-S-P+R\left(\frac{HT-H}{2}\right)f(R,R,T)$$
$$+R(HT-C)\int_0^R\frac{\partial f(x,R,T)}{\partial R}dx-R(P+\pi)\int_R^\infty\frac{\partial f(x,R,T)}{\partial R}dx$$
$$+\left(C-P-\frac{HT}{2}\right)\int_0^R x\frac{\partial f(x,R,T)}{\partial R}dx+S\int_R^\infty x\frac{\partial f(x,R,T)}{\partial R}dx$$
$$+H\int_R^\infty\frac{R}{x}\cdot\frac{\partial f(x,R,T)}{\partial R}dx+H\int_R^\infty\frac{R^2}{2x}\cdot\frac{\partial f(x,R,T)}{\partial R}dx \tag{2}$$

Proposition 1 *Equation* (2) *provides necessary condition for optimal choice of initial stock level of Model* 1.

3.3 Model 2: Time dependent sale price

In the case of price reduction we assume the sale price at time t after the start of the period is Pe^{-it}. This formula does not incorporate holding cost and it is considered separately. Demand rate during the period is assumed to be constant and is equal to x/T in which x is the whole demand amount (random variable) of the period. Hence expected sale is (see Fig. 1):

$$\int_0^R\left(\int_0^T\frac{Pxe^{-it}}{T}dt\right)f(x,R,T)dx+\int_R^\infty\left(\int_0^{TR/x}\frac{Pxe^{-it}}{T}dt\right)f(x,R,T)dx$$
$$=\frac{P}{iT}\left(\int_0^R x\left(1-e^{-iT}\right)f(x,R,T)dx+\int_R^\infty x\left(1-e^{-\frac{iTR}{x}}\right)f(x,R,T)dx\right)$$

Like Model 1 we assume transferred stock to next period is refunded at the end of current period. Although in the beginning of the next period, the sale price is Pe^{-iT} instead of P. Therefore we approximately subtract that difference from unit purchase saving. Hence $r(T)=C-(P-Pe^{-iT})$ is refunded at the end of the period per each unsold product. For Model 2 the expected profit function is:

$$E(Z) = \frac{1}{T}\left[\frac{P}{iT}\left(\int_0^R x\left(1-e^{-iT}\right)f(x,R,T)dx + \int_R^\infty x\left(1-e^{-\frac{iTR}{x}}\right)f(x,R,T)dx\right)\right.$$
$$- H\left(T\int_0^R \left(\frac{2R-x}{2}\right)f(x,R,T)dx + \int_R^\infty \left(\frac{R^2}{2x}\right)f(x,R,T)dx\right) \tag{3}$$
$$\left. - S\int_R^\infty (x-R)f(x,R,T)dx - CR + r(T)\int_0^R (R-x)f(x,R,T)dx - A\right]$$

Differentiating the profit function of Model 2, we can now conclude the first order optimality conditions respect to *R:*

$$F(R)\left(Ce^{-iT} - S\right) + \frac{P}{iT}\int_R^\infty g(x,R,T)dx = C - S - RCe^{-iT}\int_0^R \frac{\partial f(x,R,T)}{\partial R}dx$$
$$+ \left(Ce^{-iT} - \frac{P}{iT}\left(1-e^{-iT}\right)\right)\int_0^R x\frac{\partial f(x,R,T)}{\partial R}dx + S\int_R^\infty x\frac{\partial f(x,R,T)}{\partial R}dx \tag{4}$$
$$- SR\int_R^\infty \frac{\partial f(x,R,T)}{\partial R}dx$$

In which $g(x,R,T) = iTe^{-\frac{iTR}{x}}f(x,R,T) + x\left(1-e^{-\frac{iTR}{x}}\right)\frac{\partial f(x,R,T)}{\partial R}$.

Proposition 2 *Equation* (4) *provides necessary condition for optimal choice of initial stock level of Model* 1.

Ignoring the stimulation effect the optimality condition will be simplified to:

$$F(R)\left(Ce^{-iT} - \pi\right) + p\int_R^\infty e^{\frac{-iTR}{x}}f(x,R,T)dx = C - \pi \tag{5}$$

4 Demand distribution functions

Demand probability distribution has an important role on inventory management. Mostly, it is assumed that demand is formed of (large number of) individual customers. This assumption will result in continuous distributions and in many cases Normal distribution is fitting well [30]. We propose the following normal density function for *T,* as the length of period and initial inventory level of *R:*

$$f_N(x,R,T) = \frac{1}{\sigma_0\beta(R)\sqrt{2\pi T}} \exp\left(-\frac{(x-\mu_0\beta(R)T)^2}{2(\sigma_0\beta(R))^2 T}\right) \tag{6}$$

In which $\beta(R)$ is an increasing and concave function of R. Note that the mean demand is equal to $\mu_0\beta(R)T$ and stimulation and saturation conditions hold for the above distribution.

In some cases Gamma distribution gives closer fit to the demand data. It takes non-negative values and according to the amount of the parameters it varies from memoryless distribution to the normal distribution. Describing the demand over a period of time, it is quite rational to set the shape parameter equal to the period length. We consider the following Gamma distribution that maintains demand stimulation and saturation conditions:

$$f_G(x,R,T) = \frac{x^{\alpha_0 T-1}\exp(-x/\beta_0\beta(R))}{(\beta_0\beta(R))^{\alpha_0 T}\Gamma(\alpha_0 T)}, \quad T >> 0 \tag{7}$$

Since demand cannot be negative, applying normal distribution is not appropriate (e.g.$\mu < 2\sigma$). In such situations we propose normal distribution which is truncated at zero:

$$f_{TN}(x,R,T) = f_N(x,R,T) \cdot \left(1 - \int_{-\infty}^{0} f_N(x,R,T)dx\right)^{-1} \tag{8}$$

where $f_N(x,R,T)$ is the normal density function presented in (6).

Proposition 3 *Saturation and stimulation conditions hold for the above truncated normal distribution.*

Proof Let x be a random variable with PDF given in Eq. (8). From truncated normal properties the expected value of x is:

$$\mu_N(R,T) + \sigma_N(R,T) \cdot \frac{f_Z(a')}{1-F_Z(a')}$$

In which: $f_Z(.)$ and $F_Z(.)$ are PDF and CDF of standard normal distribution respectively and $a' = \frac{\mu_N(R,T)}{\sigma_N(R,T)} = \frac{\mu_0\sqrt{T}}{\sigma_0}$ hence: $\mu_{TN}(R,T) = K \cdot \beta(R)$.

In which $K = \mu_0 T + \sigma_0\sqrt{T} \cdot \frac{f_Z(a')}{1-F_Z(a')}$. Note that K does not depend on R and is always greater than zero, therefore stimulation and saturation conditions hold for proposed truncated normal distribution.

5 Numerical studies

In this section a set of numerical examples are conducted to test the effect of demand inventory dependency and demand distribution functions on optimal policy and profit function. For each example we explore the optimal decision variables (inventory position at the beginning of each period and period length) via integral and fminsearch functions in MATLAB. Throughout the section we assume $\beta(R) = \frac{aR+b}{cR+d}$ in which if $a, b, c, d \geq 0,\ \ ad \geq bc$.

Per unit purchase cost is $C=6$, fixed ordering cost $A=100$, discount rate is $i=0.01$, holding cost of per unit on hand inventory per unit time is $H=0.2$, per unit Sale price is $P=10$, per unit shortage cost is $S=10$ and $\mu_0 = \sigma_0 = \alpha_0 = \beta_0 = 1$.

We set $a=40$, $c=1$ and $b=10{*}d$ which determine the demand distribution dependency on the initial stock level i.e.$\beta(R)$. Therefore in all examples with Normal or Gamma distribution, the average demand varies from *10* to *40 T* due to the value of R. Obviously bigger b and d will cause less demand sensitivity to the stock level.

Table 1 presents the optimal revenue and optimal inventory policy for different values of d when demand follows Normal or Gamm distribution. In our examples d varies from 50 to M (big number) and consequently b varies from 500 to *10 M*. We set $d=50$ for most demand sensitivity to stock level.

In general inventory models, optimal inventory is expected to be increasing with demand rate. Figure 2 shows a different pattern, in which $R*$ is not monotone with respect to d. Similarly T^* is not monotone in d although commonly it is expected to be decreasing with demand. Note that not only demand (stochastically) but also the demand dependency decrease as b and d increase. When demand has high sensitivity to the inventory, increase in d will lead to more initial stock in order to maintain the demand rate. However for low demand sensitivity, that strategy loses its effectiveness. Such a situation occurs for $d \geq 1000$ when $R*$ decreases rapidly and T^* decreases to be aligned with lower demand. Normal and Gamma demand distribution functions are very similar especially for higher T's (consequently higher

Table 1 Optimal policy and revenue function of model 1 for different values of d

No.	d	Gamma			Normal		
		T^*	$R*$	$E^*(Z)$	T^*	$R*$	$E^*(Z)$
1	50	6.34	310	77.4	6.65	322	78.6
2	100	7.56	327	68.3	7.62	332	69.7
3	200	9.49	342	55.4	9.29	341	56.6
4	500	12.53	314	35.2	12.13	311	35.9
5	1000	12.85	226	23.1	12.45	223	23.6
6	4000	11.15	136	14.8	10.78	136	15.1
7	20 k	10.80	121	13.1	10.45	121	13.3
8	M	10.72	118	12.7	10.40	118	12.9

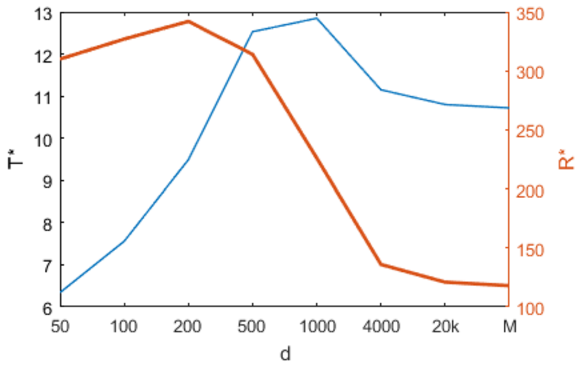

Fig. 2 Optimal inventory policy as a function of d for Model 1 when demand has Gamma distribution

shape parameter of Gamma distribution) as a result of Gamma-Normal distribution relationship.

Optimal revenue and optimal inventory policy of Model 2 is presented in Table 2. Similar results can be inferred: as demand sensitivity to inventory decreases optimal policy orders more to maintain demand rate. However for low demand sensitivity to the inventory maintaining the demand rate by raising inventory does not worth because of holding costs. In Model 2 less $R*/T*$ prevents large amount of discounted products at the end of each period. It can also be inferred form Table 2 that optimal period length is less as the discount cost of each remained item at the end of period depends on its length.

Table 3 provides optimal revenue and optimal inventory policy of Model 1 and Model 2 when demand follows truncated normal distribution. Note that the same Normal distribution (Tables 1, 2) is truncated at zero hence its expected value is more and its variance is less than the underlying Normal distribution. Obviously it will result in shorter optimal length and better objective function.

Table 2 Optimal policy and revenue function of Model 2 for different values of d

No.	d	Gamma			Normal		
		T^*	$R*$	$E^*(Z)$	T^*	$R*$	$E^*(Z)$
1	50	5.43	246	60.4	5.70	259	61.2
2	100	6.43	253	51.1	6.48	260	52.2
3	200	7.94	251	38.6	7.74	253	39.6
4	500	9.81	203	21.46	9.43	203	22.0
5	1000	9.82	146	13.5	9.45	146	13.8
6	4000	9.43	106	8.5	9.12	107	8.6
7	20 k	9.37	98	7.4	9.06	99	7.50
8	M	9.36	96	7.2	9.06	97	7.2

Table 3 Optimal policy and revenue function for truncated normal distribution demand for different values of *d*

No.	d	Model 1			Model 2		
		T^*	$R*$	$E^*(Z)$	T^*	$R*$	$E^*(Z)$
1	50	5.02	269	81.6	3.97	208	66.2
2	100	6.65	303	71.0	5.33	229	54.5
3	200	8.87	329	57.0	7.13	237	40.4
4	500	12.01	308	35.9	9.14	197	22.1
5	1000	12.29	222	23.6	9.16	142	13.9
6	4000	10.57	134	15.1	8.81	104	8.76
7	20 k	10.29	119	13.4	8.76	97	7.62
8	M	10.15	116	13.0	8.75	95	7.35

6 Summary and conclusion

To our knowledge this article is the first in literature that examines *(R, T)* periodic inventory policy under stochastic and inventory dependent demand. In this paper we study inventory policy when demand is uncertain and its probability distribution depends on both decision variables (period length and initial inventory). Some assumptions are made to capture key features of uncertain inventory-dependent demand. Considering those assumptions general forms of well fitted distributions to the demand (Gamma, Normal and Truncated Normal) are proposed. Finally optimal inventory policy that maximizes the expected profit function is explored for numerical examples. The key result is the behavior of optimal policy due to demand distribution changes. Numerical study reveals that demand sensitivity to the inventory plays an important role in optimal policy. It is also illustrated that optimal initial inventory and period lengths is not monotonic in demand sensitivity to the inventory.

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Declarations

Conflict of interest The authors declare that they have no competing interests.

Ethics approval and consent to participate Not applicable.

Consent for publication Not applicable.

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