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THEORETICAL ARTICLE

Solving the general fully neutrosophic multi-level multiobjective linear programming problems

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Abstract

This paper addresses the general fully neutrosophic multi-level multiobjective programming problems i.e. ML-MOLPPs in which not only coefficients of each objective at each level but also all coefficients and parameters in the constraints are fully neutrosophic numbers in the form of trapezoidal neutrosophic numbers (NNs). From the point of view of complexity of the problem, it is proposed to apply ranking function of NNs to convert problem into equivalent ML-MOLPPs with crisp values of neutrosophic coefficients and parameters. Then suitable membership function for each objective and decision variable are formulated using lowest and highest value of each objective and decision variables of converted ML-MOLPP. Formulation of membership functions for decision variables (using corresponding values to maximum and minimum of objectives) will avoid decision deadlock in hierarchical structure. Accordingly, simple fuzzy goal programming strategy is applied to build FGP solution models. With the help of linear programming techniques on these solution models, compromise optimal solution of original fully neutrosophic ML-MOLPP is obtained. The proposed approach is a unique and simple method to provide compromise optimal solution to decision makers of general fully neutrosophic ML-MOLPP. The proposed approach is illustrated with numerical example to show its uniqueness and simplicity as solution technique. A case study is also discussed to demonstrate its applicability on real problems.

Keywords Multi-level multiobjective programming problem (ML-MOPP) · Neutrosophic numbers · Trapezoidal NNs · Ranking function

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1 Introduction

Mathematical programming problems (MPPs) are specific planning problems with unique mathematical structured normally having three parts: objective function/s, constraints and non-negative restrictions MPPs includes LPPs, NLPPs, MOPPs, BLPPs, MLPPs etc. Among them, multi-level programming problems play a pivotal role modelling hierarchical decision making problems consisting of multiple decision makers in hierarchy like ML-LPP/ ML-NLPP (multi-level linear/non linear programming problem as: more than two levels hierarchical programming problem with linear objective functions and linear constraints or non-linear objective function/any non-linear constraint respectively. Further in more complex way, ML-MOLPP/ML-MONLPP (multi-level multiobjective linear/non-linear programming problem): More than two levels hierarchical programming problem with multiple linear objective functions and linear constraints or non-linear multiple objective function/any non-linear constraint respectively. Bi-level programming problem (BLPP) is one particular case of MLPP with two level hierarchy i.e. First and Second level. The existing literature witnessed that good research reviews have been carried out in the area of Taxanomy and classification of general MPPs and some important reviews are Lachhwani and Dwivedi [1] and Bhati et al. [2].

The research contributions in the domain of MLPPs and BLPPs have been applied in many industrial and real world problems arising in the field particularly in the field of design of transport network by (Gzara [3], Fontaine and Minner [4]), planning of product supply chain by (Kis and Kovacs [5], Wang et al. [6]), engineering problems related to facility location and design by (Camacho-Vallejo et al. [7]), Kalashnikov et al. [8]) etc. It is important to mention that most of existing academic research on MLPPs and other extension problems are focused to deterministic cases and therefore implementation of research suggestions in real problems become mostly infeasible in view of imprecise information about problems. Nowadays, in real world complex decision making problems, the precision of data is inconsistent and unpredictable in nature. Therefore, use of neutrosophic theory in real problems become very imperative for efficient and effective use of data. In order to handle vague, imprecise information, fuzzy set theory and later improved intuitionistic fuzzy set theory were popularized by Zadeh [9] and Atanassove [10] respectively. But again there were drawbacks like in fuzzy set theory only considers truthiness function of given information while intuitionistic fuzzy set theory considers truthiness and falsity function. However, both the approaches were silent on indeterministic feature of information data. In view of these shortcomings of two important approaches, Smarandache [11] introduced neutrosophy set theory in which indeterministic is a pivotal role along with truthiness and falsity function of given data. Neutrosophic set theory covers all important aspects of decision making (i.e. certain—truthiness, uncertain—indeterministic and not agree—falsity) process with imprecise information. Basic properties of neutrosophic numbers were established by Smarandache [12, 13]. Keeping in view of inapplicability of research algorithms and techniques in real world industrial problems arising in the form of multi-level structure with imprecise data, it was thought to explore some unique and simple technique for solving general fully ML-MOLPP with neutrosophic information on coefficients and parameters.

An effort has been made to propose solution methodology for solving general fully neutrosophic ML-MOLPP with trapezoidal NNs. The present simplified study is intended to provide unique method to solve complex fully neutrosophic ML-MOPPs in which not only coefficients of each objective at each level but also all coefficients and parameters in the constraints are fully neutrosophic numbers in the form of trapezoidal neutrosophic numbers (NNs). In this study, all three different cases of ML-MOPPs with NNs are proposed as: (i) coefficients of objective functions are NNs, (ii) coefficients of constraints and right hand side of constraint parameters are NNs, (iii) all coefficients and parameters in objective functions and constraints of ML-MOPP are NNs. Then simplified solution methodology is proposed which effectively solve all these cases. As a solution methodology, it is proposed to apply ranking function of NNs to convert problem into equivalent crisp ML-MOLPPs models. Then suitable membership function for each objective and decision variable are formulated using individual lowest and highest value of every objective and decision variables in such a way decision deadlock situations can be avoided in hierarchical structure. Accordingly, simple fuzzy goal programming (FGP) strategy is applied to build FGP models. And solving these solution models, compromise optimal solution of original fully neutrosophic ML-MOLPP is obtained. The content of this manuscript is structured as: in next section, we discuss on about NNs and their significance with relevant references. In Sect. 3, we have brief literature review on mathematical programming problems with the use of NNs. Some preliminary information on NNs are explained in Sect. 4. Mathematical formulation of ML-MOPPs with NNs along with its different cases are proposed in Sect. 5. In next section, a simplified solution methodology for solving full neutrosophic ML-MOPPs with NNs (all cases) is proposed with the combination of ranking function method of NNs and basic FGP approach to find solution of the original problem. This section is further sub divided into two parts as: (i) formulation of equivalent crisp model in which conversion of ML-MOPPs with NNs into ML-MOPPs with crisp values of NNs is proposed and (ii) solution process and solution models are proposed to obtain compromise optimal solution of fully neutrosophic ML-MOPP. Flow chart of proposed simplified solution approach is also suggested in the same section. In next Sect. 7, two examples are solved numerically to exhibit how proposed solution methodology can be applied. In the same section, Comparative study with other different methods to solve different ML-MOPPs (not with fully neutrosophic trapezoidal numbers) is also discussed to show uniqueness of proposed method in this domain. A real case study is discussed in Sect. 8 to demonstrate appropriateness of proposed approach in industrial problem. Concluding remarks and future directions are suggested in the last section.

2 Background: neutrosophic numbers (NN) and their significance

Neutrosophic set theory is an extended fuzzy set theory which handles incomplete and indeterministic information introduced by Smarandache [11] in 1998 to avert the shortcomings of fuzzy set theory and intuitionistic fuzzy set theory. Neutrosophic numbers (NNs) or neutrosophic set theory are used in decision making problems involving incomplete, inconsistent and indeterministic information. In general, neutrosophic set theory consists of neutrosophic numbers (NNs), neutrosophic set (N-set), neutrosophic probability and statistics, neutrosophic logic etc. Generally, neutrosophic sets are described jointly by three unrelated membership degrees (T—truthiness, I—indeterminacy and *F*—falsity), where T, I, $F \in [0,\ 1]$ as shown in Fig. 1. Neutrosophic set theory is advantageous in decision making by considering all three features of data (i) truthiness, (ii) falsity and (iii) indeterminacy and enables decision makers (DMs) to increase degree of truthiness and decrease falsity and indeterminacy according to methodology. This theory is being applied in many application areas to tackle imprecise information. The beauty of neutrosophic set is that it can handle both incomplete and indeterministic information whereas intuitionistic fuzzy set can only handle incomplete information. In other words, neutrosophic set is interpretation of fuzzy set and intuitionistic fuzzy set with truthiness, falsity and indeterministic features of information. Therefore, Use of neutrosophic set theory and neutrosophic numbers (NNs) as coefficients and parameters in formulation of decision making problems are very effective in assimilating inaccurate and inconsistent information efficiently. In literature, neutrosophic set structure, N set properties and neutrosophic statistics were introduced by Smarandache [12, 13]. Thereafter, Ye [14] first used ranking function approach for NNs in solving different type of decision making problems. Deli and Şubaş [15, 16] proposed ranking method for NNs and its utilization in solving decision making problems.

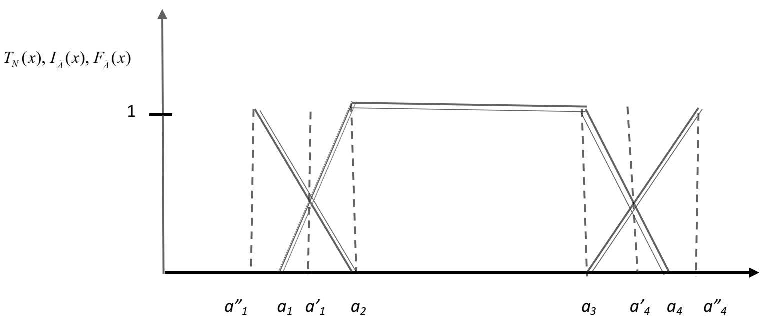

Fig. 1 Membership functions (combining truthiness, indeterminacy and falsity) of trapezoidal NN

3 Literature review

In recent decades, Fuzzy set theory is being widely adopted as a part of mathematical programming problem like fuzzy programming problem (FPP) and also as tool for solving complex MPPs like use of fuzzy membership function, fuzzy if–then rules, fuzzy goal programming, fuzzy intuitionistic set approach etc. The first introduction of fuzzy programming theory and the formulation of FLP with solution are presented by Tanaka et al. [17] and Zimmerman [18] respectively. Thereafter, many researchers have contributed in solving FPP and other extension problems with different approaches. Normally, in solving FPP and intuitionistic FPP, first of all these problems are converted into standard programming problems with the help of properties of fuzzy set theory. For example, Bharati and Singh [19] introduced the fully intuitionistic FLP problems using triangular intuitionistic fuzzy numbers. Sidhu and Kumar [20] used ranking function for solving intuitionistic FLP problems. In this continuation, some important research articles in the area ML-MOPPs and their different solution methodologies based on the use fuzzy set theory, FGP, other extension theories etc. are worth noted here as: Liu and Yang [21] developed interactive programming approach for the solution of ML-MOLPP. Pramanik et al. [22] suggested methodology for multi- level multiobjective linear plus fractional programming problem (ML-MOL + FPP) using fuzzy goal Programming (FGP) methodology. Baky [23] suggested FGP approach for obtaining solution of ML-MOLPP. Thereafter, Lachhwani [24] developed simplified FGP approach for ML-MOLPP with modifications in the work of Baky [23]. Lachhwani [25] developed modified FGP methodology for ML-MOLFP problem. Osman et al. [26] proposed solution methodology with FGP approach.

But again with the drawbacks of fuzzy and fuzzy intuitionistic set, the new era of use of neutrosophic set theory begin in recent decades. Use of neutrosophic set theory and neutrosophic numbers (NNs) in programming problems is new area of research and very limited researchers have contributed in programming problem domain with NNs. Most of them are: Pramanik et al. [27] suggested solution approach for teacher selection problem and formulation of this problem under NN environment. Ye [28] suggested solution technique for linear programming problems (LPPs) under NN environment. Ye et al. [29] also suggested solution technique for non linear programming problems (NLPP) with NN status. Pramanik and Banerjee [30] used goal programming approach to solve MOLP problem with NN status. Thereafter, Pramanik and Dey [31] developed solution methodology to bi-level programming problems (BLPPs) with NN status. Maiti et al. [32] proposed some goal programming approach for solving ML-MOPPs with NNs. Here, it is important to note NNs used in coefficients and parameters are in the the form of $c + dI$, where I represents indeterminacy and c, d are real numbers. Recently, Mohamed Abdel-Basset et al. [33] suggested a method to solve linear programming problem with trapezoidal NNs as coefficients and parameters.

The main drawback of the work of Pramanik and Banerjee [30] for solution methodology of MOLPP under NN environment, Pramanik and Dey [31] for

BLPPs under NN environment, Maiti et al. [32] for ML-MOLPP in NN environment is considering NN in only linear form $c + dI$, in which I represents indeterminacy and not taking account of the truthiness and falsity features of information. However, in imprecise and inconsistent information, truthiness and falsity are also decisive parts. This motivates us to propose a unique simplified solution approach for general fully neutrosophic ML-MOPPs with trapezoidal NNs with all three features truthiness, indeterminacy and falsity of NNs.

In this paper, an effort has been made to present a unique and simple solution methodology for general full neutrosophic ML-MOPPs with NNs. The proposed solution approach is also novel for fully neutrosophic ML-MOPPs in which trapezoidal NNs are used in either or all parameters and coefficients in the problem.

4 Preliminary information on NNs

Here, we address some important definitions and properties of neutrosophic numbers in context of solving mathematical programming problems as:

Definition 1 (Mohammed et al. [34]) A neutrosophic set N through the set of universe X is defined as a set in the form $N = \{(x,\ T_N(x),\ I_N(x),\ F_N(x))\ :\ x \in X\}$, where $T_N(x) : X \to [0,1]$, $I_N(x) : X \to [0,1]$ and $F_N(x) : X \to [0,1]$ $[0,1]$ $\forall x \in X$ represent truth, indeterminancy and falsity membership degree of x to N respectively.

Different type of neutrosophic numbers are defined to describe the real world problems. The simplest form of NNs is linear form ($N = c + dI$) where I denotes the membership function for indeterminacy and c, d are real numbers. This linear form has drawback of considering only the indeterminacy feature of given information. But trapezoidal NNs consider truthiness and falsity characteristic along with indeterminacy. Trapezoidal NNs are defined as follow:

Definition 2 (Mohamed Abdel-Basset et al. [33]) A neutrosophic set in R with the following trapezoidal format of truth, indeterminacy and falsity membership functions is defined as trapezoidal neutrosophic number (NN) $\tilde{A}$ as:

$$T_N(x) = \begin{cases} \alpha_{\tilde{A}}\left(\frac{x-a_1}{a_2-a_1}\right) & (a_1 \le x \le a_2) \\ \alpha_{\tilde{A}} & (a_2 \le x \le a_3) \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

$$I_{\tilde{A}}(x) = \begin{cases} \frac{(a_2-x+\theta_{\tilde{A}}(x-a_1^{'}))}{(a_2-a_1^{'})} & (a_1^{'} \le x \le a_2) \\ \theta_{\tilde{A}} & (a_2 \le x \le a_3) \\ \frac{(x-a_3+\theta_{\tilde{A}}(a_4^{'}-x))}{(a_4^{'}-a_3)} & (a_3 \le x \le a_4^{'}) \\ 1 & \text{Otherwise} \end{cases} \tag{2}$$

$$F_{\tilde{A}}(x) = \begin{cases} \frac{(a_2 - x + \beta_{\tilde{A}}(x - a_1^{''}))}{(a_2 - a_1^{''})} & (a_1^{''} \le x \le a_2) \\ \beta_{\tilde{A}} & (a_2 \le x \le a_3) \\ \frac{(x - a_3 + \beta_{\tilde{A}}(a_4^{''} - x))}{(a_4^{''} - a_3)} & (a_3 \le x \le a_4^{''}) \\ 1 & \text{Otherwise} \end{cases} \tag{3}$$

where $\alpha_{\tilde{A}}$, $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}}$ represent the maximum degree of truthiness, minimum degree of indeterminacy, minimum degree of falsity, respectively and $\alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} \in [0, 1]$. Also $a_1^{''} \le a_1 \le a_1^{'} \le a_2 \le a_3 \le a_4^{'} \le a_4 \le a_4^{''}$. The membership functions (combining truthiness, indeterminacy and falsity) of trapezoidal NN are depicted in Fig. 1.

Now, for converting coefficients and parameters in NN environment, we use conversion formulas to convert NNs into equivalent crisp values. Some ranking function and its different types are:

Definition 3 (Mohamed Abdel-Basset et al. [33]) A ranking function $R : N(R) \rightarrow R$, for neutrosophic numbers where N(R) is a set of NNs is a function which convert each neutrosophic number into equivalent crisp numbers.

Different types of ranking functions are used in view of complexity of data, aspired degree of confirmation (degree of truthiness, indeterminacy and falsity), suitability in given data etc. Here we describe ranking function for trapezoidal NNs used in ML-MOPPs as: let $\tilde{a} = a^l,\ a^{m_1},\ a^{m_2}, a^u, ;\ T_{\tilde{a}},\ I_{\tilde{a}},\ F_{\tilde{a}})$ be a trapezoidal neutrosophic number, where $a^l,\ a^{m_1},\ a^{m_2}, a^u$, are lower bound value, first median value, second median value and upper bound value for trapezoidal NN respectively. Also $T_{\tilde{a}},\ I_{\tilde{a}},\ F_{\tilde{a}}$ are the truth, indeterminacy and falsity membership degree of trapezoidal NN. If any expression of NN is for maximization purpose, then

$$R(\tilde{a}) = \left(\frac{a^l + a^u + 2(a^{m_1} + a^{m_2})}{2} \right) + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) \tag{4}$$

Similarly, if any expression of NN is for minimization purpose, then

$$R(\tilde{a}) = \left(\frac{a^l + a^u - 3(a^{m_1} + a^{m_2})}{2} \right) + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) \tag{5}$$

where term $(T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})$ is also known as confirmation degree. According to the type of ML-MOPP with NNs, suitable ranking function is applied to convert each trapezoidal NN into equivalent crisp values.

5 Formulation of ML-MOLPPs with NNs

Now, we discuss the mathematical formulation of ML-MOPPs with NNs in which either or all coefficients and parameters of the problem are NNs. Before this formulation, let us consider general maximization ML-MOPPs mathematically described as:

$$\left.\begin{array}{l} \underset{x_1}{Max}\left\{Z_{11}, Z_{12}, \dots, Z_{1m}\right\} \\ \underset{x_2}{Max}\left\{Z_{21}, Z_{22}, \dots, Z_{2m}\right\} \\ \quad\quad . \\ \quad\quad . \\ \underset{x_T}{Max}\left\{Z_{T1}, Z_{T2}, \dots, Z_{Tm}\right\} \end{array}\right\} \quad \text{(i)} \tag{6}$$

Subject to, $A_{l1}\, x_1 + A_{l2}\, x_2 + \dots + A_{lT}\, x_T (\leq, =, \geq)\, b_i \;\forall l = 1, 2, \dots, p$ (ii)

and $x_1, x_2, \dots, x_T \geq 0$ (iii)

where $Z_{tr}(X) = C_{tr1}\, X_1 + C_{tr2}\, X_2 + \dots + C_{trT}\, X_T \;\forall t = 1, 2, \dots, T \; r = 1, 2, \dots, m$ is the i-th objective function at i-th level decision maker (DM).

$x_1 = \left\{x_1^1, x_1^2, \dots, x_1^{N_1}\right\}'$ decision variables for the first level decision maker (DM).

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$x_T = \left\{x_T^1, x_T^2, \dots, x_T^{N_T}\right\}'$ decision variables for the t-level decision maker (DM).

Where $'$ denotes transposition, $A_{lt}\; l = 1, 2, \dots, n,\; t = 1, 2, \dots, T$ are m row vectors, each of dimension $(1 \times N_j)$. $A_{lt}\, x_t,\; t = 1, 2, \dots, T$ is a column vector of dimension $(n \times 1)$.$C_{111}, C_{211}, \dots, C_{T11}$, all are row vectors of dimension of $(1 \times N_1)$, $x = x_1 \cup x_2 \cup \dots . \cup x_T$ and $N = N_1 + N_2 + \dots + N_T$. Here one set of decision variables are located at each level like $x_t,\; t = 1, 2, \dots, T$ is t th level DM with N_t number of decision variables. Similarly, minimization type ML-MOLP problem can be depicted with minimization of objective functions. Now fully neutrosophic ML-MOPPs with NNs are defined in following different types of problem as:

First type: In this type of ML-MOPPs in which each coefficient of each objective function variables at each level are represented by trapezoidal NNs and all other parameters/coefficient have their usual crisp values. Mathematically this type of problem is defined by problem (1) except part (i) of (1) which is to be replaced with

$$Z_{tr}(X) \approx \tilde{C}_{tr1} X_1 + \tilde{C}_{tr2}\, X_2 + \dots + \tilde{C}_{trT}\, X_T \qquad \forall t = 1, 2, \dots, T \; r = 1, 2, \dots, m \tag{7}$$

where $\tilde{C}_{trT}$ represent trapezoidal neutrosophic number and '~' denotes values in NN environment. Remaining part of the problem remains unchanged.

Second type: In this type of ML-MOPPs, coefficients and parameters (right hand side) of constraints are trapezoidal NNs and all other parameters/coefficient are having their usual crisp values. Mathematically, this type of ML-MOLFP problem is presented by problem (1) with set of constraints (part (ii) of problem (1) is replaced with set of constraints:

$$\tilde{A}_{l1}\, X_1 + \tilde{A}_{l2}\, X_2 + \dots + \tilde{A}_{lT}\, X_T (\leq, =, \geq)\, \tilde{b}_i \qquad \forall l = 1, 2, \dots, p \tag{8}$$

where $\tilde{A}_{l1}, \tilde{A}_{l2}, \dots, \tilde{A}_{lT}$, $\forall l = 1, 2, \dots, p$ and $\tilde{b}_i$,$\forall i = 1, 2, \dots, m$ are trapezoidal NNs. Remaining parts of the problem remain unchanged.

Third type: This type of ML-MOPPs is general fully neutrosophic ML-MOLPP in which all coefficients and parameters of the problem are represented by trapezoidal NNs and variables are considered as usual real variables. Mathematically, the problem can be stated as:

$$\begin{aligned}
&\underset{x_1}{Max}\left\{\tilde{Z}_{11},\tilde{Z}_{12},\dots,\tilde{Z}_{1m}\right\}\\
&\underset{x_2}{Max}\left\{\tilde{Z}_{21},\tilde{Z}_{22},\dots,\tilde{Z}_{2m}\right\}\\
&.\\
&.\\
&\underset{x_T}{Max}\left\{\tilde{Z}_{T1},\tilde{Z}_{T2},\dots,\tilde{Z}_{Tm}\right\}\\
&\text{Subject to, } \tilde{A}_{l1}x_1+\tilde{A}_{l2}x_2+\dots+\tilde{A}_{lT}x_T(\le,=,\ge)\,\tilde{b}_i\ \forall l=1,2,\dots,p\\
&\text{and} \qquad x_1,\,x_2,\,\dots,\,x_T\ge 0
\end{aligned} \tag{9}$$

where $\tilde{Z}_{tr}(x)\approx\tilde{C}_{tr1}x_1+\tilde{C}_{tr2}x_2+\dots+\tilde{C}_{trT}x_T\ \forall t=1,2,\dots,T\ r=1,2,\dots,m.$

$x_1=\left\{x_1^1,x_1^2,\dots,x_1^{N_1}\right\}'$ decision variables of the first level DM.

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$x_T=\left\{x_T^1,x_T^2,\dots,x_T^{N_T}\right\}'$ decision variables for the t-level DM.

Where $\tilde{A}_{lt}$ $l=1,2,\dots,n,\ t=1,2,\dots,T$ are m row vectors, each of dimension $(1\times N_j)$. $\tilde{A}_{lt}x_t,\ t=1,2,\dots,T$ is a column vector of dimension $(n\times 1)$ under NN environment. $\tilde{C}_{111},\tilde{C}_{211},\dots,\tilde{C}_{T11}$, are row vectors of $(1\times N_1)$ dimension and $\tilde{C}_{11T},\tilde{C}_{12T},\dots,\tilde{C}_{1mT}$, are row vectors of dimension of $(1\times N_T)$. Here $x=x_1\cup x_2\cup\dots.\cup x_T$ and $N=N_1+N_2+\dots+N_T$. Also '~' denotes values of coefficients and parameters in NN environment.

6 Simplified methodology for fully neutrosophic ML-MOLFPP with NNs

In this section, a new simplified solution methodology is suggested for general fully neutrosophic ML-MOLPP in following two sub sections in steps as:

A. **Crisp equivalent model**

Step 1. First of all, we use ranking function method to convert trapezoidal NNs into equivalent crisp values. But here it is to be noted that in maximization ML-MOPPs decision maker normally try to maximize the truth degree, minimize indeterminacy and falsity degree. Then, in this way membership degree of truthiness (T) = 1, degree of indeterminacy (I) = 0 and degree of falsity (F) = 0. So, confirmation degree = 1 + 0 + 0 = 1.

Further, first and second median values can be considered same in trapezoidal neutrosophic number in view of similarity of membership functions as depicted in Fig. 1. Therefore, each trapezoidal NN can be written in following form (Mohamed Abdel-Basset et al. [33]):

$$(\tilde{a} = a^l,\ a^u,\ a^{m_1},\ a^{m_2}) = (a^l,\ a^u,\ \alpha,\ \alpha) \tag{10}$$

With unit confirmation degree. Where $a^l,\ a^u,\ a^{m_1},\ a^{m_2}$ represents the lower bound value, upper bound value, first median value and second median value of trapezoidal NN respectively. The similar notation has been suggested by Ganesan and Veeramani [35] and Ebrahimnejad and Tavana [36]. Using this format with ranking function [given in Eqs. (4) and (5)], the trapezoidal NNs (10) can be simplified as:

$$\text{For maximization of NN,} \qquad R(\tilde{a}) = \left(\frac{a^l + a^u + 4\alpha)}{2} \right) + 1 \tag{11}$$

$$\text{Similarly, for minimization of NN,} \qquad R(\tilde{a}) = \left(\frac{a^l + a^u - 6\alpha)}{2} \right) + 1 \tag{12}$$

Now, using ranking functions (11) and (12), ML-MOPPs with NNs are converted into another equivalent ML-MOPPs with crisp values of NN. This conversion of ML-MOLPPs with NNs into equivalent ML-MOPPs with crisp values (for all three types of problem) can be described as:

First type: In this type of ML-MOPPs (7) with NNs, neutrosophic coefficients of objective functions are converted into crisp values with the use of ranking functions [(11) and (12)], objective functions are replaced with equivalent crisp value of objectives as:

$$\tilde{Z}_{tr} \approx [R(\tilde{C}_{tr1})]x_1 + [R(\tilde{C}_{tr2})]x_2 + \ldots + [R(\tilde{C}_{trT})]x_T \quad \forall t = 1, 2, \ldots, T;\ r = 1, 2, \ldots, m \tag{13}$$

where $R(\tilde{C}_{trT}) = \left[\frac{(c^u_{trT} + c^l_{trT}) + 4\alpha_{c_{trT}}}{2}\right] + 1$ (for maximization problem) or $R(\tilde{C}_{trT}) = \left[\frac{(c^u_{trT} + c^l_{trT}) - 6\alpha_{c_{rtT}}}{2}\right] + 1$ (for minimization problem) and remaining part of the problem remain unchanged.

Second type: In this type of ML-MOPPs (8), neutrosophic coefficients and right-hand side parameters of constraints are converted into crisp values with ranking function variables [(11) and (12)]. Accordingly, set of constraints with parameters are replaced with the set of constraints with the equivalent crisp values as:

$$[\mathrm{R}(\tilde{\mathrm{A}}_{l1})]x_1 + [R(\tilde{\mathrm{A}}_{l2})]x_2 + \ldots + [R(\tilde{\mathrm{A}}_{lT})]x_T (\leq, =, \geq) [R(\tilde{b}_i)] \forall l = 1, 2, \ldots, p,\ \forall i = 1, 2, \ldots, m \tag{14}$$

where $R(\tilde{A}_{trT}) = \left[\frac{(a^u_{trT} + a^l_{trT}) + 4\alpha_{a_{trT}}}{2}\right] + 1$ and $R(\tilde{b}_i) = \left[\frac{(b^u_i + b^l_i) + 4\alpha_{b_i}}{2}\right] + 1$ (for maximization problem) $R(\tilde{A}_{trT}) = \left[\frac{(a^u_{trT} + a^l_{trT}) - 6\alpha_{a_{trT}}}{2}\right] + 1$ and $R(\tilde{b}_i) = \left[\frac{(b^u_i + b^l_i) - 6\alpha_{b_i}}{2}\right] + 1$ (for minimization problem) $\forall t = 1, 2, \ldots, T;\ r = 1, 2, \ldots, m$ and remaining part of the problem remain unchanged.

Third Type: For this type of ML-MOPP (9), each coefficient and each parameter of the problem are converted into their equivalent crisp values with the used of corresponding ranking functions [(11) and (12)] and converted equivalent ML-MOPP with crisp values can be represented mathematically as:

$$\underset{x_1}{Max}\left\{\tilde{Z}_{11},\tilde{Z}_{12},\dots,\tilde{Z}_{1m}\right\}$$
$$\underset{x_2}{Max}\left\{\tilde{Z}_{21},\tilde{Z}_{22},\dots,\tilde{Z}_{2m}\right\}$$
$$\underset{x_T}{Max}\left\{\tilde{Z}_{T1},\tilde{Z}_{T2},\dots,\tilde{Z}_{Tm}\right\} \tag{15}$$

Subject to, $[R(\tilde{A}_{l1})]x_1 + [R(\tilde{A}_{l2})]x_2 + \dots$
$+ [R(\tilde{A}_{lT})]x_T(\le,=,\ge)[R(\tilde{b}_i)]\forall l = 1,2,\dots,p,\ \forall i = 1,2,\dots,m$
and $x_1,\ x_2,\ \dots,\ x_T \ge 0$

where $\tilde{Z}_{tr} \approx [R(\tilde{C}_{tr1})]x_1 + [R(\tilde{C}_{tr2})]x_2 + \dots + [R(\tilde{C}_{trT})]x_T$ $R(\tilde{A}_{trT}) = \left[\frac{(a^u_{trT}+a^l_{trT})+4\alpha_{a_{trT}}}{2}\right] + 1$, $R(\tilde{C}_{trT}) = \left[\frac{(c^u_{trT}+c^l_{trT})+4\alpha_{c_{trT}}}{2}\right] + 1$ and $R(\tilde{b}_i) = \left[\frac{(b^u_i+b^l_i)+4\alpha_{b_i}}{2}\right] + 1 \quad \forall t = 1,2,\dots,T;\ r = 1,2,\dots,m.$

Similarly, for minimization ML-MOLP problem, accordingly we use the following ranking functions for these coefficients and parameters as:
$R(\tilde{A}_{trT}) = \left[\frac{(a^u_{trT}+a^l_{trT})-6\alpha_{a_{trT}}}{2}\right] + 1$, $R(\tilde{C}_{trT}) = \left[\frac{(c^u_{trT}+c^l_{trT})-6\alpha_{c_{trT}}}{2}\right] + 1$

$$\text{and}\quad R(\tilde{b}_i) = \left[\frac{(b^u_i+b^l_i)-6\alpha_{b_i}}{2}\right] + 1 \quad \forall t = 1,2,\dots,T;\ r = 1,2,\dots,m \tag{16}$$

and remaining symbols and notations have same meanings.

B. **Solution process and models**

In this sub section, we consider the third type (general full neutrosophic) of ML-MOLPP [problem (15)] in NN environment for solution process as:

Step 3. Using equivalent crisp ML-MOLPP model obtained in sub section A, we calculate $\overline{\tilde{Z}_{tr}}$ (highest or best value or maximum) and $\underline{\tilde{Z}_{tr}}$ (lowest or worst value or minimum) of each objective functions with corresponding variables without

Table 1 Membership functions for objectives and decision variables

For maximization objective functions	For minimization objective function
Membership function for objectives	
$\mu_{Z_t}(\tilde{Z}_{tr}(\overline{X})) \cong \begin{cases} 1 & if\ \tilde{Z}_{tr}(\overline{X}) \gtrsim \overline{\tilde{Z}_{tr}} \\ \frac{\tilde{Z}_{tr}(\overline{X})-\underline{\tilde{Z}_{tr}}}{\overline{\tilde{Z}_{tr}}-\underline{\tilde{Z}_{tr}}} & if\ \underline{\tilde{Z}_{tr}} \le \tilde{Z}_{tr}(\overline{X}) \le \overline{\tilde{Z}_{tr}} \\ 0 & if\ \tilde{Z}_{tr}(\overline{X}) \le \underline{\tilde{Z}_{tr}} \end{cases}$ (17)	$\mu_{Z_t}(\tilde{Z}_{tr}(\overline{X})) \cong \begin{cases} 0 & if\ \tilde{Z}_{tr}(\overline{X}) \gtrsim \overline{\tilde{Z}_{tr}} \\ \frac{\overline{\tilde{Z}_{tr}}-\tilde{Z}_{tr}(\overline{X})}{\overline{\tilde{Z}_{tr}}-\underline{\tilde{Z}_{tr}}} & if\ \underline{\tilde{Z}_{tr}} \le \tilde{Z}_{tr}(\overline{X}) \le \overline{\tilde{Z}_{tr}} \\ 1 & if\ \tilde{Z}_{tr}(\overline{X}) \le \underline{\tilde{Z}_{tr}} \end{cases}$ (17A)
Membership function for decision variables	
$\mu_{x_t}(x_t) = \begin{cases} 1 & for\ x_t \ge \overline{x_t} \\ \frac{x_t-\underline{x_t}}{\overline{x_t}-\underline{x_t}} & for\ \underline{x_t} \le x_t \le \overline{x_t} \\ 0 & for\ x_t \le \underline{x_t} \end{cases}$ (18)	$\mu_{x_t}(x_t) = \begin{cases} 0 & for\ x_t \ge \overline{x_t} \\ \frac{\overline{x_t}-x_t}{\overline{x_t}-\underline{x_t}} & for\ \underline{x_t} \le x_t \le \overline{x_t} \\ 1 & for\ x_t \le \underline{x_t} \end{cases}$ (18A)

considering the hierarchical structure of the problem. Then suitable membership function for each objective and decision variables up to (T-1) level (from 1st level to T-1th level i.e. from first level to second last level) are formulated as suggested by Lachhwani [24] for the general third type of ML-MOPP with NNs as in Table 1.

Where $\overline{x_t}$ and $\underline{x_t}$ are the corresponding decision variables at each level which yield the maximum and minimum values of the objective functions (($\overline{\tilde{Z}_t}(\overline{x})$ and $\underline{\tilde{Z}_t}(\underline{x})$ $\forall t = 1, 2, \ldots, T-1$) at each level respectively as:

$$\overline{\tilde{Z}_t} \tilde{=} \underset{\overline{x_t} \in X}{Max} \left\{ \overline{\tilde{Z}_{tr}}(\overline{x_t}),\ \forall r = 1, 2, \ldots, m \right\} \tag{19}$$

$$\underline{\tilde{Z}_t} \tilde{=} \underset{\underline{x_t} \in X}{Min} \left\{ \underline{\tilde{Z}_{tr}}(\underline{x_t}),\ \forall r = 1, 2, \ldots, m \right\} \tag{20}$$

It is worth noting that to tackle hierarchical characteristics of MLPPs, membership functions for decision variables are considered only up to (T-1) levels.

Step 4. Now, we formulate the solution models of equivalent crisp model of maximization problem (6) with the help of modified FGP approach by Lachhwani [25] in the format as:

Model I $\min \lambda = \sum_{t=1}^{T} \sum_{r=1}^{m} \dot{w}_{tr} d_{tr}^{Z-} + \sum_{t=1}^{T-1} d_t^-$

Subject to, $-\overline{\tilde{Z}_{tr}} + \tilde{Z}_{tr} + d_{tr}^{Z-}(\overline{\tilde{Z}_{tr}} - \underline{\tilde{Z}_{tr}}) \geq 0 \quad \forall t = 1, 2, \ldots, T; r = 1, 2, \ldots, m$

$$-\overline{x_t} + x_t + d_t^-(\overline{x_t} - \underline{x_t}) \geq \mathbf{0} \quad \forall t \quad up\ to\ (T-1)$$

Subject to, $[R(\tilde{A}_{l1})]x_1 + [R(\tilde{A}_{l2})]x_2 + \ldots$
$+ [R(\tilde{A}_{lT})]x_T (\leq, =, \geq) [R(\tilde{b}_i)] \forall l = 1, 2, \ldots, p,\ \forall i = 1, 2, \ldots, m$

and $x_1, x_2, \ldots, x_T \geq 0$

where $\overline{\tilde{Z}_{tr}} = Maximum\{\tilde{Z}_{tr} \approx [R(\tilde{C}_{tr1})]x_1 + [R(\tilde{C}_{tr2})]x_2 + \ldots + [R(\tilde{C}_{trT})]x_T\ \forall t = 1, 2, \ldots, T;\ r = 1, 2, \ldots, m\}$ and $\underline{\tilde{Z}_{tr}} = Minimum\{\tilde{Z}_{tr} \approx [R(\tilde{C}_{tr1})]x_1 + [R(\tilde{C}_{tr2})]x_2 + \ldots + [R(\tilde{C}_{trT})]x_T\ \forall t = 1, 2, \ldots, T;\ r = 1, 2, \ldots, m\}$ subject to satisfying set of the constraints.

$$R(\tilde{A}_{trT}) = \left[\frac{(a_{trT}^u + a_{trT}^l) + 4\alpha_{a_{trT}}}{2} \right] + 1,\ R(\tilde{C}_{trT}) = \left[\frac{(c_{trT}^u + c_{trT}^l) + 4\alpha_{c_{trT}}}{2} \right] + 1$$

$$\text{and } R(\tilde{b}_i) = \left[\frac{(b_i^u + b_i^l) + 4\alpha_{b_i}}{2} \right] + 1\ \forall t = 1, 2, \ldots, T;\ r = 1, 2, \ldots, m$$

Also λ represents the fuzzy achievement function with the weights $\dot{w}_{tr} > 0$, ($\forall t = 1, \ldots, T; r = 1, \ldots, m$) showing the corresponding comparable significance of the aspired values of fuzzy goals. If we consider unit weights for aspired value of each objective function, then another solution model can be given as:

Model II $\min \lambda = \sum_{t=1}^{T} \sum_{r=1}^{m} d_{tr}^{Z-} + \sum_{t=1}^{T-1} d_t^-$

Subject to, $-\overline{\tilde{Z}_{tr}} + \tilde{Z}_{tr} + d_{tr}^{Z-}(\overline{\tilde{Z}_{tr}} - \underline{\tilde{Z}_{tr}}) \geq 0 \quad \forall t = 1, 2, \ldots, T; r = 1, 2, \ldots, m$

$$-\overline{x_t} + x_t + d_t^-(\overline{x_t} - \underline{x_t}) \geq \mathbf{0} \quad \forall t \quad up\ to\ (T-1)$$

Subject to, $[\mathrm{R}(\tilde{\mathrm{A}}_{l1})]x_1 + [R(\tilde{\mathrm{A}}_{l2})]x_2 + \ldots$
$+ [R(\tilde{\mathrm{A}}_{l\mathrm{T}})]x_T(\leq, =, \geq)[R(\tilde{b}_i)] \, \forall l = 1, 2, \ldots, p, \ \forall i = 1, 2, \ldots, m$

and $x_1 \geq 0, x_2 \geq 0, \ldots, x_T \geq 0.$

And remaining notations have same meanings. Similarly, for minimization ML-MOLP problem, solution models can be formulated using membership functions for objectives and decision variables (as given in (17A) and (18A)) and ranking function [given in Eq. (12)] in the format as:

Model III $\min \lambda = \sum_{t=1}^{T} \sum_{r=1}^{m} \dot{w}_{tr} d_{tr}^{Z-} + \sum_{t=1}^{T-1} d_t^-$

Subject to, $\underline{\tilde{Z}_{tr}} - \tilde{Z}_{tr} + d_{tr}^{Z-}(\overline{\tilde{Z}_{tr}} - \underline{\tilde{Z}_{tr}}) \geq 0 \quad \forall t = 1, 2, \ldots, T; r = 1, 2, \ldots, m$

$$\underline{x_t} - x_t + d_t^-(\overline{x_t} - \underline{x_t}) \geq \mathbf{0} \quad \forall t \quad up\ to\ (T-1)$$

Subject to, $[\mathrm{R}(\tilde{\mathrm{A}}_{l1})]x_1 + [R(\tilde{\mathrm{A}}_{l2})]x_2 + \ldots$
$+ [R(\tilde{\mathrm{A}}_{l\mathrm{T}})]x_T(\leq, =, \geq)[R(\tilde{b}_i)] \forall l = 1, 2, \ldots, p, \ \forall i = 1, 2, \ldots, m$

and $x_1 \geq 0, x_2 \geq 0, \ldots, x_T \geq 0.$

where $\overline{\tilde{Z}_{tr}} = Maximum\{\tilde{Z}_{tr} \approx [R(\tilde{C}_{tr1})]x_1 + [R(\tilde{C}_{tr2})]x_2 + \ldots + [R(\tilde{C}_{trT})]x_T \ \forall t = 1, 2, \ldots, T; \ r = 1, 2, \ldots, m\}$
and $\underline{\tilde{Z}_{tr}} = Minimum\{\tilde{Z}_{tr} \approx [R(\tilde{C}_{tr1})]x_1 + [R(\tilde{C}_{tr2})]x_2 + \ldots + [R(\tilde{C}_{trT})]x_T \ \forall t = 1, 2, \ldots, T; \ r = 1, 2, \ldots, m\}$
subject to satisfying set of the constraints.

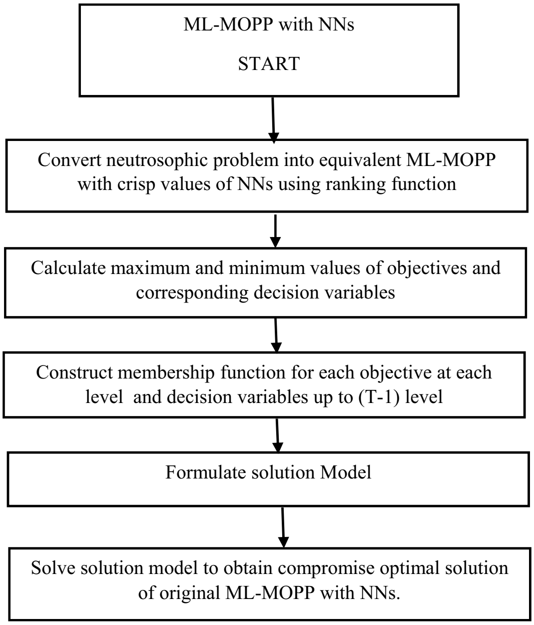
$R(\tilde{A}_{trT}) = \left[\frac{(a^u_{trT} + a^l_{trT}) - 6\alpha_{a_{trT}}}{2}\right] + 1$, $R(\tilde{C}_{trT}) = \left[\frac{(c^u_{trT} + c^l_{trT}) - 6\alpha_{c_{trT}}}{2}\right] + 1$ and $R(\tilde{b}_i) = \left[\frac{(b^u_i + b^l_i) - 6\alpha_{b_i}}{2}\right] + 1 \ \forall t = 1, 2, \ldots, T; \ r = 1, 2, \ldots, m$

Model IV $\min \lambda = \sum_{t=1}^{T} \sum_{r=1}^{m} d_{tr}^{Z-} + \sum_{t=1}^{T-1} d_t^-$

Subject to, $\underline{\tilde{Z}_{tr}} - \tilde{Z}_{tr} + d_{tr}^{Z-}(\overline{\tilde{Z}_{tr}} - \underline{\tilde{Z}_{tr}}) \geq 0 \quad \forall t = 1, 2, \ldots, T; r = 1, 2, \ldots, m$

$$\underline{x_t} - x_t + d_t^-(\overline{x_t} - \underline{x_t}) \geq \mathbf{0} \quad \forall t \quad up\ to\ (T-1)$$

Fig. 2 Flow chart of proposed technique



$$\text{Subject to,}\quad [\mathrm{R}(\tilde{\mathrm{A}}_{l1})]x_1 + [R(\tilde{\mathrm{A}}_{l2})]x_2 + \ldots + [R(\tilde{\mathrm{A}}_{l\mathrm{T}})]x_T(\leq, =, \geq)[R(\tilde{b}_i)]\forall l = 1, 2, \ldots, p,\ \forall i = 1, 2, \ldots, m$$

$$\text{and}\quad x_1 \geq 0, x_2 \geq 0, \ldots, x_T \geq 0.$$

And remaining notations have same meanings.

Step 5. Using linear programming solution techniques, we obtain the compromise optimal solution of fully neutrosophic ML-MOPPs with NNs. The flow chart of proposed methodology is given in Fig. 2.

7 Numerical illustrations

Now, this section of the article addresses illustration of following two numerical examples (maximization and minimization problem) of general fully neutrosophic ML-MOLPP (third type) to demonstrate the applicability of proposed solution technique. This illustration is also acceptable for first and second type ML-MOLPP under NN environment as first and second type cases are shortened cases of third type problem.

Numerical Example 1

Let us consider maximization type full neutrosophic three level multiobjective programming problem (third type) with trapezoidal NNs given as:

$$\underset{x_1}{Max}\left\{\tilde{Z}_{11}, \tilde{Z}_{12}\right\}\qquad \text{(First level)}$$

$$\underset{x_2}{Max}\left\{\tilde{Z}_{21},\tilde{Z}_{22}\right\} \qquad \text{(second level)}$$

$$\underset{x_2}{Max}\left\{\tilde{Z}_{31},\tilde{Z}_{32}\right\} \qquad \text{(third level)}$$

$$\text{Subject to,} \qquad (4,10,2,2)x_1 + (6,10,2,2)x_2 + (6,8,2,2)x_3 \le (475,505,6,6)$$

$$(6,8,3,3)x_1 + (6,10,2,2)x_3 \le (460,480,8,8)$$

$$(4,6,3,3)x_1 + (10,10,2,2)x_2 \le (465,495,5,5)$$

$$\text{And} \qquad x_1 \ge 0,\ \ x_2 \ge 0,\ \ x_3 \ge 0.$$

where $\tilde{Z}_{11} = (3,5,1,1)x_1 + (4,6,1,1)x_2 + (6,8,2,2)x_3$

$$\tilde{Z}_{12} = (13,15,2,2)x_1 + (12,14,3,3)x_2 + (12,14,1,1)x_3$$

$$\tilde{Z}_{21} = (6,8,2,2)x_1 + (4,6,1,1)x_2 + (3,5,1,1)x_3$$

$$\tilde{Z}_{22} = (16,18,2,2)x_1 + (14,16,1,1)x_2 + (13,15,2,2)x_3$$

$$\tilde{Z}_{31} = (16,18,2,2)x_1 + (6,8,1,1)x_2 + (7,9,1,1)x_3$$

$$\tilde{Z}_{32} = (4,8,1,1)x_1 + (3,7,1,1)x_2 + (4,6,1,1)x_3$$

Step 1. Using ranking function $R(\tilde{a}) = \left(\frac{a^l + a^u + 4(\alpha)}{2}\right) + 1$ for maximization problem, then NNs are converted into equivalent crisp values and given ML-MOPPs with NNs is converted into equivalent ML-MOPPs with crisp values as:

$$\underset{x_1}{Max}\left\{7x_1 + 8x_2 + 12x_3, 19x_1 + 20x_2 + 16x_3\right\} \qquad \text{(First level)}$$

$$\underset{x_2}{Max}\left\{12x_1 + 8x_2 + 7x_3, 22x_1 + 18x_2 + 19x_3\right\} \qquad \text{(second level)}$$

$$\underset{x_2}{Max}\left\{22x_1 + 10x_2 + 11x_3, 9x_1 + 8x_2 + 8x_3\right\} \qquad \text{(third level)}$$

$$\text{Subject to,} \qquad 12x_1 + 13x_2 + 12x_3 \le 503$$

$$14x_1 + 13x_3 \le 487$$

Table 2 $\overline{\tilde{Z}_{tr}}$ and $\underline{\tilde{Z}_{tr}}$ values with respective points

Objective	Maximum value at point	Minimum value at point
$\tilde{Z}_{11}$	482.4379 at (0, 4.1124, 37.4615)	0 at (0, 0, 0, 0)
$\tilde{Z}_{12}$	786.4339 at (32.8596, 6.4456, 2.0742)	0 at (0, 0, 0, 0)
$\tilde{Z}_{21}$	460.4004 at (32.8596, 6.4456, 2.0742)	0 at (0, 0, 0, 0)
$\tilde{Z}_{22}$	878.3442 at (32.8596, 6.4456, 2.0742)	0 at (0, 0, 0, 0)
$\tilde{Z}_{31}$	814.3333 at (34.7857, 4.9047, 0)	0 at (0, 0, 0, 0)
$\tilde{Z}_{32}$	363.8958 at (32.8596, 6.4456, 2.0742)	0 at (0, 0, 0, 0)

Table 3 Membership functions of objectives and variables (for numerical example 1)

Objective	Membership function for objective	Membership function for decision variable
$\tilde{Z}_{11}$	$\mu_{Z_1}(\tilde{Z}_{11}(\overline{X}))\cong\begin{cases}1 \ \ if\ \tilde{Z}_{11}(\overline{X})\gtrsim 482.4379\\ \frac{\tilde{Z}_{11}(\overline{X})-0}{482.4379-0}\ if\ 0\le\tilde{Z}_{tr}(\overline{X})\le 482.4379\\ 0\ if\ \tilde{Z}_{tr}(\overline{X})\le 0\end{cases}$	$\mu_{x_1}(x_1)=\begin{cases}1 \quad for\ x_1\ge 32.8596\\ \frac{x_1-0}{32.8596-0}\quad for\ 0\le x_1\le 32.8596\\ 0\quad for\ x_1\le 0\end{cases}$
$\tilde{Z}_{12}$	$\mu_{Z_1}(\tilde{Z}_{12}(\overline{X}))\cong\begin{cases}1 \ \ if\ \tilde{Z}_{12}(\overline{X})\gtrsim 786.4339\\ \frac{\tilde{Z}_{12}(\overline{X})-0}{786.4339-0}\ if\ 0\le\tilde{Z}_{12}(\overline{X})\le 786.4339\\ 0\ if\ \tilde{Z}_{12}(\overline{X})\le 0\end{cases}$	
$\tilde{Z}_{21}$	$\mu_{Z2}(\tilde{Z}_{21}(\overline{X}))\cong\begin{cases}1 \ \ if\ \tilde{Z}_{21}(\overline{X})\gtrsim 460.4004\\ \frac{\tilde{Z}_{21}(\overline{X})-0}{460.4004-0}\ if\ 0\le\tilde{Z}_{21}(\overline{X})\le 460.4004\\ 0\ if\ \tilde{Z}_{21}(\overline{X})\le 0\end{cases}$	$\mu_{x_2}(x_2)=\begin{cases}1 \quad for\ x_2\ge 6.4456\\ \frac{x_2-0}{6.4456-0}\quad for\ 0\le x_2\le 6.4456\\ 0\quad for\ x_2\le 0\end{cases}$
$\tilde{Z}_{22}$	$\mu_{Z_2}(\tilde{Z}_{22}(\overline{X}))\cong\begin{cases}1 \ \ if\ \tilde{Z}_{22}(\overline{X})\gtrsim 878.3442\\ \frac{\tilde{Z}_{22}(\overline{X})-0}{878.3442-0}\ if\ 0\le\tilde{Z}_{22}(\overline{X})\le 878.3442\\ 0\ if\ \tilde{Z}_{22}(\overline{X})\le 0\end{cases}$	
$\tilde{Z}_{31}$	$\mu_{Z_3}(\tilde{Z}_{31}(\overline{X}))\cong\begin{cases}1 \ \ if\ \tilde{Z}_{31}(\overline{X})\gtrsim 814.3333\\ \frac{\tilde{Z}_{31}(\overline{X})-0}{814.3333-0}\ if\ 0\le\tilde{Z}_{31}(\overline{X})\le 814.3333\\ 0\ if\ \tilde{Z}_{31}(\overline{X})\le 0\end{cases}$	
$\tilde{Z}_{32}$	$\mu_{Z_3}(\tilde{Z}_{32}(\overline{X}))\cong\begin{cases}1 \ \ if\ \tilde{Z}_{32}(\overline{X})\gtrsim 363.8958\\ \frac{\tilde{Z}_{32}(\overline{X})-0}{363.8958-0}\ if\ 0\le\tilde{Z}_{32}(\overline{X})\le 363.8958\\ 0\ if\ \tilde{Z}_{32}(\overline{X})\le 0\end{cases}$	

$$12x_1 + 15x_2 \le 491$$

$$\text{And} \qquad x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.$$

Step 2. Calculating $\overline{\tilde{Z}_{tr}}$ (maximum) and $\underline{\tilde{Z}_{tr}}$ (minimum) values of individual objective functions without considering the hierarchical structure with corresponding values of decision variables. These values presented in tabular form (Table 2) as:

Step 3. Construction of membership functions for each objective and decision variables (up to two levels only) using maximum and minimum values of these parameters is described in Table 3 as:

Step 4. Using simplified methodology by Lachhwani [24], we formulate the proposed solution model A (considering equal importance to each objective at each level as given in proposed solution model II for maximization problem) for given problem as:

Model A $\min \lambda = \sum_{t=1}^{3} \sum_{r=1}^{2} d_{tr}^{Z-} + \sum_{t=1}^{2} d_t^-$

$$\text{subject to,} \qquad -482.4379 + 7x_1 + 8x_2 + 12x_3 + d_{11}^{Z-}(482.4379) \ge 0$$

$$-786.4339 + 19x_1 + 20x_2 + 16x_3 + d_{12}^{Z-}(786.4339) \ge 0$$

$$-460.4004 + 12x_1 + 8x_2 + 7x_3 + d_{21}^{Z-}(460.4004) \ge 0$$

$$-878.3442 + 22x_1 + 18x_2 + 19x_3 + d_{22}^{Z-}(878.3442) \ge 0$$

$$-814.3333 + 22x_1 + 10x_2 + 11x_3 + d_{31}^{Z-}(814.3333) \ge 0$$

$$-363.8958 + 9x_1 + 8x_2 + 8x_3 + d_{32}^{Z-}(363.8958) \ge 0$$

$$-32.8596 + x_1 + d_1^-(32.8596) \ge 0$$

$$-6.4456 + x_2 + d_2^-(6.4456) \ge 0$$

$$12x_1 + 13x_2 + 12x_3 \le 503$$

$$14x_1 + 13x_3 \le 487$$

$$12x_1 + 15x_2 \le 491$$

$$\text{And} \qquad x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.$$

Step 5. Using linear programming technique (Simplex method) on solution model A, the compromise optimal solution of original ML-MOPP with NNs is obtained as: $x_1 = 32.8596,\ x_2 = 6.4456,\ x_3 = 2.0742$ with $Z_{11} = 306.4737, Z_{12} = 786.4339, Z_{21} = 460.4004, Z_{22} = 878.3442, Z_{31} = 810.1848, Z_{32} = 363.8958.$

Numerical Example 2

Now, let us consider another numerical example of minimization type full neutrosophic three level multiobjective programming problem (third type) with trapezoidal NNs given as:

$$\underset{x_1}{Min}\left\{\tilde{Z}_{11}, \tilde{Z}_{12}\right\} \qquad \text{(First level)}$$

$$\underset{x_2}{Min}\left\{\tilde{Z}_{21}, \tilde{Z}_{22}, \tilde{Z}_{23}\right\} \qquad \text{(second level)}$$

$$\underset{x_3}{Min}\left\{\tilde{Z}_{31}, \tilde{Z}_{32}\right\} \qquad \text{(third level)}$$

$$\text{Subject to,} \qquad (3,5,1,1)x_1 + (5,7,1,1)x_2 + (6,8,2,2)x_3 \le (480,520,10,10)$$

$$(6,8,1,1)x_1 + (5,7,1,1)x_2 + (6,8,2,2)x_3 \ge (420,460,10,10)$$

$$(4,6,1,1)x_1 + (10,10,2,2)x_2 \ge (465,495,5,5)$$

$$\text{And} \qquad x_1,\ x_2,\ x_3 \ge 0$$

where $\tilde{Z}_{11} = (16,18,2,2)x_1 + (2,8,1,1)x_2 + (8,10,2,2)x_3$

$$\tilde{Z}_{12} = (13,15,2,2)x_1 + (12,14,3,3)x_2 + (15,17,2,2)x_3$$

$$\tilde{Z}_{21} = (10,12,2,2)x_1 + (12,14,2,2)x_2 + (6,8,1,1)x_3$$

$$\tilde{Z}_{22} = (6,8,1,1)x_1 + (8,10,2,2)x_2 + (10,12,2,2)x_3$$

$$\tilde{Z}_{23} = (16,18,2,2)x_1 + (14,16,1,1)x_2 + (13,15,2,2)x_3$$

$$\tilde{Z}_{31} = (16,18,2,2)x_1 + (6,8,1,1)x_2 + (7,9,1,1)x_3$$

$$\tilde{Z}_{32} = (4,8,1,1)x_1 + (4,6,1,1)x_2 + (4,6,1,1)x_3$$

Step 1. Using ranking function $R(\tilde{a}) = \left(\frac{a^l + a^u - 6(\alpha)}{2}\right) + 1$ for minimization problem, then coefficients and parameters in NNs format are converted into equivalent crisp values. Therefore, converted equivalent ML-MOPPs with crisp values can be written as:

$$\underset{x_1}{Min}\left\{12x_1 + 3x_2 + 4x_3, 9x_1 + 5x_2 + 11x_3\right\} \qquad \text{(First level)}$$

$$\underset{x_2}{Min}\left\{6x_1 + 8x_2 + 5x_3, 5x_1 + 4x_2 + 6x_3,\ 12x_1 + 13x_2 + 9x_3\right\} \qquad \text{(second level)}$$

$$\underset{x_2}{Min}\left\{12x_1 + 5x_2 + 6x_3, 4x_1 + 3x_2 + 3x_3\right\} \qquad \text{(third level)}$$

$$\text{Subject to,} \qquad 2x_1 + 4x_2 + 2x_3 \le 471$$

$$5x_1 + 4x_2 + 2x_3 \ge 411$$

$$3x_1 + 5x_3 \le 466$$

$$\text{And} \qquad x_1 \ge 0,\ \ x_2 \ge 0,\ \ x_3 \ge 0.$$

With following steps 2 and 3 of methodology, the membership functions for each objective and decision variables of these parameters are described in Table 4 as:

Now with the similar arguments, the solution model B (considering equal importance to each objective at each level as given in proposed solution model IV for minimization problem) can be given as:

Model B $\min \lambda = d_{11}^{Z-} + d_{12}^{Z-} + d_{21}^{Z-} + d_{22}^{Z-} + d_{23}^{Z-} + d_{31}^{Z-} + d_{32}^{Z-} + d_1^{-}$

$$\text{Subject to,} \qquad 541.25 - 12x_1 - 3x_2 - 4x_3 + d_{11}^{Z-}(1423.447) \ge 0$$

$$1167.053 - 9x_1 - 5x_2 - 11x_3 + d_{12}^{Z-}(769.6842) \ge 0$$

$$643.3158 - 6x_1 - 8x_2 - 5x_3 + d_{21}^{Z-}(769.6842) \ge 0$$

$$641.9474 - 5x_1 - 4x_2 - 6x_3 + d_{22}^{Z-}(771.0526) \ge 0$$

$$1228.8950 - 12x_1 - 13x_2 - 9x_3 + d_{23}^{Z-}(1597.105) \ge 0$$

$$839.95 - 12x_1 - 5x_2 - 6x_3 + d_{31}^{Z-}(1986.05) \ge 0$$

$$409.6316 - 4x_1 - 3x_2 - 3x_3 + d_{32}^{Z-}(532.3684) \ge 0$$

$$-x_1 + d_1^{-}(235.5) \ge 0$$

$$2x_1 + 4x_2 + 2x_3 \le 471$$

Table 4 Membership functions of objectives and variables for numerical example 2

Objective	Membership function for objective	Membership function for decision variable
$\tilde{Z}_{11}$	$\mu_{Z_1}(\tilde{Z}_{11}(\overline{X})) \cong \begin{cases} 1 \ if \quad \tilde{Z}_{11}(\overline{X}) \le 541.25 \\ \frac{2826-\tilde{Z}_{11}(\overline{X})}{2826-541.25} \ if 541 \quad .25 \le \tilde{Z}_{11}(\overline{X}) \le 2826 \\ 0 \ if \quad \tilde{Z}_{11}(\overline{X}) \ge 2826 \end{cases}$	$\mu_{x_1}(x_1) = \begin{cases} 1 \quad for \quad x_1 \le 0 \\ \frac{235.5-x_1}{235.5-0} \quad for \quad 0 \le x_1 \le 235.5 \\ 0 \quad for \quad x_1 \ge 235.5 \end{cases}$
$\tilde{Z}_{12}$	$\mu_{Z_1}(\tilde{Z}_{12}(\overline{X})) \cong \begin{cases} 1 \ if \quad \tilde{Z}_{12}(\overline{X}) \le 1167.0530 \\ \frac{2590.50-\tilde{Z}_{12}(\overline{X})}{2590.50-1167.0530} \ if \quad 1167.0530 \le \tilde{Z}_{12}(\overline{X}) \le 2590.50 \\ 0 \ if \quad \tilde{Z}_{12}(\overline{X}) \ge 2590.5 \end{cases}$	
$\tilde{Z}_{21}$	$\mu_{Z2}(\tilde{Z}_{21}(\overline{X})) \cong \begin{cases} 1 \ if \quad \tilde{Z}_{21}(\overline{X}) \le 643.3158 \\ \frac{1413-\tilde{Z}_{21}(\overline{X})}{1413-643.3158} \ if \quad 643.3158 \le \tilde{Z}_{21}(\overline{X}) \le 1413 \\ 0 \ if \quad \tilde{Z}_{21}(\overline{X}) \ge 1413 \end{cases}$	Membership function $\mu_{x_1}(x_1)$ does not exist due to non-conformity values
$\tilde{Z}_{22}$	$\mu_{Z_2}(\tilde{Z}_{22}(\overline{X})) \cong \begin{cases} 1 \ if \quad \tilde{Z}_{22}(\overline{X}) \le 641.9474 \\ \frac{1413-\tilde{Z}_{22}(\overline{X})}{1413-641.9474} \ if \quad 641.9474 \le \tilde{Z}_{22}(\overline{X}) \le 1413 \\ 0 \ if \quad \tilde{Z}_{22}(\overline{X}) \ge 1413 \end{cases}$	
$\tilde{Z}_{23}$	$\mu_{Z_2}(\tilde{Z}_{23}(\overline{X})) \cong \begin{cases} 1 \ if \quad \tilde{Z}_{23}(\overline{X}) \le 1228.895 \\ \frac{2826-\tilde{Z}_{23}(\overline{X})}{2826-1228.895} \ if \quad 1228.895 \le \tilde{Z}_{23}(\overline{X}) \le 2826 \\ 0 \ if \quad \tilde{Z}_{23}(\overline{X}) \ge 2826 \end{cases}$	
$\tilde{Z}_{31}$	$\mu_{Z_3}(\tilde{Z}_{31}(\overline{X})) \cong \begin{cases} 1 \ if \quad \tilde{Z}_{31}(\overline{X}) \le 839.95 \\ \frac{2826-\tilde{Z}_{31}(\overline{X})}{2826-839.95} \ if \quad 839.95 \le \tilde{Z}_{31}(\overline{X}) \le 2826 \\ 0 \ if \quad \tilde{Z}_{31}(\overline{X}) \ge 2826 \end{cases}$	
$\tilde{Z}_{32}$	$\mu_{Z_3}(\tilde{Z}_{32}(\overline{X})) \cong \begin{cases} 1 \ if \quad \tilde{Z}_{32}(\overline{X}) \le 409.6316 \\ \frac{942-\tilde{Z}_{32}(\overline{X})}{942-409.6316} \ if \quad 409.6316 \le \tilde{Z}_{32}(\overline{X}) \le 942 \\ 0 \ if \quad \tilde{Z}_{32}(\overline{X}) \ge 942 \end{cases}$	

$$5x_1 + 4x_2 + 2x_3 \geq 411$$

$$3x_1 + 5x_3 \leq 466$$

$$\text{And} \qquad x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

In last step, we obtain the compromise optimal solution of ML-MOPP with NNs using linear programming technique (simplex method) on solution model B of this problem as: $x_1 = 28.2424$, $x_2 = 67.4469$, $x_3 = 0$ with $Z_{11} = 541.25$, $Z_{12} = 591.4167$,$Z_{21} = 709.0303$, $Z_{22} = 411.00$, $Z_{23} = 1215.720$,$Z_{31} = 676.1439$, $Z_{32} = 315.3106$.

ML-MOLPPs with neutrosophic environment is latest area of research. Recently, Maiti et al. [32] suggested ML-MOLPP with parameters as NNs in the form of $c + dI$, where I as indeterminacy and c, d are considered as real numbers. If we assess the work of Maiti et al. [32], it can be observed that Maiti et al. [32] considered NNs in the $c + dI$ form in which only indeterminacy part of NNs is considered and sidestepped the truthiness and falsity characteristics of information. For programming problems, truthiness and falsity features are also decisive. In our proposed methodology, coefficients and parameters are represented in trapezoidal NNs in which all three parts truthiness, falsity and indeterminacy of information are used. In this way, the proposed methodology is useful in solving general full neutrosophic ML-MOLPP. Again, there exists no other solution approach or methodology in the literature for solving fully neutrosophic ML-MOLPP with trapezoidal NNs and therefore comparison of our proposed methodology with other method on same problem does not arise. This proves the novelty and uniqueness of proposed methodology in solving fully neutrosophic ML-MOLPP with trapezoidal NNs.

8 Case study

In this section, a real problem of production planning of electronics digital gadgets manufacturing company is employed to exhibit the formulation of real problem in ML-MOLPP under NN environment and applicability of proposed methodology for real problems. Let this company and its sub production units produce five kinds of electronic items namely mobile phones, laptops, android TVs, digital tabs and related supplementary accessories for consumers. These all three companies (main company and two sub units) wish to maximize their own all profit objectives (more than one conflicting objective at each company) through production of electronics goods. The electronic gadgets company (Company 1—Level 1) play role of leader company on production of main two products namely mobile phones and laptops and thereafter the sub units (Company 2 and 3—Level 2 and 3) determine their strategies as followers on production of other three products android TVs, digital tabs and accessories. The reasons in deciding different levels in this case are: normally mobile phones and laptops (objective functions considered at 1st level) have larger market share in terms of consumers and android TVs, digitals tabs, accessories are considered as only sub sets of this market share. Therefore, in view of maximization of each objective function, First level decision making problem is decided to maximize objective functions for profits from mobile phones and laptops, then second level and third levels decision problems are considered to maximize objective functions for profits from android TVs, digital tabs and profits from accessories respectively within the feasible region of constraints. This is in agreement that first level decision maker (FLDM) (decision makers for mobile phones and laptops) sets his decision and then ask each lower level decision makers (second levels for android TVs, digital tabs and third level for accessories) for their optimal solution within the feasible region. Lower level's decisions are then submitted and modified by main company (1st level). This process continue until a satisfactory solution is reached. The decision of Leader Company is affected by the reactions of the follower companies. Let us assume that the variables involved are raw materials (x_1), labour cost (x_2) and marketing cost (x_3) in which x_1 is the most influential decision variable for first level DMs while x_2, x_3 are decision variables for the second and third level DMs respectively. Then such problem can be formulated as multi-level multiobjective linear programming problem as:

$$\underset{x_1 \to Raw\ Material}{Maximize} \left\{ z_{11} = \Pr ofit\ from\ Mobile\ Phones, \quad z_{12} = \Pr ofit\ from\ Laptops \right\}$$

$$\underset{x_2 \to Labour\ \mathrm{Cost}}{Maximize} \left\{ z_{21} = \Pr ofit\ from\ Android\ TVs, \quad z_{22} = \Pr ofit\ from\ Digital\ Tabs \right\}$$

$$\underset{x_3 \to Marketing\ \cos t}{Maximize} \left\{ z_{31} = \Pr ofit\ from\ Accessories \right\} \tag{A}$$

Subject to the set of constraints and non-negative restrictions. Taking into account the neutrosophic environments, variables such as basic electronic materials, labour cost, and packaging cost involved in the production of electronic gadgets are imprecise and fluctuating. Accordingly, parameters and coefficients of these variables also become imprecise and indeterministic. Therefore, we apply the concept of neutrosophic numbers to describe these parameters and coefficients in objective functions and constraints in the problem. Further, at first level, decision makers (DMs) wish to maximize profits simultaneously from two product supplies for mobile phones and laptops while at the second and third level DMs, their target objectives are android TVs, digital tabs and accessories respectively.

Let $z_{11} \tilde{=} \tilde{c}_{111}x_1 + \tilde{c}_{112}x_2 + \tilde{c}_{113}x_3$ represents profit per unit as first level first objective (profit from Mobile phones) where $\tilde{c}_{111}$, $\tilde{c}_{112}$, $\tilde{c}_{113}$ are neutrosophic numbers with values as: $\tilde{c}_{111} = (8, 6, 2, 2)$, $\tilde{c}_{112} = (10, 6, 2, 2)$, $\tilde{c}_{113} = (4, 6, 1, 1)$.

Similarly other objective functions are $z_{12} \tilde{=} \tilde{c}_{121}x_1 + \tilde{c}_{122}x_2 + \tilde{c}_{123}x_3 = (10, 6, 2, 2)x_1 + (10, 6, 2, 2)x_2 + (3, 5, 1, 1)x_3$

$$z_{21} \tilde{=} \tilde{c}_{211}x_1 + \tilde{c}_{212}x_2 + \tilde{c}_{213}x_3 = (4, 6, 1, 1)x_1 + (3, 5, 1, 1)x_2 + (4, 6, 2, 2)x_3$$

$$z_{22} \tilde{=} \tilde{c}_{221}x_1 + \tilde{c}_{222}x_2 + \tilde{c}_{223}x_3 = (3, 5, 1, 1)x_1 + (3, 5, 1, 1)x_2 + (5, 7, 2, 2)x_3$$

$$z_{31} \cong \tilde{c}_{311}x_1 + \tilde{c}_{312}x_2 + \tilde{c}_{313}x_3 = (4, 6, 1, 1)x_1 + (4, 6, 1, 1)x_2 + (3, 5, 1, 1)x_3$$

Subject to the set of conditions

$$(4, 10, 2, 2)x_1 + (6, 10, 2, 2)x_2 + (6, 8, 2, 2)x_3 \le (475, 505, 6, 6)$$

$$(6, 8, 3, 3)x_1 + (6, 10, 2, 2)x_3 \le (460, 480, 8, 8)$$

$$(4, 6, 3, 3)x_1 + (10, 10, 2, 2)x_2 \le (465, 495, 5, 5)$$

$$\text{And} \qquad x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0. \qquad \text{(B)}$$

Programming problem (A) together with neutrosophic values of coefficients and parameters (B) formulates full neutrosophic ML-MOLPP with trapezoidal NNs. Using the proposed methodology, solution model (model I or II) can be formulated and accordingly compromise optimal solution of this real problem (using solution model I considering each objective with equal weight) obtained is as: $x_1 = 32.8996$, $x_2 = 6.4136$,$x_3 = 2.0312$ with $Z_{11} = 453.0778$,$Z_{12} = 483.9462$,$Z_{21} = 287.0599$, $Z_{22} = 256.1915$,$Z_{31} = 287.3800$.

9 Concluding remarks and research directions

This article presents a unique and simple solution methodology for solving fully neutrosophic ML-MOLPP with trapezoidal NNs. In this methodology, it is proposed to apply ranking function of NNs to convert problem into equivalent ML-MOLPPs with equivalent crisp values of neutrosophic coefficients and parameters. Then individual best and worst values of objectives and decision variables are evaluated using equivalent converted ML-MOLPP and fuzzy membership functions for each objective and also for decision variables are constructed to avoid decision deadlock situation in hierarchical structure. Accordingly, FGP solution models are formulated and solving these models, compromise optimal solution of original fully neutrosophic ML-MOLPP is obtained. The proposed methodology is novel and unique for solving general fully neutrosophic ML-MOLPP due to non-existent of any other method in literature for solving such ML-MOLPP with trapezoidal NNs. The proposed approach can be applied to solve real world problems arising in industries and business organizations with imprecise and inconsistent information formulated in the form of ML-MOLPP. A real problem case has been discussed to show applicability of proposed approach.

For future research, one possible direction may be application of proposed approach in more complex real decision making problems such as multi-level logistic planning problem under neutrosophic environment. Furthermore, another possible direction may be extension to fully neutrosophic multi-level multiobjective non linear programming problem (ML-MONLPP) involving all coefficient and parameters in the form of trapezoidal NNs.

Declarations

Conflict of interest Author declares that there is no conflict of interest regarding the publication of this article.

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