APPLICATION ARTICLE

Inter-dependent lead-time and ordering cost reduction strategy: a supply chain model with quality control, lead-time dependent backorder and price-sensitive stochastic demand

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Abstract

This research work addresses an inter-dependent reduction strategy of lead time and ordering cost in a two-stage single vendor and single buyer supply chain model with price-sensitive stochastic demand. Buyer's backorder rate is a variable, as it depends on the variable lead time. Quality improvement is another aspect of this study. The objective of this study is three folds. Firstly, reducing the lead time and ordering cost simultaneously. Secondly, improving the quality of products and third, optimizing order lot size, lead time, process quality parameter, safety factor, ordering cost, lead time crashing cost, backorder rate, and the number of deliveries so that the joint expected total profit becomes maximum. Stackelberg game and Joint decision, both approaches are discussed. Numerical result shows that the Joint decision approach gives better result than the Stackelberg game approach. Sensitivity analysis for Case-III with respect to some key parameters has been carried out.

Keywords Supply chain · Lead time · Ordering cost reduction · Quality improvement · Backorder · Price-dependent stochastic demand

List of abbreviations
EOQ   Economic order quantity
EDI   Electronic data interchange

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1 Introduction

In the real competitive market, business competitiveness and benefits are directly or indirectly influenced by selling price, lead time and/or ordering cost reduction. Naturally, the demand of the customer is influenced by the selling price. A convenient price may help to increases the sales of the product. Lead time reduction helps to reduce the safety stock and the loss for stock-out but it improves customer service level. On the other hand, an ordering cost reduction encourages to place more orders. Ordering cost is inversely proportional to carrying cost. An increasing number of orders increases the ordering cost but it decreases the average inventory level and hence the inventory carrying cost. As a result, the total profit is increased.

In some practical situations, the lead time and ordering cost may be interdependent. The reduction of lead time may accompany the ordering cost reduction and vice versa. For example, the utilization of *Electronic Data Interchange* (EDI) technology may simultaneously reduce both the lead and also the ordering cost. Until now, a few numbers of research works have been done regarding the establishment of the relation between the lead time and ordering cost reduction. But none of them have focused on two important issues: quality control and variable backorders. The motivation of this research is to fill up this research gap and present more realistic and useful supply chain model.

The main contribution of this research work is to present an inter-dependent lead-time and ordering cost reduction strategy together with quality improvement and variable backorder rate, in order to maximize the joint expected total profit of the entire vendor–buyer supply chain system. The present work can be used in various industries such as automobiles, mobile phones, computers, textiles, footwear, and so on.

In this article, an integrated vendor–buyer supply chain model is investigated with quality control. Lead-time dependent variable ordering cost and backorder rate are taken into consideration. The demand of the buyer is stochastic and it is influenced by selling price. Investment for lead-time and quality control are considered. Lead time and ordering cost reductions act interdependently. A logarithmic function is used to state the relationship between the percentage of reductions in lead time and ordering cost. The purpose of this study is to optimize the decision variables and maximize the joint expected total profit by reducing the lead time and ordering cost interdependently.

2 Literature review

In inventory literature, several models have been investigated for price-sensitive demand. Jørgensen and Zaccour [16], Xie and Neyret [36], and Pal and Adhikari [27] are some of them who have developed their model by considering price-dependent demand. He et al. [14] have studied a supply chain model with effort

and price-sensitive stochastic demand whereas inventory models with random sales price dependent stochastic demand is discussed by Roy et al. [29] and Das Roy and Sana [8]. Also, there is a number of articles where the reduction of lead time and/or ordering cost are taken into consideration. The work of Woo et al. [35], Ouyang et al. [25], Zhang et al. [40], Yi and Sarker [38], and Das Roy [5] are significant in this regard. Chang et al. [3] have investigated an integrated vendor–buyer supply chain model where they have taken both controllable lead time and ordering cost reduction into consideration while an optimal inventory policy is proposed by Lee et al. [21] where the lead time demand is assumed to be controllable. They have included an ordering cost reduction and backorder discount in their model. Arkan and Hejazi [1] have incorporated a two-stage supply chain system with the concept of credit period in conjunction with controllable lead time and ordering cost whereas a trade credit policy in an integrated supply chain model with lead time and set up cost reduction is presented by Das Roy and Sana [9]. In all the above studies, the lead time and ordering cost are not dependent on each other. Also, they are reduced independently.

Chen et al. [4] have analyzed a continuously reviewed inventory model where they have assumed that the reductions of lead time span and ordering cost depend on each other. A periodically reviewed inventory model together with a backorder price discount is studied by Ouyang et al. [26] where they have considered that the reductions of the lead time and ordering cost act dependently. Hemapriya and Uthayakumar [15] have also investigated an inventory model with a consideration of lead time dependent ordering cost. A multi-stage cleaner production process is discussed by Kim and Sarkar [17] where they have assumed lead time sensitive ordering cost and backorder price discount. Vijayashree and Uthayakumar [34] have introduced lead time dependent ordering cost control in a two-level supply chain system. An inter-dependent reduction strategy of lead time and ordering cost together with trade credit financing and rework in an integrated supply chain model is proposed by Das Roy [6].

The quality of a product makes the image of a company. Several researchers have considered quality control in their model. Among them, Porteus [28] is recognized as the first who introduced the idea of considering an additional investment cost for quality improvement. Ouyang et al. [25] have studied an imperfect manufacturing process where setup cost reduction and quality control are taken into consideration together with stochastic lead time while an integrated inventory model with variable lead time and investment for quality improvement is considered by Yang and Pan [37]. Yoo et al. [39], Shih and Wang [32], Kim and Sarkar [17, 18], and Das Roy [7] have also taken quality improvement into consideration.

In business, backorder is a common practice. The effect of variable lead time and controllable backorder rate in an inventory model with the mixtures of distribution is discussed by Lee [20]. Lin [22] has presented an integrated vendor–buyer inventory model by including ordering cost reduction together with a backorder price discount where the backorder rate is considered as a decision variable. An economic order quantity (EOQ) model for imperfect quality products including variable backorder rate is proposed by Das Roy et al. [10]. They have assumed that the backorder rate is an exponential function of variable waiting time during the stock-out period. Moshrefi and

Jokar [24] have analyzed an integrated supply chain model with a partial backorder where the backorder rate is considered as a linearly decreasing function of shortage time, while an inventory model with stochastic demand and lead-time sensitive partial backlogging is investigated by Sana and Goyal [30]. Besides these, there are many articles that have included backorders. A few of them are Cardenas-Barron et al. [2], Teng et al. [33], Sarker et al. [31], Lashgari et al. [19], Das Roy et al. [11–13], Kim and Sarkar [17, 18], and Majumder et al. [23]. The present article has also considered a variable backorder rate. A brief comparison between the contribution of existing relevant literature and the present model is given in Table 1.

3 Notation and assumptions

The notation and assumptions used to develop the proposed model are as follows.

3.1 Notation

The notations of the proposed model are divided into the following two subsections.

3.1.1 Parameters

D	Expected demand
S	Vendor's setup cost (\$/setup)
M	Vendor's production rate (units/unit time)
A_0	Buyer's original ordering cost (\$/order)
C_v	Vendor's production cost (\$/unit)
C_r	Vendor's rework cost (\$/unit)
h_v	Vendor's holding cost (\$/unit)
h_b	Buyer's holding cost (\$/unit)
π_b	Buyer's shortage cost (\$/unit)
π_l	Buyer's lost sale cost (\$/unit)
θ_0	Initial probability of the production process that may go out-of-control state during the production run
X	Lead time demand
x^+	Max$\{x, 0\}$, where x is a random variable
$f(x)$	Density function of X
μ	Mean of X
σ	Standard deviation of X
$E(X)$	Mathematical expectation of X

3.1.2 Variables

Q	Ordering quantity (units) of the buyer

Table 1 Contribution of different authors in the related field

Authors (year)	Supply chain	Stochastic demand	Lead time dependent ordering cost	Quality control	Controllable lead time	Lead time dependent back-order rate
Porteus [28]				✓		
Chen et al. [4]		✓	✓		✓	
Ouyang et al. [25]		✓		✓	✓	
Yang and Pan [37]	✓	✓		✓	✓	
Ouyang et al. [26]		✓	✓		✓	
Sana and Goyal [30]		✓			✓	✓
Hemapriya and Uthayakumar [15]	✓	✓	✓		✓	
Kim and Sarkar [17]		✓	✓	✓	✓	
Kim and Sarkar [18]	✓	✓		✓	✓	✓
Vijayashree and Uthayakumar [34]	✓	✓	✓		✓	
Das Roy [6]	✓	✓	✓		✓	
Das Roy and Sana [9]	✓	✓			✓	
Present paper	✓	✓	✓	✓	✓	✓

R	Reorder point (units) of the buyer
n	Number of deliveries from vendor's premises to buyer's premises per production cycle
θ	Probability of the production process that may go out-of-control state during the production run
w	Purchasing cost ($/unit) of the buyer, i.e., wholesale price of the vendor
p	Selling price ($/unit) of the buyer
k	Safety factor
L	Length of lead time (days)
$A(L)$	Buyer's ordering cost ($/order), $0 < A \leq A_0$.
$\beta(L)$	Buyer's backorder rate (unit/unit time)

3.2 Assumptions

The assumptions to frame the model are as follows.

1. This study considers an integrated two-echelon supply chain model that includes a single-vendor, a single-buyer, and a single type of product.
2. Price sensitive stochastic demand is assumed. The demand of the product is $x-$ap, where x indicates the capacity of the market which is a random variable and p is the selling price of the buyer. x follows normal distribution with mean μ. Therefore, $D =$ expected demand $= \mu - ap$, where $a > 0$ is the rate of change of demand with price.
3. Quality improvement of items is taken into consideration. A logarithmic function in θ is taken into consideration as an investment cost for quality improvement (see Porteus [28], Ouyang et al. [25] and Yang and Pan [37]) which is as follows

$$I_v(\theta) = J\big(\ln \theta_0 - \ln \theta\big), 0 < \theta \leq \theta_0,$$

    where $J = \frac{1}{\xi}$, ξ is the percentage decrease in θ per dollar increase in $I_v(\theta)$.

4. The model considers lead-time sensitive variable backorder rate $\beta(L)$(see Kim and Sarkar [18]), which is as follows.

$$\beta(L) = \frac{1}{1 + \lambda\sigma\sqrt{L}\psi(k)}, \quad \lambda \text{ is a constant } 0 < \lambda < \infty.$$

    Here, $\beta'(L) = -\frac{\lambda\sigma\psi(k)}{2\sqrt{L}\left\{1+\lambda\sigma\sqrt{L}\psi(k)\right\}^2} < 0 \, \forall \, k \geq 0$. Thus, $\beta(L)$ is a strictly decreasing function in L.

5. This study assumes lead time dependent variable ordering cost. Reduction of lead time and ordering cost act interdependently and the relationship (see Chen et al. [4]) between them is

$$\rho\ln\left(\frac{L}{L_0}\right) = \frac{A(L) - A_0}{A_0},$$

where $\rho < 0$, is a constant. It is the scaling parameter which indicates the logarithmic relationship between percentages of reductions in lead time and ordering cost. The ordering cost A can be written as

$A(L) = \gamma_0 + \gamma_1 \ln L$, where $\gamma_0 = A_0 + \rho A_0 \ln L_0$ and $\gamma_1 = -\rho A_0 > 0$.

Here $A''(L) = -\frac{\omega}{L^2} \langle 0 \text{ as } \omega \rangle 0$. Thus, the ordering cost $A(L)$ is strictly a concave function in L.

6. The lead time L of the buyer has m mutually independent components. The i th component has a normal duration b_i and the minimum duration a_i with the crashing cost per unit time $c_i, i = 1, 2, 3, \ldots, m$ such that $c_1 \leq c_2 \leq c_3 \leq \cdots \leq c_m$.
7. The concept of Yang and Pan [37] and Das Roy [6] regarding lead time crashing cost is used in this model. Let $L_0 = \sum_{i=1}^{m} b_i$, and L_i be the length of lead time with components $1, 2, 3, \ldots, i$ crashed to their minimum duration, then L_i can be considered as $L_i = L_0 - \sum_{j=1}^{m}(b_j - a_j), i = 1, 2, 3, \ldots, m$; and the lead time crashing cost per cycle $C(L)$ can be expressed as

$$C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j).$$

8. To reduce the lead time, an extra cost $C(L)$ is added to the buyer's total cost.
9. The lengths of lead times are equal for all the shipment cycles, and therefore the lead time crashing costs are also the same for all the ordering cycles of the buyer.

4 Mathematical model

Here first, we calculate the individual expected total profit of the vendor and the buyer, then calculate their joint expected total profit.

4.1 Buyer's individual profit

The stock level of the buyer is continuously reviewed. The buyer places an order quantity Q whenever the level of on-hand stock reaches to the reorder point R. The average cycle time of the buyer is $\frac{Q}{(\mu - ap)}$. The ordering cost of the buyer is influenced by lead time L. So, the expected ordering cost per unit time is $\frac{A(L)(\mu - ap)}{Q}$.

If $x - ap > R, i.e., x > ap + R = \bar{R}$, then shortages take place. The expected shortage at the end of the buyer's cycle is $E(x - \bar{R})^+$. The backorder rate is $\beta(L)$. The amount of lost sale is $(1 - \beta(L))E(x - \bar{R})^+$. This amount is stored at the buyer's premises. The expected stock level of the buyer just before and immediately after the arrival of an order Q at the buyer's level are respectively $\bar{R} - (\mu - ap)L + (1 - \beta(L))E(x - \bar{R})^+$ and $Q + (\bar{R} - (\mu - ap)L) + (1 - \beta(L))E(x - \bar{R})^+$. Hence, the average inventory of the buyer's over the cycle time $\frac{Q}{(\mu - ap)}$ can be written as

$\frac{Q}{2} + (\bar{R} - (\mu - ap)L) + (1 - \beta(L))E(x - \bar{R})^+$. Therefore, the expected holding cost of the buyer per unit time is $h_b\left\{\frac{Q}{2} + \bar{R} - (\mu - ap)L + (1 - \beta(L))E(x - \bar{R})^+\right\}$ (see Majumder et al. [23]).

The expected shortage and lost sale cost per unit time are $\frac{\pi_b(\mu - ap)}{Q}E(x - \bar{R})^+$ and $\frac{(\mu - ap)\pi_l(1 - \beta(L))}{Q}E(x - \bar{R})^+$, respectively. The expected lead time crashing cost per unit time is $\frac{(\mu - ap)C(L)}{Q}$. Therefore, the expected total profit of the buyer per unit time is

$$EP_b(Q, w, p, k, L) = \text{sales revenue} - [\text{ordering cost} + \text{holding cost}$$
$$+ \text{shortage cost} + \text{lost sale cost} + \text{lead time crashing cost}]$$

$$= (p - w)(\mu - ap) - \left[\frac{A(L)(\mu - ap)}{Q}\right.$$

$$+ h_b\left\{\frac{Q}{2} + \bar{R} - (\mu - ap)L + (1 - \beta(L))E(x - \bar{R})^+\right\}$$

$$\left. + \frac{(\mu - ap)}{Q}\left\{\pi_b + \pi_l(1 - \beta(L))\right\}E(x - \bar{R})^+ + \frac{(\mu - ap)C(L)}{Q}\right]$$

$$(1)$$

The mean and standard deviation of the lead time demand are $(\mu - ap)L$ and $\sigma\sqrt{L}$, respectively. Buyer's reorder point $\bar{R} = (\mu - ap)L + k\sigma\sqrt{L}$, where k denotes the safety factor and

$$E(x - \bar{R})^+ = \int_{\bar{R}}^{\infty} (x - \bar{R})f(x)dx = \sigma\sqrt{L}\psi(k), \quad (2)$$

where $\psi(k) = \varphi(k) - k(1 - \Phi(k))$. Here $\varphi(k)$ and $\Phi(k)$ are the standard normal distribution and cumulative distribution functions of the normal distribution, respectively.

Using Eq. (2) and Assumption 4 in Eq. (1). After simplification, the expected total profit of the buyer per unit time becomes

$$EP_b(Q, w, p, k, L) = (p - w)(\mu - ap) - \frac{(\mu - ap)}{Q}\left\{A(L) + C(L) + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\}$$

$$- \frac{1}{2}h_bQ - h_b\sigma\sqrt{L}\left[k + \frac{\alpha_0\psi(k)}{(1 + \alpha_0)}\right],$$

$$(3)$$

$$\text{where } \alpha_0 = \lambda\sigma\sqrt{L}\psi(k), \quad \tilde{\pi} = \pi_b + \pi_l\frac{\alpha_0}{1 + \alpha_0}.$$

4.2 Vendor's individual profit

Let us consider that the vendor produces nQ quantities in a production cycle with a finite production rate M $(M > (\mu - ap))$ and ships them to the buyer over n times each of lot size Q. So, the average cycle time of the vendor is $\frac{nQ}{(\mu - ap)}$. The

expected set up cost per unit time is $\frac{S(\mu-ap)}{nQ}$. The average of vendor's inventory is $\frac{Q}{2}\left\{n\left(1-\frac{(\mu-ap)}{M}\right)-1+\frac{2(\mu-ap)}{M}\right\}$ (see Kim and Sarkar [18]). So, the corresponding expected holding cost per unit time is $\frac{h_vQ}{2}\left\{n\left(1-\frac{(\mu-ap)}{M}\right)-1+\frac{2(\mu-ap)}{M}\right\}$. Therefore, the expected total profit of the vendor per unit time is

$$
\begin{aligned}
EP_v(Q,w,p,n) &= \text{sales revenue} - \big[\text{setup cost} + \text{holding cost}\big] \\
&= (w-C_v)(\mu-ap) \\
&\quad - \left[\frac{S(\mu-ap)}{nQ} + \frac{h_vQ}{2}\left\{n\left(1-\frac{(\mu-ap)}{M}\right)-1+\frac{2(\mu-ap)}{M}\right\}\right]
\end{aligned}
$$

The production process produces perfect as well as defective items. The generation of the expected number of defectives in a lot size Q is approximated to $Q^2\theta/2$ (see Porteus [28]). Therefore, the expected number of defective in the lot size nQ is $n^2Q^2\theta/2$. The expected reworked cost per unit time is $\frac{C_rn\theta(\mu-ap)Q}{2}$. To improve the quality of products, the vendor invests an additional cost $I_v(\theta)$ which has been expressed in Assumption 3.

Now, the expected total profit of the vendor per unit time becomes

$$
\begin{aligned}
EP_v(Q,\theta,w,p,n) &= (w-C_v)(\mu-ap) - \frac{S(\mu-ap)}{nQ} \\
&\quad - \frac{h_vQ}{2}\left\{n\left(1-\frac{(\mu-ap)}{M}\right)-1+\frac{2(\mu-ap)}{M}\right\} \\
&\quad - \delta J\big(\ln\theta_0-\ln\theta_0\big) - \frac{C_rn\theta(\mu-ap)Q}{2},
\end{aligned}
\tag{4}
$$

where δ denotes the annual fractional opportunity cost of the vendor's capital investment.

4.3 Joint profit of the vendor and the buyer

Therefore, the joint expected total profit (*EJP*) of the entire system per unit time is the sum of the expected total profit of the vendor and the buyer per unit time and it can be written as

$$
\begin{aligned}
EJP(Q,\theta,p,k,L,n) &= EP_v(Q,\theta,w,p,n) + EP_b(Q,w,p,k,L) \\
&= (p-C_v)(\mu-ap) - \frac{(\mu-ap)}{Q}\left\{A(L)+C(L)+\frac{S}{n}+\sigma\sqrt{L}\psi(k)\tilde{\pi}\right\} \\
&\quad - \frac{Q}{2}\left[h_b+h_v\left\{n\left(1-\frac{(\mu-ap)}{M}\right)-1+\frac{2(\mu-ap)}{M}\right\}+C_rn\theta(\mu-ap)\right] \\
&\quad - \delta J\big(\ln\theta_0-\ln\theta\big) - h_b\sigma\sqrt{L}\left[k+\frac{\alpha_0\psi(k)}{(1+\alpha_0)}\right],
\end{aligned}
\tag{5}
$$

5 Solution methodology

In a supply chain model, it is often observed that all the members of the chain are not of equal power. In a vendor–buyer supply chain, sometimes the vendor (example Microsoft) dominates the buyer and sometimes the buyer (Example shopping Malls) dominates the vendor. Also, sometimes both of the members take a joint decision to maximize their integrated profit. Here, all the three cases: Case-I: Vendor as leader, Case-II: Buyer as leader, and Case-III: Joint decision approach are discussed separately.

5.1 Case-I: Vendor as leader

In this case, the vendor is considered as the leader and the buyer is the follower. Obviously, $w > C_v$. Otherwise, the vendor cannot get any profit. Again, $w < p$, otherwise, the buyer cannot get any profit. Thus, the wholesale price of the vendor must satisfy the condition $C_v < w < p$. To solve the problem, we have considered one more hypothesis as considered by Jorgensen and Zaccour [16] and Xie and Neyret [36] and it is the profit margins of the vendor and buyer are equal. It means $w - C_v = p - w$

$$\text{or, } w = \frac{p + C_v}{2}$$

By using the above relation in Eq. (3), the expected total profit of the buyer per unit time becomes

$$EP_b(Q, p, k, L) = \frac{1}{2}(p - C_v)(\mu - ap) - \frac{(\mu - ap)}{Q}\left\{A(L) + C(L) + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\}$$
$$- \frac{1}{2}h_b Q - h_b\sigma\sqrt{L}\left[k + \frac{\alpha_0\psi(k)}{(1 + \alpha_0)}\right].$$
(6)

Now, using the Stackelberg game approach and optimize $EP_b(Q, p, k, L)$. The first order partial derivatives of EP_b with respect to the decision variables $Q, p, k,$ and L are as follows.

$$\frac{\partial EP_b}{\partial Q} = \frac{(\mu - ap)}{Q^2}\left\{A(L) + C(L) + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\} - \frac{1}{2}h_b$$
(7)

$$\frac{\partial EP_b}{\partial p} = \frac{1}{2}(\mu - 2ap + aC_v) + \frac{a}{Q}\left\{A(L) + C(L) + \sigma\sqrt{L}\tilde{\pi}\psi(k)\right\}$$
(8)

$$\frac{\partial EP_b}{\partial k} = -\frac{(\mu - ap)}{Q}\sigma\sqrt{L}\left[\alpha_1\tilde{\pi} + \frac{\pi_l\alpha_0\alpha_1}{(1 + \alpha_0)^2}\right] - h_b\sigma\sqrt{L}\left[1 + \frac{\alpha_0\alpha_1}{(1 + \alpha_0)} + \frac{\alpha_0\alpha_1}{(1 + \alpha_0)^2}\right],$$
(9)

where $\alpha_1 = \Phi(k) - 1$

$$
\begin{aligned}
\frac{\partial EP_b}{\partial L} = & -\frac{(\mu - ap)}{Q}\left[\frac{\gamma_1}{L} - c_i - \frac{\sigma\psi(k)}{2\sqrt{L}}\left\{\tilde{\pi} + \frac{\alpha_0}{(1+\alpha_0)^2}\right\}\right] \\
& -\frac{h_b\sigma}{2\sqrt{L}}\left[k + \frac{\alpha_0\psi(k)}{(1+\alpha_0)} + \frac{\alpha_0}{(1+\alpha_0)^2}\right]
\end{aligned}
\tag{10}
$$

EP_b is convex with respect to the lead time L, while $Q, p,$ and k are constants. Equating Eqs. (7)–(10) equals to zero to find the optimal values of $Q, p,$ and k for a given value of $L \in [L_i, L_{i-1}]$, which are as follows.

$$
Q^* = \sqrt{\frac{2(\mu - ap)\left\{A(L) + C(L) + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\}}{h_b}}
$$

$$
p^* = \frac{1}{2a}(\mu + aC_v) + \frac{1}{Q}\left\{A(L) + C(L) + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\}
$$

$$
\Phi(k^*) = 1 - \frac{h_b(1+\alpha_0)^2 Q}{\left[h_b\alpha_0(2+\alpha_0)Q + (\mu - ap)\left\{\tilde{\pi}(1+\alpha_0)^2 + \pi_l\alpha_0\right\}\right]}
$$

Now, substituting the optimal values of $Q^*, w^*,$ and p^* into the expected total profit function of the vendor, which becomes

$$
\begin{aligned}
EP_v(Q^*, \theta, w^*, p^*, n) = & (w^* - C_v)(\mu - ap^*) - \frac{S(\mu - ap^*)}{nQ^*} \\
& -\frac{h_vQ^*}{2}\left\{n\left(1 - \frac{(\mu - ap^*)}{M}\right) - 1 + \frac{2(\mu - ap^*)}{M}\right\} \\
& -\delta J(\ln\theta_0 - \ln\theta) - \frac{C_r n\theta(\mu - ap^*)Q^*}{2},
\end{aligned}
\tag{11}
$$

where

$$
w^* = \frac{p^* + C_v}{2}.
$$

The optimal values of n and θ are obtained by equating $\frac{\partial EP_v}{\partial n} = 0$ and $\frac{\partial EP_v}{\partial \theta} = 0$ which gives

$$
n^* = \frac{1}{Q^*}\sqrt{\frac{2S(\mu - ap^*)}{h_v\left(1 - \frac{(\mu - ap^*)}{M}\right) + C_r\theta(\mu - ap^*)}}
$$

and

$$\theta^* = \frac{2\delta J}{C_r n(\mu - ap^*)Q^*}$$

5.2 Case-II: Buyer as leader

Here, the buyer is the leader and the vendor is the follower. As the profit margins of the vendor and buyer are assumed to be equal so $w - C_v = p - w$ o r, $p = 2w - C_v$. This relation is used to convert p in terms of w in the expected total profit of the vendor per unit time i.e. $EP_v(Q, \theta, w, p, n)$ in Eq. (4) and it becomes

$$EP_v(Q, \theta, w, n) = (w - C_v)(\mu - 2aw + aC_v) - \frac{S(\mu - 2aw + aC_v)}{nQ}$$
$$- \frac{h_v Q}{2}\left\{ n\left(1 - \frac{(\mu - 2aw + aC_v)}{M}\right) - 1 + \frac{2(\mu - 2aw + aC_v)}{M}\right\}$$
$$- \delta J(\ln\theta_0 - \ln\theta) - \frac{C_r n\theta(\mu - 2aw + aC_v)Q}{2},$$

(12)

Now, optimize $EP_v(Q, \theta, w, n)$ with respect to the decision variables. Differentiating EP_v with respect to n, Q, θ, and w, which gives

$$\frac{\partial EP_v}{\partial n} = \frac{S(\mu - 2aw + aC_v)}{n^2 Q} - \frac{h_v Q}{2}\left(1 - \frac{(\mu - 2aw + aC_v)}{M}\right) - \frac{C_r\theta(\mu - 2aw + aC_v)Q}{2}$$

$$\frac{\partial EP_v}{\partial Q} = \frac{S(\mu - 2aw + aC_v)}{nQ^2} - \frac{h_v}{2}\left\{ n\left(1 - \frac{(\mu - 2aw + aC_v)}{M}\right)\right.$$
$$\left. -1 + \frac{2(\mu - 2aw + aC_v)}{M}\right\} - \frac{C_r n\theta(\mu - 2aw + aC_v)}{2}$$

$$\frac{\partial EP_v}{\partial \theta} = \frac{\delta J}{\theta} - \frac{C_r n(\mu - 2aw + aC_v)Q}{2}$$

$$\frac{\partial EP_v}{\partial w} = -4aw + \mu + a\left(3C_v + \frac{2S}{nQ} + C_r n\theta - \frac{h_v(n-2)Q}{M}\right)$$

To get the optimal values of n, Q, θ, and w, equating $\frac{\partial EP_v}{\partial n} = 0$, $\frac{\partial EP_v}{\partial Q} = 0$, $\frac{\partial EP_v}{\partial \theta} = 0$, and $\frac{\partial EP_v}{\partial w} = 0$, which gives

$$n^* = \frac{1}{Q}\sqrt{\frac{2S(\mu - 2aw + aC_v)}{h_v\left(1 - \frac{(\mu - 2aw + aC_v)}{M}\right) + C_r\theta(\mu - 2aw + aC_v)}}$$

$$Q^* = \sqrt{\frac{2S(\mu - 2aw + aC_v)}{n\left[h_v\left\{n\left(1 - \frac{(\mu - 2aw + aC_v)}{M}\right) - 1 + \frac{2(\mu - 2aw + aC_v)}{M}\right\} + C_r n\theta(\mu - 2aw + aC_v)\right]}}$$

$$\theta^* = \frac{2\delta J}{C_r n(\mu - 2aw + aC_v)Q}$$

$$w^* = \frac{\mu}{4a} + \frac{1}{4}\left(3C_v + \frac{2S}{nQ} + C_r n\theta - \frac{h_v(n-2)Q}{M}\right)$$

Now, after putting the optimal values $Q^*, w^*,$ and p^*, the expected total profit function of the buyer becomes

$$EP_b(Q^*, w^*, p^*, k, L) = (p^* - w^*)(\mu - ap^*) - \frac{(\mu - ap^*)}{Q^*}\left\{A(L) + C(L) + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\}$$
$$- \frac{1}{2}h_b Q^* - h_b\sigma\sqrt{L}\left[k + \frac{\alpha_0\psi(k)}{(1+\alpha_0)}\right],$$

(13)

where $p^* = 2w^* - C_v$.

The function EP_b is convex with respect to L, when k is constant. For a given value of $L \in [L_i, L_{i-1}]$, $\frac{\partial EP_b}{\partial k} = 0$ gives

$$\Phi(k^*) = 1 - \frac{h_b(1+\alpha_0)^2 Q^*}{\left[h_b\alpha_0(2+\alpha_0)Q^* + (\mu - ap^*)\left\{\tilde{\pi}(1+\alpha_0)^2 + \pi_l\alpha_0\right\}\right]}.$$

5.3 Case-III: Joint decision approach

In this model, both of the members are assumed to be of equal power. None of them dominates the other. Here we optimize the joint expected total profit $EJP(Q, \theta, p, k, L, n)$ of the vendor and buyer. To optimize EJP, we first fix n and then obtain the first order partial derivatives of EJP with respect to $Q, \theta, w, p, k,$ and $L \in [L_i, L_{i-1}]$ respectively, which gives

$$\frac{\partial EJP}{\partial Q} = \frac{(\mu - ap)}{Q^2}\left\{A(L) + C(L) + \frac{S}{n} + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\}$$
$$- \frac{1}{2}\left[h_b + h_v\left\{n\left(1 - \frac{(\mu - ap)}{M}\right) - 1 + \frac{2(\mu - ap)}{M}\right\} + C_r n\theta(\mu - ap)\right]$$

(14)

$$\frac{\partial EJP}{\partial \theta} = \frac{\delta J}{\theta} - \frac{C_r n(\mu - ap)Q}{2} \tag{15}$$

$$\frac{\partial EJP}{\partial p} = -2ap + \mu + a\left[C_v + \frac{1}{Q}\left\{A(L) + C(L) + \frac{S}{n} + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\} + \frac{Q}{2M}\left\{C_r n\theta M - h_v(n-2)\right\}\right] \tag{16}$$

$$\frac{\partial EJP}{\partial k} = -\frac{(\mu - ap)}{Q}\sigma\sqrt{L}\left[\alpha_1\tilde{\pi} + \frac{\pi_l\alpha_0\alpha_1}{(1+\alpha_0)^2}\right] - h_b\sigma\sqrt{L}\left[1 + \frac{\alpha_0\alpha_1}{(1+\alpha_0)} + \frac{\alpha_0\alpha_1}{(1+\alpha_0)^2}\right] \tag{17}$$

where $\alpha_1 = \Phi(k) - 1$

$$\frac{\partial EJP}{\partial L} = -\frac{(\mu - ap)}{Q}\left[\frac{\gamma_1}{L} - c_i - \frac{\sigma\psi(k)}{2\sqrt{L}}\left\{\tilde{\pi} + \frac{\alpha_0}{(1+\alpha_0)^2}\right\}\right] - \frac{h_b\sigma}{2\sqrt{L}}\left[k + \frac{\alpha_0\psi(k)}{(1+\alpha_0)} + \frac{\alpha_0}{(1+\alpha_0)^2}\right] \tag{18}$$

Now, $EJP(Q, \theta, p, k, L, n)$ is convex with respect to the lead time L, when Q, θ, p, k, and n, are constants. The maximum value of $EJP(Q, \theta, p, k, L, n)$ can be determined from the end point of $[L_i, L_{i-1}]$. Thus, for a given n and $L \in [L_i, L_{i-1}]$, the values of Q, θ, p, and k can be obtained by equating Eqs. (14)–(17) equal to zero which provide

$$Q = \sqrt{\frac{2(\mu - ap)\left\{A(L) + C(L) + \frac{S}{n} + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\}}{h_b + h_v\left\{n\left(1 - \frac{(\mu-ap)}{M}\right) - 1 + \frac{2(\mu-ap)}{M}\right\} + C_r n\theta(\mu - ap)}} \tag{19}$$

$$\theta = \frac{2\delta J}{C_r n(\mu - ap)Q} \tag{20}$$

$$p = \frac{(\mu + aC_v)}{2a} + \frac{1}{2Q}\left\{A(L) + C(L) + \frac{S}{n} + \sigma\sqrt{L}\psi(k)\tilde{\pi}\right\} + \frac{Q}{4M}\left\{C_r n\theta M - h_v(n-2)\right\} \tag{21}$$

$$\Phi(k) = 1 - \frac{h_b(1+\alpha_0)^2 Q}{\left[h_b\alpha_0(2+\alpha_0)Q + (\mu - ap)\left\{\tilde{\pi}(1+\alpha_0)^2 + \pi_l\alpha_0\right\}\right]} \tag{22}$$

It is noted from Eqs. (19), (20), (21), and (22) that the expressions for Q, θ, p and k are not independent of one another. One's value is required to calculate the other. To obtain the optimal solution, a suitable solution algorithm is designed as follows.

Solution Algorithm 1:

Step 1. Set $n = 1$.

Step 2. For all integral values of $L \in [L_i, \ L_{i-1}], i = 1, 2, 3, \dots, m$ perform the

Steps (2.1) to (2.10).

Step 2.1. Set $\theta_{i1} = 0$ and $k_{i1} = 0$ [implies $\psi(k_{i1}) = 0.39894$].

Step 2.2. Utilize $\psi(k_{i1})$ to evaluate α_0 and obtain Q_{i1} from Equation (19).

Step 2.3. Put Q_{i1} into Equation (20) and Evaluate θ_{i1}.

Step 2.4. Use the values of Q_{i1} and θ_{i1} in Equation (21) to calculate p.

Step 2.5. Substitute Q_{i1} and α_0 in Equation (22) to find the value of $\Phi(k_{i2})$.

Step 2.6. For the value of $\Phi(k_{i2})$, find the value of k_{i2} from the normal table and hence calculate $\psi(k_{i2})$.

Step 2.7. Repeat Steps (2.2) to (2.6) until no changes are found in the values of Q_i, θ_i, p_i and k_i. Denote this solution as $(\widetilde{Q}_i, \widetilde{\theta}_i, \widetilde{p}_i, \widetilde{k}_i)$.

Step 2.8. If $\widetilde{\theta}_i < \theta_0$, then the solution is optimal for the given $L \in [L_i, L_{i-1}]$. Denote this solution by $(Q_i^*, \theta_i^*, p_i^*, k_i^*)$.

Step 2.9. If $\widetilde{\theta}_i \geq \theta_0$, then set $\widetilde{\theta}_i = \theta_0$ and repeat the Steps (2.2) to (2.8).

Step 2.10. Find $EJP(Q_i^*, \theta_i^*, p_i^*, k_i^*, L_i, n)$ by using the values of Q_i^*, θ_i^*, p_i^*, and k_i^* in Equation (5).

Step 3. If $EJP(Q_i^*, \theta_i^*, p_i^*, k_i^*, L_i^*, n) = \text{Max } [EJP(Q_i^*, \theta_i^*, p_i^*, k_i^*, L_i, n)]$, then $(Q_i^*, \theta_i^*, p_i^*, k_i^*, L_i^*, n)$ will be the optimal result for fixed n.

Step 4. Set $n = n + 1$ and repeat Steps (2) to (3) to get $EJP(Q_n^*, \theta_n^*, p_n^*, k_n^*, L_n^*, n)$.

Step 5. If $EJP(Q_n^*, \theta_n^*, p_n^*, k_n^*, L_n^*, n) \geq EJP(Q_{n-1}^*, \theta_{n-1}^*, p_{n-1}^*, k_{n-1}^*, L_{n-1}^*, n-1)$, then go to Step 4, otherwise go to Step 6.

Step 6. Set $EJP(Q_n^*, \theta_n^*, p_n^*, k_n^*, L_n^*, n) = EJP(Q_{n-1}^*, \theta_{n-1}^*, p_{n-1}^*, k_{n-1}^*, L_{n-1}^*, n-1)$ and $EJP(Q_n^*, \theta_n^*, p_n^*, k_n^*, L_n^*, n^*)$ is the optimum result.

6 Numerical analysis

To demonstrate the proposed supply chain model, a set of suitable parameter values are considered. Most of which are taken from Vijayashree and Uthayakumar [34] and the rest from Kim and Sarkar [18] as follows: $\mu = 1000$, $a = 5$, $M = 3200$ units/unit time, $S = \$400$ /setup, $A_0 = \$25$ /order, $C_v = \$20$ /unit, $C_r = \$10$ /unit, $h_v = \$4$ /unit, $h_b = \$5$ /unit, $\pi_b = \$6$ /unit, $\pi_l = \$8$ /unit, $\sigma = 7$ units/week, $L_0 = 56$ days, $\theta_0 = 0.00035$, $\delta = 0.5$, $J = 400$, $\lambda = 0.2$, $\rho = -0.5$. The lead time of the buyer is assumed to have three components which are shown in Table 2 with data.

The optimal solutions for the three cases are as follows.

a.   Case-I: Vendor as leader

The optimal solutions are: $n^* = 3$, $L^* = 28$ days, $Q^* = 93$ units, $\theta^* = 0.00032$, $w^* = \$65.26$, $p^* = \$110.52$, $k^* = 1.27$, $A(L^*) = \$16.34$ /order, $C(L^*) = \$18.2$, $\beta(L^*) = 0.74$ unit/unit time. The corresponding expected total profit of the buyer and vendor per unit time are $EP_b^* = \$19545$ and $EP_v^* = \$19044$, respectively. We have checked the concavity of the expected total profit of the vendor and buyer at the optimal solution.

b.   Case-II: Buyer as leader

The optimal solutions are: $n^* = 1$, $L^* = 28$ days, $Q^* = 512$ units, $\theta^* = 0.00018$, $w^* = \$65.55$, $p^* = \$111.10$, $k^* = 0.45$, $A(L^*) = \$16.34$ /order, $C(L^*) = \$18.2$, $\beta(L^*) = 0.39$ unit/unit time, $EP_v^* = \$19419$ and $EP_b^* = \$18754$. We have checked the concavity of the expected total profit of the vendor and buyer at the optimal solution.

c.   Case-III: Joint decision approach

The optimal solutions are: $n^* = 2$, $L^* = 28$ days, $Q^* = 138$ units, $\theta^* = 0.00033$, $p^* = \$111.16$, $k^* = 1.10$, $A(L^*) = \$16.34$ /order, $C(L^*) = \$18.2$, and $\beta(L^*) = 0.66$ unit/unit time. The corresponding optimum value of the joint expected total profit $EJP^* = \$38624$.

The principal minors of the Hessian matrix $H = \begin{pmatrix} \frac{\partial^2 EJP}{\partial Q^2} & \frac{\partial^2 EJP}{\partial Q \partial \theta} & \frac{\partial^2 EJP}{\partial Q \partial p} & \frac{\partial^2 EJP}{\partial Q \partial k} \\ \frac{\partial^2 EJP}{\partial \theta \partial Q} & \frac{\partial^2 EJP}{\partial \theta^2} & \frac{\partial^2 EJP}{\partial \theta \partial p} & \frac{\partial^2 EJP}{\partial \theta \partial k} \\ \frac{\partial^2 EJP}{\partial p \partial Q} & \frac{\partial^2 EJP}{\partial p \partial \theta} & \frac{\partial^2 EJP}{\partial p^2} & \frac{\partial^2 EJP}{\partial p \partial k} \\ \frac{\partial^2 EJP}{\partial k \partial Q} & \frac{\partial^2 EJP}{\partial k \partial \theta} & \frac{\partial^2 EJP}{\partial k \partial p} & \frac{\partial^2 EJP}{\partial k^2} \end{pmatrix}$ at the

optimal solution are $|H_{11}| = -0.0867066 < 0$, $|H_{22}| = 1.39509 \times 10^8 > 0$, $|H_{33}| = -1.3831 \times 10^9 < 0$, and $|H_{44}| = 8.39547 \times 10^{11} > 0$. Therefore, the Hessian matrix H is negative definite.

Hence, the required optimal solutions are $n^* = 2$, $L^* = 28$ days, $Q^* = 138$ units, $\theta^* = 0.00033$, $p^* = \$111.16$, $k^* = 1.10$, and $EJP^* = \$38624$.

Table 2 The lead time components with data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i ($/day)
1	20	6	0.1
2	20	6	1.2
3	16	9	5.0

6.1 Comparison

In this subsection, the three cases are compared based on the above optimal solutions and it is shown in Table 3.

Table 3 shows that the optimum value of the joint expected total profit in Case-III (Joint decision approach) is greater than Case-I (Vendor as leader) and Case-II (Buyer as leader). Therefore, we can say that the Joint decision approach is the best in the three cases.

6.2 Sensitivity analysis

The sensitivity analysis for Case-III (Joint Decision approach) has been performed with respect to some of the key parameters A_0, h_v, h_b, and θ_0 whose values are changed by -50%, -25%, $+25\%$, and $+50\%$ while the values of other parameters remain unchanged (see Tables 4, 5, 6 and 7).

6.3 Managerial insights

The managerial insights of the present study based on the numerical analysis are given below.

- This study shows that the joint decision (Case-III) approach helps to achieve more profit compared to the Stackelberg (Case-I and Case-II) game approach. The profit is increased by 0.09% and 1.18% if the joint decision approach is employed instead of the Case-I (Vendor as leader) or Case-II (Buyer as leader), respectively(see Table 3).

Table 3 Comparison of the three cases

Optimal decisions	Case-I (vendor as leader)	Case-II (buyer as leader)	Case-III (joint decision approach)
n^*	3	1	2
L^*	28	28	28
Q^*	93	512	138
θ^*	0.00032	0.00018	0.00033
w^*	65.26	65.55	–
p^*	110.52	111.10	111.16
k^*	1.27	0.45	1.10
EP_v^*	19044	19,419	–
EP_b^*	19,545	18,754	–
EJP^*	38,589	38,173	38,624

Table 4 Sensitivity analysis of Case-III when A_0 changes

Parameter	Parameter value	n^*	L^*	Q^*	θ^*	p^*	k^*	EJP^*
A_0	12.5	2	28	136	0.00033	111.14	1.10	38,649
	18.75	2	28	137	0.00033	111.15	1.10	38,637
	25	**2**	**28**	**138**	**0.00033**	**111.16**	**1.10**	**38,624**
	31.25	2	28	140	0.00032	111.16	1.09	38,611
	37.5	2	28	141	0.00032	111.17	1.08	38,596

The bold data indicates the optimal solutions

Table 5 Sensitivity analysis of Case-III when h_v changes

Parameter	Parameter value	n^*	L^*	Q^*	θ^*	p^*	k^*	EJP^*
h_v	2	2	28	155	0.00029	111.07	1.05	38,770
	3	2	28	146	0.00031	111.11	1.07	38,694
	4	**2**	**28**	**138**	**0.00033**	**111.16**	**1.10**	**38,624**
	5	2	28	131	0.00034	111.20	1.12	38,557
	6	2	28	126	0.00036	111.24	1.14	38,493

The bold data indicates the optimal solutions

Table 6 Sensitivity analysis of Case-III when h_b changes

Parameter	Parameter value	n^*	L^*	Q^*	θ^*	p^*	k^*	EJP^*
h_b	2.5	2	28	155	0.00029	111.02	1.34	38,920
	3.75	2	28	146	0.00031	111.09	1.20	38,766
	5	**2**	**28**	**138**	**0.00033**	**111.16**	**1.10**	**38,624**
	6.25	2	28	132	0.00034	111.22	1.02	38,489
	7.5	2	28	126	0.00036	111.28	0.95	38,359

The bold data indicates the optimal solutions

Table 7 Sensitivity analysis of Case-III when θ_0 changes

Parameter	Parameter value	n^*	L^*	Q^*	θ^*	p^*	k^*	EJP^*
θ_0	0.000175	2	28	138	0.00033	111.16	1.10	38,763
	0.0002625	2	28	138	0.00033	111.16	1.10	38,682
	0.00035	**2**	**28**	**138**	**0.00033**	**111.16**	**1.10**	**38,624**
	0.0004375	2	28	138	0.00033	111.16	1.10	38,580
	0.000525	2	28	138	0.00033	111.16	1.10	38,543

The bold data indicates the optimal solutions

- The inter-dependent reduction strategy of lead time and ordering cost help in the simultaneous reduction of lead time and ordering cost. As a result, the expected total cost is reduced. Hence, the expected total profit is increased.
- The backorder rate is increased with the reduction of lead time. Therefore, the lost sales are decreased. Consequently, the expected profit is increased.
- If the initial ordering cost of the buyer (A_0) increases (see Table 4), the order lot size (Q) and selling price of the buyer (p) increase but the probability of the production process that may go out-of-control state during the production run (θ), safety factor (k), and the joint expected total profit (EJP) are decreased fairly. The values of the number of deliveries (n) and lead time (L) remain unchanged.
- When the holding cost of the vendor (h_v) is increased, the values of θ, p, and k are increased but the values of Q and EJP are decreased significantly. The values of n and L remain unaltered (see Table 5).
- An increase in the holding cost of the buyer (h_b) causes a decrease in the values of Q, k, and EJP but an increase in the values of θ and p while n and L remain unchanged (see Table 6).
- The joint expected total profit (EJP) decreases with an increase in θ_0 but the values of all the decision variables remain unaltered (see Table 7).

7 Conclusion

This study has presented a two-stage vender-buyer supply chain model with price-sensitive stochastic demand, controllable lead time, quality improvement, and variable ordering cost and backorder rate. Lead-time dependent ordering cost and backorder rate are the major two assumptions of this model. Three cases are discussed based on the solution approach. The results recorded in Table 3 show that Case-III i.e. the Joint decision approach provides the maximum profit than Case-I (Vendor as leader) and Case-II (Buyer as leader). So, we can conclude that the supply chain members can achieve more profit if they adopt the Joint decision approach. This study has introduced different strategies to increase the expected total profit. All these strategies may help the manager of the industry to control the quality of products and increase the financial gain.

The limitations of this study are the consideration of a single item and a single-vendor-single-buyer supply chain system. In future, this model can be extended by considering multi-items and also single-vendor-multi-buyers, multi-vendors-single-buyer, and multi-vendors-multi-buyers systems with the consideration of transportation cost. One another extension can be attempted by considering a discrete investment for quality improvement instead of continuous investment.

Compliance with ethical standards

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