OPSEARCH (2021) 58:276–329
https://doi.org/10.1007/s12597-020-00477-2

APPLICATION ARTICLE

Preservation effort effects on retailers and manufacturers in integrated multi-deteriorating item discrete supply chain model

Monalisha Pattnaik[1] · **Padmabati Gahan**[2]

Accepted: 26 August 2020 / Published online: 15 September 2020

Abstract

This paper studies the multi-deteriorating item discrete supply chain to realize considerable savings by aggregating the replenishment. The present integrated replenishment policy has already been widely applied in a variety of industries. This study deals with an integrated multi-deteriorating item replenishment problem with preservation effort and discrete demand rate and discrete order quantity. Most existing studies about preservation effort focused on a single-item replenishment policy. However, integrated replenishment has been extensively applied in many industries to take advantage of economies of scale in preservation. Since it is difficult to solve this problem directly, the necessary and sufficient conditions with these properties are derived; a solution procedure and an algorithm using heuristic approach are developed to obtain the optimal solutions. Numerical examples, comparative analysis and sensitivity analyses are also provided and tested to elucidate the multi-deteriorating item discrete supply chain model with preservation efforts. The results reveal that the extensions of the model provide a wider and reasonable situation in practice, so that the annual channel profit can be maximized.

Keywords Multi-deteriorating item · Preservation effort · Pricing · Linear deterioration rate · Integrated supply chain model · Discrete demand rate

Padmabati Gahan: Mentor.

✉ Monalisha Pattnaik
monalisha_1977@yahoo.com

Padmabati Gahan
pgahan7@gmail.com

1 Department of Statistics, School of Mathematical Sciences, Sambalpur University, Jyoti Vihar, Burla 768019, India

2 Department of Business Administration, Sambalpur University, Jyoti Vihar, Burla 768019, India

1 Introduction

The company will be probably proud to create a truly efficient leanness company of operations, revamped the processes, reducing overhead and cutting out redundant activities. It should enhance the equality of the products and services, ridding the organization of mistakes and miscommunication and should break down the walls between the units, getting people to work together and share information. Maximum high-tech companies have taken more aggressive approach to restructuring work in cross-company processes. It means reshaping the economics of supply chain for their product using a host of unrelated information systems. For achieving this, it is required to hold cumbersome set of process together, at a great cost.

In multi-echelon supply chain, the company setup is to share information among all the supply chain participants by which the performance of the supply chain can be dramatically enhanced. The suppliers get benefits from this new relationship as well so, the simplicity and security of dealing with one large customer may be attracted rather than a host of small ones. Nowadays, all the companies persistently aimed at greater speed and cost effectiveness the popular grails of supply chain management. Companies' quests changed with the industrial cycle. When business was booming, executives concentrated on maximizing speed, and when economy headed south, firms desperately tried to maximize the profit. So, supply chain efficiency is necessary to optimize the supply chain's performance when they maximize their interests. Only agile, adaptable, and aligned supply chains provide companies with sustainable competitive advantage. Today, the industrial environment has become more competitive in the rapidly developing global market. A "Win–Win" supply chain management system should be significant no matter whether the position is on the supply side or buy side. Therefore, integrated supply chain model is introduced to deal with inventory problems in supportive activities between suppliers and retailers.

2 Review of literature

In the real world, procurement and inventory control are truly large scale problems, often involving more than hundreds of items. In a multi-item distribution channel, considerable savings can be realized during the replenishment by coordinating the ordering of several different items. Multi-echelon multi-item replenishment strategies are already widely applied in the real world, for example, the supplying of parts for computers and for automotive assembly Hahm and Yano [17, 18] or refrigerated goods to supermarkets Hammer [19] and Lu [31]. In these industries, a supplier normally produces different products for a single customer and ships to the customer simultaneously in a single truck. In the grocery supply industry or a fast moving consumer goods industry different types of refrigerated goods (General Mills yogurt, Derived Milk products etc.) can be shipped in the same truck to the same supermarket or retail store Hammer [19], Goyal [15], Kao [23], Graves [16], Ben-Khedher and Yano [3], Van Eijis [42], Rempala [36], Chen and Chen [6, 7], Tsao and Sheen [40], Bhattacharya [4], Chen and Chen [8, 9], Elmaghraby and Keskinocak [12], Viswanathan [43–45], Lee and Whang [28], Lee [29], Marn and Rosiello [32], Frohlich and Westbrook [13], Fung and Ma [14], Joneja [22], Kaspi and Rosenblatt [24] and Lee and Yao [26] have developed models and algorithms for solving multi-item replenishment problems for different constraints. Miranda and Garrido [33], Shinn [37], Sucky [38], Taylor [39] developed integrated models and Hsu et al. [21] developed the single item model with preservation technology for deteriorating items. Multi-echelon coordination is frequently applied in current business practice it is an essential component in supply chain model. Hence the multi-echelon multi-deteriorated item supply chain is the focus of the present study.

The coordination among channel members is important for enhancing a channel's competitiveness. It has been investigated that a company can increase its market share by aligning itself with its channel partners Narayanan and Raman [34]. However, companies in the same channel often do not act in ways that maximize the channel profit, so the whole supply chain performs inadequately. A company often thinks that whatever policy which maximizes its own profit and it will also maximize the channel profit Lee [27]. Recently Khouja [25] framed an integrated three-stage supply chain with multiple vendors and buyers. Hsu and Wee [20] discussed horizontal suppliers' coordination issue under uncertain deliveries. Chen and Chen [7] discussed the effects of joint replenishment and channel coordination for managing multiple deteriorating items. Li and Wang [30] briefed a complete literature review of supply chain system coordination. Prasad et al. [35] proposed and validated a model on decentralized production–distribution planning in multi-echelon supply chain network using intelligent agents. Banu and Mondal [2] considered an integrated inventory model that deals with one manufacturer and one retailer. The manufacturer provides warranty period for the finished product to ensure product reliability. The objective of this study is to analyze the effect of the credit period offered by the manufacturer which is functionally depended on warranty period of the product. The objective is to maximize the integrated model by optimizing product warranty, customers' credit period and cycle length.

The present study will be devoted to the issue of channel coordination of multi-items as a coordination mechanism. The goal of this paper is to develop a procedure to find the retail price, annual replenishment frequency and preservation effort factor that maximizes the annual profit consisting of setup costs, inventory holding cost, and preservation effort cost for both the perspective of the individual like retailer, manufacturer and the perspective of the channel.

The remainder of this paper as follows. In Sect. 2, the assumptions and notations used in this study are described. In Sect. 3 the mathematical models both for decentralized policy and centralized policy are developed, under decentralized policy two different policies are developed like non cooperative replenishment with individual items and with joint items respectively and under centralized policy two another policies are developed like cooperative replenishment with individual items and with joint items respectively. A discussion of the mathematical properties of the models and a solution procedure with algorithm are given in Sect. 4. Finally, a numerical analysis is presented with some comparative analysis and sensitivity analyses with testing have been performed in Sect. 5. Conclusions are drawn in Sect. 6. The major assumptions used in the above reviewed research articles are summarized in Table 1. Figure 1 depicts the environment of the multi-item integrated discrete supply chain with preservation efforts.

3 Assumptions and notations

Table 2 describes the summary of notations in detail.

The mathematical model is developed under the following assumptions:

This paper deals with a multi-echelon supply chain, consisting of one supplier, one manufacturer and one buyer (retailer), who stocks and sells multiple (k) items for the end customers. These multi-deteriorated items have short life-times and are subject to linear decaying with preservation efforts per unit time. The inventory level of the raw materials and finished products are always greater than the demand rate. The demand rate for each item is assumed to be linear price-dependent function over the selling period. The supplier adopts a lot-for-lot strategy. It means the quantity ordered by the supplier equals the demand quantity placed by the retailer.

1. The demand for item i is represented by a linear decreasing function of the retail price.
2. The deterioration rates of the raw materials and finished items are linear decreasing function of preservation efforts.
3. The supplier provides a credit period t to the retailer; a capital opportunity cost due to the period incurs. On the other hand, the retailer gains a capital opportunity benefit due to this period.
4. The preservation effort cost under policy j, $PrE_j(\tau_i, D_i) = \sum_{i=1}^{k} b(\tau_i)^2 D_i(p_{j,i})$ where, $\tau_i > 0$, b is a constant and τ_i is a preservation effort factor.

4 Mathematical model

In this section, the model is formulated from the perspectives of the retailer and supplier and channel.

4.1 The decentralized policy

In the decentralized production and replenishment decision-making policy, each individual within the supply chain aims at optimizing its own profit function with preservation effort effects, and without consideration being given to its counterpart's reaction or resulting profit. The retailer makes a replenishment decision based on an EOQ policy that includes inventory-holding cost, major and minor setup costs

Table 1 Comparisons with other recent researches

Research parameters Authors	Demand	Forward Financing	Model	Item	Echelon	Benefit	Detern	Preservation Effort Effects	Replenishment
Cohen [11]	Price dependent	No	NLP	Single	One	No		No	Finite
Ben-Khender and Yano [3]	Constant	No	NLP	Multi Item	Two	Container Effect		No	Finite
Chang et al. [5]	Linear Trend	Yes	NLP	Multi Item	One	No		No	Infinite
Abad and Jaggi [1]	Price Dependent Exponential Demand	Yes	NLP	Single	Two	No		No	Finite
Chen and Chen [6]	Constant	No	NLP	Multi	Multi	Quantity		No	Finite
Chen and Chen [7]	Constant	No	NLP	Multi	Two	Centralization		No	Finite
Tsao and Sheen [41]	Time and Price Dependent	Yes	NLP	Multi Item	One	No		Yes	Finite
Chung [10]	Constant	Yes	NLP	Single	One	No		No	Finite
Tsao and Sheen [40]	Price Dependent Linear Trend	Yes	NLP	Multi Item	Multi	Freight		No	Finite
Chen and Chen [8]	Price Dependent	No	NLP	Multi	Multi	Demand and Revenue Increments		No	Finite
Present Paper	Retail Price Dependent Linear Decreasing	No	MINLP(Discrete)	Multi Item	Multi	Integration and Preservation Effort	Preservation Effort Dependent Linear Decreasing both for Manufacturer and Retailer (Functional)	Yes	Finite

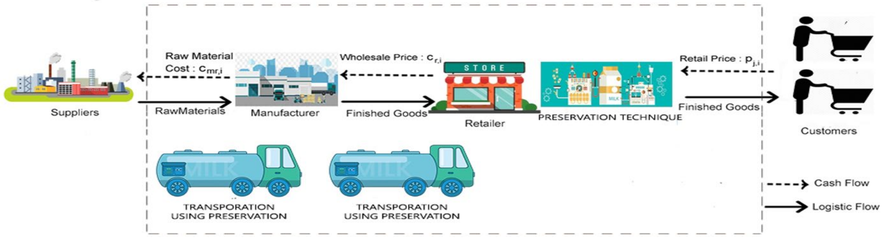

Fig. 1 Environment of the multi-deteriorated item and multi-echelon discrete supply chain with preservation efforts

and preservation effort cost. The major setup cost may represent a fixed transportation cost, regardless of its composition, and the minor setup cost may represent item-specific warehousing, material handling, and order processing costs, for each specific deteriorated item included in the order. The individual replenishment policy (policy I), and then the joint replenishment policy (policy II) are presented for the multi-item problem.

4.1.1 Policy-I: Non-cooperative replenishment for individual items

Under the individual replenishment of deteriorated multi-item, the decision problem facing the retailer with preservation effort effects is to determine the retail price, replenishment cycle and preservation effort for each individual deteriorated item. The change in inventory level for each item is due to the combined effects and preservation effort effects of deterioration over the replenishment cycle, and the model can be framed by the following differential equation.

$$\begin{aligned} \frac{dI_{r,i}(t)}{dt} &= -I_{r,i}(t)\theta_i - D_i, 0 \le t \le T_{I,i} \\ \frac{dI_{r,i}(t)}{dt} &+ I_{r,i}(t)\theta_i = -D_i, 0 \le t \le T_{I,i} \end{aligned} \tag{1}$$

After solving Eq. (1) and using the boundary condition the value of $I_{r,i}(t)$ is:

$$I_{r,i}(t) = \frac{-D_i}{\theta_i} + \frac{D_i e^{T_{I,i}\theta_i}}{\theta_i e^{t\theta_i}} \Rightarrow I_{r,i}(t) = \frac{D_i}{\theta_i}\left[e^{\theta_i(T_{I,i}-t)} - 1\right], \quad for\, 0 \le t \le T_{I,i}$$

Purchasing Cost (PC):

$$PC = \sum_{i=1}^{k} c_{r,i} \times I_{r,i}(0)$$

where, $c_{r,i}$ is the per unit purchasing cost of item i of the retailer and $I_{r,i}(0)$ *is the initial inventory level of item i* or the inventory level at the beginning of the replenishment cycle and it is also equivalent to the ordering quantity.

Table 2 Summary of notations

System	
k	The number of finished items considered
i	The index of finished items, $i = 1, 2, \ldots \ldots .k$
j	The index of the decision policies, $j \in [I, II, III, IV]$
π_j	Total profit per unit time under policy j
Manufacturer	
	Raw Material
$a_{mr,i}$	The ordering cost of raw material for finished item i per lot
$h_{mr,i}$	The inventory holding cost of raw material for finished item i
$c_{mr,i}$	The purchase cost of raw material for finished item i per unit, i.e. the selling price charged by the outlet vendor
$I_{mr,i}(t)$	Inventory level of raw material for finished item i at time t
	Finished Item
A_m	The major setup cost per lot
$a_{m,i}$	The minor setup cost for adding finished item i into the production schedule
$h_{m,i}$	The inventory holding cost of finished item i
μ_i	The production rate of finished item i
u_i	The usage rate of raw material for finished item i
$\theta_{m,i}$	The deterioration rate of raw material for finished item i, $\theta_{m,i} = \alpha_{2i} - \beta_{2i}\tau_i$ *where*, $\alpha_{2i} > 0, \beta_{2i} > 0$ and $\tau_i > 0$, b is a constant and τ_i is a preservation effort factor
$t_{m,i}$	The manufacturer's starting production time for item i over each retailer's replenishment cycle
$I_{m,i}(t)$	Inventory level of finished item i of manufacturer at time t
$Q_{f,i}$	The manufacturer's order quantity for item i
$\pi_{j,m}$	Total profit per unit time under policy j
Retailer	
A_r	The major setup cost per order
$a_{r,i}$	The minor setup cost for adding finished item i into the order
$h_{r,i}$	The inventory holding cost of finished item i
$c_{r,i}$	The purchase cost of finished item i per unit, i.e. the selling price charged by the manufacturer
$D_i(p_{j,i})$	The demand rate of finished item i under policy j in the marketplace, which is a function of retail price $p_{j,i}$, $D_i(p_{j,i}) = D_i = \alpha_i - \beta_i p_{j,i}$, *where* $\alpha_i > 0, \beta_i > 0, p_{j.i} \le \alpha_i/\beta_i$
θ_i	The deterioration rate of finished item i facing both the retailer and the manufacturer, $\theta_i = \alpha_{1i} - \beta_{1i}\tau_i$ *where*, $\alpha_{1i} > 0, \beta_{1i} > 0$ and $\tau_i > 0$, b is a constant and τ_i is a preservation effort factor
$T_{j,i}$	The individual replenishment cycle of the finished item i under policy j, $j \in [I, III]$, which is a decision variable
T_j	The common replenishment cycle of all the finished items under policy j, $j \in [II, IV]$, which is a decision variable
$I_{r,i}(t)$	Inventory level of finished item i of retailer at time t
$p_{j,i}$	The retail price of finished item i under policy $j, j \in [I, II, III, IV]$, which is a decision variable
$PrE_j(\tau_i, D_i)$	The preservation effort cost under policy j, $PrE_j(\tau_i, D_i) = \sum_{i=1}^{k} b(\tau_i)^2 D_i(p_{j,i})$, where $\tau_i > 0$, b is a constant and τ_i is a preservation effort factor, Tsao and Sheen [41]

Table 2 (continued)

$Q_{r,i}$	The retailer's order quantity for item i
$\pi_{j,r}$	Total profit per unit time under policy j

We have, $I_{r,i}(t) = \frac{D_i}{\theta_i}\left[e^{\theta_i(T_{I,i}-t)} - 1\right], for\, 0 \le t \le T_{I,i}$, putting $t = 0\, in\, I_{r,i}(t)$ it can be obtained:

$$I_{r,i}(0) = \frac{D_i}{\theta_i}\left[e^{\theta_i(T_{I,i}} - 1\right].$$

So the total purchasing cost of all items is:

$$PC = \sum_{i=1}^{k} c_{r,i} \times I_{r,i}(0) \Rightarrow PC = \sum_{i=1}^{k} \frac{c_{r,i}D_i}{\theta_i}\left[e^{\theta_i T_{I,i}} - 1\right]$$

Holding Cost (HC):

$$HC = \sum_{i=1}^{k} h_{r,i} \times \int_0^{T_{I,i}} I_{r,i}(t)dt$$

where, $h_{r,i}$ is the per unit inventory holding cost of item i and $I_{r,i}(t)$ is the inventory level of item i at timet.

$$HC = \sum_{i=1}^{k} h_{r,i} \times \int_0^{T_{I,i}} I_{r,i}(t)dt = \sum_{i=1}^{k} h_{r,i} \times \int_0^{T_{I,i}} \frac{D_i}{\theta_i}\left[e^{\theta_i(T_{I,i}-t)} - 1\right]dt.$$

So, the total inventory holding cost of all items is:

$$HC = \sum_{i=1}^{k} \frac{h_{r,i}D_i}{\theta_i T_{I,i}}\left[\frac{e^{\theta_i T_{I,i}} - 1}{\theta_i} - T_{I,i}\right]$$

Major Setup Cost (MaSC):

$$MaSC = \sum_{i=1}^{k} \frac{A_r}{T_{I,i}}$$

Minor Setup Cost (MSC):

$$MSC = \sum_{i=1}^{k} \frac{a_{r,i}}{T_{I,i}}$$

Preservation Effort Cost (PrEC):

$$PrEC = \sum_{i=1}^{k} \frac{b(\tau_i)^2 D_i(p_{j,i})}{T_{I,i}}$$

Sales Revenue (SR):

$$SR = \sum_{i=1}^{k} p_{I,i} D_i$$

The exponential term $e^{\theta_i T_{I,i}}$ can be approximated by using the Taylor series expansion, $e^{\theta_i T_{I,i}} = 1 + \theta_i T_{I,i} + \frac{(\theta_i T_{I,i})^2}{2!} + \frac{(\theta_i T_{I,i})^3}{3!} + \cdots$, for a reasonable deterioration rate of a perishable product like diary items, medicinal product and agricultural items whose lifetime is commonly less than 1 month then the third or higher order terms of the Taylor series expansion can be neglected. So the total purchasing cost of all items, PC and the total inventory holding cost, HC is given by:

$$PC = \sum_{i=1}^{k} c_{r,i} D_i + \frac{c_{r,i} D_i \theta_i T_{I,i}}{2} \; and \; HC = \sum_{i=1}^{k} \frac{h_{r,i} D_i T_{I,i}}{2}$$

The retailer's profit function per unit time; $\pi_{I,r}$ is given by:

$$\begin{aligned} \pi_{I,r} &= Sales\,Revenue - Purchasing\,Cost - Inventory\,Holding\,Cost \\ &\quad - Major\,Setup\,Cost - Minor\,Setup\,Cost - Preservation\,Effort\,Cost \\ &\Rightarrow \pi_{I,r} = SR - PC - HC - MaSC - MSC - PrEC \end{aligned}$$

$$\pi_{I,r} = \sum_{i=1}^{k} \left[p_{I,i} D_i - \left\{ c_{r,i} D_i + \frac{c_{r,i}\theta_i D_i T_{I,i}}{2} \right\} - \frac{h_{r,i} D_i T_{I,i}}{2} - \frac{A_r}{T_{I,i}} - \frac{a_{r,i}}{T_{I,i}} - \frac{b(\tau_i)^2 D_i(p_{j,i})}{T_{I,i}} \right]$$

$$\Rightarrow \pi_{I,r} = \sum_{i=1}^{k} \left[(p_{I,i} - c_{r,i}) D_i - \frac{(A_r + a_{r,i})}{T_{I,i}} - \frac{(h_{r,i} + c_{r,i}\theta_i) D_i T_{I,i}}{2} - \frac{b(\tau_i)^2 D_i(p_{j,i})}{T_{I,i}} \right] \tag{2}$$

4.1.1.1 Optimization The retailer determines the retail price for each item, individual replenishment cycle of each finished item and preservation effort factor of each item, so as to maximize his profit. We utilize the following propositions to discuss the condition for an optimal solution from the individual non-cooperative policy.

Proposition 1 *The retailer's profit function per unit time; $\pi_{I,r}$ is concave in $p_{I,i}$.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the demand function. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{I,r}}{\partial p_{I,i}} = D_i - \left(p_{I,i} - c_{r,i}\right)\beta_i + \frac{T_{I,i}\beta_i}{2}\left(h_{r,i} + c_{r,i}\theta_i\right) + \frac{b\left(\tau_i\right)^2}{T_{I,i}}$$

$$\frac{\partial \pi_{I,r}}{\partial p_{I,i}} = 0 \Rightarrow \frac{T_{I,i}\beta_i}{2}\left(h_{r,i} + c_{r,i}\theta_i\right) + \frac{b\left(\tau_i\right)^2}{T_{I,i}} + D_i = \left(p_{I,i} - c_{r,i}\right)\beta_i,$$

$$p^*_{I,i} = \frac{1}{2}\left[\alpha\beta + c_{r,i} + \frac{T_{I,i}}{2}\left(h_{r,i} + c_{r,i}\theta_i\right) + \frac{b\left(\tau_i\right)^2}{T_{I,i}}\right].$$

Hence, the second order derivative of $\pi_{I,r}$ should be negative for concavity property.

$$\frac{\partial^2 \pi_{I,r}}{\partial p^2_{I,i}} = -\left[2\beta_i\right] < 0.$$

It implies that the retailer's total profit per unit time is concave □

Proposition 2 *The retailer's profit function per unit time; $\pi_{I,r}$ is concave in $T_{I,i}$.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and time. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{I,r}}{\partial T_{I,i}} = \frac{A_r + a_{r,i} + b\left(\tau_i\right)^2}{T^2_{I,i}} - \frac{D_i}{2}\left(h_{r,i} + c_{r,i}\theta_i\right) \text{ and } \frac{\partial \pi_{I,r}}{\partial T_{I,i}} = 0 \Rightarrow T^*_{I,i} = \sqrt{\frac{2\left(A_r + a_{r,i} + b\left(\tau_i\right)^2 D_i\right)}{D_i(h_{r,i} + c_{r,i}\theta_i)}}$$

The optimal value of $T_{I,i}(T^*_{I,i})$ is obtained and the second order derivative of $\pi_{I,r}$ should be negative for concavity property.

$$\frac{\partial^2 \pi_{I,r}}{\partial T^2_{I,i}} = -\left[\frac{2\left(A_r + a_{r,i} + b\left(\tau_i\right)^2 D_i\right)}{T^3_{I,i}}\right] < 0.$$

Hence, the retailer's total profit per unit time is concave. □

Proposition 3 *The retailer's profit function per unit time; $\pi_{I,r}$ is concave in τ_i.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the preservation effort. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{I,r}}{\partial \tau_i} = \frac{\beta_{1i} D_i T_{I,i} c_{r,i}}{2} - \frac{2b\tau_i D_i}{T_{I,i}} \; and \; \frac{\partial \pi_{I,r}}{\partial \tau_i} = 0 \Rightarrow \tau_i^* = \frac{\beta_{1i} c_{r,i} T_{I,i}^2}{4b}$$

The optimal value of $\tau_i(\tau_i^*)$ is obtained and the second order derivative of $\pi_{I,r}$ should be negative for concavity property.

$$\frac{\partial^2 \pi_{I,r}}{\partial \tau_i^2} = -\left[\frac{2bD_i}{T_{I,i}}\right] < 0.$$

Hence, the retailer's total profit per unit time is concave.

The upstream manufacturer has adopted a make –to-order policy, i.e. a lot-for-lot production policy, the production is the same quantity as demanded by the downstream player of supply chain i.e. retailer. Here for each production run, a major setup cost is incurred due to such factors as changeover costs, and a minor setup cost was incurred for each additional item being produced in the line. For the manufacturer the preservation effort cost was not required for market demand but he incurred additional inventory holding and ordering costs for the raw materials required to produce the finished goods.

$$\begin{aligned} &\frac{dI_{mr,i}(t)}{dt} = -I_{mr,i}(t)\theta_{m,i} - \mu_i u_i, t_{m,i} \le t \le T_{I,i} \\ &\frac{dI_{mr,i}(t)}{dt} + I_{mr,i}(t)\theta_{m,i} = -\mu_i u_i, t_{m,i} \le t \le T_{I,i} \\ &I_{mr,i}(t) = \frac{-\mu_i u_i}{\theta_{m,i}} + \frac{\mu_i u_i e^{T_{I,i}\theta_{m,i}}}{\theta_{m,i} e^{t\theta_{m,i}}} \\ &\quad \Rightarrow I_{mr,i}(t) = \frac{\mu_i u_i}{\theta_{m,i}}\left[e^{\theta_{m,i}(T_{I,i}-t)} - 1\right], for\, t_{m,i} \le t \le T_{I,i} \end{aligned} \tag{3}$$

Finished product:

$$\begin{aligned} &\frac{dI_{m,i}(t)}{dt} = \mu_i - I_{m,i}(t)\theta_i, t_{m,i} \le t \le T_{I,i} \\ &\frac{dI_{m,i}(t)}{dt} + I_{m,i}(t)\theta_i = \mu_i, t_{m,i} \le t \le T_{I,i} \\ &I_{m,i}(t) = \frac{\mu_i}{\theta_i} - \frac{\mu_i e^{t_{m,i}\theta_i}}{\theta_i e^{t\theta_i}} \Rightarrow I_{m,i}(t) = \frac{\mu_i}{\theta_i}\left[1 - e^{\theta_i(t_{m,i}-t)}\right], for\, t_{m,i} \le t \le T_{I,i} \end{aligned} \tag{4}$$

The profit function per unit time of the manufacturer can be obtained by a procedure similar to the one developed for the retailer, can be expressed as follows:

$$\pi_{I,m} = Sales\,Revenue - Purchasing\,Cost - Inventory\,Holding\,Cost$$
$$- Major\,Setup\,Cost - Minor\,Setup\,Cost$$
$$\Rightarrow \pi_{I,m} = SR - PC - HC - MaSC - MSC$$
$$\pi_{I,m} = \sum_{i=1}^{k}\left[c_{r,i}D_i - \left\{c_{mr,i}\mu_i u_i + \frac{c_{mr,i}\theta_{m,i}\mu_i u_i T_{I,i}}{2}\right\} - \frac{(h_{m,i} + h_{mr,i}u_i)D_i T_{I,i}}{2} - \frac{(A_m + a_{m,i} + a_{mr,i})}{T_{I,i}}\right]$$
$$\Rightarrow \pi_{I,m} = \sum_{i=1}^{k}\left[c_{r,i}D_i - c_{mr,i}\mu_i u_i - \left\{\frac{c_{mr,i}\theta_{m,i}\mu_i u_i + (h_{m,i} + h_{mr,i}u_i)D_i}{2}\right\}T_{I,i} - \frac{(A_m + a_{m,i} + a_{mr,i})}{T_{I,i}}\right] \tag{5}$$

Summing up the Eqs. (2) and (5) yield the profit model for the supply chain under policy I:

$$\pi_I = \pi_{I,r} + \pi_{I,m} \tag{6}$$

□

4.1.2 Policy-II: Non-cooperative replenishment for joint items

Under the joint replenishment policy, the retailer determines a common replenishment cycle T^*_{II} for the simultaneous replenishment of all multi-items, the retail price $p^*_{II,i}$ for each item and the preservation effort factor τ^*_i for each item with an aim at maximizing its profit. In this policy, the revenue and the associated costs considered by the retailer are similar to the individual replenishment, except for the number of major replenishment setups that are reduced to one over the cycle. The total profit per unit time of the retailer $\pi_{II,r}$ is

$$\pi_{II,r} = \left[Sales\,Revenue - Purchasing\,Cost - Inventory\,Holding\,Cost\right.$$
$$\left.- Minor\,Setup\,Cost - Preservation\,Effort\,Cost\right]$$
$$- Major\,Setup\,Cost \Rightarrow \pi_{II,r} = [SR - PC - HC - MSC - PrEC] - MaSC$$
$$\pi_{II,r} = \sum_{i=1}^{k}\left[p_{II,i}D_i - \left\{c_{r,i}D_i + \frac{c_{r,i}\theta_i D_i T_{II}}{2}\right\} - \frac{h_{r,i}D_i T_{II}}{2} - \frac{a_{r,i}}{T_{II}} - \frac{b(\tau_i)^2 D_i}{T_{II}}\right] - \frac{A_r}{T_{II}}$$
$$\Rightarrow \pi_{II,r} = \sum_{i=1}^{k}\left[(p_{II,i} - c_{r,i})D_i - \frac{a_{r,i}}{T_{II}} - \frac{(h_{r,i} + c_{r,i}\theta_i)D_i T_{II}}{2} - \frac{b(\tau_i)^2 D_i}{T_{II}}\right] - \frac{A_r}{T_{II}} \tag{7}$$

4.1.2.1 Optimization The retailer determines the retail price for each item, common replenishment cycle of each finished item and publicity effort factor of each item, so as to maximize his profit. We utilize the following propositions to discuss the condition for an optimal solution from the joint non-cooperative policy.

Proposition 1 *The retailer's profit function per unit time;* $\pi_{II,r}$ *is concave in* $p_{II,i}$.

Proof The optimal property of the profit function strongly depends on the value of the parameters and the demand function. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{II,r}}{\partial p_{II,i}} = \left[D_i - \left(p_{II,i} - c_{r,i}\right)\beta_i + \frac{T_{II}\beta_i}{2}\left(h_{r,i} + c_{r,i}\theta_i\right) + \frac{b\left(\tau_i\right)^2 \beta_i}{T_{II}} \right]$$

$$\frac{\partial \pi_{II,r}}{\partial p_{II,i}} = 0 \Rightarrow p^*_{II,i} = \frac{1}{2}\left[\alpha\beta + c_{r,i} + \frac{T_{II}}{2}\left(h_{r,i} + c_{r,i}\theta_i\right) + \frac{b\left(\tau_i\right)^2}{T_{II}} \right].$$

The second order derivative of $\pi_{II,r}$ should be negative for concavity property.

$$\frac{\partial^2 \pi_{II,r}}{\partial p^2_{II,i}} = -2\beta < 0.$$

The retailer's total profit per unit time is concave. □

Proposition 2 *The retailer's profit function per unit time; $\pi_{II,r}$ is concave in T_{II}.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the time. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{II,r}}{\partial T_{II}} = \sum_{i=1}^{k} \left[\frac{a_{r,i} + bD_i\left(\tau_i\right)^2}{T^2_{II}} - \frac{D_i}{2}\left(h_{r,i} + c_{r,i}\theta_i\right) \right] + \frac{A_r}{T^2_{II}} \text{ and}$$

$$\frac{\partial \pi_{II,r}}{\partial T_{II}} = 0 \Rightarrow T^*_{II} = \sqrt{\frac{\left[2\sum_{i=1}^{k}\left(a_{r,i} + bD_i\left(\tau_i\right)^2\right)\right] + A_r}{\sum_{i=1}^{k} D_i(h_{r,i} + c_{r,i}\theta_i)}}.$$

The optimal value of $T_{II}(T^*_{II})$ is obtained and the second order derivative of $\pi_{II,r}$ should be negative for concavity property.

$$\frac{\partial^2 \pi_{II,r}}{\partial T^2_{II}} = -\left[\sum_{i=1}^{k} \left[\frac{2\left(a_{r,i} + bD_i\left(\tau_i\right)^2\right)}{T^3_{II}} \right] + \frac{2A_r}{T^3_{II}} \right] < 0.$$

Hence, the retailer's total profit per unit time is concave. □

Proposition 3 *The retailer's profit function per unit time; $\pi_{II,r}$ is concave in τ_i.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the preservation effort. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{II,r}}{\partial \tau_i} = \left[\frac{\beta_{1,i} c_{r,i} D_i T_{II}}{2} - \frac{2bD_i \tau_i}{T_{II}}\right] \text{ and } \frac{\partial \pi_{II,r}}{\partial \tau_i} = 0 \Rightarrow \tau_i^* = \frac{\beta_{1,i} c_{r,i} T_{II}^2}{4b}$$

The optimal value of $\tau_i(\tau_i^*)$ is obtained and the second order derivative of $\pi_{II,r}$ should be negative for concavity property.

$$\frac{\partial^2 \pi_{II,r}}{\partial \tau_i^2} = -\frac{2bD_i}{T_{II}} < 0.$$

Hence, the retailer's total profit per unit time is concave.

Accordingly, the total profits per unit time for the manufacturer and for the system are

$$\begin{aligned} \pi_{II,m} &= \left[\mathit{Sales\,Revenue} - \mathit{Purchasing\,Cost} - \mathit{Inventory\,Holding\,Cost} \right.\\ &\quad \left. - \mathit{Minor\,Setup\,Cost}\right] - \mathit{Major\,Setup\,Cost} \Rightarrow \pi_{II,m} \\ &= [SR - PC - HC - MSC] - MaSC \end{aligned}$$

$$\pi_{II,m} = \sum_{i=1}^{k} \left[c_{r,i} D_i \rho_i - \frac{\left(a_{m,i} + a_{mr,i}\right)}{T_{II}} - c_{mr,i} u_i \mu_i - \left\{c_{mr,i} \theta_{m,i} u_i \mu_i + \left(h_{m,i} + h_{mr,i} u_i\right) D_i \rho_i\right\} \frac{T_{II}}{2}\right] - \frac{A_m}{T_{II}} \tag{8}$$

and Summing up the Eqs. (7) and (8) yield the profit model for the supply chain under policy I:

$$\pi_{II} = \pi_{II,r} + \pi_{II,m} \tag{9}$$

□

4.2 The centralized policy

In contrast to the decentralized decision process, the centralized policy simultaneously determines the retail price, the replenishment cycle and preservation effort by considering the total profit incurred by the retailer and the manufacturer, so that the system is maximized. The individual replenishment policy (policy III) and then the joint replenishment policy (policy IV) are framed for the problem.

4.2.1 Policy-III: Cooperative replenishment for individual items

In this policy, the retail price, replenishment cycle and preservation effort for each item are determined jointly by the individuals in supply chain. The system profit of policy III is:

$$\pi_{III} = \pi_{I,r} + \pi_{I,m} = \left[\sum_{i=1}^{k}\left[\left(p_{III,i} - c_{r,i}\right)D_i - \frac{(A_r + a_{r,i})}{T_{III,i}} - \frac{\left(h_{r,i} + c_{r,i}\theta_i\right)}{2}D_iT_{III,i} - \frac{b\left(\tau_i\right)^2D_i}{T_{III,i}}\right]\right]$$
$$+\left[\sum_{i=1}^{k}\left[\left(c_{r,i}D_i\right) - \frac{(A_m + a_{m,i} + a_{mr,i})}{T_{III,i}} - c_{mr,i}u_i\mu_i - \left[c_{mr,i}u_i\mu_i\theta_{m,i} + \left(h_{m,i} + h_{mr,i}u_i\right)D_i\right]\frac{T_{III,i}}{2}\right]\right]$$
$$\pi_{III} = \pi_{I,r} + \pi_{I,m} = \sum_{i=1}^{k}\left[\left(p_{III,i}D_i\right) - \frac{(A_r + A_m + a_{r,i} + a_{m,i} + a_{mr,i})}{T_{III,i}} - c_{mr,i}u_i\mu_i\right.$$
$$\left.-\left[\left[c_{mr,i}u_i\mu_i\theta_{m,i} + \left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right)D_i\right]\frac{T_{III,i}}{2} + \frac{b\left(\tau_i\right)^2D_i}{T_{III,i}}\right]\right] \tag{10}$$

Proposition 1 *The system profit function per unit time; π_{III} is concave in $p_{III,i}$.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the demand function. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial\pi_{III}}{\partial p_{III,i}} = D_i - p_{III,i}\beta_i - \frac{T_{III,i}\beta_i}{2}\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right) - \frac{b\tau_i^2\beta_i}{T_{III,i}}$$
$$\frac{\partial\pi_{III}}{\partial p_{III,i}} = 0 \Rightarrow p_{III,i}\beta_i + \frac{T_{III,i}}{2}\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right)\beta_i + \frac{b\tau_i^2\beta_i}{T_{III,i}} = D_i \text{ and}$$
$$p^*_{III,i} = \frac{1}{2}\left[\alpha\beta_i + \frac{T_{III,i}}{2}\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right) - \frac{b\tau_i^2}{T_{III,i}}\right]$$

So, the optimal value of $p_{III,i}(p^*_{III,i})$ is obtained and the second order derivative of π_{III} should be negative for concavity property.

$$\frac{\partial^2\pi_{III}}{\partial p^2_{III,i}} = -\left[2\beta_i\right] < 0.$$

Hence the system profit per unit time is concave in $p_{III,i}$. □

Proposition 2 *The system profit function per unit time; π_{III} is concave in $T_{III,i}$.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the time. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{III}}{\partial T_{III,i}} = \frac{(a_{r,i} + a_{m,i} + a_{mr,i} + A_r + A_m + b\tau_i^2 D_i)}{T_{III,i}^2} - \frac{1}{2}\left[D_i\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right) + c_{mr,i}u_i\mu_i\theta_{m,i}\right]$$

$$\frac{\partial \pi_{III}}{\partial T_{III,i}} = 0 \Rightarrow \frac{(a_{r,i} + a_{m,i} + a_{mr,i} + A_r + A_m + b\tau_i^2 D_i}{T_{III,i}^2} = \frac{1}{2}\left[D_i\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right) + c_{mr,i}u_i\mu_i\theta_{m,i}\right]$$

$$\Rightarrow T_{III,i}^* = \sqrt{\frac{2\left[a_{r,i} + a_{m,i} + a_{mr,i} + A_r + A_m + b\tau_i^2 D_i\right]}{\left[D_i\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right) + c_{mr,i}u_i\mu_i\theta_{m,i}\right]}}$$

The optimal value of $T_{III,i}(T_{III,i}^*)$ is obtained and the second order derivative of π_{III} should be negative for concavity property.

$$\frac{\partial^2 \pi_{III}}{\partial T_{III,i}^2} = -\left[\frac{2\left(A_r + A_m + a_{r,i} + a_{m,i} + a_{mr,i} + b\tau_i^2 D_i\right)}{T_{III,i}^3}\right] < 0.$$

Hence, the system profit per unit time is concave in $T_{III,i}$. □

Proposition 3 *The system profit function per unit time; π_{III} is concave in τ_i.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the preservation effort. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{III}}{\partial \tau_i} = \frac{\beta_{2,i}T_{III,i}c_{mr,i}u_i\mu_i}{2} + \frac{\beta_{1,i}T_{III,i}c_{r,i}D_i}{2} - \frac{2b\tau_i D_i}{T_{III,i}} \text{ and}$$

$$\frac{\partial \pi_{III}}{\partial \tau_i} = 0 \Rightarrow \tau_i^* = \frac{T_{III,i}^2}{4bD_i}\left[\beta_{1,i}c_{r,i}D_i + \beta_{2,i}c_{mr,i}u_i\mu_i\right].$$

The optimal value of $\tau_i(\tau_i^*)$ is obtained and the second order derivative of π_{III} should be negative for concavity property.

$$\frac{\partial^2 \pi_{III}}{\partial \tau_i^2} = -\left[\frac{2bD_i}{T_{III,i}}\right] < 0.$$

Hence, the system profit per unit time; π_{III} is concave in τ_i. □

4.2.2 Policy-IV: Cooperative replenishment for joint items

If both cooperation and joint replenishment are employed, the decision facing the retailer and the manufacturer is to jointly determine a common replenishment cycle, the retail price for each item and the preservation effort for each item. The system profit per unit time is:

$$\begin{aligned}\pi_{IV} &= \pi_{II,r} + \pi_{II,m} \\ &= \left[\sum_{i=1}^{k}\left[\left(p_{IV,i} - c_{r,i}\right)D_i - \frac{a_{r,i}}{T_{IV}} - \frac{\left(h_{r,i} + c_{r,i}\theta_i\right)}{2}D_iT_{IV} - \frac{b\left(\tau_i\right)^2 D_i}{T_{IV}}\right] - \frac{A_r}{T_{IV}}\right] \\ &+ \left[\sum_{i=1}^{k}\left[\left(c_{r,i}D_i\right) - \frac{(a_{m,i} + a_{mr,i})}{T_{IV}} - c_{mr,i}u_i\mu_i\right.\right. \\ &- \left.\left.\left[c_{mr,i}u_i\mu_i\theta_{m,i} + \left(h_{m,i} + h_{mr,i}u_i\right)D_i\right]\frac{T_{IV}}{2}\right] - \frac{A_m}{T_{IV}}\right]\end{aligned}$$

After simplifying:

$$\begin{aligned}\pi_{IV} &= \pi_{II,r} + \pi_{II,m} \\ &= \sum_{i=1}^{k}\left[\left(p_{IV,i}D_i\right) - \frac{(a_{r,i} + a_{m,i} + a_{mr,i})}{T_{IV}} - c_{mr,i}u_i\mu_i\right. \\ &- \left.\left[\left[c_{mr,i}u_i\mu_i\theta_{m,i} + \left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right)D_i\right]\frac{T_{IV}}{2} + \frac{b\left(\tau_i\right)^2 D_i}{T_{IV}}\right] - \frac{\left(A_r + A_m\right)}{T_{IV}}\right]\end{aligned} \tag{11}$$

Proposition 1 *The system profit function per unit time; π_{IV} is concave in $p_{IV,i}$.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the demand function. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{IV}}{\partial p_{IV,i}} = \sum_{i=1}^{k}\left[D_i - p_{IV,i}\beta_i + \frac{T_{IV}\beta_i}{2}\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right) + \frac{b\left(\tau_i\right)^2\beta_i}{T_{IV}}\right]$$

$$\frac{\partial \pi_{IV}}{\partial p_{IV,i}} = 0 \Rightarrow p^*_{IV,i} = \frac{1}{2}\sum_{i=1}^{k}\left[\alpha_i\beta_i + \frac{T_{IV}}{2}\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right) + \frac{b\left(\tau_i\right)^2}{T_{IV}}\right]$$

So, the optimal value of $p_{IV,i}(p^*_{IV,i})$ is obtained and the second order derivative of π_{IV} should be negative for concavity property.

$$\frac{\partial^2 \pi_{IV}}{\partial p_{IV,i}^2} = -\sum_{i=1}^{k} 2\beta_i < 0.$$

Hence, the system profit per unit time is concave in $p_{IV,i}$. □

Proposition 2 *The system profit function per unit time; π_{IV} is concave in T_{IV}.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the time. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{IV}}{\partial T_{IV}} = \sum_{i=1}^{k} \left[\frac{(a_{r,i} + a_{m,i} + a_{mr,i} + b(\tau_i)^2 D_i)}{T_{IV}^2} - \frac{1}{2}\left[D_i\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right) + c_{mr,i}u_i\mu_i\theta_{m,i}\right]\right] + \frac{(A_r + A_m)}{T_{IV}^2}$$

$$\frac{\partial \pi_{IV}}{\partial T_{IV}} = 0 \Rightarrow T_{IV}^* = \sqrt{\frac{2\sum_{i=1}^{k}\left[a_{r,i} + a_{m,i} + a_{mr,i} + b(\tau_i)^2 D_i\right] + 2(A_r + A_m)}{\sum_{i=1}^{k}\left[D_i\left(h_{r,i} + c_{r,i}\theta_i + h_{m,i} + h_{mr,i}u_i\right) + c_{mr,i}u_i\mu_i\theta_{m,i}\right]}}.$$

The optimal value of $T_{IV}(T_{IV}^*)$ is obtained and the second order derivative of π_{IV} should be negative for concavity property.

$$\frac{\partial^2 \pi_{IV}}{\partial T_{IV}^2} = -2\left[\frac{(A_r + A_m)}{T_{IV}^3} + \sum_{i=1}^{k}\left[\frac{\left(a_{r,i} + a_{m,i} + a_{mr,i} + b(\tau_i)^2 D_i\right)}{T_{IV}^3}\right]\right] < 0.$$

Hence, the system profit per unit time is concave in T_{IV}. □

Proposition 3 *The system profit function per unit time; π_{IV} is concave in τ_i.*

Proof The optimal property of the profit function strongly depends on the value of the parameters and the preservation effort. The concavity property of an optimization is differentiating with respect to the decision parameter, equating to zero and putting the value of the parameter in the second order differentiation then test for optimality i.e. negative for maximization.

$$\frac{\partial \pi_{IV}}{\partial \tau_i} = \frac{\beta_{2,i} T_{IV,i} c_{mr,i} u_i \mu_i}{2} + \frac{\beta_{1,i} T_{IV,i} c_{r,i} D_i}{2} - \frac{2b\tau_i D_i}{T_{IV,i}}$$

$$\frac{\partial \pi_{IV}}{\partial \tau_i} = 0 \Rightarrow \tau_i^* = \frac{T_{IV,i}^2}{4bD_i}\left[\beta_{1,i} c_{r,i} D_i + \beta_{2,i} c_{mr,i} u_i \mu_i\right].$$

Table 3 Algorithm of integrated supply chain models

Policy I and Policy II	
Step 1.	Determine the local maximum points solving for $\frac{d\pi_{Ir}}{dp_{Ii}} = \frac{d\pi_{Ir}}{d\tau_i} = \frac{d\pi_{Ir}}{dT_{Ii}} = 0$ and $\frac{d\pi_{IIr}}{dp_{IIi}} = \frac{d\pi_{IIr}}{d\tau_i} = \frac{d\pi_{IIr}}{dT_{II}} = 0$. Verify that $\frac{d^2\pi_{Ir}}{dp_{Ii}^2} < 0, \frac{d^2\pi_{Ir}}{d\tau_i} < 0\, and\, \frac{d^2\pi_{Ir}}{dT_{Ii}^2} < 0$ and $\frac{d^2\pi_{IIr}}{dp_{IIi}^2} < 0, \frac{d^2\pi_{IIr}}{d\tau_i} < 0\, and\, \frac{d^2\pi_{IIr}}{dT_{II}^2} < 0$ for satisfying the concavity property of theory of calculus.
Step 2.	$\pi^*_{Ir}\left(p^*_{Ii}, \tau^*_i, T^*_{I,i}\right)$ and $\pi^*_{IIr}\left(p^*_{IIi}, \tau^*_i, T^*_{II}\right)$ associated with the local maximum points, which give the highest values of π_{Ir} and π_{IIr} respectively. By putting all the optimal values in Eq. (5) to obtain the value of π_{Im}.
Step 3.	Determine $Q^*_r, Q^*_f, D^*, \theta^*, \theta^*_m$ for $i = 1, 2, \ldots\, kitems$. Where, $Q^*_r = \mu_i u_i T_{Ii}\left[1 + \frac{\theta^*_{mi} T_{Ii}}{2}\right]$ and $Q^*_f = D^* T_{Ii}\left[1 + \frac{\theta^*_i T_{Ii}}{2}\right]$ and $Q^*_r = \mu_i u_i T_{II}\left[1 + \frac{\theta^*_{mi} T_{II}}{2}\right]$ and $Q^*_f = D^* T_{II}\left[1 + \frac{\theta^*_i T_{II}}{2}\right]$ for policy I and Policy II respectively.
Step 4.	Adjust Q^*_r, Q^*_f, D^* to get the nearest integer values or discrete values for $i = 1, 2, \ldots\, k\, items$ of policy I and policy II respectively.
Step 5.	Obtain $\pi^*_I = \pi^*_{Ir} + \pi^*_{Im}$ and $\pi^*_{II} = \pi^*_{IIr} + \pi^*_{IIm}$ respectively.
Policy III and Policy IV	
Step 1.	Determine the local maximum points solving the Eqs. (10) and (11) for $\frac{d\pi_{III}}{dp_{IIIi}} = \frac{d\pi_{III}}{d\tau_i} = \frac{d\pi_{III}}{dT_{IIIi}} = 0$ and $\frac{d\pi_{IV}}{dp_{IVi}} = \frac{d\pi_{IV}}{d\tau_i} = \frac{d\pi_{IV}}{dT_{IV}} = 0$. Verify that $\frac{d^2\pi_{III}}{dp_{IIIi}^2} < 0, \frac{d^2\pi_{III}}{d\tau_i} < 0\, and\, \frac{d^2\pi_{III}}{dT_{IIIi}^2} < 0$ and $\frac{d^2\pi_{IV}}{dp_{IVi}^2} < 0, \frac{d^2\pi_{IV}}{d\tau_i} < 0\, and\, \frac{d^2\pi_{IV}}{dT_{IV}^2} < 0$ for satisfying the concavity property of theory of calculus.
Step 2.	$Obtain \pi^*_{III}\left(p^*_{IIIi}, \tau^*_i, T^*_{III,i}\right)$ and $\pi^*_{IV}\left(p^*_{IVi}, \tau^*_i, T^*_{IV}\right)$ associated with the local maximum points, which give the highest values of π_{III} and π_{IV} respectively.
Step 3.	Determine $Q^*_r, Q^*_f, D^*, \theta^*, \theta^*_m$ for $i = 1, 2, \ldots\, kitems$. Where, $Q^*_r = \mu_i u_i T_{IIIi}\left[1 + \frac{\theta^*_{mi} T_{IIIi}}{2}\right]$ and $Q^*_f = D^* T_{IIIi}\left[1 + \frac{\theta^*_i T_{IIIi}}{2}\right]$ and $Q^*_r = \mu_i u_i T_{IV}\left[1 + \frac{\theta^*_{mi} T_{IV}}{2}\right]$ and $Q^*_f = D^* T_{IV}\left[1 + \frac{\theta^*_i T_{IV}}{2}\right]$ for policy III and Policy IV respectively.
Step 4.	Adjust Q^*_r, Q^*_f, D^* to get the nearest integer values or discrete values for $i = 1, 2, \ldots\, kitems$ of policy III and policy IV respectively.
Step 5.	Obtain $\pi^*_{III} = \pi^*_{IIIr} + \pi^*_{IIIm}$ and $\pi^*_{IV} = \pi^*_{IVr} + \pi^*_{IVm}$ respectively.

The optimal value of $\tau_i(\tau^*_i)$ is obtained and the second order derivative of π_{IV} should be negative for concave property.

$$\frac{\partial^2 \pi_{IV}}{\partial \tau_i^2} = -\left[\frac{2bD_i}{T_{IV,i}}\right] < 0.$$

Hence, the system profit per unit time; π_{IV} is concave in τ_i. □

Table 4 Numerical data for the example

Item	1	2	3
Retailer			
Demand parameters (α_i, β_i)	[350, 2.0]	[500, 2.5]	[450, 3.0]
Deterioration parameters (α_{1i}, β_{1i})	[0.07, 0.005]	[0.08, 0.006]	[0.09, 0.007]
Preservation effort parameter (b)	0.1	0.1	0.1
Major setup cost (A_r)	250	250	250
Minor setup cost ($a_{r,i}$)	45	60	40
Holding cost of finished item ($h_{r,i}$)	5.2	5.1	4.8
Purchase cost of finished item ($c_{r,i}$)	47	45	40
Manufacturer			
Production rate (μ_i)	250	350	300
Deterioration parameters (α_{2i}, β_{2i})	[0.02, 0.003]	[0.01, 0.002]	[0.005, 0.001]
Usage rate of the raw material (u_i)	0.3	0.2	0.25
Major setup cost (A_m)	450	450	450
Minor setup cost ($a_{m,i}$)	65	80	55
Ordering cost of raw material ($a_{mr,i}$)	35	50	30
Holding cost of finished item ($h_{m,i}$)	3.8	3.5	2.6
Holding cost of raw material ($h_{mr,i}$)	1.8	1.7	1.2
Purchase cost of raw material ($c_{mr,i}$)	20	18	16

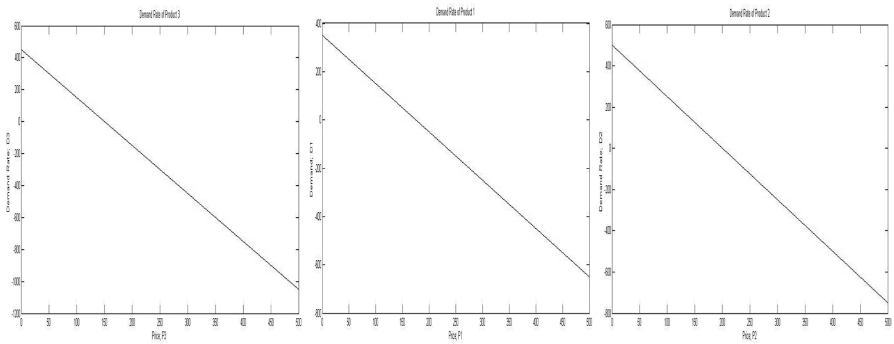

Fig. 2 2-D plot of retail price and demand function of product 1, 2 and 3 respectively

Box 1 Four replenishment policies of the integrated supply chain models

		Policy	
		Individual	Joint
Policy	Uncooperative	I	II
	Cooperative	III	IV

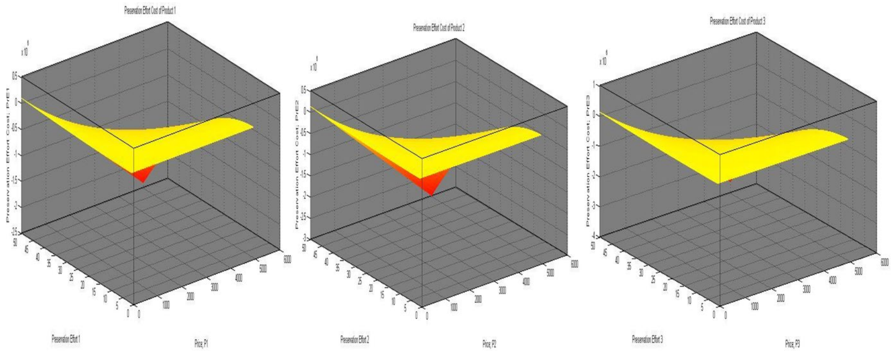

Fig. 3 3-D Mesh plot of retail price, preservation effort and preservation effort cost of product 1, 2 and 3 respectively

5 Algorithm

Table 3 shows the solution procedure of the four integrated supply chain models.

6 Numerical analysis

The multi-echelon supply chain profit models can be applied in solving multi-item problems using the arbitrary number of items, to illustrate the effect of the models three items are taken into consideration in the supply chain. The four different policies with preservation effort effects and without preservation effort effects incorporated with the proposed mixed integer heuristic approach are implemented on a personal computer using Lingo 14.0.

In this section, some managerial insight by considering the following interesting issues:

- The supplier's and retailer's profit (attained from the individual and channel perspectives) are compared and the mixed integer channel coordination are shown.
- The impact of preservation efforts for deteriorating items under four different policies is shown.

We consider ten different items that need to be replenished jointly, namely items 1-3. Using the data of Table 4 the model is illustrated. Figure 2 shows 2-Dimension plot of retail price and demand function of product 1, 2 and 3 respectively. Figure 3 shows 3-Dimension mesh plot of retail price, preservation effort and preservation effort cost of product 1, 2 and 3 respectively. Figure 4 shows the three dimensional mesh plotting of price, replenishment time and total profit of product 1, 2 and 3 for policy I, II, III and IV respectively. Table 5 shows the optimal values of different

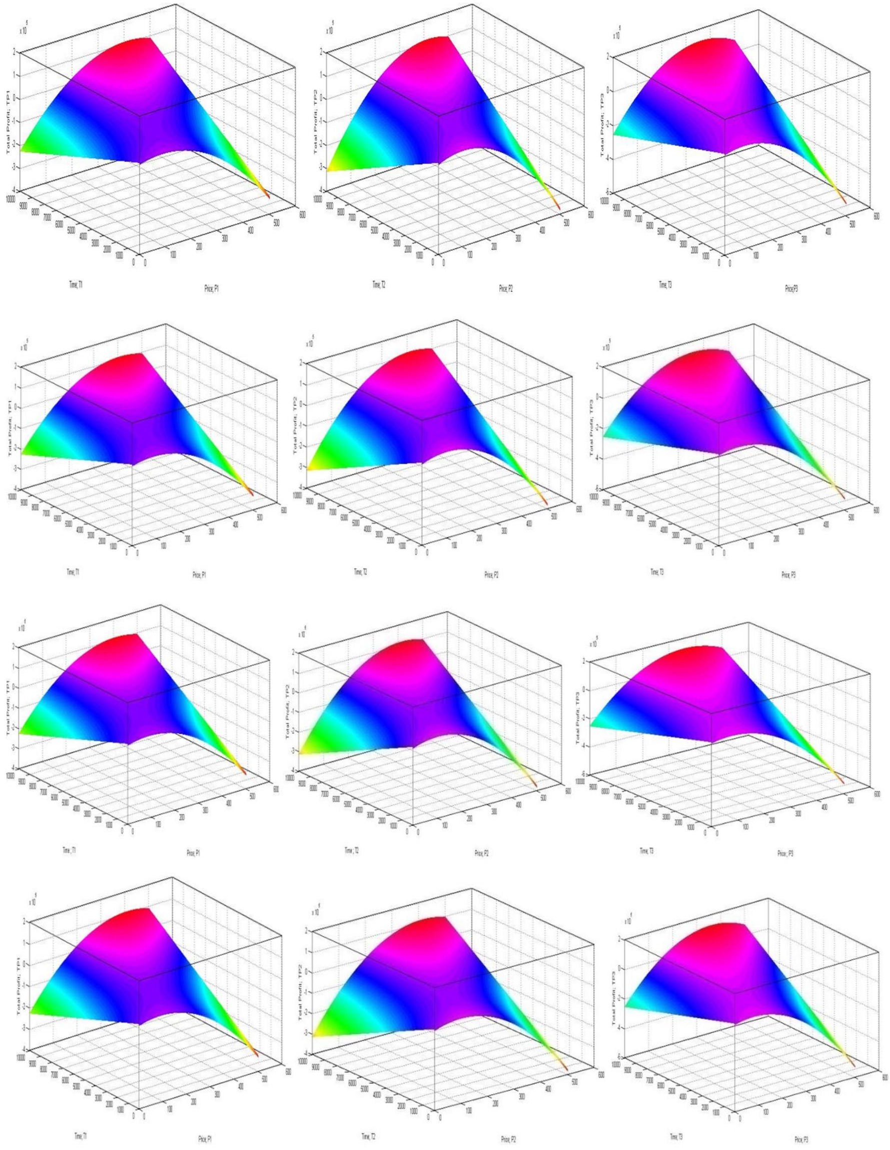

Fig. 4 Three dimensional mesh plotting of: price, replenishment time and Total Profit of product 1, 2 and 3 for policy I, II, III and IV respectively

Table 5 Numerical results of the four supply chain models

Channel co-ordination	Joint replenishment	
	No	Yes
No	Policy I Iteration – 140 $p_{I,1}^* = 112.58, T_{I,1}^* = 0.75, \tau_1^* = 0.09$ $p_{I,2}^* = 123.83, T_{I,2}^* = 0.61, \tau_2^* = 0.09$ $p_{I,3}^* = 96.38, T_{I,3}^* = 0.66, \tau_3^* = 0.11$ $\theta_1^* = 0.07, \theta_{m,1}^* = 0.02$ $\theta_2^* = 0.08, \theta_{m,2}^* = 0.01$ $\theta_3^* = 0.09, \theta_{m,3}^* = 0.05$ $Q_{r,1}^* = 56.49, Q_{f,1}^* = 95.76$ $Q_{r,2}^* = 43.04, Q_{f,2}^* = 119.59$ $Q_{r,3}^* = 49.34, Q_{f,3}^* = 108.75$ $D_1^* = 124.83$ $D_2^* = 190.42$ $D_3^* = 160.87$ $PrEC = 0.45$ $\pi_{I,r}^* = 29579.97$ $\pi_{I,m}^* = 13817.88$ $\pi_I^* = 43397.84$	Policy II Iteration – 133 $p_{II,1}^* = 111.93, T_{II}^* = 0.44, \tau_1^* = 0.04$ $p_{II,2}^* = 123.46, \tau_2^* = 0.04$ $p_{II,3}^* = 95.92, \tau_3^* = 0.05$ $\theta_1^* = 0.07, \theta_{m,1}^* = 0.02$ $\theta_2^* = 0.08, \theta_{m,2}^* = 0.01$ $\theta_3^* = 0.09, \theta_{m,3}^* = 0.05$ $Q_{r,1}^* = 33.10, Q_{f,1}^* = 56.28$ $Q_{r,2}^* = 30.83, Q_{f,2}^* = 85.57$ $Q_{r,3}^* = 32.99, Q_{f,3}^* = 72.70$ $D_1^* = 126.14$ $D_2^* = 191.36$ $D_3^* = 162.23$ $PrEC = 0.09$ $\pi_{II,r}^* = 30477.26$ $\pi_{II,m}^* = 14932.52$ $\pi_{II}^* = 45409.78$
Yes	Policy III	Policy IV
	Iteration-113 $p_{III,1}^* = 90.31, T_{III,1}^* = 0.88, \tau_1^* = 0.16$ $p_{III,2}^* = 102.39, T_{III,2}^* = 0.76, \tau_2^* = 0.15$ $p_{III,3}^* = 77.31, T_{III,3}^* = 0.82, \tau_1^* = 0.16$ $\theta_1^* = 0.07, \theta_{m,1}^* = 0.02$ $\theta_2^* = 0.08, \theta_{m,2}^* = 0.01$ $\theta_3^* = 0.09, \theta_{m,3}^* = 0.01$ $Q_{r,1}^* = 66.41, Q_{f,1}^* = 153.23$ $Q_{r,2}^* = 53.61, Q_{f,2}^* = 191.81$ $Q_{r,3}^* = 61.59, Q_{f,3}^* = 185.26$ $D_1^* = 169.37$ $D_2^* = 244.03$ $D_3^* = 218.07$ $PrEC = 1.49$ $\pi_{III,r}^* = 26193.71$ $\pi_{III,m}^* = 20706.03$ $\pi_{III}^* = 46899.73$	Iteration – 103 $p_{IV,1}^* = 89.59, T_{IV}^* = 0.65, \tau_1^* = 0.09$ $p_{IV,2}^* = 102.05, \tau_2^* = 0.09$ $p_{IV,3}^* = 76.84, \tau_3^* = 0.10$ $\theta_1^* = 0.07, \theta_{m,1}^* = 0.02$ $\theta_2^* = 0.08, \theta_{m,2}^* = 0.01$ $\theta_3^* = 0.09, \theta_{m,3}^* = 0.01$ $Q_{r,1}^* = 49.36, Q_{f,1}^* = 114.23$ $Q_{r,2}^* = 45.92, Q_{f,2}^* = 164.28$ $Q_{r,3}^* = 49.12, Q_{f,3}^* = 147.69$ $D_1^* = 170.81$ $D_2^* = 244.88$ $D_3^* = 219.47$ $PrEC = 0.62$ $\pi_{IV,r}^* = 26192.26$ $\pi_{IV,m}^* = 21922.78$ $\pi_{IV}^* = 48115.04$

Table 6 Comparative analysis

Policy	Model	Iteration	$T^*_{I,i}$	$p^*_{I,i}$	τ^*_i	θ^*_i	$\theta^*_{m,i}$	$Q^*_{r,i}$	$Q^*_{f,i}$	$D^*_i(p_{I,i})$	PrEC	$\pi^*_{I,r}$	$\pi^*_{I,m}$	π^*_I
I	With Preservation Effort (PrE) Effect	140	0.75, 0.61, 0.66	112.58, 123.83, 96.38	0.09, 0.09, 0.11	0.07, 0.08, 0.09	0.02, 0.01, 0.05	56.49, 43.04, 49.34	95.76, 119.59, 108.75	124.83, 190.42, 160.87	0.45	29579.97	13817.88	43397.84
	Without Preservation Effort (WPrE) Effect	109	0.75, 0.61, 0.66	112.58, 123.83, 96.38	–	0.07, 0.08, 0.09	0.02, 0.01, 0.005	56.38, 42.95, 49.22	95.57, 119.34, 108.50	124.83, 190.42, 160.87	–	29577.96	13813.63	43391.58
	Mixed Integer Q_r, Q_f, D_i, PrE	133938	0.75, 0.63, 0.67	113, 124, 96	0.06, 0.27, 0.23	0.07, 0.08, 0.09	0.02, 0.01, 0.01	57, 44, 50	96, 122, 111	124, 190, 162	2.33	29574.91	13834.85	43409.76
	Mixed Integer (D_i), PrE	966	0.75, 0.61, 0.66	112.5, 124, 96.33	0.09, 0.09, 0.1	0.07, 0.08, 0.09	0.02, 0.01, 0.01	56.45, 43.09, 49.32	95.83, 119.46, 108.80	125, 190, 161	0.45	29579.88	13812.39	43392.27
	Mixed Integer (D_i), WPrE	315	0.75, 0.61, 0.65	112.5, 124.0, 96.33	–	0.07, 0.08, 0.09	0.02, 0.01, 0.005	5634, 43.0, 49.2	95.63, 119.21, 108.54	125, 190, 161	–	29577.87	13807.82	43385.68
II	With Preservation Effort (PrE) Effect	133	0.44	111.93, 123.46, 95.92	0.04, 0.04, 0.05	0.07, 0.08, 0.09	0.02, 0.01, 0.05	33.10, 30.83, 32.99	56.28, 85.57, 72.70	126.14, 191.36, 162.23	0.09	30477.26	14932.52	45409.78

Table 6 (continued)

Policy	Model	Iteration	$T^*_{I,i}$	$p^*_{I,i}$	τ^*_i	θ^*_i	$\theta^*_{m,i}$	$Q^*_{r,i}$	$Q^*_{f,i}$	$D^*_i(p_{I,i})$	PrEC	$\pi^*_{I,r}$	$\pi^*_{I,m}$	π^*_I
	Without Preservation Effort (WPrE) Effect	101	0.44	111.93, 124.45, 95.92	–	0.07, 0.08, 0.09	0.02, 0.01, 0.005	33.07, 30.80, 32.96	56.23, 85.49, 72.63	126.14, 191.37, 162.24	–	30476.66	14931.21	45407.87
	Mixed Integer Q_r, Q_f, D_i, PrE	312993	2.04	112, 110.8, 98.67	5.92, 3.19, 2.76	0.04, 0.07, 0.07	0.01, 0.01, 0.01	153, 143, 153	267, 482, 336	126, 223, 154	784.95	26662.13	15887.87	42550.00
	Mixed Integer (D_i), PrE	821	0.44	112, 123.6, 96	0.04, 0.05, 0.05	0.07, 0.08, 0.09	0.02, 0.01, 0.01	33.13, 30.85, 33.02	56.27, 85.48, 72.65	126, 191, 162	0.09	30477.18	14902.14	45379.32
	Mixed Integer (D_i), WPrE	275	0.44	112, 123.6, 96	–	0.07, 0.08, 0.09	0.02, 0.01, 0.005	33.09, 30.82, 32.99	56.21, 85.39, 72.59	126, 191, 162	–	30476.58	14900.74	45377.32
III	With Preservation Effort (PrE) Effect	113	0.88, 0.76, 0.82	90.31, 102.39, 77.31	0.16, 0.15, 0.16	0.07, 0.08, 0.09	0.02, 0.01, 0.01	66.41, 53.61, 61.59	153.23, 191.81, 185.26	169.37, 244.03, 218.07	1.49	26193.71	20706.03	46899.73

Table 6 (continued)

Policy	Model	Iteration	$T^*_{I,i}$	$p^*_{I,i}$	τ^*_i	θ^*_i	$\theta^*_{m,i}$	$Q^*_{r,i}$	$Q^*_{f,i}$	$D^*_i(p_{I,i})$	PrEC	$\pi^*_{I,r}$	$\pi^*_{I,m}$	π^*_I
	Without Preservation Effort (WPrE) Effect	100	0.88, 0.76, 0.82	90.31, 102.39, 77.31	–	0.07, 0.08, 0.09	0.02, 0.01, 0.005	66.26, 53.48, 61.42	152.89, 191.39, 184.78	169.38, 244.03, 218.07	–	26190.84	20703.46	46894.30
	Mixed Integer Q_r, Q_f, D_i PrE	23155	0.87, 0.13E11, 0.82	91, 200, 76.67	0.30, 5, 0.22	0.07, 0.05, 0.09	0.02, 0.01, 0.01	66, 0.89E12, 61	151, 0, 185	168, 0, 220	2.60	13395.44	10854.79	24250.24
	Mixed Integer (D_i), PrE	344	0.88, 0.13E11, 0.82	90.5, 200, 77.33	0.24, 5, 0.25	0.07, 0.05, 0.09	0.02, 0.81E13, 0.01	66.57, 0.89E12, 61.71	153.26, 0, 185.5	169, 0, 218	2.36	13423.49	10829.21	24252.70
	Mixed Integer (D_i), WPrE	339	0.88, 0.76, 0.82	90.50, 102.40, 77.33	–	0.07, 0.08, 0.09	0.02, 0.01, 0.005	66.34, 53.48, 61.43	152.73, 191.38, 184.75	169, 244, 218	–	26211.92	20682.30	46894.23
IV	With Preservation Effort (PrE) Effect	103	0.65	89.59, 102.05, 76.84	0.09, 0.09, 0.1	0.07, 0.08, 0.09	0.02, 0.01, 0.01	49.36, 45.92, 49.12	114.23, 164.28, 147.69	170.81, 244.88, 219.47	0.62	26192.26	21922.78	48115.04

Table 6 (continued)

Policy	Model	Iteration	$T^*_{I,i}$	$p^*_{I,i}$	τ^*_i	θ^*_i	$\theta^*_{m,i}$	$Q^*_{r,i}$	$Q^*_{f,i}$	$D^*_i(p_{I,i})$	PrEC	$\pi^*_{I,r}$	$\pi^*_{I,m}$	π^*_I
	Without Preservation Effort (WPrE) Effect	56	0.65	89.59, 102.05, 76.844	–	0.07, 0.08, 0.09	0.02, 0.01, 0.005	49.28, 45.84, 49.04	114.05, 164.03, 147.47	170.81, 244.88, 219.47	–	26189.93	21922.27	48112.20
	Mixed Integer Q_r, Q_f, D_i, PrE	332292	0.63	175, 96, 65.33	6.57, 0.01, 4.72	0.38, 0.08, 0.06	0.01, 0.01, 0.01	47, 44, 47	0, 167, 162	0, 260, 254	565.61	14286.57	16131.38	30417.95
	Mixed Integer (D_i), PrE	831	0.65	89.5, 102, 77	0.09, 0.01, 0.1	0.07, 0.08, 0.09	0.02, 0.01, 0.01	49.36, 45.92, 49.12	114.36, 164.38, 147.39	16, 171, 245	0.62	26196.33	21918.61	48114.94
	Mixed Integer (D_i), WPrE	262	0.65	89.5, 102, 77	–	0.07, 0.08, 0.09	0.02, 0.01, 0.005	49.28, 45.85, 49.04	114.19, 164.13, 147.17	171, 245, 219,	–	26194.42	21917.68	48112.11

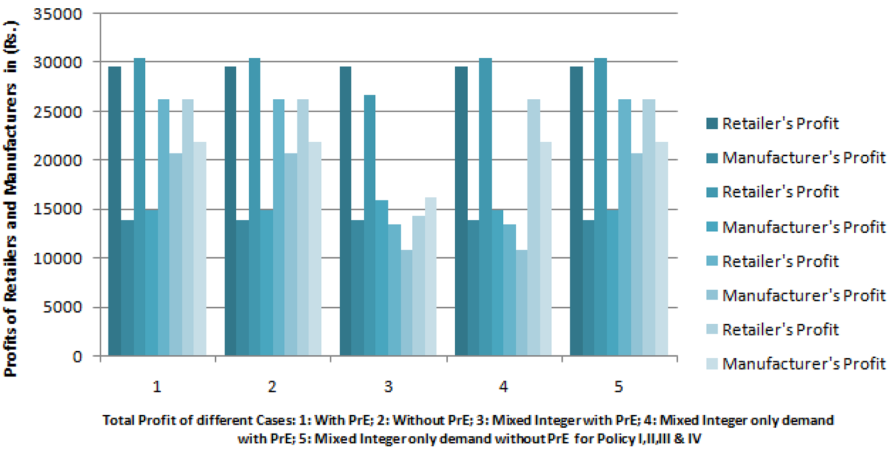

Fig. 5 Profit of the players with four policies in different scenario

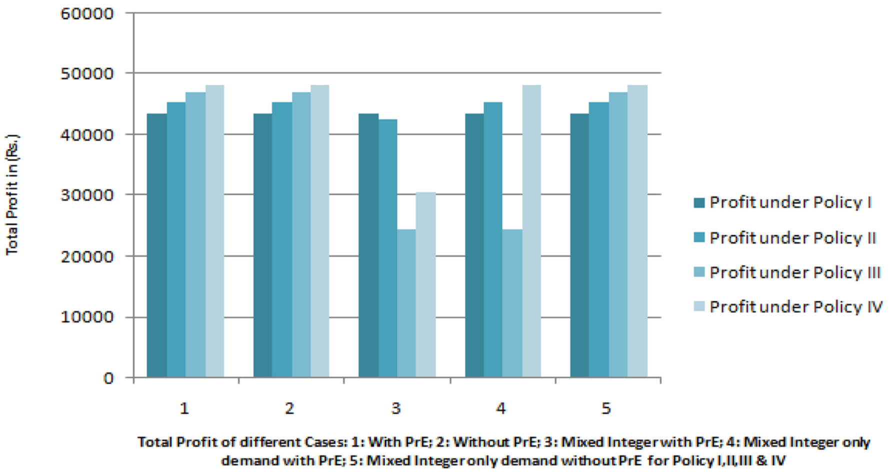

Fig. 6 Total profit with four policies in different scenario

models and comparative analysis of different models with different cases are shown in Table 6.

6.1 Interpretation

From the Table 6 for the channel perspective point of view, it is observed that from all the policies the profit of cooperative model with joint items (policy IV)

Table 7 Sensitivity analysis of Policy-IV with respect to the parameters

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
b	Iteration	99	116	103	96	107
	$T^*_{IV,i}$	0.655	0.654	0.65	0.653	0.653
	$p^*_{IV,i}$	89.597, 102.049, 76.845	89.596, 102.048, 76.845	89.59, 102.05, 76.84	89.595, 102.047, 76.844	89.595, 102.047, 76.844
	τ^*_i	0.186, 0.200, 0.204	0.133, 0.142, 0.145	0.09, 0.09, 0.10	0.0715, 0.0767, 0.781	0.0620, 0.0665, 0.0677
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	49.438, 45.994, 49.202	49.392, 45.951, 49.155	49.36, 45.92, 49.12	49.339, 45.901, 49.101	49.331, 45.894, 49.093
	$Q^*_{f,\mathrm{i}}$	114.408, 164.534, 147.918	114.305, 164.389, 147.790	114.23, 164.28, 147.69	114.188, 164.223, 147.644	114.170, 164.198, 147.621
	$D^*_i\left(p_{IV,i}\right)$	170.804, 244.877, 219.462	170.806, 244.879, 219.464	170.81, 244.88, 219.47	170.809, 244.881, 219.465	170.809, 244.881, 219.466
	PrEC	1.247	0.887	0.62	0.475	0.412
	$\pi^*_{IV,r}$	26194.60	26193.26	26192.26	26191.72	26191.48
	$\pi^*_{IV,m}$	21923.29	21923.00	21922.78	21922.66	21922.61
	π^*_{IV}	48117.88	48116.26	48115.04	48114.38	48114.09

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
α_{1i}	Iteration	92	98	103	100	110
	$T^*_{IV,i}$	0.706	0.681	0.65	0.627	0.611
	$p^*_{IV,i}$	89.472, 101.893, 76.674	89.516, 101.981, 76.739	89.59, 102.05, 76.84	89.666, 102.135, 76.940	89.712, 102.191, 77.001
	τ^*_i	0.108, 0.116, 0.118	0.101, 0.108, 0.110	0.09, 0.09, 0.10	0.0857, 0.0920, 0.0937	0.0815, 0.0874, 0.0890
	θ^*_i	0.04, 0.04, 0.05	0.05, 0.06, 0.06	0.07, 0.08, 0.09	0.09, 0.11, 0.12	0.11, 0.12, 0.13
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	53.340, 49.611, 53.063	51.473, 47.880, 51.215	49.36, 45.92, 49.12	47.363, 44.068, 47.143	46.161, 42.954, 45.953
	$Q^*_{f,\mathrm{i}}$	122.284, 175.633, 157.791	118.480, 170.435, 153.010	114.23, 164.28, 147.69	110.158, 158.539, 142.582	107.695, 155.063, 139.486
	$D^*_i(p_{IV,i})$	171.054, 245.265, 219.976	170.966, 245.046, 219.782	170.81, 244.88, 219.47	170.666, 244.662, 219.178	170.574, 244.521, 218.994
	PrEC	0.844	0.732	0.62	0.525	0.473
	$\pi^*_{IV,r}$	26499.03	26359.60	26192.26	26022.91	25915.12
	$\pi^*_{IV,m}$	21995.54	21964.48	21922.78	21877.64	21847.23
	π^*_{IV}	48494.57	48324.08	48115.04	47900.56	47762.35

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
β_{1i}	Iteration	107	101	103	88	95
	$T^*_{IV,i}$	0.653	0.653	0.65	0.654	0.655
	$p^*_{IV,i}$	89.594, 102.046, 76.844	89.594, 102.047, 76.844	89.59, 102.05, 76.84	89.596, 102.048, 76.845	89.598, 102.049, 76.845
	τ^*_i	0.0511, 0.0516, 0.0517	0.0595, 0.0708, 0.0716	0.09, 0.09, 0.10	0.118, 0.129, 0.131	0.127, 0.148, 0.152
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,i}$	49.299, 45.864, 49.060	49.316, 45.879, 49.077	49.36, 45.92, 49.12	49.412, 45.969, 49.174	49.451, 46.005, 49.213
	$Q^*_{f,i}$	114.100, 164.100, 147.535	114.138, 164.151, 147.580	114.23, 164.28, 147.69	114.346, 164.446, 147.840	114.433, 164.562, 147.942
	$D^*_i\left(p_{IV,i}\right)$	170.811, 244.882, 219.466	170.810, 244.882, 219.466	170.81, 244.88, 219.47	170.806, 244.878, 219.463	170.803, 244.877, 219.463
	PrEC	0.168	0.296	0.62	1.029	1.324
	$\pi^*_{IV,r}$	26190.52	26191.01	26192.26	26193.86	26195.02
	$\pi^*_{IV,m}$	21922.46	21922.55	21922.78	21923.04	21923.21
	π^*_{IV}	48112.98	48113.56	48115.04	48116.90	48118.23

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
α_{2i}	Iteration	102	101	103	97	99
	$T^*_{IV,i}$	0.654	0.654	0.65	0.653	0.652
	$p^*_{IV,i}$	89.599, 102.051, 76.847	89.597, 102.049, 76.846	89.59, 102.05, 76.84	89.593, 102.045, 76.843	89.592, 102.044, 76.842
	τ^*_i	0.0934, 0.100, 0.102	0.0932, 0.100, 0.101	0.09, 0.09, 0.10	0.0929, 0.0996, 0.101	0.0928, 0.0995, 0.101
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.01, 0.01, 0.01	0.01, 0.01, 0.01	0.02, 0.01, 0.01	0.01, 0.01, 0.01	0.01, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	49.276, 45.917, 49.158	49.309, 45.918, 49.143	49.36, 45.92, 49.12	49.406, 45.919, 49.097	49.439, 45.920, 49.082
	$Q^*_{f,\mathrm{i}}$	114.410, 164.544, 147.931	114.338, 164.439, 147.836	114.23, 164.28, 147.69	114.120, 164.124, 147.554	114.048, 164.020, 147.460
	$D^*_i(p_{IV,i})$	170.801, 244.872, 219.456	170.804, 244.875, 219.459	170.81, 244.88, 219.47	170.812, 244.885, 219.470	170.815, 244.888, 219.473
	PrEC	0.623	0.621	0.62	0.617	0.615
	$\pi^*_{IV,r}$	26192.62	26192.48	26192.26	26192.03	26191.88
	$\pi^*_{IV,m}$	21930.36	21927.33	21922.78	21918.24	21915.21
	π^*_{IV}	48122.99	48119.80	48115.04	48110.27	48107.10

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
β_{2i}	Iteration	103	81	103	103	114
	$T^*_{IV,i}$	0.653	0.653	0.65	0.653	0.653
	$p^*_{IV,i}$	89.595, 102.047, 76.844	89.595, 102.047, 76.844	89.59, 102.05, 76.84	89.595, 102.047, 76.845	89.595, 102.048, 76.845
	τ^*_i	0.0884, 0.0980, 0.100	0.0902, 0.0987, 0.101	0.09, 0.09, 0.10	0.0959, 0.100, 0.102	0.0978, 0.101, 0.102
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.01, 0.01, 0.01	0.01, 0.01, 0.01	0.02, 0.01, 0.01	0.01, 0.01, 0.01	0.01, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	49.356, 45.917, 49.117	49.357, 45.917, 49.118	49.36, 45.92, 49.12	49.358, 45.920, 49.122	49.359, 45.921, 49.123
	$Q^*_{f,\mathrm{i}}$	114.221, 164.269, 147.684	114.224, 164.274, 147.688	114.23, 164.28, 147.69	114.234, 164.289, 147.702	114.237, 164.294, 147.706
	$D^*_i(p_{IV,i})$	170.808, 244.880, 219.465	170.808, 244.880, 219.465	170.81, 244.88, 219.47	170.808, 244.880, 219.464	170.808, 244.880, 219.464
	PrEC	0.591	0.602	0.62	0.636	0.648
	$\pi^*_{IV,r}$	26192.24	26192.25	26192.26	26192.26	26192.27
	$\pi^*_{IV,m}$	21922.67	21922.71	21922.78	21922.85	21922.90
	π^*_{IV}	48114.91	48114.96	48115.04	48115.11	48115.17

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
α_i	Iteration	122	97	103	117	105
	$T^*_{IV,i}$	0.983	0.787	0.65	0.571	0.531
	$p^*_{IV,i}$	51.503, 57.500, 44.229	63.773, 72.465, 54.721	89.59, 102.05, 76.84	115.582, 131.790, 99.112	132.953, 151.664, 113.999
	τ^*_i	0.229, 0.232, 0.234	0.141, 0.147, 0.148	0.09, 0.09, 0.10	0.0694, 0.0756, 0.0773	0.0593, 0.0651, 0.0667
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	74.461, 69.166, 73.933	59.537, 55.353, 59.194	49.36, 45.92, 49.12	43.105, 40.117, 42.923	40.058, 37.289, 39.900
	$Q^*_{f,\mathrm{i}}$	73.201, 108.533, 94.732	95.048, 137.145, 122.985	114.23, 164.28, 147.69	130.469, 187.344, 168.601	140.209, 201.199, 181.136
	$D^*_i(p_{IV,i})$	71.993, 106.249, 92.312	117.452, 168.835, 150.835	170.81, 244.88, 219.47	223.835, 320.524, 287.661	259.093, 370.838, 333.000
	PrEC	1.460	0.935	0.62	0.463	0.396
	$\pi^*_{IV,r}$	0.000	6224.459	26192.26	56579.89	82600.29
	$\pi^*_{IV,m}$	6610.701	13573.04	21922.78	30270.36	35838.99
	π^*_{IV}	6610.701	19797.50	48115.04	86850.24	118439.3

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
β_i	Iteration	96	88	103	99	95
	$T^*_{IV,i}$	0.650	0.651	0.65	0.656	0.657
	$p^*_{IV,i}$	177.083, 202.036, 151.834	127.088, 144.897, 108.981	89.59, 102.05, 76.84	69.410, 78.978, 59.543	60.441, 68.726, 51.855
	τ^*_i	0.0919, 0.0986, 0.100	0.0924, 0.0991, 0.100	0.09, 0.09, 0.10	0.0938, 0.100, 0.102	0.0943, 0.101, 0.102
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	49.070, 45.652, 48.835	49.184, 45.757, 48.948	49.36, 45.92, 49.12	49.534, 46.083, 49.295	49.654, 46.194, 49.414
	$Q^*_{f,\mathrm{i}}$	114.954, 165.022, 148.677	114.665, 164.727, 148.287	114.23, 164.28, 147.69	113.788, 163.831, 147.097	113.491, 163.528, 146.694
	$D^*_i\left(p_{IV,i}\right)$	172.916, 247.455, 222.248	172.076, 246.428, 221.139	170.81, 244.88, 219.47	169.531, 243.320, 217.779	168.675, 242.274, 216.648
	PrEC	0.611	0.614	0.62	0.624	0.627
	$\pi^*_{IV,r}$	83049.99	50502.87	26192.26	13194.86	7463.420
	$\pi^*_{IV,m}$	22237.89	22112.28	21922.78	21731.94	21603.94
	π^*_{IV}	105287.9	72615.15	48115.04	34926.80	29067.36

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
A_r	Iteration	131	118	103	96	100
	$T^*_{IV,i}$	0.574	0.607	0.65	0.697	0.725
	$p^*_{IV,i}$	89.340, 101.798, 76.620	89.446, 101.902, 76.713	89.59, 102.05, 76.84	89.735, 102.184, 76.967	89.823, 102.270, 77.045
	τ^*_i	0.0718, 0.0770, 0.0784	0.0803, 0.0861, 0.0877	0.09, 0.09, 0.10	0.105, 0.113, 0.115	0.114, 0.122, 0.125
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	43.315, 40.313, 43.132	45.822, 42.639, 45.617	49.36, 45.92, 49.12	52.671, 48.991, 52.401	54.776, 50.942, 54.485
	$Q^*_{f,\mathrm{i}}$	100.352, 144.207, 129.668	106.113, 152.537, 137.149	114.23, 164.28, 147.69	121.825, 175.284, 157.573	126.646, 182.272, 163.845
	$D^*_i\left(p_{IV,i}\right)$	171.318, 245.502, 220.137	171.106, 245.244, 219.858	170.81, 244.88, 219.47	170.529, 244.539, 219.097	170.352, 244.323, 218.863
	PrEC	0.369	0.461	0.62	0.800	0.934
	$\pi^*_{IV,r}$	26794.04	26538.76	26192.26	25879.58	25686.09
	$\pi^*_{IV,m}$	21931.70	21933.08	21922.78	21902.46	21885.08
	π^*_{IV}	48725.74	48471.84	48115.04	47782.04	47571.17

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
$a_{r,i}$	Iteration	106	98	103	102	106
	$T^*_{IV,i}$	0.639	0.645	0.65	0.662	0.668
	$p^*_{IV,i}$	89.548, 102.002, 76.803	89.567, 102.020, 76.820	89.59, 102.05, 76.84	89.623, 102.074, 76.869	89.641, 102.092, 76.885
	τ^*_i	0.0889, 0.0954, 0.0972	0.0906, 0.0972, 0.0990	0.09, 0.09, 0.10	0.0956, 0.102, 0.104	0.0972, 0.104, 0.106
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	48.245, 44.887, 48.018	48.693, 45.302, 48.461	49.36, 45.92, 49.12	50.014, 46.527, 49.770	50.447, 46.929, 50.199
	$Q^*_{f,\mathrm{i}}$	111.676, 160.586, 144.377	112.704, 162.073, 145.713	114.23, 164.28, 147.69	115.735, 166.462, 149.653	116.728, 167.901, 150.944
	$D^*_i\left(p_{IV,i}\right)$	170.902, 244.994, 219.588	170.864, 244.948, 219.539	170.81, 244.88, 219.47	170.753, 244.812, 219.392	170.716, 244.768, 219.344
	PrEC	0.566	0.587	0.62	0.652	0.675
	$\pi^*_{IV,r}$	26299.76	26256.32	26192.26	26129.44	26088.24
	$\pi^*_{IV,m}$	21927.41	21925.69	21922.78	21919.50	21917.13
	π^*_{IV}	48227.17	48182.01	48115.04	48048.95	48005.36

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
$h_{r,i}$	Iteration	77	101	103	106	79
	$T^*_{IV,i}$	0.733	0.698	0.65	0.617	0.596
	$p^*_{IV,i}$	89.372, 101.827, 76.627	89.464, 101.918, 76.717	89.59, 102.05, 76.84	89.719, 102.169, 76.964	89.798, 102.247, 77.039
	τ^*_i	0.116, 0.125, 0.127	0.106, 0.113, 0.115	0.09, 0.09, 0.10	0.0830, 0.0890, 0.0906	0.0774, 0.0830, 0.0845
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	55.373, 51.495, 55.075	52.710, 49.027, 52.440	49.36, 45.92, 49.12	46.580, 43.342, 46.368	44.972, 41.850, 44.774
	$Q^*_{f,\mathrm{i}}$	128.729, 185.134, 166.622	122.304, 175.892, 158.231	114.23, 164.28, 147.69	107.549, 154.681, 138.987	103.688, 149.132, 133.955
	$D^*_i(p_{IV,i})$	171.255, 245.430, 220.116	171.070, 245.203, 219.846	170.81, 244.88, 219.47	170.561, 244.576, 219.107	170.403, 244.381, 218.880
	PrEC	0.980	0.805	0.62	0.491	0.426
	$\pi^*_{IV,r}$	26641.87	26452.96	26192.26	25953.25	25803.75
	$\pi^*_{IV,m}$	22025.17	21985.46	21922.78	21858.08	21814.42
	π^*_{IV}	48667.04	48438.43	48115.04	47811.32	47618.18

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
$c_{r,i}$	Iteration	110	91	103	83	82
	$T^*_{IV,i}$	0.705	0.683	0.65	0.628	0.612
	$p^*_{IV,i}$	89.471, 101.892, 76.673	89.521, 101.956, 76.744	89.59, 102.05, 76.84	89.667, 102.135, 76.940	89.714, 102.192, 77.002
	τ^*_i	0.0595, 0.0601, 0.0602	0.0742, 0.0775, 0.0783	0.09, 0.09, 0.10	0.109, 0.118, 0.121	0.118, 0.129, 0.133
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	53.253, 49.530, 52.975	51.582, 47.981, 51.322	49.36, 45.92, 49.12	47.406, 44.109, 47.187	46.231, 43.019, 46.023
	$Q^*_{f,\mathrm{i}}$	123.579, 177.798, 160.008	119.568, 171.997, 154.723	114.23, 164.28, 147.69	109.552, 157.525, 141.544	106.737, 153.460, 137.845
	$D^*_i(p_{IV,i})$	171.057, 245.268, 219.978	170.956, 245.109, 219.767	170.81, 244.88, 219.47	170.664, 244.660, 219.177	170.571, 244.518, 218.992
	PrEC	0.229	0.376	0.62	0.872	1.040
	$\pi^*_{IV,r}$	40434.59	34727.45	26192.26	17684.86	12027.40
	$\pi^*_{IV,m}$	8057.376	13610.09	21922.78	30217.35	35737.70
	π^*_{IV}	48491.97	48337.54	48115.04	47902.20	47765.10

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
μ_i	Iteration	98	123	103	100	101
	$T^*_{IV,i}$	0.654	0.654	0.65	0.653	0.652
	$p^*_{IV,i}$	89.598, 102.050, 76.847	89.597, 102.049, 76.846	89.59, 102.05, 76.84	89.593, 102.046, 76.843	89.592, 102.044, 76.842
	τ^*_i	0.0886, 0.0983, 0.101	0.0904, 0.0989, 0.101	0.09, 0.09, 0.10	0.0957, 0.100, 0.102	0.0975, 0.101, 0.102
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	24.716, 22.994, 24.597	34.582, 32.172, 34.415	49.36, 45.92, 49.12	64.106, 59.640, 63.798	73.924, 68.774, 73.569
	$Q^*_{f,\mathrm{i}}$	114.402, 164.532, 147.920	114.333, 164.432, 147.830	114.23, 164.28, 147.69	114.125, 164.132, 147.561	114.056, 164.032, 147.472
	$D^*_i\left(p_{IV,i}\right)$	170.802, 244.872, 219.456	170.804, 244.875, 219.460	170.81, 244.88, 219.47	170.812, 244.885, 219.470	170.814, 244.888, 219.473
	PrEC	0.595	0.604	0.62	0.634	0.644
	$\pi^*_{IV,r}$	26192.61	26192.47	26192.26	26192.04	26191.89
	$\pi^*_{IV,m}$	23910.25	23115.26	21922.78	20730.31	19935.33
	π^*_{IV}	50102.86	49307.73	48115.04	46922.35	46127.23

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
u_i	Iteration	84	99	103	101	103
	$T^*_{IV,i}$	0.660	0.657	0.65	0.650	0.647
	$p^*_{IV,i}$	89.570, 102.038, 76.837	89.580, 102.042, 76.840	89.59, 102.05, 76.84	89.610, 102.053, 76.849	89.620, 102.056, 76.852
	τ^*_i	0.0900, 0.0998, 0.102	0.0913, 0.0998, 0.102	0.09, 0.09, 0.10	0.0948, 0.0998, 0.101	0.0960, 0.0998, 0.100
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	$Q^*_{r,\mathrm{i}}$	34.744, 32.323, 34.576	49.36, 45.92, 49.12	63.811, 59.366, 63.505	73.359, 68.250, 73.009
	$Q^*_{f,\mathrm{i}}$	$Q^*_{f,\mathrm{i}}$	114.900, 165.230, 148.551	114.23, 164.28, 147.69	113.569, 163.349, 146.854	113.135, 162.736, 146.300
	$D^*_i\left(p_{IV,i}\right)$	170.858, 244.902, 219.487	170.838, 244.893, 219.478	170.81, 244.88, 219.47	170.778, 244.867, 219.451	170.759, 244.858, 219.442
	PrEC	0.614	0.616	0.62	0.622	0.624
	$\pi^*_{IV,r}$	26184.29	26187.52	26192.26	26196.86	26199.86
	$\pi^*_{IV,m}$	23958.24	23143.96	21922.78	20701.89	19888.13
	π^*_{IV}	50142.53	49331.48	48115.04	46898.76	46087.99

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
A_m	Iteration	104	101	103	113	99
	$T^*_{IV,i}$	0.607	0.626	0.65	0.680	0.697
	$p^*_{IV,i}$	89.446, 101.902, 76.713	89.507, 101.961, 76.767	89.59, 102.05, 76.84	89.680, 102.130, 76.919	89.735, 102.184, 76.967
	τ^*_i	0.0803, 0.0861, 0.0877	0.0854, 0.0916, 0.0933	0.09, 0.09, 0.10	0.100, 0.108, 0.110	0.105, 0.113, 0.115
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,i}$	45.822, 42.639, 45.617	47.266, 43.979, 47.048	49.36, 45.92, 49.12	51.369, 47.784, 51.112	52.671, 48.991, 52.401
	$Q^*_{f,i}$	106.113, 152.537, 137.149	109.429, 157.334, 141.457	114.23, 164.28, 147.69	118.842, 170.963, 153.693	121.825, 175.284, 157.573
	$D^*_i(p_{IV,i})$	171.106, 245.244, 219.858	170.984, 245.095, 219.697	170.81, 244.88, 219.47	170.639, 244.673, 219.241	170.529, 244.539, 219.097
	PrEC	0.461	0.521	0.62	0.725	0.800
	$\pi^*_{IV,r}$	26168.28	26179.94	26192.26	26199.56	26202.16
	$\pi^*_{IV,m}$	22303.56	22146.00	21922.78	21713.11	21579.88
	π^*_{IV}	48471.84	48325.94	48115.04	47912.68	47782.04

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
$a_{m,i}$	Iteration	74	115	103	89	105
	$T^*_{IV,i}$	0.633	0.641	0.65	0.665	0.673
	$p^*_{IV,i}$	89.530, 101.984, 76.787	89.557, 102.010, 76.810	89.59, 102.05, 76.84	89.633, 102.085, 76.878	89.658, 102.109, 76.900
	τ^*_i	0.0874, 0.0937, 0.0954	0.0897, 0.0962, 0.0979	0.09, 0.09, 0.10	0.0965, 0.103, 0.105	0.0988, 0.105, 0.107
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	47.816, 44.489, 47.593	48.438, 45.066, 48.209	49.36, 45.92, 49.12	50.261, 46.756, 50.015	50.855, 47.307, 50.603
	$Q^*_{f,\mathrm{i}}$	110.692, 159.163, 143.099	112.120, 161.229, 144.954	114.23, 164.28, 147.69	116.301, 167.282, 150.389	117.663, 169.254, 152.160
	$D^*_i(p_{IV,i})$	170.938, 245.039, 219.636	170.885, 244.975, 219.567	170.81, 244.88, 219.47	170.732, 244.787, 219.364	170.682, 244.726, 219.298
	PrEC	0.546	0.575	0.62	0.665	0.697
	$\pi^*_{IV,r}$	26183.68	26187.47	26192.26	26196.05	26198.09
	$\pi^*_{IV,m}$	22086.70	22020.18	21922.78	21828.05	21766.29
	π^*_{IV}	48270.38	48207.65	48115.04	48024.10	47964.38

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
$a_{mr,i}$	Iteration	111	105	103	100	100
	$T^*_{IV,i}$	0.642	0.646	0.65	0.660	0.665
	$p^*_{IV,i}$	89.558, 102.011, 76.812	89.573, 102.026, 76.825	89.59, 102.05, 76.84	89.617, 102.069, 76.864	89.632, 102.083, 76.877
	τ^*_i	0.0898, 0.0963, 0.0981	0.0911, 0.0977, 0.0995	0.09, 0.09, 0.10	0.0950, 0.101, 0.103	0.0964, 0.103, 0.105
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,i}$	48.477, 45.102, 48.248	48.831, 45.430, 48.598	49.36, 45.92, 49.12	49.879, 46.402, 49.636	50.224, 46.722, 49.978
	$Q^*_{f,i}$	112.208, 161.357, 145.069	113.021, 162.533, 146.125	114.23, 164.28, 147.69	115.425, 166.013, 149.250	116.215, 167.158, 150.278
	$D^*_i(p_{IV,i})$	170.882, 244.971, 219.563	170.852, 244.934, 219.523	170.81, 244.88, 219.47	170.764, 244.826, 219.407	170.735, 244.791, 219.368
	PrEC	0.576	0.593	0.62	0.645	0.663
	$\pi^*_{IV,r}$	26187.69	26189.63	26192.26	26194.55	26195.91
	$\pi^*_{IV,m}$	22016.07	21978.45	21922.78	21868.00	21831.95
	π^*_{IV}	48203.76	48168.08	48115.04	48062.55	48027.86

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
$h_{m,i}$	Iteration	97	104	103	90	97
	$T^*_{IV,i}$	0.702	0.681	0.65	0.629	0.614
	$p^*_{IV,i}$	89.416, 101.891, 76.752	89.490, 101.955, 76.790	89.59, 102.05, 76.84	89.696, 102.136, 76.898	89.761, 102.193, 76.933
	τ^*_i	0.107, 0.115, 0.117	0.101, 0.108, 0.110	0.09, 0.09, 0.10	0.0862, 0.0925, 0.0942	0.0822, 0.0881, 0.0898
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,i}$	53.028, 49.322, 52.754	51.463, 47.870, 51.205	49.36, 45.92, 49.12	47.494, 44.190, 47.274	46.365, 43.143, 46.155
	$Q^*_{f,i}$	123.123, 177.023, 159.132	119.327, 171.585, 154.251	114.23, 164.28, 147.69	109.723, 157.827, 141.900	106.994, 153.918, 138.391
	$D^*_i(p_{IV,i})$	171.166, 245.271, 219.742	171.019, 245.111, 219.629	170.81, 244.88, 219.47	170.606, 244.659, 219.304	170.477, 244.516, 219.198
	PrEC	0.824	0.731	0.62	0.531	0.482
	$\pi^*_{IV,r}$	26111.44	26146.81	26192.26	26230.21	26252.06
	$\pi^*_{IV,m}$	22355.47	22176.25	21922.78	21685.12	21534.30
	π^*_{IV}	48466.91	48323.06	48115.04	47915.33	47786.35

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
$h_{mr,i}$	Iteration	91	85	103	93	92
	$T^*_{IV,i}$	0.658	0.656	0.65	0.650	0.648
	$p^*_{IV,i}$	89.567, 102.035, 76.834	89.578, 102.040, 76.838	89.59, 102.05, 76.84	89.612, 102.055, 76.851	89.623, 102.059, 76.855
	τ^*_i	0.0945, 0.101, 0.103	0.0939, 0.100, 0.102	0.09, 0.09, 0.10	0.0922, 0.0989, 0.100	0.0917, 0.0983, 0.100
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	49.745, 46.278, 49.503	49.589, 46.133, 49.349	49.36, 45.92, 49.12	49.130, 45.707, 48.894	48.979, 45.568, 48.745
	$Q^*_{f,\mathrm{i}}$	115.177, 165.615, 148.900	114.795, 165.077, 148.414	114.23, 164.28, 147.69	113.671, 163.497, 146.986	113.303, 162.980, 146.519
	$D^*_i(p_{IV,i})$	170.864, 244.910, 219.496	170.842, 244.898, 219.483	170.81, 244.88, 219.47	170.775, 244.862, 219.446	170.752, 244.850, 219.434
	PRE	0.639	0.631	0.62	0.608	0.600
	$\pi^*_{IV,r}$	26183.97	26187.32	26192.26	26197.09	26200.26
	$\pi^*_{IV,m}$	21970.67	21951.44	21922.78	21894.33	21875.48
	π^*_{IV}	48154.64	48138.76	48115.04	48091.42	48075.74

Table 7 (continued)

Parameters	Decision Parameters	−50%	−30%	0%	+30%	50%
$c_{mr,i}$	Iteration	98	113	103	91	93
	$T^*_{IV,i}$	0.654	0.654	0.65	0.653	0.652
	$p^*_{IV,i}$	89.598, 102.050, 76.847	89.597, 102.049, 76.846	89.59, 102.05, 76.84	89.593, 102.046, 76.843	89.592, 102.044, 76.842
	τ^*_i	0.0886, 0.0983, 0.101	0.0904, 0.0989, 0.101	0.09, 0.09, 0.10	0.0957, 0.100, 0.102	0.0975, 0.101, 0.102
	θ^*_i	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09	0.07, 0.08, 0.09
	$\theta^*_{m,i}$	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01	0.02, 0.01, 0.01
	$Q^*_{r,\mathrm{i}}$	49.433, 45.988, 49.194	49.403, 45.960, 49.165	49.36, 45.92, 49.12	49.312, 45.877, 49.075	49.282, 45.849, 49.046
	$Q^*_{f,\mathrm{i}}$	114.402, 164.532, 147.920	114.333, 164.432, 147.830	114.23, 164.28, 147.69	114.125, 164.132, 147.561	114.056, 164.032, 147.472
	$D^*_i\left(p_{IV,i}\right)$	170.802, 244.872, 219.456	170.804, 244.875, 219.460	170.81, 244.88, 219.47	170.812, 244.885, 219.470	170.814, 244.888, 219.473
	PrEC	0.595	0.604	0.62	0.634	0.644
	$\pi^*_{IV,r}$	26192.61	26192.47	26192.26	26192.04	26191.89
	$\pi^*_{IV,m}$	23910.25	23115.26	21922.78	20730.31	19935.33
	π^*_{IV}	50102.86	49307.73	48115.04	46922.35	46127.23

Table 8 Sensitivity analysis report of Table 7

Parameters	$T^*_{IV,i}$	$p^*_{IV,i}$	τ^*_i	θ_i	$\theta_{m,i}$	$Q_{r,i}$	$Q_{f,i}$	D^*_i	$PrEC$	$\pi^*_{IV,r}$	$\pi^*_{IV,m}$	π^*_{IV}
b	IS	IS	S	IS	IS	IS	IS	IS	S	IS	IS	IS
α_{1i}	S	IS	S	S	IS	S	S	S	S	S	S	S
β_{1i}	IS	IS	MS	IS	IS	IS	IS	IS	S	S	S	S
α_{2i}	IS	IS	MS	IS	IS	IS	IS	IS	S	IS	S	S
β_{2i}	IS	IS	IS	IS	IS	IS	IS	IS	S	IS	IS	IS
α_i	S	HS	MS	IS	IS	HS	HS	HS	S	HS	HS	HS
β_i	IS	HS	S	IS	IS	S	S	S	S	HS	S	HS
A_r	S	S	MS	IS	IS	HS	HS	S	S	S	S	HS
$a_{r,i}$	S	IS	S	IS	IS	S	S	S	S	S	S	S
$h_{r,i}$	S	S	S	IS	IS	HS	IS	S	HS	HS	S	HS
$c_{r,i}$	S	S	MS	IS	IS	HS	IS	S	HS	HS	HS	S
μ_i	IS	IS	S	S	S	HS	IS	IS	S	S	HS	HS
u_i	IS	IS	S	IS	IS	HS	HS	IS	IS	S	HS	HS
A_m	S	S	MS	IS	IS	HS	HS	S	HS	S	HS	HS
$a_{m,i}$	S	S	S	IS	IS	HS	HS	IS	S	S	HS	HS
$a_{mr,i}$	S	IS	S	IS	IS	HS	HS	IS	S	S	S	S
$h_{m,i}$	S	S	MS	IS	IS	HS	HS	S	S	S	HS	HS
$h_{mr,i}$	IS	IS	MS	IS	IS	S	S	IS	S	S	S	S
$c_{mr,i}$	IS	IS	MS	IS	IS	IS	IS	IS	S	IS	S	HS

S sensitive, *MS* moderately sensitive, *HS* highly sensitive, *IS* insensitive

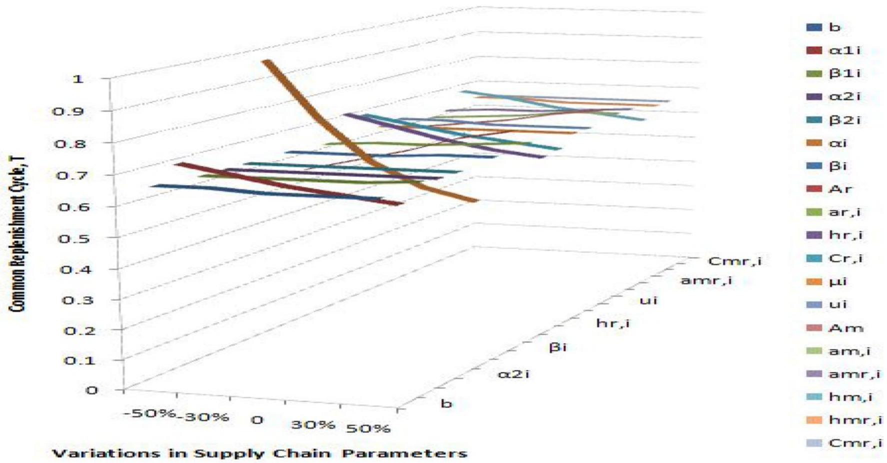

Fig. 7 Changes in common replenishment cycle T with variations in supply chain parameters

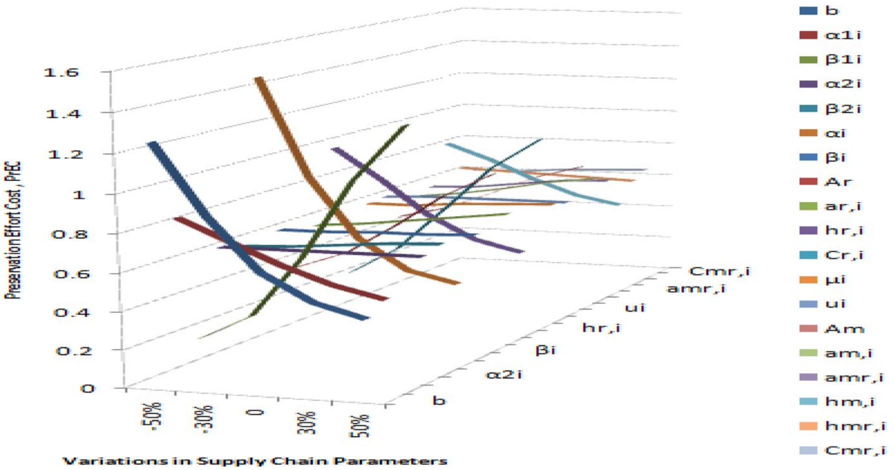

Fig. 8 Changes in preservation effort cost with variations in supply chain parameters

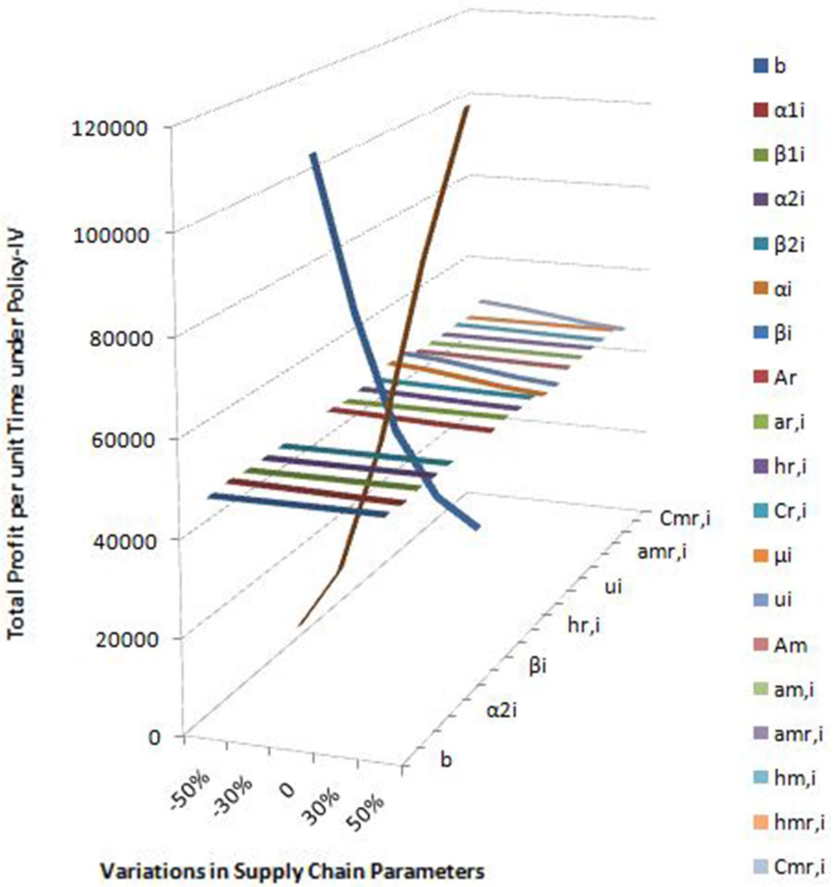

Fig. 9 Changes in total profit per unit time under policy IV with variations in supply chain parameters

Table 9 Analysis of Variance (ANOVA)

Tests of between-subjects effects

Dependent variable: TP

Source	Sum of squares		Degrees of freedom	Mean square	F	Sig.
A_m	Sum of squares due to A_m effects	2787.359	4	696.840	104145.815	0.000
A_r	Sum of squares due to A_R effects	25083.691	4	6270.923	937217.576	0.000
Error	Sum of squares due to errors	0.107	16	0.007	–	–
Total	Total sum of squares	27871.157	24	–	–	–

Table 10 Effect of A_m and A_r on system annual profit per unit time with preservation effort

$A_m \backslash A_r$	260	265	270	275	280
460	48054.23	48031.62	48009.11	47986.69	47964.38
465	48046.68	48024.10	48001.62	47979.24	47956.96
470	48039.15	48016.60	47994.15	47971.80	47949.55
475	48031.62	48009.11	47986.69	47964.38	47942.16
480	48024.10	48001.62	47979.24	47956.96	47934.77

has a maximum value whereas the profit of cooperative model with individual item (policy III) has the minimum value. Similarly, for the preservative effort point of view it is investigated that policy IV has a maximum value whereas policy III has a minimum value. So, it is concluded that channel perspective annual profit for joint items with preservative effort is maximum than any other annual profit of channel for multi-item and multi-echelon supply chain model but the channel perspective model of cooperative replenishment for joint items with mixed integer and preservation effort is 48114.94 which is very much approaching to the profit of policy IV model i.e. 48115.04. Figure 5 shows the profits of the players like retailers and manufacturers with four different policies like I, II, III and IV under the cases like with preservation efforts, without preservation efforts, mixed integer of demand rate and order quantity and preservation efforts, mixed integer of demand rate and preservation efforts and mixed integer of demand rate and without preservation efforts. It is observed that the retailer as an individual player has more profit as compared to manufacturer. This suggests that the retailer will like be the leading player of the business connections of supply chain. To increase manufacturer profit, the strategy of profit sharing for deteriorated items will be profitable to both the players which results the better profit of the integrated supply chain. Figure 6 shows the profits of the four different policies like I, II, III and IV with different cases like with preservation efforts, without preservation efforts, mixed integer of demand rate and order quantity and

Table 11 Analysis of variance (ANOVA)

Tests of between-subjects effects

Dependent variable: TP

Source	Sum of squares		Degrees of freedom	Mean square	F	Sig.
A_m	Sum of squares due to A_m effects	2797.223	4	699.306	105851.158	0.000
A_r	Sum of squares due to A_R effects	25172.012	4	6293.003	952547.190	0.000
Error	Sum of squares due to errors	0.106	16	0.007	–	–
Total	Total sum of squares	27969.341	24	–	–	–

Table 12 Effect of A_m and A_r on system annual profit per unit time without preservation effort

$A_m \backslash A_r$	260	265	270	275	280
460	48051.29	48028.64	48006.09	47983.64	47961.28
465	48043.73	48021.11	47998.59	47976.17	47953.85
470	48036.18	48013.60	47991.11	47968.72	47946.43
475	48028.64	48006.09	47983.64	47961.28	47939.02
480	48021.11	47998.59	47976.17	47953.85	47931.62

preservation efforts, mixed integer of demand rate and preservation efforts and mixed integer of demand rate and without preservation efforts. It is observed that the policy IV has maximum profit as compared to other three policies except the third case. It suggests that cooperative policy for joint deteriorating items gives more profit in discrete demand rate supply chain.

6.2 Sensitivity analysis

It is interesting to investigate the influence of the significant parameters, $b, \alpha_{1i}, \beta_{1i}, \alpha_{2i}, \beta_{2i}, \alpha_i, \beta_i, A_r, a_{r,i}, h_{r,i}, c_{r,i}, \mu_i, u_i, A_m, a_{m,i}, a_{mr,i}, h_{m,i}, h_{mr,i}\, and\, c_{mr,i}$ on policy IV profit model. The computational results of Table 7 are summarized in Table 8. Figure 7 shows the changes in common replenishment cycle T with variations in supply chain parameters and it is observed that the common replenishment cycle T is highly sensitive to the parameters $\alpha_{2i}, A_r, h_{r,i}\, and\, a_{mr,i}$. Figure 8 shows the changes in preservation effort cost with variations in supply chain parameters and it is observed that the preservation effort cost is highly sensitive to the parameters $b, \alpha_{1i}, \beta_{1i}, \alpha_i, h_{r,i}, c_{r,i}, A_m\, and\, h_{m,i}$. Figure 9 shows the changes in total profit per unit time under policy IV with variations in supply chain parameters and it is observed that the total profit per unit time under policy IV is highly sensitive to the parameters $\alpha_{1i}\, and\, \beta_{1i}$.

6.3 Sensitivity analysis through ANOVA testing

The sensitivity of the system profit per unit time with publicity effort with respect to the important parameters like A_m and A_r has been presented using the Analysis of Variance (ANOVA) method. The main conclusions drawn from the sensitivity analysis are as follows:

H_{01} : Null Hypothesis: the optimal joint replenishment profit with preservation effort under co-operative policy is insignificant for different values of $A_m\, and\, A_r$.
H_{11} : Alternative Hypothesis: the optimal joint replenishment profit with preservation effort under co-operative policy is differs significantly for different values of $A_m\, and\, A_r$.

The ANOVA in Table 9 is constructed for the data values of Table 10, it is seen that the p-values of $F_{0.05;5,25}$ and $F_{0.05;5,25}$ (i.e. F-distribution at 5% level), are less than 0.05 respectively. So, the null hypothesis is rejected. Hence the system profit per unit time with preservation effort is significantly differ for different values of $A_m\, and\, A_r$.

The sensitivity of the system profit per unit time without preservation effort with respect to the important parameters like A_m and A_r has been presented using the Analysis of Variance (ANOVA) method. The main conclusions drawn from the sensitivity analysis are as follows:

H_{01} : Null Hypothesis: the optimal joint replenishment profit without preservation effort under co-operative policy is insignificant for different values of $A_m\, and\, A_r$.
H_{11} : Alternative Hypothesis: the optimal joint replenishment profit without preservation effort under co-operative policy is differs significantly for different values of $A_m\, and\, A_r$.

The ANOVA in Table 11 is constructed for the data values of Table 12, it is seen that the p-values of $F_{0.05;5,25}$ and $F_{0.05;5,25}$ (i.e. F-distribution at 5% level), are less than 0.05 respectively. So, the null hypothesis is rejected. Hence the system profit per unit time with preservation effort is significantly differ for different values of $A_m\, and\, A_r$.

7 Conclusion

This study considers four profit optimization models, with discrete demand rate, discrete order quantity and preservation efforts for economies of scale, on the individual and joint items. We explore the preservation efforts benefits the retailer's cooperative replenishment with joint items model but for the channel perspective model preservation efforts play economies of scale where discrete (integer) demand rates benefit the retailers for multi-item integrated mixed integer supply chain model.

Numerical analysis is conducted to clarify managerial insights about the impact of preservation efforts, mixed integer modeling on the decisions and profits of both the parties. From sensitivity analysis it is tested that the system profit per unit time with and without preservation efforts is significant for different values of A_m *and* A_r. The major parameters, α_i, β_i, A_r, $h_{r,i}$, μ_i, u_i, A_m, $a_{m,i}$, $h_{m,i}$ *and* $c_{mr,i}$ on cooperative replenishment for joint items model are highly sensitive for channel profit so for choosing the values of these parameters at managerial level is a very crucial decision for optimizing profit. This work can be extended in several ways like deterioration, backlogging, forward financing, partially forward financing, two components demand etc. for further research work.

References

1. Abad, P.L., Jaggi, C.K.: A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. Int. J. Prod. Econ. **83**, 115–122 (2003)
2. Banu, A., Mondal, S.K.: An integrated inventory model with warranty dependent credit period under two policies of a manufacturer. J. Oper. Res. Soc. India **55**, 677 (2018). https://doi.org/10.1007/s12597-018-0345-x
3. Ben-Khedher, N., Yano, C.A.: The multi-item joint replenishment problem with transportation and container effects. Transp. Sci. **28**(1), 37–54 (1994)
4. Bhattacharya, D.K.: On multi-item inventory. Eur. J. Oper. Res. **162**, 786–791 (2005)
5. Chang, H.J., Hung, C.H., Dye, C.Y.: An inventory model for deteriorating items with linear demand under condition of permissible delay in payments. Prod. Plan. Control **12**, 274–282 (2001)
6. Chen, J.M., Chen, T.H.: The multi-item replenishment problem in a two-echelon supply chain: the effect of centralization versus decentralization. Comput. Oper. Res. **32**, 3191–3207 (2005)
7. Chen, J.M., Chen, T.H.: Effects of joint replenishment and channel coordination for managing multiple deteriorating products in a supply chain. J. Oper. Res. Soc. **56**(10), 1224–1234 (2005)
8. Chen, J.M., Chen, T.H.: The profit maximization model for a multi-item distribution channel. Transp. Res. Part-E Logist Transp. Rev. **43**, 338–354 (2007)
9. Chen, T.H., Chen, J.M.: Optimizing supply chain collaboration based on joint replenishment and channel coordination. Transp. Res. Part E Logist. Transp. Rev. **41**(4), 261–285 (2005)
10. Chung, K.J.: A theorem on determination of economic order quantity under conditions of permissible in payments. Comput. Oper. Res. **25**(1), 49–52 (1998)
11. Cohen, M.A.: Joint pricing and ordering policy for exponentially decaying inventory with known demand. Naval Res. Logist. Q. **24**, 257–268 (1977)
12. Elmaghraby, W., Keskinocak, P.: Dynamic pricing in the presence of inventory considerations: research overview, current practices, and future directions. Manag. Sci. **49**, 1287–3091 (2003)
13. Frohlich, M.T., Westbrook, R.: Demand chain management in manufacturing and services: web-based integration, drivers and performance. J. Oper. Manag. **20**, 729–745 (2002)
14. Fung, R.Y.K., Ma, X.: A new method for joint replenishment problems. J. Oper. Res. Soc. **52**(3), 358–362 (2001)
15. Goyal, S.K.: Economic policy for jointly replenished items. Int. J. Prod. Res. **26**(7), 1237–1240 (1988)
16. Graves, S.C.: On the deterministic demand multi-product single-machine lot scheduling problem. Manag. Sci. **25**(3), 276–280 (1979)
17. Hahm, J., Yano, C.A.: The economic lot and delivery scheduling problem: the common cycle case. IIE Trans. **27**, 113–125 (1995)
18. Hahm, J., Yano, C.A.: The economic lot and delivery scheduling problem: models for nested schedules. IIIE Trans. **27**, 126–139 (1995)
19. Hammer, M.: The superefficient company. Harv. Bus. Rev. **79**, 82–91 (2001)
20. Hsu, P.H., Wee, H.M.: Horizontal suppliers coordination with uncertain suppliers deliveries. Int. J. Oper. Res. **2**(2), 17–30 (2005)

21. Hsu, P.H., Wee, H.M., Teng, H.M.: Preservation technology investment for deteriorating inventory. Int. J. Prod. Econ. **124**, 388–394 (2010)
22. Joneja, D.: The joint replenishment problem: new heuristics and worst case performance bounds. Oper. Res. **38**(4), 711–723 (1990)
23. Kao, E.P.C.: A multi-product dynamic lot-size model with individual and joint set-up costs. Oper. Res. **27**(2), 279–289 (1979)
24. Kaspi, M., Rosenblatt, M.J.: The effectiveness of heuristic algorithms for multi-item inventory systems with joint replenishment costs. Int. J. Prod. Res. **23**(1), 109–116 (1985)
25. Khouja, M.: Optimizing inventory decisions in a multi-stage multi-customer supply chain. Transp. Res. Part E Logist. Transp. Rev. **39**(3), 193–208 (2003)
26. Lee, F.C., Yao, M.J.: A global optimum search algorithm for the joint replenishment problem under power-of-two policy. Comput. Oper. Res. **30**(9), 1319–1333 (2003)
27. Lee, H.L.: The triple-a supply chain. Harv. Bus. Rev. **82**(10), 102–112 (2004)
28. Lee, H.L., Whang, S.: Demand chain excellence. Supply Chain Manag. Rev. **44**(3), 40–46 (2001)
29. Lee, W.: A joint economic lot size model for raw material ordering, manufacturing setup, and finished goods delivering. Omega **33**(2), 163–174 (2005)
30. Li, X., Wang, Q.: Coordination mechanisms of supply chain systems. Eur. J. Oper. Res. **179**(1), 1–16 (2007)
31. Lu, L.: A one-vendor multi-buyer integrated inventory model. Eur. J. Oper. Res. **81**(2), 312–323 (1995)
32. Marn, M.V., Rosiello, R.L.: Managing price gaining profits. Harv. Bus. Rev. **70**(5), 84–94 (1992)
33. Miranda, P.A., Garrido, R.A.: Incorporating inventory control decisions into a strategic distribution network design model with stochastic demand. Transp. Res. Part E Logist. Transp. Rev. **40**(3), 183–207 (2004)
34. Narayanan, V.G., Raman, A.: Aligning incentives in supply chains. Harv. Bus. Rev. **82**(11), 94–102 (2004)
35. Prasad, T.V.S.R.K., Srinivas, K., Srinivas, C.: Decentralized production–distribution planning in multi-echelon supply chain network using intelligent agents. J. Oper. Res. Soc. India **54**, 217 (2017). https://doi.org/10.1007/s12597-016-0277-2
36. Rempala, R.: Joint replenishment multiproduct inventory problem with continuous and discrete demands. Int. J. Prod. Econ. **81–82**, 495–511 (2003)
37. Shinn, S.W.: Determining optimal retail price and lot size under day-term supplier credit. Comput. Ind. Eng. **33**(3–4), 717–720 (1997)
38. Sucky, E.: Coordinated order and production policies in supply chains. OR Spectrum **26**, 493–520 (2004)
39. Taylor, T.A.: Supply chain coordination under channel rebates with sales effort effects. Manag. Sci. **48**(8), 992–1007 (2002)
40. Tsao, Y.C., Sheen, G.J.: A multi-item supply chain with credit periods and weight freight cost discounts. Int. J. Prod. Econ. **135**, 106–115 (2012)
41. Tsao, Y.C., Sheen, G.J.: Joint pricing and replenishment decisions for deteriorating items with lot-size and time dependent purchasing cost under credit period. Int. J. Syst. Sci. **38**(7), 549–561 (2007)
42. Van Eijis, M.J.G.: Multi-item inventory systems with joint ordering and transportation decisions. Int. J. Prod. Econ. **35**, 285–292 (1994)
43. Viswanathan, S.: Note: periodic review policies for joint replenishment inventory systems. Manag. Sci. **43**(10), 1447–1454 (1997)
44. Viswanathan, S.: On optimal algorithms for the joint replenishment problem. J. Oper. Res. Soc. **53**, 1286–1290 (2002)
45. Viswanathan, S., Piplani, R.: Coordinating supply chain inventories through common replenishment epochs. Eur. J. Oper. Res. **129**(2), 277–286 (2001)