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APPLICATION ARTICLE

A production inventory model with deteriorating items and retrial demands

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Abstract

This paper considers a single server perishable inventory system in which customers arrive in a homogeneous Poisson stream. The system has a production unit which produces a single item in an exponentially distributed time interval. At the time of arrival, a customer leads to service if the server is available with a positive level of inventory. Otherwise, the customer goes to a waiting place(orbit) of infinite capacity with pre-determined probability or exits the system with complementary probability. Each customer in the orbit tries to access the server in an exponentially distributed time interval. After every unsuccessful retrial, the customer returns to the orbit with a pre-allotted probability or is lost forever with complementary probability. An algorithmic solution to the problem is obtained using Matrix Analytic Method. The mean number of customer loss before and after entering the system, the rate of successful retrials among overall retrials and some other performance measures of the system are derived. The impacts of system parameters on different measures are numerically studied. A suitable cost function is constructed and the optimum control policy is numerically obtained.

Keywords Matrix analytic method · Perishable inventory · Retrials

Mathematics Subject Classification 60G99 · 60K25 · 90B05

1 Introduction

Most of the existing perishable inventory models in the literature assume that items are purchased from an outside source. Due to this, industries/firms would lose their business without goods/items on hand. Primarily, a firm should be able to meet the demands of customers, by confirming the availability of adequate stock of items, by

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which the firm can suitably avoid loss/backlogged cases. In this work, we propose a production inventory model of deteriorating items. The often quoted review articles [3, 11, 14] give an extensive summary of the modelling of perishable inventory. An inventory system with positive service time and retrial of customers has been received a small scale of attention in the literature. In all the stochastic inventory models prior to Sigman and Simchi-Levi [17], it is assumed that the service time is negligible. This was followed by Berman et al. [2] with an inventory model of deterministic service time. The first published work on retrial inventory is by Artalejo et al. [1]. This paper introduces an alternative to classical approaches based either on backlogged or on lost sale cases. Authors considered a continuous review (s, S) inventory system in which primary customer arrives in stock out period, leaves the server and retries after some random time.

Krishnamoorthy and Viswanath [7] studied a production inventory system where the demand process is assumed to be Poisson. The duration of each service and the time required to produce each item is distributed as exponential random variable. Customers are not allowed to join when the inventory level is zero. Under this assumption, an explicit product form solution for the steady state probability vector is obtained. Ravichandran [15] investigated a continuous review perishable inventory system of (s, S) type with positive lead time. The demands arrive according to a Poisson process. The usable age of items is distributed as Erlangian. Krishnamoorthy and Jose [6] compared three production inventory systems with the assumption that all the underlying processes are independent exponential distribution. Infinite orbit facility is provided for customers who arrive at the stock out period or server busy or buffer full. Each customer retries from the orbit according to linear retrial policy depending on the number of customers in the orbit. Unsuccessful customers may rejoin the orbit or lose forever.

Sivakumar [18] considered a perishable inventory system under continuous review (s, Q) policy with a finite number of demands. The lifetime of each item and lead time are assumed to be exponential. Also, assume that customers who arrive during the stock-out period enter into an orbit and these customers send out signals to access the server. Reshmi and Jose [16] studied a queueing inventory system with perishable items and all underlying processes are assumed as exponential. Items in the inventory perish in a linear rate. Periyasamy [13] analysed a continuous review perishable inventory system with a single server and zero lead time. If the demand occurrs during busy period, it is directed to an obit and may retry from there. Also, the server searches for customers with a pre-assigned probability. Some important joint probability distributions are obtained in the steady state. Yadavalli [20] designed a finite source perishable inventory system with two servers such that one server is always available and the other one undergoes interruptions. Primary customers are directed to orbit with preassigned probabilities and retry to find a free server. Kumar and Elango [8] considered a single server queueing system of perishable items with finite waiting space. All the underlying processes are assumed as exponential. They modelled the problem as a Markov decision problem by using the value iteration algorithm to obtain the minimal average cost of the service. Laxmi and Soujanya [9] studied a perishable system in which customers arrive according to Markovian arrival process. An orbit of finite size is arranged for retrying customers and the server goes for multiple working vacations during stock outs. Melikov and Shahmaliyev [10] developed a model with perishable inventory in which customers are provided the facility of repeated attempts. During system stock out, primary customers either enter the queue or the orbit according to the Bernoulli scheme. Recently, Ko [4] proposed a perishable retrial inventory system with (s, S) control policy. The lead time is assumed to be more generalized phase-type distribution. Krishnamoorthy and Islam [5] introduced perishability in retrial inventory model with a production unit. When the inventory level reaches zero, arriving demands are sent to the orbit with finite capacity and tries for their luck. Customers, who find the orbit full and inventory level zero, lose the system. Demands arriving from the orbital customers are exponentially distributed with a linear rate.

This paper assumes a continuous review perishable inventory system with a production unit and retrial facility. In detail, the model provides a retrial facility so that customer loss during stock out can be greatly reduced. In the real world, most of the inventoried items have a random lifetime. So, we assume that items have an exponential lifetime with a linear rate. If the system has its own production unit then the firm can be smoothly run without shortages.

The remaining portion of the paper is organized as follows: in Sect. 2, mathematical modelling of the system is provided. Sections 3 and 4 discuss the computation of the stationary distribution and some important performance measures respectively. Section 5 deals with the numerical experiment of the effect of parameters on different measures. In Sect. 6, a suitable cost function is defined and the optimal (s, S) pair is obtained. Finally concluding remarks are included in Sect. 7.

2 Modelling and assumptions

Consider an (s, S) production inventory system with perishable items. The lifetime of an item in the inventory is exponentially distributed with parameter $j\omega$, when there are j items in the inventory. Customers arrive at a single server counter according to a Poisson process of rate λ and they demand a single item. If the server is idle at an arrival epoch then that customer is taken for service immediately. Service time duration follows a negative exponential distribution with parameter μ. When on-hand inventory level drops to s, the production is switched to *ON* mode and it continues until the inventory level reaches S. The production process follows an exponential distribution with parameter β and it adds one unit to the inventory at a time. Any arriving customer, when the inventory level zero or server busy, is offered the choice of either to join a waiting space of infinite capacity called orbit with probability γ or to exit the system with probability $1 - \gamma$. All customers who enter the orbit, independently generate requests for service at exponentially distributed time intervals with mean $\frac{1}{\theta}$. The retrial customers who find the inventory level out of stock or server busy, return to the orbit with probability δ and exit the system with probability $1 - \delta$.

Let $N(t)$ and $I(t)$ denote the number of customers in the orbit at time t and the inventory level at time t respectively. Further, let

$$C(t):\begin{cases}0, & \text{if the server idle at time } t\\ 1, & \text{if the server busy at time } t\end{cases}$$

$$K(t):\begin{cases}0, & \text{if the production is OFF at time } t\\ 1, & \text{if the production is ON at time } t\end{cases}$$

Now, $\mathbf{X} = \{(N(t), C(t), K(t), I(t))|t \geq 0\}$ constitutes a continuous time Markov chain with state space $l(0) \cup l(1)$, where

$$l(0) = \{(i,k,0,j)|i \geq 0; k = 0,1; s+1 \leq j \leq S\} \quad \text{and}$$
$$l(1) = \{(i,k,1,j)|i \geq 0; k = 0,1; k \leq j \leq S-1\}$$

In the sequel, $\mathbf{e}$ denotes a column vector of $1'$s of appropriate order and $\mathbf{0}$ denotes a zero matrix of appropriate order. The generator matrix of the process is

$$Q = \begin{bmatrix} A_{10} & A_0 & & & & \\ A_{21} & A_{11} & A_0 & & & \\ & A_{22} & A_{12} & A_0 & & \\ & & A_{23} & A_{13} & A_0 & \\ & & & \ddots & \ddots & \ddots \end{bmatrix}$$

where each element in Q has size $(4S-2s-1) \times (4S-2s-1)$.

Transitions of A_0

- $(i,0,1,0) \xrightarrow{\lambda\gamma} (i+1,0,1,0); i \geq 0$
- $(i,1,0,j) \xrightarrow{\lambda\gamma} (i+1,1,0,j); i \geq 0, s+1 \leq j \leq S$
- $(i,1,1,j) \xrightarrow{\lambda\gamma} (i+1,1,1,j); i \geq 0, 1 \leq j \leq S-1$

Transitions of A_{2i}

- $(i,0,0,j) \xrightarrow{i\theta} (i-1,1,0,j); s+1 \leq j \leq S$
- $(i,0,1,0) \xrightarrow{i\theta(1-\delta)} (i-1,0,1,0)$
- $(i,0,1,j) \xrightarrow{i\theta} (i-1,1,1,j); 1 \leq j \leq S-1$
- $(i,1,0,j) \xrightarrow{i\theta(1-\delta)} (i-1,1,0,j); s+1 \leq j \leq S$
- $(i,1,1,j) \xrightarrow{i\theta(1-\delta)} (i-1,1,1,j); 1 \leq j \leq S-1$

Transitions of A_{1i}

- $(i,0,0,j) \xrightarrow{\lambda} (i,1,0,j); s+1 \leq j \leq S$
- $(i,0,1,j) \xrightarrow{\lambda} (i,1,1,j); 1 \leq j \leq S-1$
- $(i,1,0,j) \xrightarrow{\mu} (i,0,1,j-1); j = s+1$
- $(i,1,0,j) \xrightarrow{\mu} (i,0,0,j-1); s+2 \leq j \leq S$
- $(i,1,1,j) \xrightarrow{\mu} (i,0,1,j-1); 1 \leq j \leq S-1$
- $(i,0,1,j) \xrightarrow{\beta} (i,0,1,j+1); 0 \leq j \leq S-1$
- $(i,0,1,j) \xrightarrow{\beta} (i,0,0,j+1); j = S-1$
- $(i,1,1,j) \xrightarrow{\beta} (i,1,1,j+1); 1 \leq j \leq S-2$
- $(i,0,0,j) \xrightarrow{j\omega} (i,0,0,j-1); s+2 \leq j \leq S$

- $(i,0,0,j) \xrightarrow[j\omega]{(s+1)\omega} (i,0,1,j-1); j = s+1$
- $(i,0,1,j) \xrightarrow{j\omega} (i,0,1,j-1); 1 \leq j \leq S-1$
- $(i,1,0,j) \xrightarrow{(s+1)\omega} (i,1,1,j-1); j = s+1$
- $(i,0,1,j) \xrightarrow{j\omega} (i,1,0,j-1); s+2 \leq j \leq S$
- $(i,1,1,j) \xrightarrow{j\omega} (i,1,1,j-1); 2 \leq j \leq S-1$
- diagonal elements of A_{1i} are the negative of the sum of other elements in the corresponding row of Q.

3 Steady state distribution

The system under consideration is stable, one can verify it by Tweedie [19]. Since $\mathbf{X}$ is a level dependent quasi-birth–death process, to calculate the steady state probability vector, Neuts–Rao [12] truncation method is used. The steady state probability vector $\mathbf{x} = (x_0, x_1, x_2, \dots)$ of Q, where

$$x_i = (\phi_{i,0,0,s+1}, \phi_{i,0,0,s+2} \dots, \phi_{i,0,0,S}, \phi_{i,0,1,0}, \phi_{i,0,1,1}, \dots, \phi_{i,0,1,S-1}, \phi_{i,1,0,s+1}, \phi_{i,1,0,s+2}, \dots, \phi_{i,1,0,S}, \phi_{i,1,1,1}, \phi_{i,1,1,2}, \dots, \phi_{i,1,1,S-1})(i \geq 0)$$

satisfies the relation

$$x_{N+k-1} = x_{N-1} R^k, \ k \geq 1$$

where the matrix R is the unique non-negative solution of the matrix quadratic equation

$$R^2 A_2 + R A_1 + A_0 = \mathbf{0}$$

with $A_1 = A_{1N}, A_2 = A_{2N}$ and $R = \lim_{n \to \infty} R_n$, where $\{R_n\}$ is defined such that $R_{n+1} = -A_0 A_1^{-1} - R_n A_2 A_1^{-1}; n \geq 0$ and $R_0 = \mathbf{0}$. The components $x_0, x_1, \dots, x_{N-1}$ corresponding to boundary portion of Q are obtained using Gauss–Siedel method. Finally, the vector is normalized by dividing $\sum_{i=0}^{\infty} x_i \mathbf{e}$.

4 Performance measures

1. Average inventory level in the system,

$$E_{inv} = \sum_{i=0}^{\infty} \sum_{k=0}^{1} \sum_{j=s+1}^{S} j\phi_{i,k,0,j} + \sum_{i=0}^{\infty} \sum_{k=0}^{1} \sum_{j=1}^{S-1} j\phi_{i,k,1,j}$$

2. Mean number of customers in the orbit,

$$E_{orbit} = \left(\sum_{i=1}^{\infty} i x_i \right) \mathbf{e}$$

3. Average rate at which production is switched *ON*,

$$E_{ON} = \mu \sum_{i=0}^{\infty} \phi_{i,1,0,s+1} + (s+1)\omega \left(\sum_{i=0}^{\infty} \phi_{i,0,0,s+1} + \sum_{i=0}^{\infty} \phi_{i,1,0,s+1} \right)$$

4. Average perishable rate,

$$E_p = \omega \left(\sum_{i=0}^{\infty} \sum_{k=0}^{1} \sum_{j=s+1}^{S} j\phi_{i,k,0,j} + \sum_{i=0}^{\infty} \sum_{k=0}^{1} \sum_{j=1}^{S-1} j\phi_{i,k,1,j} \right)$$

5. Mean number of departures after service completion,

$$E_{ds} = \mu \sum_{i=0}^{\infty} \left(\sum_{j=s+1}^{S} \phi_{i,1,0,j} + \sum_{j=1}^{S-1} \phi_{i,1,1,j} \right)$$

6. Mean number of customers lost before entering the orbit,

$$E_{la} = \lambda(1-\gamma) \sum_{i=0}^{\infty} \left(\phi_{i,0,1,0} + \sum_{j=s+1}^{S} \phi_{i,1,0,j} + \sum_{j=1}^{S-1} \phi_{i,1,1,j} \right)$$

7. Mean number of customers lost due to retrials,

$$E_{lr} = \theta(1-\delta) \sum_{i=1}^{\infty} i \left(\phi_{i,0,1,0} + \sum_{j=s+1}^{S} \phi_{i,1,0,j} + \sum_{j=1}^{S-1} \phi_{i,1,1,j} \right)$$

8. Overall rate of retrials,

$$\theta_1^* = \theta \left(\sum_{i=1}^{\infty} i x_i \right) \mathbf{e}$$

9. Successful rate of retrials,

$$\theta_2^* = \theta \sum_{i=0}^{\infty} i \left(\sum_{j=s+1}^{S} \phi_{i,0,0,j} + \sum_{j=1}^{S-1} \phi_{i,0,1,j} \right)$$

Table 1 Effect of arrival rate λ on various performance measures

λ	E_{inv}	E_{orbit}	E_{ON}	E_p	E_{ds}	E_{la}	E_{lr}	θ_1^*	θ_2^*
2.0	5.2852	0.7461	10.227e−05	1.5856	1.4205	0.2999	0.2796	1.1191	0.4202
2.1	5.1481	0.8183	8.7394e−05	1.5444	1.4626	0.3256	0.3118	1.2274	0.4480
2.2	5.0187	0.8931	7.5099e−05	1.5056	1.5024	0.3519	0.3457	1.3397	0.4753
2.3	4.8966	0.9704	6.4897e−05	1.4690	1.5400	0.3787	0.3814	1.4556	0.5022
2.4	4.7816	1.0501	5.6401e−05	1.4345	1.5754	0.4059	0.4187	1.5751	0.5285
2.5	4.6732	1.1320	4.9298e−05	1.4020	1.6089	0.4336	0.4575	1.6980	0.5543
2.6	4.5712	1.2160	4.3336e−05	1.3714	1.6405	0.4617	0.4978	1.8240	0.5794

$\mu = 3; \omega = 0.3; \beta = 3; \theta = 1.5; \gamma = 0.7; \delta = 0.6$

Table 2 Effect of service rate μ on various performance measures

μ	E_{inv}	E_{orbit}	E_{ON}	E_p	E_{ds}	E_{la}	E_{lr}	θ_1^*	θ_2^*
3.0	5.2852	0.7461	10.227e−05	1.5856	1.4205	0.2999	0.2796	1.1191	0.4202
3.1	5.2334	0.7274	9.7257e−05	1.5700	1.4362	0.2946	0.2692	1.0911	0.4181
3.2	5.1844	0.7097	9.2773e−05	1.5553	1.4511	0.2894	0.2595	1.0646	0.4158
3.3	5.1381	0.6930	8.8745e−05	1.5414	1.4651	0.2845	0.2505	1.0395	0.4133
3.4	5.0942	0.6771	8.5114e−05	1.5283	1.4784	0.2797	0.2419	1.0156	0.4108
3.5	5.0526	0.6619	8.1831e−05	1.5158	1.4909	0.2752	0.2339	0.9929	0.4082
3.6	5.0131	0.6476	7.8853e−05	1.5039	1.5029	0.2708	0.2264	0.9714	0.4055

$\lambda = 2; \omega = 0.3; \beta = 3; \theta = 1.5; \gamma = 0.7; \delta = 0.6$

5 Numerical experiments

In this section, we provide results of numerical illustration that has been carried out for studying the effects of variation of different parameters on various performance measures. Numerical experiments are conducted by considering some artificial data. Assume that the production switch on level, $s = 7$ and the maximum permissible inventory level, $S = 20$. To study the variation of each parameter on system performances, we consider the following cases 5.1 to 5.7 with table representations.

5.1 Effect of the arrival rate λ

As the arrival rate λ increases, the number of customers in the orbit E_{orbit} also increases which in turn leads to the lost of arriving customers as well as retrying customers. The increase in E_{orbit} results in the increase of E_{ds}, θ_1^* and θ_2^* (see Table 1). The decrease in the expected inventory level can be seen due to a decrease in expected production switching rate.

Table 3 Effect of perishable rate ω on various performance measures

ω	E_{inv}	E_{orbit}	E_{ON}	E_p	E_{ds}	E_{la}	E_{lr}	θ_1^*	θ_2^*
0.1	11.427	0.7121	29674e−06	1.1427	1.4492	0.2903	0.2604	1.0681	0.4171
0.2	7.6916	0.7219	2362e−06	1.5383	1.4410	0.2931	0.2659	1.0828	0.4180
0.3	5.2852	0.7461	102.27e−06	1.5856	1.4205	0.2999	0.2796	1.1191	0.4202
0.4	4.0548	0.7769	5.8595e−06	1.6219	1.3945	0.3085	0.2970	1.1653	0.4229
0.5	3.3252	0.8093	0.4758e−06	1.6626	1.3671	0.3176	0.3153	1.2140	0.4256
0.6	2.8426	0.8417	0.0052e−06	1.7056	1.3401	0.3264	0.3338	1.2618	0.4281
0.7	2.4994	0.8716	0.0008e−06	1.7496	1.3143	0.3348	0.3509	1.3075	0.4302

$\lambda = 2;\mu = 3;\beta = 3;\theta = 1.5;\gamma = 0.7;\delta = 0.6$

Table 4 Effect of replenishment rate β on various performance measures

β	E_{inv}	E_{orbit}	E_{ON}	E_p	E_{ds}	E_{la}	E_{lr}	θ_1^*	θ_2^*
2.6	4.0925	0.7878	1.1145e−05	1.2278	1.3851	0.3112	0.3038	1.1817	0.4222
2.7	4.3822	0.7746	2.0411e−05	1.3147	1.3963	0.3076	0.2961	1.1619	0.4218
2.8	4.6781	0.7634	3.6082e−05	1.4034	1.4058	0.3046	0.2896	1.1451	0.4212
2.9	4.9794	0.7540	6.1702e−05	1.4938	1.4138	0.3021	0.2841	1.1310	0.4207
3.0	5.2852	0.7461	10.227e−05	1.5856	1.4205	0.2999	0.2796	1.1191	0.4202
3.1	5.5947	0.7395	16.458e−05	1.6784	1.4261	0.2981	0.2758	1.1092	0.4198
3.2	5.9071	0.7341	25.758e−05	1.7721	1.4307	0.2966	0.2727	1.1011	0.4193

$\lambda = 2;\mu = 3;\omega = 0.3;\theta = 1.5;\gamma = 0.7;\delta = 0.6$

5.2 Effect of the service rate μ

Intuitively, as the service rate increases lead to a greater number of service completion. Therefore, E_{ds} also increases and the number of customers in the orbit E_{orbit} decreases. The overall and successful rate of retrials decreases because E_{orbit} is decreasing. Expected inventory level E_{inv} get decreased when more and more customers get served, leading to a decrease in E_p. So the production process need not have to switch *ON* frequently. The number of unsatisfied customers decrease, that is E_{la} and E_{lr} in Table 2 support the intuition.

5.3 Effect of the perishable rate ω

When decay rate increases, obviously E_p increases, which leads to decrease in expected inventory level E_{inv} as well as in expected departure from service E_{ds}. The production switch on rate is decreasing but it is very negligible. As E_{inv} decreases, more customers joins the orbit ie E_{orbit} increases. When E_{orbit} increases, we expect increase in measures like E_{la}, E_{lr}, θ_1^* and θ_2^*. Table 3 supports these intuitions.

Table 5 Effect of retrial rate θ on various performance measures

θ	E_{inv}	E_{orbit}	E_{ON}	E_p	E_{ds}	E_{la}	E_{lr}	θ_1^*	θ_2^*
1.1	5.4232	0.7434	1.1576e−04	1.6270	1.3781	0.2894	0.3325	0.81775	0.3427
1.2	5.4325	0.6814	1.1712e−04	1.6298	1.3752	0.2887	0.3360	0.81773	0.3377
1.3	5.4416	0.6290	1.1842e−04	1.6325	1.3725	0.2881	0.3394	0.81770	0.3328
1.4	5.4503	0.5841	1.1969e−04	1.6351	1.3698	0.2875	0.3427	0.81768	0.3281
1.5	5.4588	0.5451	1.2091e−04	1.6376	1.3672	0.2869	0.3459	0.81765	0.3235
1.6	5.4671	0.5110	1.2210e−04	1.6401	1.3647	0.2863	0.3490	0.81763	0.3191
1.7	5.4751	0.4809	1.2325e−04	1.6425	1.3623	0.2858	0.3520	0.81760	0.3148

$\lambda = 2;\mu = 3;\omega = 0.3;\beta = 3;\gamma = 0.7;\delta = 0.3$

Table 6 Effect of probability γ on various performance measures

γ	E_{inv}	E_{orbit}	E_{ON}	E_p	E_{ds}	E_{la}	E_{lr}	θ_1^*	θ_2^*
0.1	5.9509	0.0840	1.9354e−05	1.7853	1.2167	0.7552	0.0281	0.1261	0.0558
0.2	5.8444	0.1748	1.7529e−05	1.7533	1.2492	0.6912	0.0596	0.2622	0.1132
0.3	5.7358	0.2727	1.5829e−05	1.7207	1.2824	0.6228	0.0948	0.4091	0.1721
0.4	5.6252	0.3784	1.4252e−05	1.6876	1.3163	0.5497	0.1341	0.5675	0.2324
0.5	5.5131	0.4922	1.2794e−05	1.6539	1.3506	0.4717	0.1777	0.7382	0.2939
0.6	5.3996	0.6146	1.1454e−05	1.6199	1.3854	0.3885	0.2261	0.9219	0.3566
0.7	5.2852	0.7461	1.0227e−05	1.5856	1.4205	0.2999	0.2796	1.1191	0.4202

$\lambda = 2;\mu = 3;\omega = 0.3;\beta = 3;\theta = 1.5;\delta = 0.6$

5.4 Effect of the replenishment rate β

As the replenishment rate β increases, the expected inventory E_{inv} increases and hence the expected perishable rate E_p increases. The production switch on rate also increases with increase in β. When the inventory available to customers increases the service completion becomes faster, so E_{ds}. Accordingly, expected number of customers in the orbit E_{orbit} decreases, due to this, the measures $E_{la}, E_{lr}, \theta_1^*$ and θ_2^* decreases (see Table 4).

5.5 Effect of the retrial rate θ

As retrial rate θ increases, one would expect decrease in expected number of customers in the orbit E_{orbit}. Which is the reason for decrease in E_{ds}, θ_1^* and θ_2^*. As the production switch on rate increases, expected inventory level E_{inv} and E_p increases. When θ increases, the number of service completion increases, that is E_{ds}. The decrease in E_{la} is very negligible because E_{inv} is increasing. From Table 5, as θ increases most of the retrying customers fail to access a free server so E_{lr} increases.

5.6 Effect of the probability γ

When the probability γ increases, unsatisfied customers move to orbit, hence E_{orbit} increases. This in turns leads to the reduced loss of customers upon arrival, E_{la} decreases. As E_{orbit} increases retrials become unsuccessful that force to increase in E_{lr}. As E_{orbit} increases, we expect increase in E_{ds}, θ_1^* and θ_2^*. Table 6 supports these intuitions. As expected production switch on rate decreases, inventory level also decreases which leading to a decrease in E_p.

5.7 Effect of the probability δ

As δ increases, the unsuccessful retrying customers return to the orbit faster, so E_{orbit} increases. This leads to the decrease in expected loss of retrying customers. Since the number of orbiting customers increases it makes the server busy so the expected loss upon arrival E_{la} increases. The increase in E_{orbit} leads to increase in E_{ds}, θ_1^* and θ_2^*. From Table 7, the production switch on rate increases with increase in δ which results the increase in E_{inv}.

6 Cost analysis

The objective is to obtain an adaptive (s, S) policy subject to some cost criteria. Since the objective cost function is not known explicitly, we define it as a combination of relevant system characteristics. One can determine the optimum values of (i) s, the point at which 'switch ON' the production unit and (ii) S, the amount of inventory to be stored by minimizing the total cost. For this, the long-run cost function for this model is defined as

$$CF = k_1 * E_{ON} + k_2 E_{inv} + k_3 E_{orbit} + k_4(E_{la} + E_{lr}) + k_5 E_{ds} + k_6 E_p,$$

where k_1 = production switch on cost per unit per unit time; k_2 = inventory holding cost per unit per unit time; k_3 = customer holding cost per unit per unit time;

Table 7 Effect of probability δ on various performance measures

δ	E_{inv}	E_{orbit}	E_{ON}	E_p	E_{ds}	E_{la}	E_{lr}	θ_1^*	θ_2^*
0.1	5.5341	0.4654	13.003e−05	1.6602	1.3442	0.2814	0.3745	0.6981	0.2820
0.2	5.4992	0.5019	12.572e−05	1.6498	1.3548	0.2839	0.3612	0.7528	0.3012
0.3	5.4588	0.5451	12.091e−05	1.6376	1.3672	0.2869	0.3459	0.8177	0.3235
0.4	5.4114	0.5975	11.550e−05	1.6234	1.3818	0.2904	0.3278	0.8962	0.3498
0.5	5.3547	0.6625	10.935e−05	1.6064	1.3992	0.2947	0.3062	0.9937	0.3813
0.6	5.2852	0.7461	10.227e−05	1.5856	1.4205	0.2999	0.2796	1.1191	0.4202
0.7	5.1969	0.8591	9.3969e−05	1.5591	1.4477	0.3067	0.2456	1.2887	0.4699

$\lambda = 2; \mu = 3; \omega = 0.3; \beta = 3; \theta = 1.5; \gamma = 0.7$

Table 8 Effect of s and S on total cost

$S \backslash s$	1	2	3	4	5	6	7
14	332.67	331.05	329.13	327.67	330.24	333.08	335.87
15	331.34	330.36	328.25	326.85	329.50	332.09	334.46
16	330.26	329.54	327.69	325.12	328.37	331.50	332.85
17	329.37	328.46	326.91	324.35	327.37	330.46	331.13
18	328.75	327.15	325.77	**323.46**	326.92	329.68	330.31
19	330.02	328.93	327.11	325.01	327.90	330.76	332.22
20	331.81	330.39	328.52	326.95	328.85	331.83	333.90
21	332.98	331.27	329.28	328.25	329.98	333.03	335.38
22	333.78	332.49	331.00	330.40	331.25	334.91	337.04

Bold value indicate the minimal cost for the optimal (s, S) pair

k_4 = cost of customer loss per unit per unit time; k_5 = cost due to service per unit per unit time; k_6 = cost of decay per unit per unit time.

6.1 Optimal (s, S) pair

This section explores the behaviour of the cost function by varying s and S, fixing other parameters fixed. Assume the parameter values as $\lambda = 2; \mu = 3; \omega = 0.3; \beta = 3; \theta = 1.5; \gamma = 0.7; \delta = 0.6$ and the different cost assumed are $k_1 = 110.5; k_2 = 2.5; k_3 = 20; k_4 = 2, k_5 = 1; k_6 = 1.3$. Using the above defined cost function, the total cost is tabulated for some set of (s, S) pair. From Table 8, the optimal (s, S) pair is (4, 18) and the corresponding optimal cost is 323.46.

7 Concluding remarks

In this paper, we studied a perishable inventory system with an infinite orbit for accommodating retrial customers. Exponential distribution is considered for inter-arrival time as well as the service time. The production process added single item exponentially to the inventory and is governed by an (s, S) policy. The customer would be allowed to join the orbit if the inventory level zero or server busy. Matrix Geometric Method is used to find the stationary probability vector, which make it easier to obtain some key performance measures. A suitable cost function is constructed and the optimal (s, S) pair is obtained. The results are numerically illustrated to show the effect of change of values of parameters. Furthermore, extended works of this model can be done by considering, a finite buffer or varying production rate, vacation to the server, etc.

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