THEORETICAL ARTICLE

Joint pricing, advertisement, preservation technology investment and inventory policies for non-instantaneous deteriorating items under trade credit

Chaitanyakumar N. Rapolu[1] · Deepa H. Kandpal[1]

Abstract

To maximize profit in a competitive market environment, for retailers, it became necessary to optimize preservation, pricing, and marketing strategies together with inventory ordering policies. This study deals with the problem of optimizing price, advertisement frequency, preservation technology (PT) investment and ordering policies simultaneously for non-instantaneous deteriorating items whose deterioration rate can be reduced by investing in PT, while demand depends on both selling price and frequency of advertisement. The supplier allows some credit period to settle the account, and under this policy, three possible cases considered separately. We adopt three-parameter Weibull distribution deterioration and partial backlogs of shortages in a general framework to formulate the model. An iterative algorithm is provided to obtain the optimal solution, then the proposed model is illustrated through numerical examples. The concavity of the total profit function with respect to decision variables shown graphically. Sensitivity analysis has been conducted to investigate the impact of each parameter. PT investment and credit period are beneficial for the retailer, and also can earn more profit through advertisement. Value-added food products, such as bottled fruit juice, soft drinks, packed fruits, bread, cake, processed meat, etc., needs preservation technology and their demand depends on the price as well as marketing. Profit maximization of such items can be studied with the help of new model developed in this paper.

Keywords Inventory · Trade credit · Preservation technology investment · Non-instantaneous deterioration · Frequency of advertisement

✉ Chaitanyakumar N. Rapolu
   rapolu123@gmail.com

   Deepa H. Kandpal
   dhk052000@yahoo.com

[1] Department of Statistics, The Maharaja Sayajirao University of Baroda, Vadodara, Gujarat 390 002, India

✆ Springer

1 Introduction

Deteriorating inventory systems are studied extensively in the past few years. In traditional economic order quantity (EOQ) model the deteriorating nature of the items was not considered. But, deterioration is a characteristic of almost all commodities, which means damage, decay, spoilage, loss of utility, vaporization, etc. Nahmias [30], Rafat [33], Goyal and Giri [15], Li et al. [23], Bakker et al. [1], and Janssen et al. [19] time to time provided up to the date literature review on deteriorating inventory problems. Ghare and Schreder [13] proposed an exponential deteriorating inventory model with constant demand. Covert and Philip [5] derived an economic order quantity model for items having two-parameter Weibull distribution deterioration with constant demand, and Philip [32] extended the work of Covert and Philip to the case of three-parameter Weibull distribution. Most of the earlier developed models assumed constant demand, but several factors affect the demand. Selling price and advertisement are the major factors for the demand. Cohen [4], Mukhopadhyay et al. [29], Dye [7], Maihami and Kamalabadi [26] and Mahmoodi [25], considered price-dependent demand and derived joint pricing and replenishment policies for deteriorating inventory systems. Together with price, advertisement also plays a very crucial role in sales. A regular frequency of advertisement through different mediums such as banners, newspaper, magazine, internet, radio, and television significantly increase the demand of the product. Till now, very few researchers studied the effect of advertisement policies on inventory. Kotler [20] first incorporated marketing policies into inventory and derived an optimal marketing policy but not EOQ. Subramanyam and Kumaraswamy [39] further extended the problem of inventory by incorporating demand as a function of price and frequency of advertisement. Also Urban [41], Bhunia and Maiti [3], Goyal and Gunasekaran [16] and Pal et al. [31] studied the impact of pricing and advertisement policies on inventory policies.

Most of the researchers consider that the deterioration rate is exogenous variable, which is not subject to control. But in practice, this phenomenon can be controlled and reduced by procedural changes, cooling storages or specialized types of equipment. When deterioration rate of a product is significantly high, it is possible to reduce the economic losses due to deterioration by investing in PT. Hsu et al. [18] first proposed a model incorporating preservation technology investment to reduce the deterioration. They considered that the reduced deterioration rate is a function of PT investment cost and the resultant deterioration rate is the difference between the original deterioration rate and reduced deterioration rate. After that, Lee and Dye [21], Dye and Hsieh [9], Hsieh and Dye [17] and Mishra [27], etc. studied the effect of preservation technology investment on inventory policies assuming controllable deterioration situation through investing in PT. Zang et al. [43] designed an effective algorithm to address the problem of pricing, preservation technology investment, and inventory control for deteriorating items. Liu et al. [24], Dye and Yang [10], Dye et al. [11] and Zang et al. [44] studied the joint pricing and preservation technology investment strategies. They assumed that the deterioration process starts as soon as the commodities enter

into the inventory system. But, some commodities such as fruits, vegetables, processed foodstuffs, etc. does not deteriorate at the beginning stage, they maintain fresh quality for some time duration. This phenomenon of products is known as non-instantaneous-deterioration. Dye [8], Mishra [28], Tsao [40], and Bardhan et al. [2] studied the effect of PT investment on inventory for non-instantaneous deteriorating items. Li et al. [22] presented a model for joint pricing, PT investment for non-instantaneous deteriorating items. Shah et al. [37] derived a model to maximize profit through optimizing price, advertisement frequency and inventory ordering policies.

One of the best practices in businesses is the supplier allows some credit period to the retailer to settle the account. If the retailer can't settle the account before the given credit period, then the supplier will charge interest at some rate on the remaining amount. This policy is beneficial for both the supplier and retailer. By allowing the credit period, the supplier can increase sales and potential customers by attracting and motivating new customers. The retailer can take advantage of it because it is not always true that the retailer has adequate capital. Also, the retailer can earn interest on sales revenue till settlement. Goyal [14] derived economic order quantity considering permissible delay in payments. Dave [6] considered delay in payments for deteriorating items. Geetha and Uthayakumar [12] proposed a model for non-instantaneous deteriorating items with permissible delay in payments. Shaikh et al. [38] presented an EOQ model with PT, trade credit and partial backlogging for deteriorating items. Yang et al. [42] studied the joint problem of preservation technology investment and trade credit as a dynamic programming problem. Shah et al. [36] developed a joint pricing, PT investment model under two level trade credit financing. Rathore [34] proposed a deteriorating inventory model considering time, price and advertisement dependent demand, with preservation technology investment and trade credit. A brief summary of the literature review is given in Table 1.

Researchers are still studying more and more practical inventory systems to fit into real-world situations, and they are trying to optimize all possible strategies to minimize cost or maximize profit. Recently, Sarkar [35] proposed an EPQ model for better management of defective items in a multistage production system by reworking imperfect items. He developed the model for manufacturing unit wherein imperfect products are inspected in each stage or at the end of the cycle, and they are reworked. So there is no wastage of the products and which helps in minimizing the cost. While in our paper, we have developed the model for perishable products by investing in preservation technology. Which helps in reducing the wastage and that way our profit is maximized.

In our model, we optimize price, advertisement frequency, preservation technology investment, together with inventory ordering policies under trade credit. Till now, no study deals with the optimization of all these major strategies together. But, our study is closely related to Rathore [34]. Our work differs from his work, mainly in five aspects. First, he considered the frequency of advertisement as a continuous variable, while in our model, we considered the frequency of advertisement as a positive integer which is more realistic. Also, we provided an iterative algorithm to obtain the optimal frequency of advertisement. Second, in his model price is not a decision variable, while in our model, price

Table 1 Brief literature review

#	Inventory model	Deterioration	Non-instantaneous deterioration	Preservation technology investment	Price dependent demand	Advertisement dependent demand	Shortages	Trade credit
[2]	EOQ	✓	✓	✓				
[3]	EOQ	✓			✓	✓	✓	
[4]	EOQ	✓			✓		✓	
[5]	EOQ	✓						
[6]	EOQ	✓						✓
[7]	EOQ	✓			✓		✓	
[8]	EOQ	✓	✓	✓			✓	
[9]	EOQ	✓		✓				✓
[10]	EOQ	✓		✓	✓			
[11]	EOQ	✓		✓	✓			
[12]	EOQ	✓	✓				✓	✓
[13]	EOQ	✓						✓
[14]	EOQ	✓						
[16]	EPQ	✓			✓	✓		
[17]	EPQ	✓						
[18]	EOQ	✓		✓				
[21]	EOQ	✓		✓				
[22]	EOQ	✓	✓	✓	✓		✓	
[24]	EOQ	✓		✓	✓		✓	
[25]	EOQ	✓			✓		✓	
[26]	EOQ	✓	✓		✓		✓	
[27]	EOQ	✓		✓			✓	
[28]	EOQ	✓		✓				
[29]	EOQ	✓			✓	✓		
[31]	EOQ	✓			✓		✓	

Table 1 (continued)

#	Inventory model	Deterioration	Non-instantaneous deterioration	Preservation technology investment	Price dependent demand	Advertisement dependent demand	Shortages	Trade credit
[34]	EOQ	✓		✓	✓	✓		✓
[36]	EOQ	✓		✓	✓			✓
[37]	EOQ	✓	✓		✓	✓		
[38]	EOQ	✓		✓			✓	✓
[40]	EOQ	✓	✓	✓				✓
[41]	EOQ	✓		✓	✓	✓		
[42]	EOQ	✓		✓				✓
[43]	EOQ	✓		✓	✓			
[44]	EOQ	✓		✓	✓			
This model	EOQ	✓	✓	✓	✓	✓	✓	✓

is one of the decision variables. Third, his model is developed for instantaneous deterioration case, while we considered non-instantaneous deterioration which is also applicable for instantaneous deterioration case by fixing no deterioration period zero. Fourth, in his model, shortages are not allowed, while in our model, shortages are allowed and partially backlogged. Fifth, in his model, deterioration cost is constant, while we considered three-parameter Weibull distribution deterioration.

2 Notations

C_O	Ordering cost (\$/order).
C	Purchase cost (\$/unit).
C_d	Deterioration cost (\$/unit).
C_h	Holding cost (\$/unit/unit time).
C_a	Advertisement cost (\$/advertisement).
C_s	Lost sale cost (\$/unit).
P	Selling price (\$/unit).
T_d	Length of time during which there is no deterioration.
T_1	Time point at which inventory becomes zero.
T	Length of inventory order cycle.
M	Credit period to settle the account.
I_c	Rate of interest charged on the remaining amount.
I_e	Rate of interest earned on sales revenue.
A	Frequency of advertisement per cycle.
$D(A,P)$	Demand function.
a, b, m	Demand parameters.
α, β	Deterioration parameters.
$I_1(t)$	Inventory level during $t \in [0, T_d]$.
$I_2(t)$	Inventory level during $t \in [T_d, T_1]$.
$I_3(t)$	Inventory level during $t \in [T_1, T]$.
I_0	Initial inventory.
δ	Inventory Backlog coefficient.
I_B	Inventory Backlogged.
ξ	Preservation technology investment per unit time.
$m(\xi)$	Proportion of reduced deterioration rate ($0 \leq m(\xi) \leq 1$).
k	PT investment coefficient.
ξ'	Maximum PT investment budget.
Q.	Order quantity per order cycle T.
$TP_i(A, T_1, T, P, \xi)$.	Total profit function for case i, ($i = 1, 2, 3$).
$(A^*, T_1^*, T^*, P^*, \xi^*)$	Optimal solution.

3 Assumptions

- The inventory system involves a single non-instantaneous deteriorating item.
- Demand is a function of selling price and advertisement frequency. We assume the demand function as (described by Kotler [20]) follows:

$$D(A, P) = A^m a P^{-b}$$

  where $A(> 0)$ is the frequency of advertisement, $P(> C)$ is the selling price, $a(> 0)$ is the scaling factor, $b(\geq 1)$ is the index of price elasticity and m is the shape parameter, where $0 \leq m < 1$. Since $\frac{\partial D(A,P)}{\partial A} > 0$ and $\frac{\partial D(A,P)}{\partial P} < 0$, the demand function is an increasing function of the advertisement frequency (A) and decreasing function of price (P), which reflect a real situation.

- The lifetime (t) of the product follows three-parameter Weibull distribution $f(t) = \alpha\beta(t - T_d)^{\beta-1}e^{-\alpha(t-T_d)^\beta}$, where $\alpha(> 0)$ is the scale parameter, $\beta(> 0)$ is the shape parameter and $T_d(\geq 0)$ (deterioration free life) is the location parameter. The cumulative distribution function is $F(t) = \int_{T_d}^{t} f(t)dt = 1 - e^{-a(t-T_d)^\beta}$, hence the deterioration rate is $\frac{f(t)}{1-F(t)} = \alpha\beta(t - T_d)^{\beta-1}$.

- The deterioration rate can be reduced by investing in preservation technology. The proportion of reduced deterioration rate (as in Hsu et al. [18]) is $m(\xi) = 1 - e^{-k\times\xi}$, where, $k(\geq 0)$ is the simulation coefficient representing the percentage increase in $m(\xi)$ per dollar increase in ξ. When $\xi = 0$, the reduced deterioration rate $m(\xi) = 0$, and for $\xi \to \infty$, $\lim_{\xi\to\infty} m(\xi) = 1$. But we set constraint $0 \leq \xi \leq \xi'$, where, ξ' is the maximum PT investment allowed.

- Instantaneous replenishment and infinite replenishment rate.

- Shortages are allowed and partially backlogged. The fraction of unsatisfied demand backlogged is $D(A, P)e^{-\delta(T-t)}$ for $t \in [T_1, T]$, where backlogging parameter δ is a positive constant and $(T - t)$ is the waiting time.

- The supplier will allow some credit period to the retailer, and the retailer must settle the account before placing the next order.

4 Model development

As shown in Fig. 1, initially the inventory system has I_0 units. During the time interval $[0, T_d]$ there will be no deterioration and hence the inventory level decrease in this period due to demand only. During the interval $[T_d, T_1]$ the inventory level decrease due to demand and as well as deterioration, but in this period the deterioration rate will be reduced by investing in preservation technology. At time T_1 the inventory reaches to zero and the demand will be partially backlogged during $[T_1, T]$. If the supplier allow credit period M units of the time to settle the account then three cases are possible (1) $0 \leq M \leq T_d$, (2) $T_d \leq M \leq T_1$ and (3) $T_1 \leq M \leq T$ (See Figs. 2, 3 and 4).
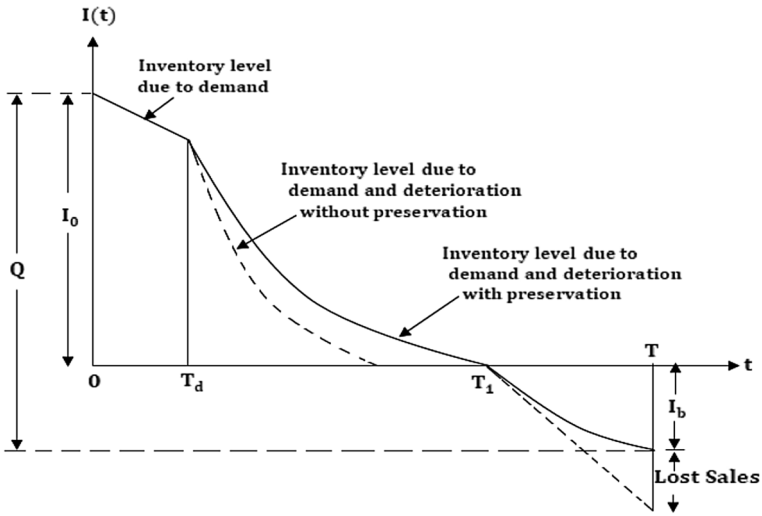
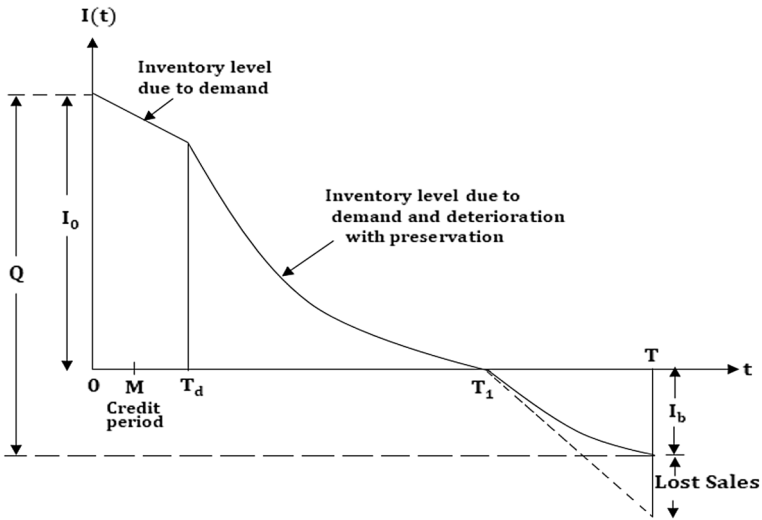**Fig. 1** Graphical representation of the inventory system

Fig. 2 $0 \le M \le T_d$

According to the above description, the differential equations representing the inventory status within different time intervals given below.

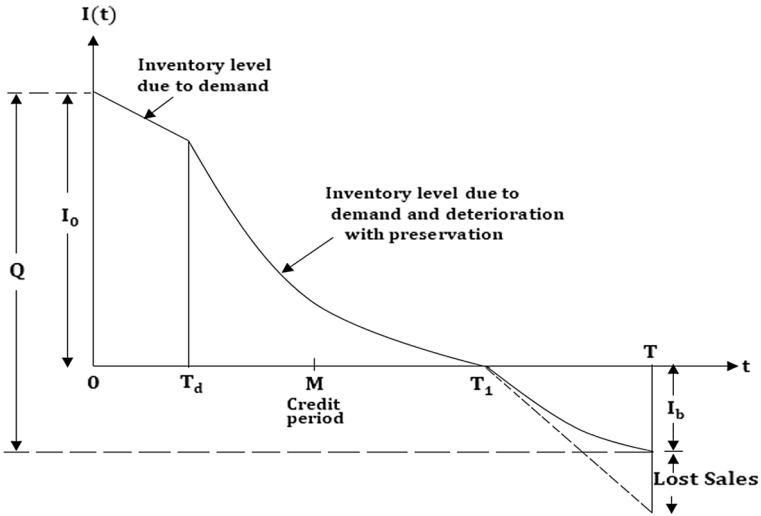$$\frac{dI_1(t)}{dt} = -D(A, P), \quad 0 \le t \le T_d \tag{1}$$
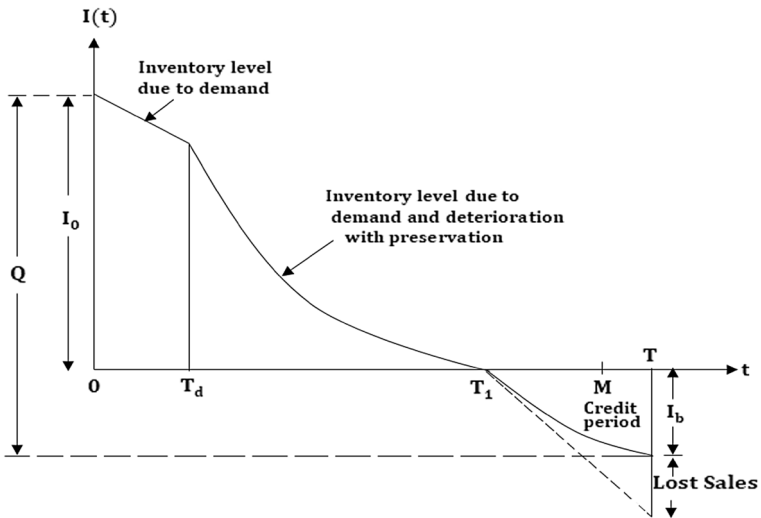
Fig. 3 $T_d \le M \le T_1$

Fig. 4 $T_1 \le M \le T$

$$\frac{dI_2(t)}{dt} = -\alpha\beta\left(t - T_d\right)^{\beta-1}(1 - m(\xi))I_2(t) - D(A, P), T_d \le t \le T_1 \tag{2}$$

$$\frac{dI_3(t)}{dt} = -D(A, P)e^{-\delta(T-t)}, \quad T_1 \le t \le T \tag{3}$$

Using the boundary conditions $I_1(0) = I_0, I_2(T_1) = 0$ and $I_3(T_1) = 0$, we get the solution of Eqs. (1), (2) and (3) respectively as follows.

$$I_1(t) = -D(A, P)t + I_0 \tag{4}$$

$$I_2(t) = D(A, P)\left[T_1 - t + \frac{\alpha(1 - m(\xi))}{(\beta + 1)}\left\{(T_1 - T_d)^{(\beta+1)} - (T - T_d)^{(\beta+1)}\right\}\right]$$
$$\times \left[1 - \alpha(1 - m(\xi))(t - T_d)\right] \tag{5}$$

$$I_3(t) = \frac{-D(A, P)}{\delta}\left[e^{-\delta(T-t)} - e^{-\delta(T-T_1)}\right] \tag{6}$$

Using the condition $I_1(T_d) = I_2(T_d)$

$$I_0 = D(A, P)\left[T_1 + \frac{\alpha(1 - m(\xi))}{(\beta + 1)}(T_1 - T_d)^{(\beta+1)}\right] \tag{7}$$

The maximum amount of demand backlogged per cycle is obtained by putting $t = T$ in Eq. (6) and considering positive quantity.

$$I_B = \frac{D(A, P)}{\delta}\left[1 - e^{-\delta(T-T_1)}\right] \tag{8}$$

Order quantity per cycle:

$$Q = I_0 + I_B = D(A, P)\left[T_1 + \frac{\alpha(1 - m(\xi))}{\beta + 1}(T_1 - T_d)^{\beta+1} + \frac{1}{\delta}\left[1 - e^{-\delta(T-T_1)}\right]\right] \tag{9}$$

Purchase cost:

$$PC = C \cdot Q \tag{10}$$

Lost sale cost:

$$LSC = C_s \int_{T_1}^{T} \left[D(A, P) - D(A, P)e^{-\delta(T-t)}\right]dt$$
$$= C_s D(A, P)\left[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T-T_1)}}{\delta}\right] \tag{11}$$

Deterioration cost:

$$DC = C_d \left[I_2(T_d) - \int_{T_d}^{T_1} D(A,P)dt \right]$$

$$= \frac{C_d D(A,P)\alpha(1 - m(\xi))}{\beta + 1} (T_1 - T_d)^{\beta+1} \tag{12}$$

Holding cost:

$$HC = C_h \left[\int_0^{T_d} I_1(t)dt + \int_{T_d}^{T_1} I_2(t)dt \right]$$

$$= C_h D(A,P) \left[\frac{T_1^2}{2} + \frac{\alpha(1 - m(\xi))}{\beta + 1} T_d (T_1 - T_d)^{\beta+1} + \frac{\alpha\beta(1 - m(\xi))}{(\beta + 1)(\beta + 2)} (T_1 - T_d)^{\beta+2} \right.$$

$$\left. - \frac{\alpha^2(1 - m(\xi))^2}{2(\beta + 1)^2} (T_1 - T_d)^{2(\beta+1)} \right] \tag{13}$$

Total sales revenue:

$$SR = P \left[\int_0^{T_1} D(A,P)dt + \int_{T_1}^{T} D(A,P)e^{-\delta(T-t)}dt \right]$$

$$= PD(A,P) \left[T_1 + \frac{1}{\delta} \left(1 - e^{-\delta(T-T_1)} \right) \right] \tag{14}$$

Preservation technology investment:

$$PTI = (T_1 - T_d)\xi \tag{15}$$

Advertisement cost:

$$AC = C_a A \tag{16}$$

Case 1 $0 \leq M \leq T_d$
Interest Charged:

$$IC_1 = CI_c \left[\int_M^{T_d} I_1(t)dt + \int_{T_d}^{T_1} I_2(t)dt \right]$$

$$= CI_c D(A,P) \left[\frac{(T_1 - M)^2}{2} + \frac{\alpha(1 - m(\xi))}{\beta + 1} (T_d - M)(T_1 - T_d)^{\beta+1} \right. \tag{17}$$

$$\left. + \frac{\alpha\beta(1 - m(\xi))}{(\beta + 1)(\beta + 2)} (T_1 - T_d)^{\beta+2} - \frac{\alpha^2(1 - m(\xi))^2}{2(\beta + 1)^2} (T_1 - T_d)^{2(\beta+1)} \right]$$

Interest earned:

$$IE_1 = PI_e \int\limits_0^M D(A,P)t\,dt$$

$$= \frac{PI_e D(A,P)M^2}{2} \tag{18}$$

Total profit per unit time:

$$TP_1(A,T_1,T,P,\xi) = \frac{1}{T}\big[SR - PC - DC - LSC - HC - OC - PTI - AC - IC_1 + IE_1\big]$$

$$TP_1(A,T_1,T,P,\xi) = \frac{1}{T}\Bigg[PD(A,P)\Big[T_1 + \frac{1}{\delta}\Big(1 - e^{-\delta(T-T_1)}\Big)\Big]$$

$$- CD(A,P)\Big[T_1 + \frac{\alpha(1-m(\xi))}{\beta+1}\big(T_1 - T_d\big)^{\beta+1} + \frac{1}{\delta}\Big[1 - e^{-\delta(T-T_1)}\Big]\Big]$$

$$- \frac{C_d D(A,P)\alpha(1-m(\xi))}{\beta+1}\big(T_1 - T_d\big)^{\beta+1} - C_s D(A,P)\Big[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T-T_1)}}{\delta}\Big]$$

$$- C_h D(A,P)\Bigg[\frac{T_1^2}{2} + \frac{\alpha(1-m(\xi))}{\beta+1}T_d\big(T_1 - T_d\big)^{\beta+1} + \frac{\alpha\beta(1-m(\xi))}{(\beta+1)(\beta+2)}\big(T_1 - T_d\big)^{\beta+2}$$

$$- \frac{\alpha^2(1-m(\xi))^2}{2(\beta+1)^2}\big(T_1 - T_d\big)^{2(\beta+1)}\Bigg] - C_O - \big(T_1 - T_d\big)\xi - C_a A$$

$$- CI_c D(A,P)\Bigg[\frac{(T_1 - M)^2}{2} + \frac{\alpha(1-m(\xi))}{\beta+1}\big(T_d - M\big)\big(T_1 - T_d\big)^{\beta+1}$$

$$+ \frac{\alpha\beta(1-m(\xi))}{(\beta+1)(\beta+2)}\big(T_1 - T_d\big)^{\beta+2} - \frac{\alpha^2(1-m(\xi))^2}{2(\beta+1)^2}\big(T_1 - T_d\big)^{2(\beta+1)}\Bigg] + \frac{PI_e D(A,P)M^2}{2}\Bigg] \tag{19}$$

So, in this case, the objective is to maximize $Z_1 = TP_1(A,T_1,T,P,\xi)$.

Subject to

$$\left.\begin{array}{l} T_d \leq T_1 \\ T_1 \leq T \\ C \leq P \\ \xi \leq \xi' \\ \text{and } T_1 \geq 0, T \geq 0, P \geq 0, \xi \geq 0, A \text{ is a positive integer } (A > 0) \end{array}\right\} \tag{20}$$

When $M \geq T_d$ there are two possibilities either $T_d \leq M \leq T_1$ or $T_1 \leq M \leq T$.
Case 2 $T_d \leq M \leq T_1$
Interest Charged:

$$IC_2 = CI_c \int_M^{T_1} I_2(t)dt$$

$$= CI_c D(A,P)\left[\frac{(T_1 - M)^2}{2} - \frac{\alpha(1 - m(\xi))}{\beta + 1}(T_1 - M)\left\{(T_1 - T_d)^{\beta+1} + (M - T_d)^{\beta+1}\right\}\right]$$

$$- \frac{2\alpha(1 - m(\xi))}{(\beta + 1)(\beta + 2)}\left\{(T_1 - T_d)^{\beta+2} - (M - T_d)^{\beta+2}\right\}$$

$$- \frac{\alpha^2(1 - m(\xi))^2}{2(\beta + 1)^2}\left\{(T_1 - T_d)^{\beta+1} - (M - T_d)^{\beta+1}\right\}^2$$

$$(21)$$

Interest earned:

$$IE_2 = PI_e \int_0^M D(A,P)t\,dt$$

$$(22)$$

$$= \frac{PI_e D(A,P)M^2}{2}$$

Total profit per unit time:

$$TP_2(A, T_1, T, P, \xi) = \frac{1}{T}\left[SR - PC - DC - LSC - HC - OC - PTI - AC - IC_2 + IE_2\right]$$

$$TP_2(A, T_1, T, P, \xi) = \frac{1}{T}\left[PD(A,P)\left[T_1 + \frac{1}{\delta}\left(1 - e^{-\delta(T-T_1)}\right)\right]\right.$$

$$- CD(A,P)\left[T_1 + \frac{\alpha(1 - m(\xi))}{\beta + 1}(T_1 - T_d)^{\beta+1} + \frac{1}{\delta}\left[1 - e^{-\delta(T-T_1)}\right]\right]$$

$$- \frac{C_d D(A,P)\alpha(1 - m(\xi))}{\beta + 1}(T_1 - T_d)^{\beta+1} - C_s D(A,P)\left[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T-T_1)}}{\delta}\right]$$

$$- C_h D(A,P)\left[\frac{T_1^2}{2} + \frac{\alpha(1 - m(\xi))}{\beta + 1}T_d(T_1 - T_d)^{\beta+1} + \frac{\alpha\beta(1 - m(\xi))}{(\beta + 1)(\beta + 2)}(T_1 - T_d)^{\beta+2}\right.$$

$$\left. - \frac{\alpha^2(1 - m(\xi))^2}{2(\beta + 1)^2}(T_1 - T_d)^{2(\beta+1)}\right] - C_O - (T_1 - T_d)\xi - C_a A$$

$$- CI_c D(A,P)\left[\frac{(T_1 - M)^2}{2} - \frac{\alpha(1 - m(\xi))}{\beta + 1}(T_1 - M)\left\{(T_1 - T_d)^{\beta+1} + (M - T_d)^{\beta+1}\right\}\right.$$

$$- \frac{2\alpha(1 - m(\xi))}{(\beta + 1)(\beta + 2)}\left\{(T_1 - T_d)^{\beta+2} - (M - T_d)^{\beta+2}\right\}$$

$$\left. - \frac{\alpha^2(1 - m(\xi))^2}{2(\beta + 1)^2}\left\{(T_1 - T_d)^{\beta+1} - (M - T_d)^{\beta+1}\right\}^2\right] + \frac{PI_e D(A,P)M^2}{2}\right]$$

$$(23)$$

So, in this case, the objective is to maximize $Z_2 = TP_2(A, T_1, T, P, \xi)$.

Subject to

$$
\left.\begin{array}{l}
M \leq T_1 \\
T_1 \leq T \\
C \leq P \\
\xi \leq \xi' \\
\text{and } T_1 \geq 0, T \geq 0, P \geq 0, \xi \geq 0, A \text{ is a positive integer } (A > 0).
\end{array}\right\}
\tag{24}
$$

Case 3 $T_1 \leq M \leq T$

Interest charged: In this case there is no interest charged

$$
IC_3 = 0
\tag{25}
$$

Interest earned:

$$
\begin{aligned}
IE_3 &= PI_e \left[\int_0^{T_1} D(A, P)t dt + \left(M - T_1\right) \int_0^{T_1} D(A, P) dt \right] \\
&= PI_e D(A, P) T_1 \left(M - \frac{T_1}{2}\right)
\end{aligned}
\tag{26}
$$

Total profit per unit time:

$$
\begin{aligned}
TP_3(A, T_1, T, P, \xi) &= \frac{1}{T}\left[SR - PC - DC - LSC - HC - OC - PTI - AC - IC_3 + IE_3\right] \\
TP_2(A, T_1, T, P, \xi) &= \frac{1}{T}\left[PD(A,P)\left[T_1 + \frac{1}{\delta}\left(1 - e^{-\delta(T-T_1)}\right)\right]\right. \\
&\quad - CD(A,P)\left[T_1 + \frac{\alpha(1 - m(\xi))}{\beta + 1}(T_1 - T_d)^{\beta+1} + \frac{1}{\delta}\left[1 - e^{-\delta(T-T_1)}\right]\right] \\
&\quad - \frac{C_d D(A,P)\alpha(1 - m(\xi))}{\beta + 1}(T_1 - T_d)^{\beta+1} - C_s D(A,P)\left[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T-T_1)}}{\delta}\right] \\
&\quad - C_h D(A,P)\left[\frac{T_1^2}{2} + \frac{\alpha(1 - m(\xi))}{\beta + 1}T_d(T_1 - T_d)^{\beta+1} + \frac{\alpha\beta(1 - m(\xi))}{(\beta + 1)(\beta + 2)}(T_1 - T_d)^{\beta+2}\right. \\
&\quad \left. - \frac{\alpha^2(1 - m(\xi))^2}{2(\beta + 1)^2}(T_1 - T_d)^{2(\beta+1)}\right] - C_O - (T_1 - T_d)\xi - C_a A \\
&\quad \left. - 0 + PI_e D(A,P)T_1\left(M - \frac{T_1}{2}\right)\right]
\end{aligned}
\tag{27}
$$

So, in this case, the objective is to maximize $Z_3 = TP_3(A, T_1, T, P, \xi)$.

Subject to

$$
\left.\begin{array}{l}
T_1 \leq M \\
M \leq T \\
C \leq P \\
\xi \leq \xi' \\
\text{and } T_1 \geq 0, T \geq 0, P \geq 0, \xi \geq 0, A \text{ is a positive integer } (A > 0).
\end{array}\right\}
\tag{28}
$$

The optimal order quantity corresponding to the optimal solution $(A^*, T_1^*, T^*, P^*, \xi^*)$ is

$$Q^* = D(A^*, P^*)\left[T_1^* + \frac{\alpha(1 - m(\xi^*))}{\beta + 1}\left(T_1^* - T_d\right)^{\beta+1} + \frac{1}{\delta}\left[1 - e^{-\delta(T^* - T_1^*)}\right]\right] \quad (29)$$

5 Solution methodology

For fixed $T_1, T, P,$ and ξ the second order partial derivative of $TP_i(A, T_1, T, P, \xi)$ with respect to A gives,

$$\frac{\partial^2 TP_i}{\partial A^2} = \frac{m(m-1)D(A, P)}{TA^2}\left[P\left[T_1 + \frac{1}{\delta}\left(1 - e^{-\delta(T - T_1)}\right)\right]\right.$$
$$- \frac{C_d\alpha(1 - m(\xi))}{\beta + 1}\left(T_1 - T_d\right)^{\beta+1} - C_s\left[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T - T_1)}}{\delta}\right]$$
$$- C_h\left[\frac{T_1^2}{2} + \frac{\alpha(1 - m(\xi))}{\beta + 1}T_d\left(T_1 - T_d\right)^{\beta+1} + \frac{\alpha\beta(1 - m(\xi))}{(\beta + 1)(\beta + 2)}\left(T_1 - T_d\right)^{\beta+2}\right.$$
$$\left.\left. - \frac{\alpha^2(1 - m(\xi))^2}{2(\beta + 1)^2}\left(T_1 - T_d\right)^{2(\beta+1)}\right] - X_i\right] \quad (i = 1, 2, 3)$$

where

$$X_1 = - CI_c\left[\frac{(T_1 - M)^2}{2} + \frac{\alpha(1 - m(\xi))}{\beta + 1}\left(T_d - M\right)\left(T_1 - T_d\right)^{\beta+1}\right.$$
$$\left. + \frac{\alpha\beta(1 - m(\xi))}{(\beta + 1)(\beta + 2)}\left(T_1 - T_d\right)^{\beta+2} - \frac{\alpha^2(1 - m(\xi))^2}{2(\beta + 1)^2}\left(T_1 - T_d\right)^{2(\beta+1)}\right] + \frac{PI_eM^2}{2}$$
$$X_2 = - CI_cD(A, P)\left[\frac{(T_1 - M)^2}{2} - \frac{\alpha(1 - m(\xi))}{\beta + 1}\left(T_1 - M\right)\left\{\left(T_1 - T_d\right)^{\beta+1} + \left(M - T_d\right)^{\beta+1}\right\}\right.$$
$$- \frac{2\alpha(1 - m(\xi))}{(\beta + 1)(\beta + 2)}\left\{\left(T_1 - T_d\right)^{\beta+2} - \left(M - T_d\right)^{\beta+2}\right\}$$
$$\left. - \frac{\alpha^2(1 - m(\xi))^2}{2(\beta + 1)^2}\left\{\left(T_1 - T_d\right)^{\beta+1} - \left(M - T_d\right)^{\beta+1}\right\}^2\right] + \frac{PI_eM^2}{2}$$

and $X_3 = PI_eT_1\left(M - \frac{T_1}{2}\right)$

Because of $0 \leq m < 1, \frac{\partial^2 TP_i}{\partial A^2} < 0$. Therefore, $TP_i(A, T_1, T, P, \xi)$ is a concave function of A and hence the problem of finding the global optimal solution of the advertisement frequency (A^*), reduces to find the local optimum solution. Concavity of the total profit function, with respect to other decision variables, has been shown graphically by means of numerical examples in concavity section. Since the variable A is a positive integer, we suggest the following algorithm to find the optimal solution of the proposed inventory system.

Algorithm:

Step 1   Assign numerical values to all the parameters in appropriate units.

Step 2   Set $A = 1$.

Step 3   Compare M and T_d. If $M \leq T_d$ then go to step 4. Otherwise go to step 8.

Step 4   Find the optimal solution of $TP_1\left(T_1, T, P, \xi | A\right)$ subject to the constraints in Eq. (20).

Then obtain the corresponding total profit $TP_1\left(A, T_1^*, T^*, P^*, \xi^*\right)$ and go to next step.

Step 5   Set $A' = A + 1$ and repeat step 4 to get $TP_1\left(A', T_1^*, T^*, P^*, \xi^*\right)$ and go to next step.

Step 6   If $TP_1\left(A', T_1^*, T^*, P^*, \xi^*\right) \geq TP_1\left(A, T_1^*, T^*, P^*, \xi^*\right)$ then set $A = A'$ and go to step 4.

Otherwise go to next step.

Step 7   Set the optimal solution $\left(A^*, T_1^*, T^*, P^*, \xi^*\right) = \left(A, T_1^*, T^*, P^*, \xi^*\right)$. Go to step 18.

Step 8   Find the optimal solution of $TP_2\left(T_1, T, P, \xi | A\right)$ subject to the constraints in Eq. (24).

Then obtain the corresponding total profit $TP_2\left(A, T_1^*, T^*, P^*, \xi^*\right)$ and go to next step.

Step 9   Set $A' = A + 1$ and repeat step 8 to get $TP_2\left(A', T_1^*, T^*, P^*, \xi^*\right)$ and goto next step.

Step 10   If $TP_2\left(A', T_1^*, T^*, P^*, \xi^*\right) \geq TP_2\left(A, T_1^*, T^*, P^*, \xi^*\right)$ then set $A = A'$ and go to step 8.

Otherwise go to next step.

Step 11   Set the optimal solution $\left(A^*, T_1^*, T^*, P^*, \xi^*\right) = \left(A, T_1^*, T^*, P^*, \xi^*\right)$ and go to next step.

Step 12   Set $A = 1$.

Step 13   Find the optimal solution of $TP_3\left(T_1, T, P, \xi | A\right)$ subject to the constraints in Eq. (28).

Then obtain the corresponding total profit $TP_3\left(A, T_1^*, T^*, P^*, \xi^*\right)$ and go to next step.

Step 14   Set $A' = A + 1$ and repeat step 13 to get $TP_3\left(A', T_1^*, T^*, P^*, \xi^*\right)$ and go to next step.

Step 15   If $TP_3\left(A', T_1^*, T^*, P^*, \xi^*\right) \geq TP_3\left(A, T_1^*, T^*, P^*, \xi^*\right)$ then set $A = A'$ and go to step 13.

Otherwise goto next step.

Step 16   Set the optimal solution $\left(A^*, T_1^*, T^*, P^*, \xi^*\right) = \left(A, T_1^*, T^*, P^*, \xi^*\right)$ and go to next step.

Table 2 Numerical examples

Example	T_d	M	α	β	a	b	m	δ	C	C_O	C_a	C_h	C_d	C_s	I_c	I_e	ξ'	k
1 (Case 1)	0.15	0.0822 (30 days)	0.4	2	500,000	2	0.04	0.5	10	300	80	1.5	10	8	0.12	0.09	500	0.03
2 (Case 2/3)	0.15	0.274 (100 days)	0.4	2	500,000	2	0.04	0.5	10	300	80	1.5	10	8	0.12	0.09	500	0.03
3 (Case 2/3)	0.15	0.5754 (210 days)	0.4	2	500,000	2	0.04	0.5	10	300	80	1.5	10	8	0.12	0.09	500	0.03

Table 3 Computational results of Example 1

Case 1					
A	T_1	T	P	ξ	TP_1
1	0.40547	0.52982	20.83804	61.15874	11,171.47
2	0.44466	0.58097	20.93221	71.35787	11,361.63
3	0.48206	0.62986	21.02127	79.67207	11,428.31
4	**0.51765**	**0.67619**	**21.10711**	**86.58933**	**11,448.15**
5	0.55151	0.72026	21.18769	92.60817	11,444.61

Table 4 Computational results of Example 2

Case 2					
A	T_1	T	P	ξ	TP_2
1	0.40135	0.49986	20.49055	59.98374	11,434.47
2	0.44106	0.55193	20.58261	70.33576	11,625.06
3	0.47903	0.60165	20.67203	78.71668	11,690.14
4	**0.51493**	**0.64856**	**20.75582**	**85.64372**	**11,707.97**
5	0.54894	0.69295	20.83572	91.57059	11,702.40
Case 3					
A	T_1	T	P	ξ	TP_3
1	0.27400	0.37900	20.25039	14.04226	11,359.87
2	0.27400	0.39566	20.27674	14.87656	11,500.40
3	**0.27400**	**0.41231**	**20.30663**	**15.30414**	**11,509.10**
4	0.27400	0.42853	20.33929	15.55682	11,466.92

Step 17 If Max$\{ TP_2^*, TP_3^* \} = TP_2^*$ then the solution obtained in step 11 is the optimal.

If Max$\{ TP_2^*, TP_3^* \} = TP_3^*$ then the solution obtained in step 16 is the optimal.

Step 18 Compute the corresponding optimal order quantity Q^* from Eq. (29). Stop.

While executing the above algorithm, for fixed A, we can obtain the optimal solution which maximizes the total profit function with constraints using software like MATLAB, MATHEMATICA, R, MATHCAD, etc.

For fixed value of the variable A the necessary and sufficient conditions to maximize the total profit function $TP_i(T_1, T, P, \xi | A)$ are as follows:

$$\frac{\partial TP_i}{\partial T_1} = 0, \frac{\partial TP_i}{\partial T} = 0, \frac{\partial TP_i}{\partial P} = 0, \frac{\partial TP_i}{\partial T_\xi} = 0;$$

Table 5 Computational results of Example 3

Case 2

A	T_1	T	P	ξ	TP_2
1	0.57540	0.63138	20.36428	96.25084	11,884.84
2	0.57540	0.64106	20.36417	97.13897	12,111.84
3	0.57540	0.65107	20.36685	97.68159	12,198.74
4	**0.57540**	**0.66098**	**20.37083**	**98.03803**	**12,228.23**
5	0.57540	0.67085	20.37542	98.30022	12,226.64

Case 3

A	T_1	T	P	ξ	TP_3
1	0.49264	0.57540	20.20050	82.23272	11,944.11
2	0.49292	0.57540	20.20008	83.16379	12,160.83
3	0.49312	0.57540	20.20015	83.78138	12,234.52
4	**0.49634**	**0.57980**	**20.20653**	**84.78092**	**12,248.58**
5	0.52522	0.61943	20.27329	90.22666	12234.73

Table 6 Results of numerical examples

Solution	A^*	T_1^*	T^*	P^*	ξ^*	Q^*	Profit	Result from
Example 1	4	0.51765	0.67619	21.10711	86.58933	795.4887	11,448.15	Case 1
Example 2	4	0.51493	0.64856	20.75582	85.64372	790.9034	11,707.97	Case 2
Example 3	4	0.49634	0.57980	20.20653	84.78092	748.8343	12,248.58	Case 3

Provided that the Hessian matrix $H = \begin{bmatrix} \dfrac{\partial^2 TP_i}{\partial T_1^2} & \dfrac{\partial^2 TP_i}{\partial T_1 T} & \dfrac{\partial^2 TP_i}{\partial T_1 P} & \dfrac{\partial^2 TP_i}{\partial T_1 \xi} \\ \dfrac{\partial^2 TP_i}{\partial T T_1} & \dfrac{\partial^2 TP_i}{\partial T^2} & \dfrac{\partial^2 TP_i}{\partial T P} & \dfrac{\partial^2 TP_i}{\partial T \xi} \\ \dfrac{\partial^2 TP_i}{\partial P T_1} & \dfrac{\partial^2 TP_i}{\partial P T} & \dfrac{\partial^2 TP_i}{\partial P^2} & \dfrac{\partial^2 TP_i}{\partial P \xi} \\ \dfrac{\partial^2 TP_i}{\partial \xi T_1} & \dfrac{\partial^2 TP_i}{\partial \xi T} & \dfrac{\partial^2 TP_i}{\partial \xi P} & \dfrac{\partial^2 TP_i}{\partial \xi^2} \end{bmatrix}$ is a negative

definite.

6 Numerical examples

In Table 2 consider the parameter values in appropriate units.

While executing the proposed algorithm, for fixed A the solution has been obtained using the DEoptimR package in R. This package uses the differential evolution stochastic algorithm and gives the approximate global optimum solution. The computational results of each example for different values of A are given in Tables 3, 4 and 5. In Tables 3, 4 and 5, the bold values
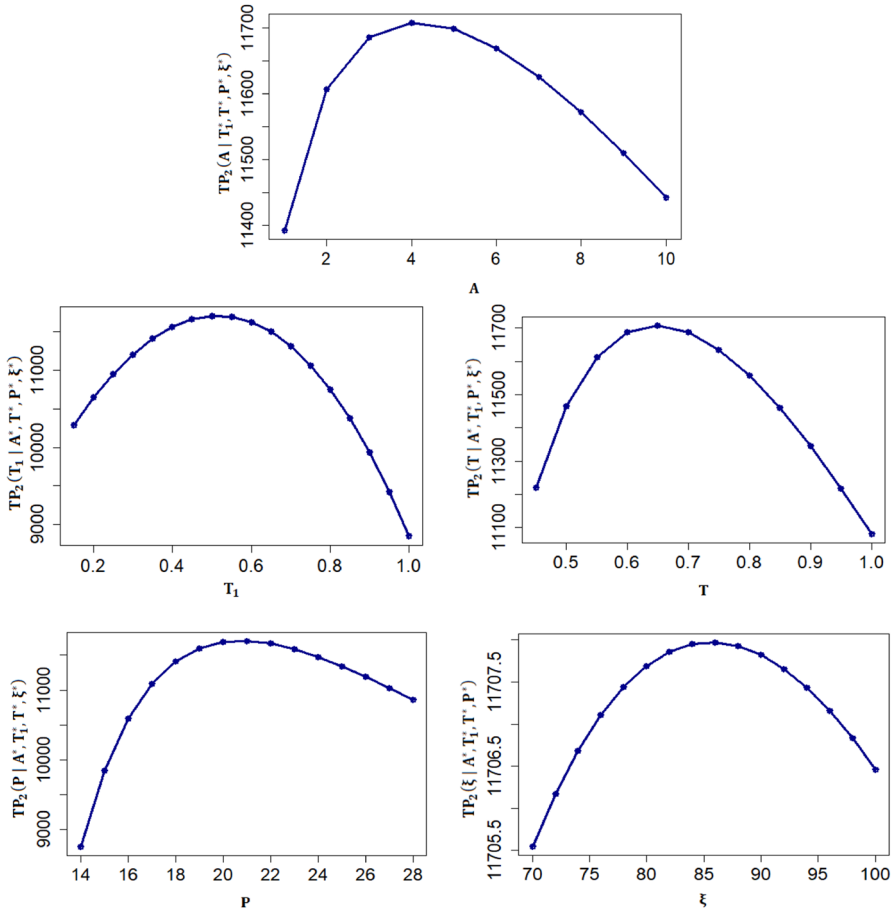
Fig. 5 Concavity of the total profit function $TP_2(A, T_1, T, P, \xi)$ (example 2) with respect to different variables when other variables are fixed
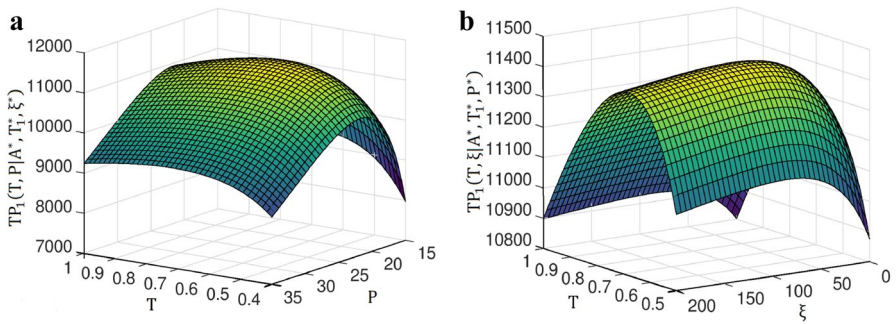

Fig. 6 Total profit $TP_1(A, T_1, T, P, \xi)$ (example 1) with respect to **a** T and P, **b** T and ξ when other variables are fixed
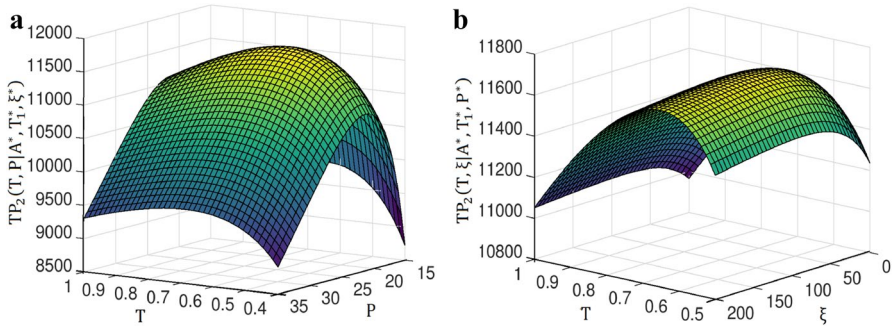
Fig. 7 Total profit $TP_2\left(A, T_1, T, P, \xi\right)$ (example 2) with respect to **a** T and P, **b** T and ξ when other variables are fixed
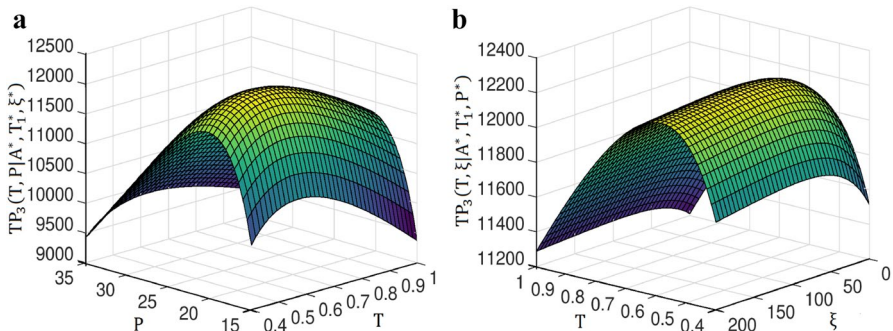

Fig. 8 Total profit $TP_3\left(A, T_1, T, P, \xi\right)$ (example 3) with respect to **a** T and P, **b** T and ξ when other variables are fixed

Table 7 Computational results for different values of T_d and M

T_d	M	A^*	T_1^*	T^*	P^*	ξ^*	Profit	Remark
0	0.0822	4	0.52540	0.68804	21.13795	107.99995	11,406.13	Case 2
	0.2740	4	0.52237	0.66026	20.78676	109.06544	11,662.81	Case 2
	0.5754	4	0.50286	0.59084	20.23584	109.37871	12,198.00	Case 3
0.15	0.0822	4	0.51765	0.67619	21.10711	86.58933	11,448.15	Case 1
	0.2740	4	0.51493	0.64856	20.75582	85.64372	11,707.97	Case 2
	0.5754	4	0.49634	0.57980	20.20653	84.78092	12,248.58	Case 3
0.25	0.0822	4	0.51620	0.67213	21.09448	66.03877	11,473.96	Case 1
	0.2740	4	0.51278	0.64387	20.74182	64.05000	11,735.01	Case 2
	0.5754	4	0.49462	0.57541	20.19282	61.78226	12,278.63	Case 3

Table 8 Effect of different parameters on optimal results

Parameter	A^*	T_1^*	T^*	P^*	ξ^*	Q^*	Profit
α							
0.2	4	0.51790	0.65031	20.76223	63.05848	792.66	11,721.01
0.3	4	0.51617	0.64930	20.75790	76.26838	791.69	11,713.38
0.5	4	0.51399	0.64801	20.75408	92.88461	790.33	11,703.79
0.6	4	0.51315	0.64751	20.75300	98.86090	789.77	11,700.38
a							
250,000	3	0.67607	0.87466	21.17994	84.93050	504.67	5455.00
375,000	3	0.55340	0.70356	20.86018	81.94273	629.27	8549.42
625,000	5	0.49005	0.61310	20.68966	88.91187	949.48	14,908.81
750,000	6	0.47242	0.58816	20.64307	91.88617	1106.4	18,142.06
m							
0.02	2	0.44417	0.55620	20.59090	70.53449	661.88	11,453.08
0.03	3	0.48176	0.60534	20.67796	78.85866	727.62	11,552.16
0.05	6	0.57595	0.72792	20.89890	96.51408	905.07	11,915.11
0.06	8	0.62832	0.79570	21.02130	104.9526	1012.04	12,170.46
C_h							
0.75	5	0.65400	0.77681	20.65993	107.00374	966.72	11,929.48
1.125	5	0.59544	0.72950	20.75430	98.88853	898.42	11,808.78
1.875	4	0.47917	0.62128	20.82310	78.77847	751.80	11,618.78
2.25	4	0.44894	0.59868	20.88266	72.43956	719.43	11,538.32
C_O							
150	3	0.39846	0.49552	20.48280	60.68171	614.60	11,963.73
225	4	0.47883	0.60124	20.67111	79.03436	739.66	11,828.01
375	5	0.58131	0.73531	20.91277	96.47637	890.23	11,597.36
450	5	0.61231	0.77575	20.98555	100.80439	932.22	11,498.08
C							
5	5	0.30949	0.38881	10.35216	68.21520	1927.28	24,047.64
7.5	5	0.43665	0.54992	15.57850	84.86723	1201.93	15,805.03
12.5	4	0.60908	0.76943	26.01698	89.41070	596.57	9259.72
15	4	0.69472	0.88014	31.30487	91.59664	470.91	7632.69
C_a							
40	9	0.52473	0.66094	20.77808	88.41013	830.67	12,065.72
60	6	0.52905	0.66686	20.78864	88.59798	823.72	11,854.81
100	3	0.50841	0.64019	20.74150	84.10810	772.91	11,593.50
120	3	0.53651	0.67701	20.80780	88.88356	811.78	11,502.40
C_s							
4	4	0.50116	0.66773	20.66845	83.40869	818.35	11748.60
6	4	0.50878	0.65704	20.71847	84.59645	802.96	11,726.19
10	4	0.51990	0.64159	20.78439	86.46362	781.12	11,692.86
12	4	0.52424	0.63600	20.80809	87.15592	773.21	11,680.10
C_d							
5	4	0.51622	0.64939	20.75854	76.25896	791.96	11,713.38

Table 8 (continued)

Parameter	A^*	T_1^*	T^*	P^*	ξ^*	Q^*	Profit
7.5	4	0.51556	0.64896	20.75715	81.23853	791.40	11,710.48
12.5	4	0.51436	0.64824	20.75481	89.43995	790.50	11,705.76
15	4	0.51395	0.64799	20.75335	92.88081	790.24	11,703.79
k							
0.015	4	0.48714	0.62433	20.72722	114.79927	763.51	11,669.26
0.0225	4	0.50513	0.64014	20.74521	99.04411	781.44	11,693.12
0.0375	4	0.52119	0.65388	20.76329	75.37330	796.82	11,718.12
0.045	4	0.52545	0.65744	20.76798	67.32116	800.80	11,725.52

Fig. 9 Effect of $\alpha, C_h, C_o, C_a, C_s, C_d$ and k on total profit
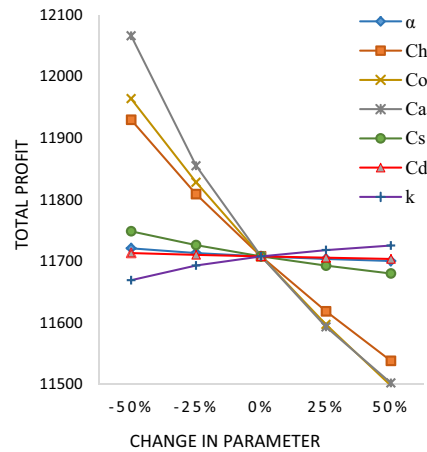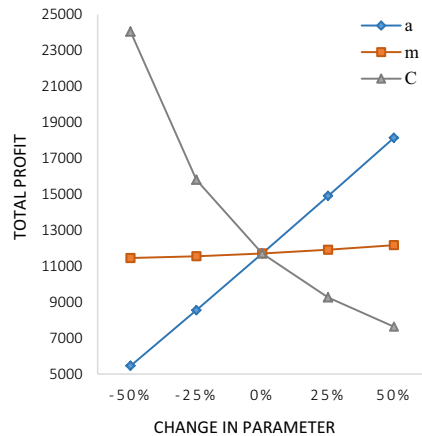

indicate optimal values for that particular case. The final results are shown in Table 6.

7 Concavity and optimality

For example 2 of the above section, the total profit is plotted against each variable fixing other variables in Fig. 5. From Fig. 5, it is obvious that the total profit function TP_2 is concave with respect to each variable. Figures 6, 7 and 8 also reveals that the total profit functions TP_1, TP_2 and TP_3 are concave functions.

Fixing $A = 4$ in example 2, at the solution $(T_1^*, T^*, P^*, \xi^*) = (0.51493, 0.64856, 20.75582, 85.64372)$ the gradient is $(-0.205, 0.125, 0.006, 0.000)$, which is close to zero.

Fig. 10 Effect of a, m, and C on total profit

The eigenvalues of H are $-0.0153, -55.6244, -270.7990, -3641.7120$. Therefore, the Hessian matrix is negative definite, and hence the solution is global maximum.

Hessian matrix is $H = \begin{bmatrix} -22528.8597 & 16592.7893 & 109.5509 & 3.0548 \\ 16592.7893 & -16592.86 & 29.8043 & -0.00006 \\ 109.5509 & 29.8043 & -59.01698 & -0.0542 \\ 3.0548 & 0.00006 & -0.0542 & -0.01688 \end{bmatrix};$

8 Sensitivity analysis

Table 7 reveals that when the supplier allows more credit period (M), the retailer earns more profit. The model assumes non-instantaneous deterioration, but it is also applicable for instantaneous deterioration case by taking $T_d = 0$. That is, the instantaneous deterioration case is a particular case of Non-instantaneous deterioration case. Table 7 shows that instantaneous deteriorating items need more PT investment.

Table 8 shows the computational results obtained by increasing each parameter of example 2 by $-50\%, -25\%, +25\%$ and $+50\%$. Figures 9 and 10 shows how sensitive the total profit with respect to different parameters.

Observations and managerial insights:

- The total profit is less sensitive with the change in parameters α, C_d, and C_s. An increment in α increase the deterioration rate and increment in C_d increase the total deterioration cost but, PT investment reduce the deterioration rate (number of deteriorating units) significantly, and hence profit is ineffective with the change in α and C_d. Hence, retailers are suggested to invest in PT to reduce losses incurring due to deterioration. As the shortage cost C_s increases, our model decreases the shortage period ($T^* - T_1^*$) (see Table 8), which reduce lost sales and hence, profit is less effective with the change in C_s.

- Increment in different cost parameters C_h, C_o, and C_a results in a decrement in total profit. In Table 8, increment in holding cost (C_h) decreases the optimal order cycle T^* while increment in ordering cost (C_o) increases the optimal order cycle (T^*). Hence, when the holding cost raises, the retailer is suggested to decrease the order cycle, and when the ordering cost rises, the retailer is suggested to increase the order cycle. As the advertisement cost (C_a) increase, the frequency of advertisement and total profit decreases. To increase the total profit, the retailer is suggested to increase the frequency of advertisement (A) when the advertisement cost (C_a) is less.
- As the value of k increase, the total profit increases. Since the reduced deterioration rate is $(1 - m(\xi)) = e^{-k \times \xi}$, an increment in k will reduce the deterioration rate greatly, which results in a less PT investment and more profit. The retailer need to invest more in PT for smaller value of k.
- The total profit is very sensitive with the change in parameters a and C. Increased value of the scale parameter (a) of the demand function will increase the demand, and hence increase the total profit. As purchase cost (C) increase, the optimal value of selling price (P^*) drastically increases. But, increased selling price (P^*) decrease the demand, and hence the total profit is decreasing drastically as C increases. As the shape parameter of demand (m) increase the total profit increases. In Fig. 10, it seems that the profit is less sensitive with the change in the parameter (m) this is due to assigning a smaller value to m $(m = 0.04)$. The profit will drastically increase for the assignment of higher value to m.

9 Conclusion

In today's competitive market environment, every business organization wants to optimize all possible strategies to get maximum revenue. Our proposed model is beneficial to the retailers to maximize the total profit by optimizing the pricing, marketing, preservation, and inventory ordering policies. The PT investment reduces faster deteriorations, which is beneficial to businesses based on agricultural products, bakery products, dairy products, and meat and fish products. The retailer earns more profit through credit period. Instantaneous deteriorating items need more PT investment and the profit for the non-instantaneous deteriorating items is more than the profit for instantaneous deteriorating items. When the advertisement cost is less, the retailer can earn more profit through increasing advertisement frequency. This model can be extended to the production order quantity model. Can be further extended for two warehouse problem wherein preservation technology applied in either of the warehouses.

References

1. Bakker, M., Riezebos, J., Teunter, R.H.: Review of inventory systems with deterioration since 2001. Eur. J. Oper. Res. **221**(2), 275–284 (2012)

2. Bardhan, S., Pal, H., Giri, B.C.: Optimal replenishment policy and preservation technology investment for a non-instantaneous deteriorating item with stock-dependent demand. Oper. Res. **19**(2), 347–368 (2019). https://doi.org/10.1007/s12351-017-0302-0

3. Bhunia, A.K., Maiti, M.: An inventory model for decaying items with selling price, frequency of advertisement and linearly time-dependent demand with shortages. IAPQR Trans. **22**, 41–50 (1997)

4. Cohen, M.A.: Joint pricing and ordering policy for exponentially decaying inventory with known demand. Nav. Res. Logist. Q. **24**(2), 257–268 (1977)

5. Covert, R.P., Philip, G.C.: An EOQ model for items with Weibull distribution deterioration. AIIE Trans. **5**(4), 323–326 (1973)

6. Dave, U.: An EOQ model for deteriorating items subject to permissible delay in payments. Optimization **18**(3), 433–437 (1987)

7. Dye, C.Y.: Joint pricing and ordering policy for a deteriorating inventory with partial backlogging. Omega Int. J. Manag. Sci. **35**(2), 184–189 (2007)

8. Dye, C.Y.: The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. Omega Int. J. Manag. Sci. **41**(5), 872–880 (2013)

9. Dye, C.Y., Hsieh, T.P.: A particle swarm optimization for solving lot-sizing problem with fluctuating demand and preservation technology cost under trade credit. J. Glob. Optim. **55**(3), 655–679 (2013)

10. Dye, C.Y., Yang, C.T.: Optimal dynamic pricing and preservation technology investment for deteriorating products with reference price effects. Omega **62**, 52–67 (2016)

11. Dye, C.Y., Yang, C.T., Wu, C.C.: Joint dynamic pricing and preservation technology investment for an integrated supply chain with reference price effects. J. Oper. Res. Soc. **69**(6), 811–824 (2018)

12. Geetha, K.V., Uthayakumar, R.: Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments. J. Comput. Appl. Math. **233**(10), 2492–2505 (2010)

13. Ghare, P.N., Schrader, G.F.: A model for exponentially decaying inventories. J. Ind. Eng. **15**, 238–243 (1963)

14. Goyal, S.K.: Economic order quantity under conditions of permissible delay in payments. J. Oper. Res. Soc. **36**(4), 335–338 (1985)

15. Goyal, S.K., Giri, B.C.: Recent trends in modeling of deteriorating inventory. Eur. J. Oper. Res. **134**(1), 1–16 (2001)

16. Goyal, S.K., Gunasekaran, A.: An integrated production-inventory-marketing model for deteriorating items. Comput. Ind. Eng. **28**(4), 755–762 (1995)

17. Hsieh, T.P., Dye, C.Y.: A production–inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. J. Comput. Appl. Math. **239**, 25–36 (2013)

18. Hsu, P.H., Wee, H.M., Teng, H.M.: Preservation technology investment for deteriorating inventory. Int. J. Prod. Econ. **124**(2), 388–394 (2010)

19. Janssen, L., Claus, T., Sauer, J.: Literature review of deteriorating inventory models by key topics from 2012 to 2015. Int. J. Prod. Econ. **182**, 86–112 (2016)

20. Kotler, P.: Marketing Decision Making: A Model Building Approach. Holt, Rinehart, Winston, New York (1972)

21. Lee, Y.P., Dye, C.Y.: An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. Comput. Ind. Eng. **63**(2), 474–482 (2012)

22. Li, G., He, X., Zhou, J., Wu, H.: Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items. Omega **84**, 114–126 (2019)

23. Li, R., Lan, H., Mawhinney, J.R.: A review on deteriorating inventory study. J. Serv. Sci. Manag. **3**(01), 117 (2010)

24. Liu, G., Zhang, J., Tang, W.: Joint dynamic pricing and investment strategy for perishable foods with price-quality dependent demand. Ann. Oper. Res. **226**(1), 397–416 (2015)

25. Mahmoodi, A.: Joint pricing and inventory control of duopoly retailers with deteriorating items and linear demand. Comput. Ind. Eng. (2019). https://doi.org/10.1016/j.cie.2019.04.017

26. Maihami, R., Kamalabadi, I.N.: Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand. Int. J. Prod. Econ. **136**(1), 116–122 (2012)

27. Mishra, V.K.: An inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost. J. Ind. Eng. Manag. **6**(2), 495 (2013)

28. Mishra, V.K.: Deteriorating inventory model with controllable deterioration rate for time-dependent demand and time-varying holding cost. Yugosl. J. Oper. Res. **24**(1), 87–98 (2014)

29. Mukhopadhyay, S., Mukherjee, R.N., Chaudhuri, K.S.: Joint pricing and ordering policy for a deteriorating inventory. Comput. Ind. Eng. **47**(4), 339–349 (2004)

30. Nahmias, S.: Perishable inventory theory: a review. Oper. Res. **30**(4), 680–708 (1982)

31. Pal, A.K., Bhunia, A.K., Mukherjee, R.N.: A marketing-oriented inventory model with three-component demand rate dependent on displayed stock level (DSL). J. Oper. Res. Soc. **56**(1), 113–118 (2005)

32. Philip, G.C.: A generalized EOQ model for items with Weibull distribution deterioration. AIIE Trans. **6**(2), 159–162 (1974)

33. Raafat, F.: Survey of literature on continuously deteriorating inventory models. J. Oper. Res. Soc. **42**(1), 27–37 (1991)

34. Rathore, H.: A preservation technology model for deteriorating items with advertisement-dependent demand and trade credit. In: Logistics, Supply Chain and Financial Predictive Analytics, pp. 211–220. Springer, Singapore (2019)

35. Sarkar, B.: Mathematical and analytical approach for the management of defective items in a multi-stage production system. J. Clean. Prod. **218**, 896–919 (2019)

36. Shah, N.H., Shah, D.B., Patel, D.G.: Optimal preservation technology investment, retail price and ordering policies for deteriorating items under trended demand and two level trade credit financing. J. Math. Model. Algorithms Oper. Res. **14**(1), 1 (2015)

37. Shah, N.H., Soni, H.N., Patel, K.A.: Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Omega Int. J. Manag. Sci. **41**(2), 421–430 (2013)

38. Shaikh, A.A., Panda, G.C., Sahu, S., Das, A.K.: Economic order quantity model for deteriorating item with preservation technology in time dependent demand with partial backlogging and trade credit. Int. J. Logist. Syst. Manag. **32**(1), 1–24 (2019)

39. Subramanyam, E.S., Kumaraswamy, S.: EOQ formula under varying marketing policies and conditions. AIIE Trans. **13**(4), 312–314 (1981)

40. Tsao, Y.C.: Joint location, inventory, and preservation decisions for non-instantaneous deterioration items under delay in payments. Int. J. Syst. Sci. **47**(3), 572–585 (2016)

41. Urban, T.L.: Deterministic inventory models incorporating marketing decisions. Comput. Ind. Eng. **22**(1), 85–93 (1992)

42. Yang, C.T., Dye, C.Y., Ding, J.F.: Optimal dynamic trade credit and preservation technology allocation for a deteriorating inventory model. Comput. Ind. Eng. **87**, 356–369 (2015)

43. Zhang, J.X., Bai, Z.Y., Tang, W.S.: Optimal pricing policy for deteriorating items with preservation technology investment. J. Ind. Manag. Optim. **10**(4), 1261–1277 (2014)

44. Zhang, J., Wei, Q., Zhang, Q., Tang, W.: Pricing, service and preservation technology investments policy for deteriorating items under common resource constraints. Comput. Ind. Eng. **95**, 1–9 (2016)