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APPLICATION ARTICLE

Uncertain transportation model with rough unit cost, demand and supply

Subhakanta Dash[1] · S. P. Mohanty[2]

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Abstract In a transportation problem, the parameters like unit cost of transportation of goods or services from source to destination, supplies from the sources and demands at destinations depend on many factors which may not be deterministic in nature. To deal with the uncertainty of the parameters, random variables and fuzzy variables were used previously. Sometimes, in absence of sufficient sample observations, the uncertain parameters are estimated by the belief degree of the experts. In this paper, the aim is to investigate the transportation problem where the unit cost of transportation, supplies, demands are initially taken as rough variables based on subjective estimation of experts. Further, these rough estimates are suitably approximated as uncertain normal variables and the conceptual uncertain programming model has been developed. The model is then transformed to a deterministic linear programming model by minimizing the expected value of the uncertain objective function under the constraints at certain confidence level.

Keywords Transportation problem · Rough variable · Uncertain measure · Uncertain programming · Uncertainty theory

1 Introduction

The transportation problem involves the distribution of goods or services from a set of sources to a set of destinations through a network. There are different routes and different transportation costs for the routes. The aim of transportation problem is to determine the number of units of goods or services to be transported so that all

✉ Subhakanta Dash
skdashmath@gmail.com

[1] Department of Mathematics, Silicon Institute of Technology, Patia, Bhubaneswar 751 024, India

[2] Department of Mathematics, SOA University, Bhubaneswar, India

demands and supplies are satisfied with the minimum transportation cost. The traditional transportation problem consists of one objective function and two constraints, namely, source constraints with the supply and destination constraints with the demand which was first initiated by Hitchcock [4] and later developed by Koopmans [5]. Dantzig [2] proposed simplex method and applied to solve transportation problem as a linear programming problem. Since then, many researchers have developed different algorithms to solve transportation problems. In traditional transportation problems, the parameters like unit cost of transportation, demand at destinations, supply at sources are taken as deterministic value. Due to various complexities in the real world, such as unpredictable weather conditions, road conditions, traffic conditions in the road, change in sale, change in attitude of the customers etc., it is not appropriate to regard the unit transportation cost, the supplies and the demands as deterministic. They should be considered as variables. Williams [16] has developed a stochastic model of transportation problem considering the parameters as random variables. Since then many researchers have studied stochastic models of transportation problem.

Later it was observed that the input data are often imprecise owing to incomplete or unobtainable information. So, researchers tried to consider the parameters as fuzzy variables. Chanas et al. [1] presented a fuzzy linear programming model to solve transportation problem with crisp cost, fuzzy supply and fuzzy demand.

Further it is observed that, in many situations, no investigated data are available to estimate appropriate probability distribution of the assumed random variables. In this situation, some domain experts are invited to give their subjective estimates of the above parameters. Pawlak [13] introduced rough set theory to deal with uncertainty. Liu [7, 8, 11, 12] has developed uncertainty theory which has become a powerful tool to deal with human belief degree. Using uncertain variables, some authors have developed models for transportation problems. Guo et al. [3] has developed a transportation model considering the supply as random variable and the cost and the demand as uncertain variables. Yuhong Shang and Kai Yao [14, 15] has developed the transportation model considering the cost, supply and demand as uncertain variables.

Practically, it is very likely that the subjective estimates of the parameters by the experts are given in certain range of values which can be characterized by rough variables. In this paper, a transportation model is developed considering the unit cost of transportation, the supply and the demand as rough variables.

The rest of the paper is organized as follows. In Sect. 2, some basic concepts of rough variable and its properties are presented, In Sect. 3, a transportation model with rough cost, demand and supply is developed. One numerical example is given in Sect. 4 for illustration of the model. Finally the conclusion is given in Sect. 5.

2 Preliminaries

In this section some concepts and notions of uncertain variable and rough variable are presented. The following definitions are based on Liu [7, 8].

2.1 Rough variable and its properties

Definition 1 Let Λ be a non empty set, A be σ-algebra of subsets of Λ, Δ be an element in A, and π be a non negative, real-valued, additive set function on A. The quadruple (Λ, Δ, A, π) is called a rough space.

Definition 2 A rough variable ξ on the rough space (Λ, Δ, A, π) is a measurable function from Λ to the set of real numbers $\Re$ such that for every Borel set B of $\Re$, we have $\{\lambda \in \Lambda | \xi(\lambda) \in B\} \in A$.

Then the lower and upper approximation of the rough variable ξ are defined as follows

$$\bar{\zeta} = \{\zeta(\lambda) | \lambda \in \Lambda\} \quad \text{(Upper approximation)}$$

$$\underline{\zeta} = \{\zeta(\lambda) | \lambda \in \Delta\} \quad \text{(Lower approximation)}$$

Definition 3 ([a, b], [c, d]) with $c \le a < b \le d$ is a rough variable, where $\xi(\lambda) = \lambda$ from the rough space to the set of real numbers and $\Lambda = \{\lambda | c \le \lambda \le d\}$ and $\Delta = \{\lambda | a \le \lambda \le b\}$, A is the Borel algebra on Λ, π is the Lebesgue measure.

Definition 4 Let (Λ, Δ, A, π) be a rough space. Then the upper and lower trust of event A is defined by $\mathrm{T\underline{r}(A)} = \frac{\pi\{A\}}{\pi\{\Lambda\}}$ and $\mathrm{T\bar{r}(A)} = \frac{\pi\{A \cap \Delta\}}{\pi\{\Lambda\}}$

The trust of the event A is defined as

$$\mathrm{Tr(A)} = \frac{1}{2}(\mathrm{T\underline{r}(A)} + \mathrm{T\bar{r}(A)})$$

Definition 5 Let ξ be a rough variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} \mathrm{Tr}\{\xi \ge r\} dr - \int_{-\infty}^0 \mathrm{Tr}\{\xi \le r\} dr$$

provided that at least one of the two integrals is finite.

Definition 6 The trust distribution ϕ: $[-\infty, \infty] \to [0, 1]$ of a rough variable ξ is defined by

$$\Phi(\mathrm{x}) = \mathrm{Tr}\{\lambda \in \Lambda | \xi(\lambda) \le \mathrm{x}\}$$

Definition 7 The trust density function $f:\ \mathrm{R} \to [0, \infty)$ of a rough variable ξ is a function such that $\mathrm{f(x)} = \int_{-\infty}^{\infty} \phi(y) dy$ holds for all $\mathrm{x} \in (-\infty, \infty)$, where ϕ is trust distribution of ξ.

Definition 8 If ξ=([a,b], $[c, d]$) is a rough variable such that c $\leq$ a $<$ b $\leq$ d, then the trust distribution ϕ(x) = Tr$\{\xi \leq x\}$ is

$$\phi(x) = \begin{cases} 0 & \textit{if } x \leq c \\ \dfrac{x-c}{2(d-c)} & \textit{if } c \leq x \leq a \\ \dfrac{[(b-a)+(d-c)]x + 2ac - ad - bc}{2(b-a)(d-c)} & \textit{if } a \leq x \leq b \\ \dfrac{x+d-2c}{2(d-c)} & \textit{if } b \leq x \leq d \\ 1 & \textit{if } x \geq d \end{cases}$$

and the trust density function is defined as

$$f(x) = \begin{cases} \dfrac{1}{2(d-c)} & \textit{if } c \leq x \leq a \quad \textit{or} \quad b \leq x \leq d \\ \dfrac{1}{2(b-c)} + \dfrac{1}{2(d-c)} & \textit{if } a \leq x \leq b \\ 0 & \textit{otherwise} \end{cases}$$

Definition 9 For a given value r and ξ=([a,b], $[c, d]$) trust of rough events characterized by $\xi \leq r$ and $\xi \geq r$ is, respectively, given by the following expressions (Liu [6])

$$\mathrm{Tr}\{\xi \geq r\} = \begin{cases} 0, & \text{if } r \leq d, \\ \dfrac{d-r}{2(d-c)}, & \text{if b} \leq r \leq d \\ \dfrac{1}{2}\left(\dfrac{d-r}{d-c} + \dfrac{b-r}{b-a}\right), & \text{if a} \leq r \leq b \\ \dfrac{1}{2}\left(\dfrac{d-r}{d-c} + 1\right), & \text{if c} \leq r \leq a \\ 1, & \text{if } r \leq c \end{cases}$$

and

$$\mathrm{Tr}\{\xi \geq r\} = \begin{cases} 0, & \text{if } r \leq c, \\ \dfrac{r-c}{2(d-c)}, & \text{if c} \leq r \leq a \\ \dfrac{1}{2}\left(\dfrac{r-a}{b-a} + \dfrac{r-c}{d-c}\right), & \text{if a} \leq r \leq b \\ \dfrac{1}{2}\left(\dfrac{r-c}{d-c} + 1\right), & \text{if b} \leq r \leq d \\ 1, & \text{if } r \leq d \end{cases}$$

Definition 10 Let ξ be a rough variable whose trust density function f exist. If the Lebesgue integral $\int_{-\infty}^{\infty} x f(x)\, dx$ is finite, then the expected value of ξ is defined as

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

2.2 Uncertain variable and its properties

Definition 11 Uncertain measure

Let L be a σ-algebra on a nonempty set Γ. A set function M: L $\rightarrow$ [0,1] is called an uncertain measure if it satisfies the following axioms

Axiom 1: (Normality axiom) M (Γ) = 1 for the universal set Γ

Axiom 2: (Duality axiom) $\mathrm{M}(\Lambda) + \mathrm{M}(\Lambda^{\mathrm{c}}) = 1$ for every event Λ

Axiom 3: (Sub additive axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have

$$\mathrm{M}\left\{\bigcup_{\mathrm{i}=1}^{\infty} \Lambda_{\mathrm{i}}\right\} \leq \sum_{i=1}^{\infty} \mathrm{M}(\Lambda_{\mathrm{i}})$$

The triplet (Γ, L, M) is called an uncertainty space.

Axiom 4: (Product measure) Let $(\Gamma_k, L_k, \mathrm{M}_k)$ be uncertainty spaces for k = 1, 2,…. The product uncertain measure is an uncertain measure satisfying

$$\mathrm{M}\left\{\prod_{k=1}^{\infty} \Lambda_{\mathrm{k}}\right\} = \bigcap_{k=1}^{\infty} \mathrm{M}(\Lambda_{\mathrm{k}})$$

where, Λ_k an arbitrary chosen events for L_k for k = 1, 2,…, respectively.

Definition 12 Uncertain variable

An uncertain variable ξ is essentially a measurable function from an uncertainty space to the set of real numbers. Let ξ be an uncertain variable. Then the uncertainty distribution of ξ is defined as $\phi(\mathrm{x}) = M\{\xi \leq x\}$ for any real number x.

An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\phi(x) = \left(1 + \exp\left(\frac{\pi(e - x)}{\sqrt{3}\,\sigma}\right)\right)^{-1} \quad for\, x \in R$$

Normal uncertainty distribution is denoted by N(e, σ), where e and σ are real numbers with $\sigma > 0$.

An uncertain distribution ϕ is said to be regular if its inverse function $\phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0, 1)$.

The normal uncertainty distribution N(e, σ) is also regular and its inverse uncertainty distribution is

$$\phi(x) = e + \frac{\sigma\sqrt{3}}{\pi} \ln \frac{\alpha}{1 - \alpha}$$

3 Problem description

In the transportation problem there are three parameters to be considered: unit cost of transportation of goods or services from the sources to the destinations, supplies available at sources and demands at destinations. The transportation problem has been earlier solved by considering the parameters as deterministic. But many a times, it is found that the parameters are uncertain due to various factors. Previously, the parameters are taken as random variables or fuzzy variables to deal with uncertainty. In this paper, a new approach has been presented by considering the parameter as rough variables. The appropriate probability distribution of random variable can be taken if sufficient sample observations are available. In absence of sample observations which is practically happening in many situations, estimation of parameters are made by the degree of belief of subject experts, where consideration of parameters as random variables are not ideal. Further, it is very likely that the experts give their estimations in range of values instead of a definite value which can be characterized by rough variables. Further, rough variables do not possess any membership function like fuzzy variables which gives an advantage by considering the parameter as rough variable. Based on this, an uncertain model has been developed.

3.1 Deterministic transportation model

Suppose that there are m sources and n destinations in a transportation problem. Let c_{ij} denote the cost of transporting one unit from source i to destination j, and x_{ij} denote the amount transported from source i to destination j, i = 1, 2,…, m, j = 1, 2,…, n. Then the objective of the problem is to make a transportation planning so that the total transportation cost is minimized. Let a_i denote the availability of source i, and b_j denote the requirement at destination j. Then the transportation problem can be described as

$$\begin{cases} Minimize \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ Subject\ to \\ \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1,2,\ldots,m \\ \sum_{i=1}^{m} x_{ij} \ge b_j, \quad j = 1,2,\ldots,n \\ x_{ij} \ge 0, \quad i = 1,2,\ldots,m\ and\ j = 1,2,\ldots,n \end{cases} \tag{1}$$

The above model is deterministic model if the quantities c_{ij}, a_i, b_j are all assumed to be crisp numbers. There are many standard techniques to solve the model.

3.2 Uncertain transportation model

In reality, the transportation planning is made in advance sometimes. Due to many uncertain factors like weather conditions, road conditions, changes in sales due to attitude of customers, the supply, demand and cost of transportation may not fixed rather uncertain. To deal with these uncertainties, the quantities c_{ij}, a_i, b_j are all assumed to be rough variables which are characterized by the subjective judgment of domain experts. Then the model (1) becomes

$$\begin{cases} \text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} x_{ij} \\ \text{Subject to} \\ \sum_{j=1}^{n} x_{ij} \le \xi_i, \quad i = 1, 2, \ldots, m. \\ \sum_{i=1}^{m} x_{ij} \ge \eta_j, \quad j = 1, 2, \ldots, n. \\ x_{ij} \ge 0, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n. \end{cases} \tag{2}$$

$$\text{where } \tilde{c}_{ij} = \left(\left[c_{ij}^1, c_{ij}^2\right]\left[c_{ij}^3, c_{ij}^4\right]\right),\ c_{ij}^3 \le c_{ij}^1 < c_{ij}^2 \le c_{ij}^4$$

$$\xi_i = ([a_i, b_i][c_i, d_i]),\ c_i \le a_i < b_i \le d_i$$

$$\eta_j = \left(\left[a_j', b_j'\right]\left[c_j', d_j'\right]\right),\ c_j' \le a_j' < b_j' \le d_j'$$

These rough variables can be approximated by uncertain normal variables by taking the mean and standard deviation of the rough variables as the mean and standard deviation of the corresponding uncertain normal variables by using the following theorem.

Theorem 1 *If* $\xi=([a,b],[c,d])$ *be a rough variable with* $c \le a < b \le d$, *then the expected value and variance of* ξ *are* $E(\xi) = \frac{a+b+c+d}{4}$ *and* $\mathrm{Var}[\xi] = \frac{a^2+b^2+c^2+d^2+ab+cd-6e^2}{6}$, where $e = E[\xi]$

$$\textit{Proof}: \quad E[\xi] = \int_{-\infty}^{\infty} xf(x)dx$$

$$= \int_c^a \frac{x}{2(d-c)}dx + \int_a^b \left[\frac{1}{2(b-a)} + \frac{1}{2(d-c)}\right]xdx + \int_b^d \frac{x}{2(d-c)}dx$$

$$= \frac{1}{4}(a+b+c+d)$$

$$\mathrm{Var}[\xi] = E(\xi - E(\xi))^2$$

$$= \int_c^a \frac{(x-e)^2}{2(d-c)}dx + \int_a^b \left[\frac{1}{2(b-a)} + \frac{1}{2(d-c)}\right](x-e)^2dx + \int_b^d \frac{(x-e)^2}{2(d-c)}dx$$

$$= \frac{a^2+b^2+c^2+d^2+ab+cd-6e^2}{6}, \text{ where } e = E[\xi]$$

Let $\mathrm{d_{ij}}$, γ_i, ζ_j be the normal uncertain variables approximating the rough variables $\tilde{\mathrm{c}}_{\mathrm{ij}}, \xi_\mathrm{i}, \eta_j$ with mean and SD as $\left(e_{ij}, \sigma_{ij}\right), (e_i, \sigma_i)$ and $\left(e_j^{'}, \sigma_j^{'}\right)$ respectively.

Considering the above theorem, the mean and SD can be obtained for these variables.

Then we obtain the equivalent form of the model (2) as

$$\begin{cases} \text{Minimize} \sum_{\mathrm{i=1}}^{\mathrm{m}} \sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{d_{ij}} x_{ij} \\ \text{subject to} \\ \sum_{\mathrm{j=1}}^{\mathrm{n}} x_{ij} \leq \gamma_i, \quad i = 1, 2, \ldots, \mathrm{m}. \\ \sum_{\mathrm{i=1}}^{\mathrm{m}} x_{ij} \geq \zeta_j, \quad j = 1, 2, \ldots, \mathrm{n}. \\ x_{ij} \geq 0, \quad i = 1, 2, \ldots, \mathrm{m}, \quad j = 1, 2, \ldots, n. \end{cases} \tag{3}$$

The model (3) becomes conceptual as in absence of natural order of uncertain variables, the objective function defined in (3) becomes invalid. Thus, the expected value criterion ([9, 10], ch. 7) for the objective function and confidence level on the constraint functions are taken. Then we obtain the equivalent form of the model (3) as

$$\begin{cases} \text{Minimize}\, \mathrm{E}\left[\sum_{\mathrm{i=1}}^{\mathrm{m}} \sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{d_{ij}} x_{ij}\right] \\ \text{Subject to} \\ \mathrm{M}\left[\sum_{\mathrm{j=1}}^{\mathrm{n}} x_{ij} \leq \gamma_i\right] \geq \alpha_i, \quad i = 1, 2, \ldots, \mathrm{m}. \\ \mathrm{M}\left[\sum_{\mathrm{i=1}}^{\mathrm{m}} x_{ij} \geq \zeta_j\right] \geq \beta_j, \quad j = 1, 2, \ldots, \mathrm{n}. \\ x_{ij} \geq 0, \quad i = 1, 2, \ldots, \mathrm{m}, \quad j = 1, 2, \ldots, n. \end{cases} \tag{4}$$

where α_i, β_j are some predetermined uncertainty confidence levels for i = 1, 2,…, m, j = 1, 2,…, n.

The model (4) can be solved once it is converted to its crisp equivalent form. For the conversion to crisp equivalent form, the following theorems are stated.

Theorem 2 (Measure Inversion Theorem [8]) *Let ξ be an uncertain variable with continuous uncertainty distribution ϕ. Then for any real number x, we have $M\{\xi \leq x\} = \phi(x)$, $M\{\xi \geq x\} = 1 - \phi(x)$.*

Theorem 3 (Liu [8]) *Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variable s with regular uncertainty distributions $\phi_1, \phi_2, \ldots, \phi_n$, respectively. If the function $f(\xi_1, \xi_2, \ldots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \ldots, \xi_m$ and strictly decreasing with respect to $\xi_{\mathrm{m}+1}, \xi_{m+2}, \ldots, \xi_n$, then $M\{f(\xi_1, \xi_2, \ldots, \xi_n) \leq 0\} \geq \alpha$ if and only if $f(\phi_1^{-1}(\alpha), \ldots, \phi_m^{-1}(\alpha), \phi_{m+1}^{-1}(1 - \alpha), \ldots, \phi_n^{-1}(1 - \alpha)) \leq 0$.*

Theorem 4 (Liu [8]) *Let ξ be an uncertain variable with regular uncertainty distribution ϕ.*

$$\text{Then } \mathrm{E}[\xi] = \int_0^1 \phi^{-1}(\alpha)d\alpha$$

Theorem 5 *Let* $\mathrm{d_{ij}}$ *are uncertain normal uncertain variable with regular uncertainty distribution ϕ_{ij}, then*

$$\mathrm{E}\left[\sum_{\mathrm{i=1}}^{\mathrm{m}}\sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{d_{ij}}x_{ij}\right] = \sum_{\mathrm{i=1}}^{\mathrm{m}}\sum_{\mathrm{j=1}}^{\mathrm{n}} e_{ij}x_{ij}$$

Proof It follows from the linearity of expected value operator is

$$\mathrm{E}\left[\sum_{\mathrm{i=1}}^{\mathrm{m}}\sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{d_{ij}}x_{ij}\right] = \sum_{\mathrm{i=1}}^{\mathrm{m}}\sum_{\mathrm{j=1}}^{\mathrm{n}} x_{ij}\mathrm{E}\left[\mathrm{d_{ij}}\right]$$

For independent uncertain normal uncertain variables $\mathrm{d_{ij}}$, $\mathrm{i} = 1, 2,\ldots,\mathrm{m}$, $\mathrm{j} = 1, 2,\ldots,\mathrm{n}$.

$$\begin{aligned}\mathrm{E}\left[\mathrm{d_{ij}}\right] &= \int_0^1 \phi_{ij}^{-1}(\alpha)d\alpha \\ &= \int_0^1 \left(e_{ij} + \frac{\sigma_{ij}\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}\right)d\alpha \\ &= \int_0^1 e_{ij}d\alpha + \frac{\sigma_{ij}\sqrt{3}}{\pi}\int_0^1 (\ln\alpha - \ln(1-\alpha))d\alpha = e_{ij}\end{aligned}$$

Using Theorems (2) and (3)

$M\left[\sum_{\mathrm{j=1}}^{\mathrm{n}} x_{ij} \le \gamma_i\right] \ge \alpha_i$ is equivalent to $\sum_{\mathrm{j=1}}^{\mathrm{n}} x_{ij} \le \psi_i^{-1}(1-\alpha_i), i{=}1, 2, \ldots, \mathrm{m}$. and $M\left[\sum_{\mathrm{i=1}}^{\mathrm{m}} x_{ij} \ge \zeta_j\right] \ge \beta_j$ is equivalent to $\sum_{\mathrm{i=1}}^{\mathrm{m}} x_{ij} \ge \varphi_j^{-1}(\beta_j),\ j = 1, 2, \ldots, \mathrm{n}$.

Then the model (4) has an equivalent crisp mathematical model

$$\begin{cases} \text{Minimize} \left[\sum_{\mathrm{i=1}}^{\mathrm{m}} \sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{e_{ij}} x_{ij} \right] \\ \text{Subject to} \\ \sum_{\mathrm{j=1}}^{\mathrm{n}} x_{ij} \le \psi_i^{-1}(1-\alpha_i), \quad i = 1,2,\ldots,\mathrm{m}. \\ \sum_{\mathrm{i=1}}^{\mathrm{m}} x_{ij} \ge \varphi_j^{-1}(\beta_j), \quad j = 1,2,\ldots,\mathrm{n}. \\ x_{ij} \ge 0, \quad i = 1,2,\ldots,\mathrm{m}\ , \quad j = 1,2,\ldots,n. \end{cases} \tag{5}$$

Using the inverse normal uncertainty distribution, the model (5) has the equivalent linear programming problem

$$\begin{cases} \text{Minimize} \left[\sum_{\mathrm{i=1}}^{\mathrm{m}} \sum_{\mathrm{j=1}}^{\mathrm{n}} x_{ij} e_{ij} \right] \\ \text{Subject to} \\ \sum_{\mathrm{j=1}}^{\mathrm{n}} x_{ij} \le e_i + \frac{\sigma_i \sqrt{3}}{\pi} \ln \frac{1-\alpha_i}{\alpha_i}, \quad i = 1,2,\ldots,\mathrm{m}. \\ \sum_{\mathrm{i=1}}^{\mathrm{m}} x_{ij} \ge e_j^1 + \frac{\sigma_j^1 \sqrt{3}}{\pi} \ln \frac{\beta_j}{1-\beta_j}, \quad j = 1,2,\ldots,\mathrm{n}. \\ x_{ij} \ge 0, \quad i = 1,2,\ldots,\mathrm{m}, \quad j = 1,2,\ldots,n. \end{cases} \tag{6}$$

4 Numerical example

Consider a problem with three sources $\mathrm{A_i}(\mathrm{i} = 1, 2, 3, 4)$ and three destinations $\mathrm{X_j}(\mathrm{j} = 1, 2, 3, 4, 5)$. The unit transportation cost, the supply at each source, demands of each destination assume rough variables are given in Table 1.

Rough supply (ξ_i) and rough demands (η_j) are given as

$$\begin{aligned} &\xi_1 = ([20, 22][19, 23]), \quad \xi_2 = ([17, 18][16, 19]), \\ &\xi_3 = ([24, 25][23, 27]), \quad \xi_4 = ([32, 34][30, 36]) \\ &\eta_1 = ([11, 13][10, 14]), \quad \eta_2 = ([24, 26][23, 27]), \quad \eta_3 = ([19, 20][18, 21]), \\ &\eta_4 = ([10, 12][8, 14]), \quad \eta_5 = ([12, 15][10, 18]), \end{aligned}$$

Table 1 Unit rough cost of transportation

	X_1	X_2	X_3	X_4	X_5
A_1	([9,12][5,15])	([22,25][18,28])	([33,35][28,40])	([14,16][12,18])	([20,23][18,24])
A_2	([28,32][25,35])	([28,32][25,35])	([12,15][9,16])	([8,10][7,12])	([16,17][15,20])
A_3	([38,42][35,45])	([22,25][18,28])	([33,35][28,40])	([22,24][20,26])	([12,15][10,18])
A_4	([32,34][30,35])	([18,20][16,24])	([20,22][18,24])	([7,9][6,12])	([12,14][10,15])

Table 2 Unit uncertain cost of transportation in terms of mean and SD

	X_1	X_2	X_3	X_4	X_5
A_1	(10.25,2.15)	(23.25,2.15)	(34,2.48)	(15,1.67)	(21.25,1.94)
A_2	(30,2.2)	(30,2.2)	(13,2.67)	(9.25,1.27)	(17,1.33)
A_3	(40,2.2)	(23.25,2.15)	(34,2.48)	(23,1.67)	(13.75,3.10)
A_4	(32.75,1.27)	(19.5,3.08)	(21,1.67)	(8.5,1.92)	(12.75,1.27)

The mean and SD of equivalent normal uncertain variable of rough costs, supplies and demands are calculated are given in Table 2.

If γ_j and ζ_i are the uncertain normal variables approximating ξ_i and η_j respectively, then

$$\gamma_1 \sim N(21,\ 0.91), \gamma_2 \sim N(17.5,\ 0.65), \gamma_3 \sim N(24.75,\ 0.88), \gamma_4 \sim N(33,\ 1.67)$$

$$\begin{aligned}&\zeta_1 \sim N(12,\ 0.91), \quad \zeta_2 \sim N(25,\ 0.91), \quad \zeta_3 \sim N(19.5,\ 0.65),\\ &\zeta_4 \sim N(11,\ 1.67), \quad \zeta_5 \sim N(13.75,\ 3.104)\end{aligned}$$

Then model (6) becomes

$$\begin{cases} \text{Minimize } \left[\sum_{i=1}^{4}\sum_{j=1}^{5} x_{ij}e_{ij}\right] \\ \text{Subject to} \\ \sum_{j=1}^{5} x_{ij} \le e_i + \dfrac{\sigma_i\sqrt{3}}{\pi}\ln\dfrac{1-\alpha_i}{\alpha_i} \\ \sum_{i=1}^{4} x_{ij} \ge e_j^1 + \dfrac{\sigma_j^1\sqrt{3}}{\pi}\ln\dfrac{\beta_j}{1-\beta_j} \\ x_{ij} \ge 0, \quad i = 1,2,3,4 \quad j = 1,2,3,4,5. \end{cases} \tag{7}$$

Assume the confidence levels are $\alpha_i = 0.9$, $\beta_j = 0.9$, $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4, 5$.

The above model becomes

$$\begin{cases} \text{Minimize Z} = 10.25x_{11} + 23.25x_{12} + 34x_{13} + 15x_{14} + 21.25x_{15} + 30x_{21} + 30x_{22} + 13x_{23} \\ \qquad + 9.25x_{24} + 17x_{25} + 40x_{31} + 23.25x_{32} + 34x_{33} + 23x_{34} + 13.75x_{35} \\ \qquad + 32.75x_{41} + 19.5x_{42} + 21x_{43} + 8.5x_{44} + 12.75x_{45} \\ \text{Subject to} \\ x_{11} + x_{12} + x_{13} + x_{14} + x_{15} \le 19.89415 \\ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \le 16.71805 \\ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \le 23.68643 \\ x_{41} + x_{42} + x_{43} + x_{44} + x_{45} \le 30.98101 \\ x_{11} + x_{21} + x_{31} + x_{41} \ge 13.10585 \\ x_{12} + x_{22} + x_{32} + x_{42} \ge 26.10585 \\ x_{13} + x_{23} + x_{33} + x_{43} \ge 20.28195 \\ x_{14} + x_{24} + x_{34} + x_{44} \ge 13.01899 \\ x_{15} + x_{25} + x_{35} + x_{45} \ge 17.51038 \\ \text{and } x_{ij} \ge 0,\ i = 1,2,3,4, \quad j = 1,2,3,4,5. \end{cases}.$$

The optimal solution is $x_{11} = 13.10585, x_{12} = 5.53168, x_{23} = 16.71805$, $x_{32} = 6.17605, x_{35} = 17.51038, x_{42} = 14.39812, x_{43} = 3.5639, x_{44} = 13.01899$ and the minimum transportation cost is 1330.909.

5 Conclusions

This paper mainly investigates a transportation problem in rough environment basing on the subjective estimation of the parameters. The rough estimates are converted to normal uncertain values to get a new transportation model based on uncertain theory. It was then transformed into crisp mathematical model by taking expected value on objective function with certain confidence level of the constraint functions. One numerical example is given to illustrate the model.

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