OPSEARCH (Oct–Dec 2015) 52(4):827–860
DOI 10.1007/s12597-015-0210-0

THEORETICAL ARTICLE

Portfolio rebalancing model with transaction costs using interval optimization

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Accepted: 28 March 2015 / Published online: 13 May 2015

Abstract In this paper, we discuss portfolio rebalancing model wherein some parameters like expected return, risk, transaction cost etc., of the objective function and constraints lie in intervals. A methodology has been proposed to find an efficient portfolio in this scenario. Further, the proposed methodology is illustrated in a numerical example with the hypothetical data to show the applicability of the results.

Keywords Closed Interval · Optimization · Portfolio selection · Rebalancing · Transaction cost

1 Introduction

The portfolio selection problem is primarily concerned with finding a combination of assets/securities that gives an investor maximum return with minimum risk. Before an investment decision is taken, the investor takes into consideration of several factors such as risk, expected return, liquidity, transaction costs, etc. Over a period of time, the different classes of assets produce different returns. Therefore, to get back the portfolio's original return and risk, it must be rebalanced by buying and selling of

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the assets. A rebalancing of portfolio is nothing but controlling the risk and return by (i) periodically monitoring the portfolio, (ii) taking care the deviation of the return of an allocated asset from its original targeted return before it is rebalanced, and (iii) verifying whether periodic rebalancing restores a portfolio to its aimed target, or some intermediate allocation is needed. For these purposes, in rebalancing of portfolio, an investor adjusts his existing portfolio by buying and /or selling of the stocks according to the market behaviour. However, each trading of stock is associated with some transaction cost, which is a fundamental factor in rebalancing of the portfolio. The transaction cost could be either fixed or variable and depends on various factors such as government tax, service tax, maintenance expenditure, security transaction tax (STT) etc. The variable transaction cost can be either a V-shaped or a concave function. A V-shaped function is directly related to absolute difference between number of units (or the proportion) corresponding to the new and current portfolio (Liu et al. [12], Best & Hlouskova [1, 2]), while concave function variable transaction cost is concave function of the number of units traded, Konno & Yamamoto [9]. In recent years, the rebalancing of portfolio with transaction cost has attracted the attention of several researchers. Some important contribution in this direction are due to Patel & Subrahmanyam [15], Konno & Wijayanayake [8], Kellerer et al. [7], Fang et al. [4], Choi et al. [3], Glen [5], Zhang et al. [18–20], Woodside-Oriakhi et al. [17].

Due to the presence of complexity in the financial market, some parameters associated with the portfolio rebalancing model are uncertain and hence these cannot be estimated exactly. In the above developments, while constructing an optimal portfolio for investment with rebalancing, the parameters like expected return, risk, proportion of total invested money, liquidity, etc. are generally estimated using probability theory, fuzzy set theory, possibility theory etc. However, sometimes it is difficult to identify the distribution function or membership function or possibility distribution function of such financial parameters. In such cases an investor can state these parameters in the form of closed intervals whose lower and upper bound can be either found from historical data or based on expert knowledge.

Some authors, see e.g., Ida [6], Lai et al. [11], Liu [13] and Kumar et al. [10] have introduced the portfolio selection model with interval parameters but they have not considered the realistic factors such as transaction cost, liquidity, market impact cost etc. Tan [16] proposed a model for portfolio selection wherein parameters are closed intervals and liquidity is a constraint. He developed a methodology to find an optimal portfolio for this model. Recently, Liu et al. [14] also proposed a portfolio optimization model for multiple time period by considering the return, risk, and liquidity in the form of closed intervals with degree of diversification, and obtained an optimal portfolio using particle swarm optimization algorithm for the maximum terminal wealth. However, both the authors have not discussed the rebalancing of the portfolio.

In this paper, we present a model which can be used to create an initial (or first time) portfolio, as well as the rebalancing of this (or existing) portfolio over an investment time horizon. This is done by considering the expected return on each asset, variance and covariance of returns of the assets, and fixed or/and variable transaction cost to lie in closed intervals. We also assume that the part of money which is consumed in transaction cost is considered as “asset zero” with constant return “-1” and risk (variance) “zero”. In order to show the effectiveness of our model, we demonstrate our methodology by considering a hypothetical numerical example as well as with the real data taken from Bombay Stock Exchange, India.

Rest of the paper is organized as follow: some preliminaries about the interval analysis are discussed in Section 2. In Section 3, a portfolio rebalancing modal with interval parameters is formulated and its solution procedure is discussed. In Section 4, we explain our procedure in form of an easily implementable algorithm. Further the whole solution procedure is demonstrated through a numerical example in Section 5. Finally, Section 6 contains some concluding remarks.

2 Preliminaries for interval analysis

The following notations are used throughout this paper.
An interval $A = [a^L, a^U]$ is the set, $\{x \in \Re |\ a^L \leq x \leq a^U\}$. $A \geq 0$ means that every element of A is positive, i.e., $a^L \geq 0$.
Any real number a can be represented as a degenerate interval $[a, a]$ and denoted by $\hat{a}$. An n-dimensional vector with real components is represented as $\mathbf{a} = (a_1, a_2, \ldots, a_n)$.

For two intervals A, B, and $* \in \{+, -, \times, \div\}$, $\circledast$ represent the operations between A and B.

$$A \circledast B = \left[\min_{a \in A,\, b \in B} (a * b),\ \max_{a \in A,\, b \in B} (a * b) \right],\quad cA(Ac) = \begin{cases} [ca^L, ca^U]\left([a^Lc, a^Uc]\right), & \text{if } c \geq 0; \\ \left[ca^U, ca^L\right]\left(\left[a^Uc, a^Lc\right]\right), & \text{if } c < 0, \end{cases}$$

where $cA = Ac$.
For n intervals $A_1, A_2, \ldots, A_n$, $\sum_{j=1}^{n} A_j = A_1 \oplus A_2 \oplus \ldots \oplus A_n$.
The set of intervals is not a totally ordered set. Several partial order relations exist in the set of intervals. Here we consider the following partial order relation ($\preceq$) between two intervals A and B

$$A \preceq B \text{ iff } a^L \leq b^L \text{ and } a^U \leq b^U, \quad A \prec B \text{ iff } a^L < b^L \text{ and } a^U < b^U. \quad (1)$$

Hence, $A \preceq \hat{b} \equiv a \leq b,\ \forall a \in A,$ and $\hat{a} \preceq B \equiv a \leq b,\ \forall b \in B.$

3 Portfolio rebalancing model

In this section, we discuss the assumption and notation, constraints, objective function and model formulation with the solution procedure.

3.1 Assumption and notations

Consider an investor who wants to invest his wealth among n risky assets/stocks, offering rate of return which lies in a closed interval. In order to develop a portfolio rebalancing model, we use following notations.

n	total number of assets, $\Lambda_n = \{1, 2, \ldots, n\}$.
p_j	the current price (value) of one unit (share) of j^{th} asset, $\forall\, j \in \Lambda_n$.

ξ_j	the number of units (shares) currently holding in portfolio of j^{th} asset.
$\mathrm{R}_j(\mu_j = \mathrm{E}(\mathrm{R}_j))$	the random rate of return (expected return) on j^{th} asset.
$\mu_j^L(\mu_j^U)$	lower (upper) bound of the expected return of j^{th} asset such that $\mu_j^L \leq \mu_j \leq \mu_j^U$, i.e., $\mu_j \in [\mu_j^L, \mu_j^U] \triangleq \bar{\mu}_j$
σ_{ij}	covariance between R_i and R_j, $i, j \in \Lambda_n$.
$\sigma_{ij}^L(\sigma_{ij}^U)$	lower(upper) bound of the covariance, wherein the covariance of returns between i^{th} and j^{th} assets varies i.e., $\sigma_{ij}^L \leq \sigma_{ij} \leq \sigma_{ij}^U$; $\sigma_{ij} \in [\sigma_{ij}^L, \sigma_{ij}^U] \triangleq \bar{\sigma}_{ij}$
$\mathrm{R}(\mu = \mathrm{E}(\mathrm{R}))$	random (expected) return (time independent) per period on the total investment.
$\bar{\mu} = [\mu^L, \mu^U]$	an interval where the expected return of investment lies, i.e., $\mu^L \leq \mu \leq \mu^U$.
γ	variance of R.
$\bar{\gamma} = [\gamma^L, \gamma^U]$	possible range for the variance of R, such that $\gamma^L \leq \gamma \leq \gamma^U$.
υ	amount of new cash that can be added or taken out from the current portfolio, if $\upsilon > 0$, then it means that cash is added in the portfolio, and if $\upsilon < 0$, then cash is taken out of the portfolio.
h	the investment horizon (number of period).
$f_{bj}\{f_{bj}^L(f_{bj}^U)\}$	the fixed transaction cost {lower (upper) bound of it} if j^{th} asset is bought, where $f_{bj}^L \leq f_{bj} \leq f_{bj}^U$, i.e., $f_{bj} \in [f_{bj}^L, f_{bj}^U] \triangleq F_{bj}$.
$f_{sj}\{f_{sj}^L(f_{sj}^U)\}$	the fixed transaction cost {lower (upper) bound of it} if j^{th} asset is sold, where $f_{sj}^L \leq f_{sj} \leq f_{sj}^U$, i.e., $f_{sj} \in [f_{sj}^L, f_{sj}^U] \triangleq F_{sj}$.
$v_{bj}\{v_{bj}^L(v_{bj}^U)\}$	the variable transaction cost {lower (upper) bound of it} if j^{th} asset is bought with $v_{bj}^L \leq v_{bj} \leq v_{bj}^U$, i.e., $v_{bj} \in [v_{bj}^L, v_{bj}^U] \triangleq V_{bj}$.
$v_{sj}\{v_{sj}^L(v_{sj}^U)\}$	the variable transaction cost {lower (upper) bound of it} if j^{th} asset is sold with $v_{sj}^L \leq v_{sj} \leq v_{sj}^U$, i.e., $v_{sj} \in [v_{sj}^L, v_{sj}^U] \triangleq V_{sj}$.
d	maximum total transaction cost for trading of stocks of portfolio,
$b_j^L(b_j^U)$	the minimum (maximum) number of units ($\geq$ 1) of j^{th} asset investors must buy if he carry out any buying of j^{th} asset.
$s_j^L(s_j^U)$	the minimum (maximum) number of units ($\geq$ 1) of j^{th} asset investors must sell if he carry out any selling of j^{th} asset.

The decision variables are:

x_j	the number of units of j^{th} asset in the rebalanced portfolio, $x_j \in \mathbb{Z}^+$.
$\alpha_j^b(\alpha_j^s)$	1 if j^{th} asset bought (sold); 0 otherwise.
$y_j^b(y_j^s)$	the number of units brought (sold) for j^{th} asset, $y_j^b, y_j^s \in \mathbb{Z}^+$.
$w_j(0 \leq w_j \leq 1)$	the proportion of total investment in j^{th} asset.
$w_0(0 \leq w_0 \leq 1)$	the proportion of total investment consumed in transaction cost.

3.2 Constraints

Before presenting mathematical model for construction of portfolio and its rebalancing, following basic constraints must be satisfied for every $j \in \Lambda_n$.

$$b_j^L \alpha_j^b \le y_j^b \le b_j^U \alpha_j^b, \tag{2}$$

$$s_j^L \alpha_j^s \le y_j^s \le s_j^U \alpha_j^s. \tag{3}$$

The above two equations represent that the number of units of j^{th} asset bought (sold) by an investor must lie in the interval $[b_j^L, b_j^U]$ ($[s_j^L, s_j^U]$). Further a single operation over the j^{th} asset is permitted, i.e., either one can buy or sell or do nothing. This can be controlled by

$$\alpha_j^b + \alpha_j^s \le 1, \quad j \in \Lambda_n. \tag{4}$$

Therefore, total number of units present in the portfolio after transaction of j^{th} asset is

$$x_j = \xi_j + y_j^b - y_j^s, \qquad j \in \Lambda_n, \tag{5}$$

with total transaction cost

$$\sum_{j=1}^{n} (f_{bj}\alpha_j^b + f_{sj}\alpha_j^s + v_{bj}y_j^b + v_{sj}y_j^s) \triangleq tc. \tag{6}$$

Due to the presence of uncertainty in the financial market, we assume that fixed and variable transaction costs vary in closed interval, i.e., $f_{bj} \in F_{bj}$, $f_{sj} \in F_{sj}$, $v_{bj} \in V_{bj}$, and $v_{sj} \in V_{sj}$. Consequently, the total transaction cost also varies in a closed interval, and it can be represented mathematically as

$$\sum_{j=1}^{n} (F_{bj}\alpha_j^b \oplus F_{sj}\alpha_j^s \oplus V_{bj}y_j^b \oplus V_{sj}y_j^s) \triangleq \text{TC}. \tag{7}$$

We also assume that money consumed in total transaction cost should not cross the maximum limit d. That is

$$\text{TC} \preceq \hat{d}. \tag{8}$$

Note that in the above inequality, the left hand side is an interval, but the right hand side is a fixed real number, i.e., $\hat{d}$ is a degenerate interval. Therefore, the inequality (8) indicates that every element of the closed interval must be less than d.

After the rebalancing of the portfolio, the monetary value of rebalanced portfolio must be equal to the sum of value of the current portfolio and new cash(υ) minus total transaction cost incurred in trading. Hence,

$$\sum_{j=1}^{n} p_j x_j = \sum_{j=1}^{n} p_j \xi_j + \upsilon - tc, \tag{9}$$

where $\sum_{j=1}^{n} p_j x_j$ is the total money invested in the rebalanced portfolio, and $\sum_{j=1}^{n} p_j \xi_j$ is the total money available from the existing portfolio. The Eq. 9 controls the investment in the rebalanced portfolio. The proportion of total available money invested in j^{th} asset is

$$\frac{p_j x_j}{\sum_{k=1}^{n} p_k \xi_k + \upsilon} \triangleq w_j, \quad j \in \Lambda_n. \tag{10}$$

Further the proportion of total available money consumed in the transaction cost is

$$\frac{tc}{\sum_{k=1}^{n} p_k \xi_k + \upsilon} \triangleq w_0. \tag{11}$$

The relations (10) and Eq. 11 implicitly satisfy $w_0 + \sum_{j=1}^{n} w_j = 1$.

As $f_{b\,j} \in F_{b\,j}$, $f_{s\,j} \in F_{s\,j}$, $v_{b\,j} \in V_{b\,j}$, and $v_{s\,j} \in V_{s\,j}$, so the proportion of total available money consumed in transaction cost w_0 lie in W_0, where

$$\begin{aligned} W_0 &\triangleq \left[\min_{f_{b\,j},\, f_{s\,j},\, v_{b\,j},\, v_{s\,j}} \left\{ \frac{tc}{\sum_{k=1}^{n} p_k \xi_k + \upsilon} \right\}, \max_{f_{b\,j},\, f_{s\,j},\, v_{b\,j},\, v_{s\,j}} \left\{ \frac{tc}{\sum_{k=1}^{n} p_k \xi_k + \upsilon} \right\} \right] \\ &= \left[\frac{\sum_{j=1}^{n} (f_{b\,j}^{\,L} \alpha_j^b + f_{s\,j}^{\,L} \alpha_j^s + v_{b\,j}^{\,L} y_j^b + v_{s\,j}^{\,L} y_j^s)}{\sum_{k=1}^{n} p_k \xi_k + \upsilon}, \right. \\ &\qquad \left. \frac{\sum_{j=1}^{n} (f_{b\,j}^{\,U} \alpha_j^b + f_{s\,j}^{\,U} \alpha_j^s + v_{b\,j}^{\,U} y_j^b + v_{s\,j}^{\,U} y_j^s)}{\sum_{k=1}^{n} p_k \xi_k + \upsilon} \right] \\ &= \frac{\mathrm{TC}}{\sum_{k=1}^{n} p_k \xi_k + \upsilon}. \end{aligned}$$

Hence, there exists $w_0 \in W_0$ such that $\sum_{j=0}^{n} w_j = 1$, which also balances the monetary value of rebalanced portfolio, money available from current portfolio and the new cash.

3.3 Objective function

In financial investment the main objective of an investor is to maximize the return for a minimum risk. Here we measure the risk as variance of the portfolio return.

3.3.1 Return and risk

As we consider the holding period of the portfolio as h, the amount invested after rebalancing of portfolio in j^{th} asset is $p_j x_j$. Then the value of investment in j^{th} asset at the end of the period h with random return rate $\mathtt{R}_j$ is $p_j x_j (1 + \mathtt{R}_j)^h$. Hence the value of portfolio at the end of the period h will be given by $\mathtt{W} = \sum_{j=1}^{n} p_j x_j (1 + \mathtt{R}_j)^h$.

In order to apply Markowitz approach, we have to calculate the expected value of $\mathtt{W}$, $\mathrm{E}(\mathtt{W})$ and the risk (variance of $\mathtt{W}$, $\mathrm{Var}(\mathtt{W})$). To find the $\mathrm{E}(\mathtt{W})$ and $\mathrm{Var}(\mathtt{W})$ involving holding period h, we adopt Woodside-Oriakhi et al. [17] approach. Accordingly, we approximate $(1+\mathtt{R}_j)^h$ by $(1+h\mathtt{R}_j)$, when $h \geq 2$, obviously for $h = 1$, it will be exact. Thus, the value of portfolio at end of time horizon h becomes $\sum_{j=1}^{n} p_j x_j (1+h\mathtt{R}_j) =$

$\mathtt{W}$. Therefore,

$$\mathrm{E}(\mathtt{W}) = \sum_{j=1}^{n} p_j x_j (1 + h\mathrm{E}(\mathtt{R}_j)) = \sum_{j=1}^{n} p_j x_j (1 + h\mu_j), \tag{12}$$

$$\begin{aligned}\mathrm{Var}(\mathtt{W}) &= \mathrm{Var}\left[\sum_{j=1}^{n} p_j x_j (1 + h\mathtt{R}_j)\right] = h^2 \mathrm{Var}\left[\sum_{j=1}^{n} p_j x_j \mathtt{R}_j\right] \\ &= h^2 \sum_{i=1}^{n}\sum_{j=1}^{n} p_i x_i p_j x_j \mathrm{Cov}(\mathtt{R}_i, \mathtt{R}_j) = h^2 \sum_{i=1}^{n}\sum_{j=1}^{n} p_i x_i p_j x_j \sigma_{ij}. \end{aligned} \tag{13}$$

As we assume that the expected return (μ_j) of j^{th} asset lies in closed interval ($\bar{\mu}_j$) i.e., μ_j =E($\mathtt{R}_j$) $\in \bar{\mu}_j$, $\forall\, j$, and the variance ($\sigma_{ij}, i = j$) and covariance ($\sigma_{ij}, i \neq j$) of the assets also vary in closed interval, i.e., $\sigma_{ij} \in \bar{\sigma_{ij}}$, $\forall\, i, j$, consequently the expected return and variance of the portfolio will also vary in closed interval. The interval form of expected return and variance of portfolio can be obtained replacing μ_j by $\bar{\mu}_j$, and σ_{ij} by $\bar{\sigma}_{ij}$, $\forall\, i, j$ in Eqs. 12 and 13, respectively. These are mathematically expressed as

$$\sum_{j=1}^{n} p_j x_j (\hat{1} \oplus h\bar{\mu}_j) \triangleq \bar{\mathtt{W}}, \ \text{ and } \ h^2 \sum_{i=1}^{n}\sum_{j=1}^{n} p_i x_i p_j x_j \bar{\sigma}_{ij} \triangleq \Gamma.$$

Now our aim is to convert E($\mathtt{W}$) and Var($\mathtt{W}$) in form of closed interval on per period return basis.

Let us assume that the initial investment is $\left(\sum_{k=1}^{n} p_i \xi_i + \upsilon\right)$, and $\mathtt{R}$ represents a time independent random return (per period) on this investment. Therefore, the total value of the investment at the end of h period will become $\left(\sum_{j=1}^{n} p_i \xi_i + \upsilon\right)(1 + \mathtt{R})^h = \mathtt{W}^*$. Similar to approximation of $\mathtt{W}$, we approximate $\mathtt{W}^*$ by $\left(\sum_{j=1}^{n} p_i \xi_i + \upsilon\right)(1 + h\mathtt{R})$. Hence

$$\mathrm{E}(\mathtt{W}^*) = \left(\sum_{j=1}^{n} p_i \xi_i + \upsilon\right)(1 + h\mu), \ \text{ and } \ \mathrm{Var}(\mathtt{W}^*) = \left(\sum_{j=1}^{n} p_i x_i + \upsilon\right)^2 h^2 \mathrm{Var}(\mathtt{R}).$$

As we have shown that the expected return of portfolio varies in the close interval, so we consider a closed interval $\bar{\mu}$, such that $\mu \in \bar{\mu}$. Accordingly, the variance of $\mathtt{R}$, γ also lies in a closed interval $\bar{\gamma}$, i.e., $\gamma \in \bar{\gamma}$. Thus, E($\mathtt{W}^*$) and Var($\mathtt{W}^*$) will also lie in closed intervals, and hence, replacing μ and γ by $\bar{\mu}$ and $\bar{\gamma}$, respectively, we obtain

$$\left(\sum_{j=1}^{n} p_i \xi_i + \upsilon\right)(\hat{1} \oplus h\bar{\mu}) = \bar{\mathtt{W}}^* \ \text{ and } \ \left(\sum_{j=1}^{n} p_i x_i + \upsilon\right)^2 h^2 \bar{\gamma} = \Gamma^*.$$

Since, $\bar{\mathtt{W}}$ and $\bar{\mathtt{W}}^*$, and Γ and Γ^* represent same intervals. Therefore, $\bar{\mathtt{W}} = \bar{\mathtt{W}}^*$ yields

$$\begin{aligned}
\left(\sum_{j=1}^{n} p_j \xi_j + \upsilon\right)(\hat{1} \oplus h\bar{\mu})) &= \sum_{j=1}^{n} p_j x_j (\hat{1} \oplus h\bar{\mu}_j), \\
(\hat{1} \oplus h\bar{\mu}) &= \sum_{j=1}^{n} w_j (\hat{1} \oplus h\bar{\mu}_j), \quad \text{using the relation (10)} \\
h\bar{\mu} &= \sum_{j=1}^{n} w_j (\hat{1} \oplus h\bar{\mu}_j) \ominus \hat{1}, \\
\bar{\mu} &= \sum_{j=1}^{n} \bar{\mu}_j w_j \ominus \frac{\left(1 - \sum_{j=1}^{n} w_j\right)}{h} \hat{1}, \\
\bar{\mu} &= \sum_{j=1}^{n} \bar{\mu}_j w_j \ominus \frac{w_0}{h} \hat{1}, \ \text{since } 1 - \sum_{j=1}^{n} w_j = w_0, \quad (14)
\end{aligned}$$

for some $w_0 \in W_0$. Note that the right hand side of Eq. 14 contains two terms, the first term represents the weighted sum of interval form of the expected return of n-assets, while the second term $-\frac{w_0}{h}$ represents the weighted combination of return on total transaction cost per period, which means the total money consumed in transaction cost is actually invested in an "asset" with constant return $\frac{-1}{h}$ per period.

Remark 1 The total money consumed in transaction cost is considered as an "asset" and we call it as "*asset zero*" with return -1, because the return on any asset is equal to (current value - previous value) / previous value. In this case current value is "zero"(as the comeback value on the transaction cost is zero), but the previous value is equal to the total transaction cost.

Now $\Gamma = \Gamma^*$, gives

$$\begin{aligned}
\left(\sum_{k=1}^{n} p_k \xi_k + \upsilon\right)^2 h^2 \bar{\gamma} &= h^2 \sum_{j=1}^{n} \sum_{j=1}^{n} p_i x_i p_j x_j \bar{\sigma}_{ij} \\
\bar{\gamma} &= \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} p_i x_i p_j x_j \bar{\sigma}_{ij}}{\left(\sum_{k=1}^{n} p_k \xi_k + \upsilon\right)^2} \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \bar{\sigma}_{ij} \qquad \text{using (10).}
\end{aligned}$$

Therefore,

$$\bar{\gamma} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \bar{\sigma}_{ij} = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}^L, \ \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}^U\right] = [\gamma^L, \gamma^U]. (15)$$

Note that the above expression does not involve h, nor does it involve w_0. Therefore, it is clear that the total money consumed in transaction cost has risk equal to zero.

3.4 Model

The portfolio rebalancing model with interval parameters for given tolerance level of expected return of portfolio $\left(\bar{\mu}_{fix} = [\mu^L_{fix}, \mu^U_{fix}]\right)$ takes the following form:

$$\textbf{(IMV)} \qquad \min\ \bar{\gamma} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \bar{\sigma}_{ij}, \tag{16}$$

$$\text{subject to} \sum_{j=1}^{n} \bar{\mu}_j w_j \ominus \frac{w_0}{h}\hat{1} \succeq \bar{\mu}_{fix}, \tag{17}$$

$$\sum_{j=1}^{n}(F_{bj}\alpha^b_j \oplus F_{sj}\alpha^s_j \oplus V_{bj}y^b_j \oplus V_{sj}y^s_j) \preceq \hat{d}, \tag{18}$$

$$\sum_{j=0}^{n} w_j = 1,\ w_0 \in W_0,\ \ 0 \leq w_j \leq 1, \tag{19}$$

$$b^L_j\alpha^b_j \leq y^b_j \leq b^U_j\alpha^b_j,\ \ s^L_j\alpha^s_j \leq y^s_j \leq s^U_j\alpha^s_j,\ \ \forall\, j, \tag{20}$$

$$\alpha^b_j + \alpha^s_j \leq 1, \quad j \in \Lambda_n, \tag{21}$$

$$x_j = \xi_j + y^b_j - y^s_j, \qquad j \in \Lambda_n, \tag{22}$$

$$x_j,\ \alpha^b_j,\ \alpha^s_j,\ y^b_j,\ y^s_j \geq 0\ \ \forall\, j \in \Lambda_n, \tag{23}$$

where $w_j,\ \forall\, j$ and w_0 are defined in Expressions 10 and 11, respectively.

Remark 2 μ^L_{fix} and μ^U_{fix}, are two given fixed values: μ^L_{fix} represents the tolerated expected return of portfolio when it is predicted pessimistically, and μ^U_{fix} represents the tolerated expected return of portfolio when it is predicted optimistically. These values are often chosen based on investor's experience.

IMV is a quadratic programming problem with interval parameters. For every feasible point of **IMV**, the objective function $\bar{\gamma}$ is an interval. Since the set of intervals is not totally ordered, so exact optimum value of **IMV** in general, may not exist. However, we may find a compromise optimal value with respect to a partial ordering as in vector optimization problems. Moreover, as in vector optimization problems, the compromise optimal value of **IMV** is not necessarily unique. Here we consider the partial ordering ($\preceq$) as defined in Eq. 1, and call the solution of **IMV** as $\preceq$-optimal solution.

Definition 1 A feasible solution $(\mathbf{x}^*, \mathbf{y}^{b*}, \mathbf{y}^{s*}, \boldsymbol{\alpha}^{b*}, \boldsymbol{\alpha}^{s*})$ of **IMV** with objective value $\bar{\gamma}^*$ is said to be an $\preceq$-optimal solution of **IMV** if there is no other feasible solution $(\mathbf{x}, \mathbf{y}^b, \mathbf{y}^s, \boldsymbol{\alpha}^b, \boldsymbol{\alpha}^s)$ with objective value $\bar{\gamma}$ such that $\bar{\gamma} \prec \bar{\gamma}^*$, where

$\mathbf{x}^* = (x_1^*, x_2^*, \dots x_n^*)$, $\mathbf{y}^{b*} = (y_1^{b*}, y_2^{b*}, \dots, y_n^{b*})$, $\mathbf{y}^{s*} = (y_1^{s*}, y_2^{s*}, \dots y_n^{s*})$, $\boldsymbol{\alpha}^{b*} = (\alpha_1^{b*}, \alpha_2^{b*}, \dots, \alpha_n^{b*})$, $\boldsymbol{\alpha}^{s*} = (\alpha_1^{s*}, \alpha_2^{s*}, \dots, \alpha_n^{s*})$, $\mathbf{x} = (x_1, x_2, \dots x_n)$, $\mathbf{y}^b = (y_1^b, y_2^b, \dots, y_n^b)$, $\mathbf{y}^s = (y_1^s, y_2^s, \dots y_n^s)$, $\boldsymbol{\alpha}^b = (\alpha_1^b, \alpha_2^b, \dots, \alpha_n^b)$, and $\boldsymbol{\alpha}^s = (\alpha_1^s, \alpha_2^s, \dots, \alpha_n^s)$.

3.5 Existence of solution

Since some parameters of **IMV** are closed intervals, so an $\preceq$-optimal solution of **IMV** can not be found directly using general optimization technique. So we consider the following parametric programming problem $\mathbf{IMV}_\lambda$ for $\lambda \in [0, 1]$.

$$
\begin{aligned}
(\mathbf{IMV}_\lambda) \qquad \min \quad & \Upsilon_\lambda\left(\mathbf{x}, \mathbf{y}^b, \mathbf{y}^s, \boldsymbol{\alpha}^b, \boldsymbol{\alpha}^s\right) \\
& = \lambda \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}^L + (1-\lambda) \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}^U \\
\text{subject to} \quad & \sum_{j=1}^{n} \mu_j w_j - \frac{w_0}{h} \geq \mu_{fix}, \ \mu_j \in \bar{\mu}_j, \ \mu_{fix} \in \bar{\mu}_{fix}, \\
& \sum_{j=1}^{n} (f_{bj} \alpha_j^b + f_{sj} \alpha_j^s + v_{bj} y_j^b + v_{sj} y_j^s) \leq d, \\
& f_{bj} \in F_{bj}, \ f_{sj} \in F_{sj}, \ v_{bj} \in V_{bj}, \ v_{sj} \in V_{sj}, \\
& \sum_{j=0}^{n} w_j = 1, \\
& b_j^L \alpha_j^b \leq y_j^b \leq b_j^U \alpha_j^b, \ \ s_j^L \alpha_j^s \leq y_j^s \leq s_j^U \alpha_j^s, \quad j \in \Lambda_n, \\
& \alpha_j^b + \alpha_j^s \leq 1, \quad j \in \Lambda_n, \\
& x_j = \xi_j + y_j^b - y_j^s, \qquad j \in \Lambda_n, \\
& w_0, \ w_j \geq 0, \ \ w_0 \in W_0, \ x_j, \ \alpha_j^b, \ \alpha_j^s, \ y_j^b, \ y_j^s \geq 0 \ \forall j \in \Lambda_n,
\end{aligned}
$$

where w_j, $\forall j$ and w_0 are defined in Expressions 10 and 11, respectively.

$\mathbf{IMV}_\lambda$ is a general quadratic programming problem with mixed integer variables whose solution can be obtained using nonlinear integer programming technique. Corresponding to each $\lambda \in [0, 1]$, $\mathbf{IMV}_\lambda$ has an optimal solution denoted by $(\mathbf{x}^*, \mathbf{y}^{b^*}, \mathbf{y}^{s*}, \boldsymbol{\alpha}^{b^*}, \boldsymbol{\alpha}^{s*})$. Such an optimal solution of $\mathbf{IMV}_\lambda$ can be obtained using any mathematical software tool which supports the quadratic and integer programming, such as LINGO, CPLEX, etc. Here one may note that when $\lambda = 0$, it means the investor estimates a desirable objective value of the problem that he is ready to accept, in other words he estimates the objective value pessimistically. However, when $\lambda = 1$, it means that the investor estimates the most desirable objective value, i.e., he estimates the objective value optimistically. In case when $\lambda = 0.5$, it means that the investor is neutral in estimating the objective value of the problem. Therefore it is recommended to select $\lambda > 0.5$ in order to find a optimum solution of $\mathbf{IMV}_\lambda$.

In the next theorem, we justify that for every parameter $\lambda \in [0, 1]$, optimum solution of $\mathbf{IMV}_\lambda$ is one $\preceq$-optimal solution of **IMV**.

Theorem 1 *(Sufficient condition for existence of solution of* **IMV***)*
An optimal solution of $\mathbf{IMV}_\lambda$ *is an* $\preceq$*-optimal solution of* ***IMV****, for* $0 \leq \lambda \leq 1$.

Proof Suppose that $(\mathbf{x}^*, \mathbf{y}^{b^*}, \mathbf{y}^{s*}, \boldsymbol{\alpha}^{b^*}, \boldsymbol{\alpha}^{s*})$ is an optimal solution of $\mathbf{IMV}_\lambda$ for some $\lambda \in [0, 1]$, with objective value

$$\Upsilon_\lambda\left(\mathbf{x}^*, \mathbf{y}^{b^*}, \mathbf{y}^{s*}, \boldsymbol{\alpha}^{b^*}, \boldsymbol{\alpha}^{s*}\right) = \lambda \sum_{i=1}^{n}\sum_{j=1}^{n} w_i^* w_j^* \sigma_{ij}^L + (1-\lambda) \sum_{i=1}^{n}\sum_{j=1}^{n} w_i^* w_j^* \sigma_{ij}^U,$$

where $w_i^* = \frac{p_i x_i^*}{\sum_{k=1}^n p_k \xi_k + \upsilon}$ and $w_j^* = \frac{p_j x_j^*}{\sum_{k=1}^n p_k \xi_k + \upsilon}$.

If possible, suppose $(\mathbf{x}^*, \mathbf{y}^{b^*}, \mathbf{y}^{s*}, \boldsymbol{\alpha}^{b^*}, \boldsymbol{\alpha}^{s*})$ is not an $\preceq$-optimal solution of **IMV**. Let the objective value of **IMV** at $(\mathbf{x}^*, \mathbf{y}^{b^*}, \mathbf{y}^{s*}, \boldsymbol{\alpha}^{b^*}, \boldsymbol{\alpha}^{s*})$ be denoted by $\bar{\gamma}^*$. Then there exists a feasible solution $(\mathbf{x}, \mathbf{y}^b, \mathbf{y}^s, \boldsymbol{\alpha}^b, \boldsymbol{\alpha}^s)$ of **IMV** with objective value $\bar{\gamma}$ such that $\bar{\gamma} \prec \bar{\gamma}^*$. This implies $\gamma^L < \gamma^{L^*}$ and $\gamma^U < \gamma^{U^*}$.

Hence, for $0 \leq \lambda \leq 1$, $\lambda\gamma^L + (1-\lambda)\gamma^U < \lambda\gamma^{L^*} + (1-\lambda)\gamma^{U^*}$. That is,

$$\lambda \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}^L + (1-\lambda) \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}^U < \lambda \sum_{i=1}^{n}\sum_{j=1}^{n} w_i^* w_j^* \sigma_{ij}^L + (1-\lambda) \sum_{i=1}^{n}\sum_{j=1}^{n} w_i^* w_j^* \sigma_{ij}^U$$

Since the feasible set of **IMV** and $\mathbf{IMV}_\lambda$ are same, so the above inequality implies that there exists a feasible solution $(\mathbf{x}, \mathbf{y}^b, \mathbf{y}^s, \boldsymbol{\alpha}^b, \boldsymbol{\alpha}^s)$ of $\mathbf{IMV}_\lambda$ such that $\Upsilon_\lambda\left(\mathbf{x}, \mathbf{y}^b, \mathbf{y}^s, \boldsymbol{\alpha}^b, \boldsymbol{\alpha}^s\right) < \Upsilon_\lambda\left(\mathbf{x}^*, \mathbf{y}^{b^*}, \mathbf{y}^{s*}, \boldsymbol{\alpha}^{b^*}, \boldsymbol{\alpha}^{s*}\right)$, which is not possible. Hence the result. □

4 Algorithm

In this section, the whole procedure for rebalancing of portfolio is described in form of an algorithm.

Step 1: Input data:

a) Number of asset (n).
b) The current price per unit of each asset (p_j).
c) The number of stocks currently holding in existing portfolio (ξ_j) of each assets.
d) (i) The bounds of fixed transaction cost $(f_{b_j}^L, f_{b_j}^U)/(f_{s_j}^L, f_{s_j}^U)$ for bought/sold,
   (ii) the bounds of variable transaction cost $(v_{b_j}^L, v_{b_j}^U)/(v_{s_j}^L, v_{s_j}^U)$ for bought/sold.
e) (i) The minimum number of units should be allowed to bought/sold (b_j^L/s_j^L),

(ii) the maximum number of units should be allowed to bought/sold (b_j^U/s_j^U),

these values are decided by the investor according to the market condition and his own potential.

f) New cash υ, maximum allowable total transaction cost d, time horizon h, and the minimum and maximum tolerance levels of expected return μ_{fix}^L and μ_{fix}^U, respectively, of portfolio such that $0 < \mu_{fix}^L,\ \mu_{fix}^U < 1$.

g) The lower and upper bounds of expected return (μ_j^L, μ_j^U), and variance and covariance of return $(\sigma_{ij}^L, \sigma_{ij}^U)$ of assets if known, otherwise estimate them from historical data on returns of the assets using Step 2 and 3, else go to step 4.

Step 2: Obtain the estimate of μ_j^L and μ_j^U using the data on return of opening- $({r_{jt}}^{open})$, maximum- $({r_{jt}}^{max})$, minimum- $({r_{jt}}^{min})$ and closing - price (r_{jt}^{close}) of each asset at the time t, for all $t = 1, 2, \ldots, T$,

a) calculate $\hat{\mu}_j^{open} = \frac{1}{T}\sum_{t=1}^{T} r_{jt}^{open}$, $\hat{\mu}_j^{close} = \frac{1}{T}\sum_{t=1}^{T} r_{jt}^{close}$, $\hat{\mu}_j^{max} = \frac{1}{T}\sum_{t=1}^{T} r_{jt}^{max}$, $\hat{\mu}_j^{min} = \frac{1}{T}\sum_{t=1}^{T} r_{jt}^{min}$, for all $j = 1, 2, \ldots, n$,

b) calculate $\hat{\mu}_j^L = \min\{\hat{\mu}_j^{open}, \hat{\mu}_j^{max}, \hat{\mu}_j^{min}, \hat{\mu}_j^{close}\}$,
$\hat{\mu}_j^U = \max\{\hat{\mu}_j^{open}, \hat{\mu}_j^{max}, \hat{\mu}_j^{min}, \hat{\mu}_j^{close}\}$, for all $j = 1, 2, \ldots, n$.

Step 3: a) Compute the estimate of σ_{ij}^L and σ_{ij}^U, $1 \le i, j \le n$ using

$$\hat{\sigma}_{ij}^L = \frac{1}{T}\sum_{t=1}^{T}\left(r_{it}^{close} - \hat{\mu}_i^L\right)\left(r_{jt}^{close} - \hat{\mu}_j^L\right) \text{ and } \hat{\sigma}_{ij}^U = \frac{1}{T}\sum_{t=1}^{T}\left(r_{it}^{close} - \hat{\mu}_i^U\right)\left(r_{jt}^{close} - \hat{\mu}_j^U\right),$$

b) if $\hat{\sigma}_{ij}^L > \hat{\sigma}_{ij}^U$ then set $\hat{\sigma}_{ij}^L = \hat{\sigma}_{ij}^U$ and $\hat{\sigma}_{ij}^U = \hat{\sigma}_{ij}^L$.

Step 4: Choose $\lambda \in [0, 1]$ and solve $\mathbf{IMV}_\lambda$ using LINGO 11 (or any other software package which support the non-linear optimization problem) and obtain the optimal solution $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)$, and $\mathbf{w}^* = (w_1^*, w_2^*, \ldots w_n^*)$, which is a initial portfolio or rebalanced portfolio with optimal range of risk $\bar{\gamma}^*$ ($\mathbf{y}^{b*}$, $\mathbf{y}^{s*}$, $\boldsymbol{\alpha}^{b*}$ and $\boldsymbol{\alpha}^{s*}$ are other necessary outputs).

Note : Convert all the interval inequalities in deterministic form by introducing a new variable corresponding to each element of the inequality. For example the equivalent deterministic form of

$$\sum_{j=1}^{n}\bar{\mu}_j w_j \ominus \frac{w_0}{h}\hat{1} \succeq \bar{\mu}_{fix},$$

is equal to

$$\sum_{j=1}^{n}\chi_j - \frac{w_0}{h} \ge \mu_{fix},\quad \mu_j^L w_j \le \chi_j \le \mu_j^U w_j,\quad \mu_{fix}^L \le \mu_{fix} \le \mu_{fix}^U,$$

where $\chi_j,\ 1 \le j \le n$ are new variables. Carry out similar conversion for other interval inequalities and than solve $\mathbf{IMV}_\lambda$.

Table 1 Current price (in Rs.) of four stocks

Stocks	Stock 1	Stock 2	Stock 3	Stock 4
Current price (p_j)	288.10	1954.42	949.54	1584

Step 5: Update the portfolio.

Remark 3
In several instances investor may not have any existing portfolio. In such situation he can first create a portfolio (known as initial portfolio), and then he can rebalance it over a fixed time horizon. In this case one can use same algorithm except the following changes in input data: set ${fs}_j^L$, ${fs}_j^U$, ${vs}_j^L$, ${vs}_j^U$, s_j^L and s_j^U equal to "zero", and consider $h = 1$.

5 Numerical example

In order to demonstrate the procedure discussed in the previous section, we consider a numerical example, wherein the total number of stocks available for investment is equal to four. We first create an initial portfolio using these four stocks and then we go for rebalancing it over a time horizon.

Step 1: Input data:

a) Total number of stocks is equal to four, i.e. $n = 4$.
b) The current price (p_j) (in Rs.) for all four stocks as given in the Table 1
c) The number of currently holding share $\xi_j = 0$, $j = 1, 2, \ldots, 4$, since we are creating a portfolio for the first time i.e. $h = 1$.
d) The lower and upper bounds of the transaction costs (fixed/variable) are given in Table 2.
e) The minimum and maximum numbers of stocks which can be bought/sold are given in Table 3.

Table 2 Lower- and upper- bounds of fixed and variable transaction costs of stocks

Transaction Cost		Stock 1	Stock 2	Stock 3	Stock 4
Fixed / Buy	Lower (${fb}_j^L$)	20	20	20	20
	Upper (${fb}_j^U$)	30	30	30	30
Fixed / Sell	Lower (${fs}_j^L$)	0	0	0	0
	Upper (${fs}_j^U$)	0	0	0	0
Variable / Buy	Lower (${vb}_j^L$)	0.667	0.6	0.7	0.333
	Upper (${vb}_j^U$)	0.7	0.677	0.9	0.6
Variable/ Sell	Lower (${vs}_j^L$)	0	0	0	0
	Upper (${vs}_j^U$)	0	0	0	0

Table 3 Minimum and maximum number of stocks bought and sold for initial portfolio

Stocks		Stock 1	Stock 2	Stock 3	Stock 4
Buy	Lower (b_j^L)	5	10	6	5
	Upper (b_j^U)	25	25	24	10
Sell	Lower (s_j^L)	0	0	0	0
	Upper (s_j^U)	0	0	0	0

f) To calculate the lower and upper bounds of the expected return, we consider the monthly return corresponding to opening-, maximum-, minimum- and closing-price, denote r_{jt}^{open}, r_{jt}^{max}, r_{jt}^{min} and r_{jt}^{close} as the rate of return of j^{th} stock at time $t = 1, 2, \ldots, 12$, respectively. This is given in Table 4. Such data is usually available in any share market.

g) The value of new cash is υ = Rs. 10000, maximum allowable total transaction cost d = Rs. 100, $h = 1$. Minimum and maximum tolerance level of expected return of portfolio are chosen as $\mu_{fix}^L = 0.013$ and $\mu_{fix}^U = 0.060$, respectively.

Step 2: Using the returns given in Table 4, we obtain the estimate (μ_j^L) and (μ_j^U), which are given in the Table 5.

Step 3: Estimates of σ_{ij}^L and σ_{ij}^U obtained using the data from Tables 4 and 5, and these are given in Table 6.

Step 4: We solve the $\mathbf{IMV}_\lambda$ and this provides an optimal portfolio in form of number of shares of each stock $\mathbf{x}^* = (7, 0, 0, 5)$, as well as proportion of the investment $\mathbf{w}^* = (0.202, 0, 0, 0.792)$ with a optimum range of risk $[0.005456, 0.005464]$ for $\lambda = 0.6$. We also obtain $\mathbf{y}^{b*} = (7, 0, 0, 5)$, $\mathbf{y}^{s*} = (0, 0, 0, 0)$, $\boldsymbol{\alpha}^{b*} = (1, 0, 0, 1)$, and $\boldsymbol{\alpha}^{s*} = (0, 0, 0, 0)$. One may observe that the initial portfolio consists of stock 1 and stock 4 with the number of shares 7 and 5 as well as the proportion of total investment in these shares as 0.202 and 0.792, respectively. The vector $\mathbf{y}^{b*}$ gives us the number of shares 7 and 5 purchased for the stocks 1 and 4, respectively, and zero for stocks 2 and 3. But $\mathbf{y}^{s*}$ indicates that no shares has been sold for any stocks.

Step 5: Hence the optimal initial portfolio is $\mathbf{x}^* = (7, 0, 0, 5)$, with $\mathbf{w}^* = (0.202, 0, 0, 0.792)$.

Next, we rebalance the above portfolio (obtained in step 5) for $h = 2$. For this, we keep the lower and upper bounds of expected return (in Table 5), variance and covariance (in Table 6) of stocks, range of the transactions cost for buying (in Table 2), and the maximum and minimum numbers of stocks (in Table 3) that can be bought, same as the initial investment. In order to find an optimal rebalanced portfolio using the algorithm discuss in Section 4, we discuss the whole procedure by providing other required input data. The number of shares available at current price of existing stocks in the portfolio are given in Table 7.

Table 4 Returns corresponding to opening-, maximum-, minimum- and closing-price of four stocks

Stocks	Stock 1				Stock 2				Stock 3				Stock 4			
S. No.	Open	Max	Min	Close	Open	Max	Min	Close	Open	Max	Min	Close	Open	Max	Min	Close
1	0.134	−0.002	0.005	−0.074	0.204	0.396	0.250	0.463	0.129	0.187	0.134	0.185	0.144	0.316	0.166	0.310
2	−0.074	0.055	0.028	0.136	0.442	0.023	0.316	−0.068	0.176	0.174	0.107	0.206	0.300	0.055	0.219	0.034
3	0.137	0.086	0.104	0.088	−0.073	−0.049	−0.062	0.041	0.190	0.044	0.217	0.051	0.030	−0.020	−0.015	0.005
4	0.072	0.011	0.048	−0.015	0.050	0.025	0.104	−0.039	0.051	0.018	0.052	−0.028	0.003	−0.015	0.015	−0.020
5	−0.012	−0.044	0.014	0.030	−0.036	0.184	0.024	0.260	−0.016	0.198	−0.092	0.243	−0.018	0.084	0.049	0.118
6	0.023	0.087	0.041	0.029	0.239	0.119	0.198	−0.002	0.209	0.017	0.281	0.021	0.112	0.051	0.113	−0.013
7	0.018	0.081	0.035	0.113	0.005	−0.042	0.005	0.022	0.037	0.106	0.139	0.119	−0.026	0.041	0.001	0.093
8	0.125	0.094	0.156	0.048	0.029	−0.008	0.033	0.014	0.118	0.084	0.054	0.013	0.094	0.016	0.038	−0.041
9	0.058	−0.028	−0.041	−0.055	0.005	−0.025	−0.080	−0.093	0.002	0.011	−0.018	−0.016	−0.035	−0.022	−0.055	−0.041
10	−0.065	−0.065	−0.005	−0.006	−0.097	−0.110	−0.048	−0.040	−0.015	−0.036	0.024	0.016	−0.045	−0.034	−0.001	0.045
11	0.003	0.075	0.029	0.069	−0.027	0.029	0.062	0.052	0.028	0.089	0.051	0.117	0.065	0.146	0.101	0.134
12	0.065	−0.007	0.036	0.016	0.048	0.094	0.019	0.105	0.113	−0.017	0.026	−0.011	0.127	0.012	0.109	0.031

Table 5 Estimated lower and upper value of expected return of four stocks

Stocks		Stock 1	Stock 2	Stock 3	Stock 4
Return	Lower ($\hat{\mu}_j$)	0.029	0.053	0.073	0.052
	Upper ($\hat{\mu}_j$)	0.040	0.068	0.085	0.063

Since some of the stocks should be sold at the time of rebalancing. Therefore, both the transaction cost (fixed / variable) must be applied at the time of selling of any share, also it is necessary to fix the minimum and maximum numbers of stocks that can be allowed for selling. In Table 8, we give the lower and upper bounds of fixed and variable transaction cost, and the minimum and maximum number of shares that can be sold .

We add an extra amount v = Rs. 1300 in the existing portfolio to be used in rebalancing of the portfolio. In addition to this we also assume that money consumed in total transaction cost should not exceed d = Rs. 100.

Based on these information, and the fixed minimum and maximum tolerance levels of return $\mu^L_{fix} = 0.013$, and $\mu^U_{fix} = 0.060$, respectively, we obtain an optimal rebalanced portfolio by solving $\mathbf{IMV}_\lambda$. Thus the optimum solution is $\mathbf{x}^* = (17, 0, 0, 4)$, $\mathbf{w}^* = (0.428, 0, 0, 0.568)$ with $w_0 = 0.004$, $\mathbf{y}^{b*} = (10, 0, 0, 0)$, $\mathbf{y}^{s*} = (0, 0, 0, 1)$, $\boldsymbol{\alpha}^{b*} = (1, 0, 0, 0)$ and $\boldsymbol{\alpha}^{s*} = (0, 0, 0, 1)$ with optimal risk $\bar{\gamma}^* = [0.003016, 0.003034]$ for $\lambda = 0.6$. One may observe from these results that the stocks 1 and 4 will form the rebalanced portfolio with number of shares 17 and 4, respectively. This means stock 1 should contain 17 shares but there are only 7 in the existing portfolio that means 10 more shares should be purchased for the rebalanced portfolio. Similarly, stock 4 contains 5 shares in existing portfolio, but after the rebalancing it should have only 4 shares and therefore one share should be sold.

In order to show the applicability of the model to real life example, we consider the data sets from Bombay Stock Exchange, India, and obtain efficient portfolio for initial and rebalanced portfolio over a time horizon of 1, 3, 6, and 12 months. This is discussed below.

Table 6 Estimate of lower- and upper- bound of variance and covariance of stocks

Stocks		Stock 1	Stock 2	Stock 3	Stock 4
Stock 1	Lower ($\hat{\sigma}^L_{1j}$)	0.00374	−0.00333	0.00177	−0.00123
	Upper ($\hat{\sigma}^U_{1j}$)	0.00381	−0.00327	0.00184	−0.00122
Stock 2	Lower ($\hat{\sigma}^L_{2j}$)	−0.00333	0.02269	0.00768	0.01229
	Upper ($\hat{\sigma}^U_{2j}$)	−0.00327	0.02272	0.00773	0.01231
Stock 3	Lower ($\hat{\sigma}^L_{3j}$)	0.00177	0.00768	0.00823	0.00590
	Upper ($\hat{\sigma}^U_{3j}$)	0.00184	0.00773	0.00830	0.00591
Stock 3	Lower ($\hat{\sigma}^L_{4j}$)	−0.00123	0.01229	0.00590	0.00908
	Upper ($\hat{\sigma}^U_{4j}$)	−0.00122	0.01231	0.00591	0.00909

Table 7 Number of shares currently holding and current price (in Rs.) of four stocks in the existing portfolio

Stocks	Stock 1	Stock 2	Stock 3	Stock 4
Number of share currently holding (ξ_j)	7	0	0	5
Current price per unit (p_j),	287.75	2191	1009.15	1621

5.1 Empirical example

We have collected the data for monthly opening-, maximum-, minimum- and closing-price of thirty stocks listed in Bombay Stock Exchange-30 (BSE-30), India for a period September 1, 2010 to October 30, 2014. From these data sets, first we find rate of returns and then calculate the simple average corresponding to each prices for each stocks. Thereafter, using step 2 of the algorithm we estimate the bounds of expected return of each stocks, which is given in Table 9 including scrip code, scrip name and current price of the stocks. The closing price on October 31, 2014 of the stocks is considered as current price of each stock.

Next the lower (σ_{ij}^L) and upper (σ_{ij}^U) bounds of variance and covariance of rate of returns are calculated using the relation given in step 3 of the algorithm. These are given in Table 10.

As far as fixed and variable transaction costs are concerned, the upper and lower bounds of both transaction costs are taken based on expert knowledge. We consider these bounds associated with each trading as provided in the Table 11.

The minimum and maximum numbers of stocks which can be bought/sold are decided by the investor according to the market condition and his own potential. We choose these values from the one given in Table 12.

Since our aim is to create first an optimal portfolio for the first time investment, we keep the number units of stock in current portfolio (ξ_j) as zero.

Based on the above information, and given value of $\upsilon =$ Rs. 100000, $d =$ Rs. 1000 and $h = 1$, our objective is to find the optimal investment strategy with minimum risk for a given tolerance level of expected return $\mu_{fix} = [0.013, 0.030]$ of the portfolio. For this, **IMV**$_\lambda$ is solved using LINGO 11 for $\lambda = 0.6$. The optimal investment

Table 8 Lower- and upper- bounds of fixed and variable transaction costs, and minimum and maximum number of share allowed for selling

Stocks		Stock 1	Stock 2	Stock 3	Stock 4
Fixed / Sell	Lower ($f{s}_j^L$)	7	7	7	7
	Upper ($f{s}_j^U$)	10	10	10	10
Variable/ Sell	Lower ($v{s}_j^L$)	0.35	0.58	0.23	0.13
	Upper ($v{s}_j^U$)	0.48	0.76	0.327	0.41
Sell	Minimum (s_j^L)	3	2	1	1
	Maximum (s_j^U)	7	9	11	5

Table 9 The scrip code, scrip name, lower and upper bounds of expected return and current price of 30 stocks

Stock	Scrip Code	Scrip Name	Bounds of Returns		Current price
			lower bound(μ_j^L)	upper bound(μ_j^U)	in Rs.(p_j)
Stock 1	500010	HDFC	0.008	0.011	1105.95
Stock 2	500087	CIPLA	0.015	0.018	667.10
Stock 3	500103	BHEL	−0.029	−0.023	255.75
Stock 4	500112	SBIN	0.000	0.003	2701.65
Stock 5	500124	DRREDDY	0.016	0.020	3161.70
Stock 6	500180	HDFCBANK	−0.002	0.001	912.20
Stock 7	500182	HEROMOTOCO	0.013	0.013	3061.60
Stock 8	500209	INFY	0.008	0.011	4051.45
Stock 9	500295	SSLT	0.000	0.005	255.75
Stock 10	500312	ONGC	−0.009	−0.007	405.05
Stock 11	500325	RELIANCE	0.001	0.003	999.20
Stock 12	500400	TATAPOWER	−0.024	−0.020	93.75
Stock 13	500440	HINDALCO	0.000	0.005	163.20
Stock 14	500470	TATASTEEL	−0.004	0.005	489.35
Stock 15	500510	LT	0.001	0.004	1654.85
Stock 16	500520	M&M	0.015	0.019	1303.40
Stock 17	500570	TATAMOTORS	0.001	0.008	535.65
Stock 18	500696	HINDUNILVR	0.020	0.023	738.35
Stock 19	500875	ITC	0.015	0.018	355.25
Stock 20	507685	WIPRO	0.008	0.012	563.45
Stock 21	524715	SUNPHARMA	0.003	0.006	845.55
Stock 22	532155	GAIL	0.001	0.004	529.15
Stock 23	532174	ICICIBANK	0.010	0.013	1625.45
Stock 24	532215	AXISBANK	−0.007	−0.002	438.75
Stock 25	532454	BHARTIARTL	0.004	0.008	398.30
Stock 26	532500	MARUTI	0.020	0.023	3338.35
Stock 27	532540	TCS	0.023	0.026	2604.55
Stock 28	532555	NTPC	−0.006	−0.004	149.95
Stock 29	532977	BAJAJ-AUTO	0.002	0.014	2609.05
Stock 30	533278	COALINDIA	0.003	0.007	369.35

strategies in terms of units that can be purchased, and proportion of total investment for each 30 stocks are provided in Table 13 with other decision variables.

One may observe from Table 13 that the stocks HDFC, HEROMOTOCO, RELIANCE, HINDALCO, M&M, HINDUNILVR, ITC, WIPRO, SUNPHARMA, GAIL, ICICIBANK, BHARTIARTL, TCS and COALINDIA are selected for initial

Table 10 Covariance matrix of rate of returns of 30 stocks

Stocks		HDFC	CIPLA	BHEL	SBIN	DRREDDY	HDFCBANK	HEROMOTOCO	INFY	SSLT	ONGC	RELIANCE
HDFC	Lower (σ_{1j}^L)	0.002	0.001	0.002	0.002	0.000	0.002	0.000	0.000	0.001	0.002	0.001
	Upper (σ_{1j}^U)	0.003	0.001	0.002	0.003	0.001	0.003	0.000	0.001	0.001	0.003	0.002
CIPLA	Lower (σ_{2j}^L)	0.001	0.003	0.001	0.000	0.001	0.002	0.000	0.001	0.000	0.002	0.000
	Upper (σ_{2j}^U)	0.001	0.005	0.002	0.001	0.002	0.002	0.000	0.001	0.002	0.002	0.001
BHEL	Lower (σ_{3j}^L)	0.002	0.001	0.023	0.003	0.000	0.002	0.001	−0.002	0.003	−0.001	0.000
	Upper (σ_{3j}^U)	0.002	0.002	0.027	0.004	0.002	0.003	0.001	−0.002	0.006	0.004	0.001
SBIN	Lower (σ_{4j}^L)	0.002	0.000	0.003	0.009	0.000	0.004	0.001	−0.001	0.004	0.003	0.003
	Upper (σ_{4j}^U)	0.003	0.001	0.004	0.011	0.000	0.005	0.001	0.000	0.007	0.005	0.005
DRREDDY	Lower (σ_{5j}^L)	0.000	0.001	0.000	0.000	0.002	0.000	0.000	0.001	−0.001	0.001	0.000
	Upper (σ_{5j}^U)	0.001	0.002	0.002	0.000	0.004	0.001	0.001	0.001	0.000	0.001	0.000
HDFCBANK	Lower (σ_{6j}^L)	0.002	0.002	0.002	0.004	0.000	0.015	−0.002	0.001	0.002	−0.001	0.002
	Upper (σ_{6j}^U)	0.003	0.002	0.003	0.005	0.001	0.018	−0.002	0.001	0.004	0.003	0.004
HEROMOTOCO	Lower (σ_{7j}^L)	0.000	0.000	0.001	0.001	0.000	−0.002	0.006	−0.001	0.000	−0.002	0.000
	Upper (σ_{7j}^U)	0.000	0.000	0.001	0.001	0.001	−0.002	0.006	0.000	0.001	0.003	0.001
INFY	Lower (σ_{8j}^L)	0.000	0.001	−0.002	−0.001	0.001	0.001	−0.001	0.005	0.000	0.001	0.001
	Upper (σ_{8j}^U)	0.001	0.001	−0.002	0.000	0.001	0.001	0.000	0.008	0.000	0.001	0.001
SSLT	Lower (σ_{9j}^L)	0.001	0.000	0.003	0.004	−0.001	0.002	0.000	0.000	0.011	0.002	0.002
	Upper (σ_{9j}^U)	0.001	0.002	0.006	0.007	0.000	0.004	0.001	0.000	0.018	0.006	0.005
ONGC	Lower (σ_{10j}^L)	0.002	0.002	−0.001	0.003	0.001	−0.001	−0.002	0.001	0.002	0.016	0.002
	Upper (σ_{10j}^U)	0.003	0.002	0.004	0.005	0.001	0.003	0.003	0.001	0.006	0.018	0.002
RELIANCE	Lower (σ_{11j}^L)	0.001	0.000	0.000	0.003	0.000	0.002	0.000	0.001	0.002	0.002	0.004
	Upper (σ_{11j}^U)	0.002	0.001	0.001	0.005	0.000	0.004	0.001	0.001	0.005	0.002	0.006

Table 10 (continued)

Stocks		TATAPOWER	HINDALCO	TATASTEEL	LT	M&M	TATAMOTORS	HINDUNILVR	ITC	WIPRO	SUNPHARMA	GAIL
HDFC	Lower (σ_{1j}^{L})	0.001	0.001	0.001	0.002	0.001	0.001	0.000	0.000	−0.001	0.002	0.000
	Upper (σ_{1j}^{U})	0.003	0.003	0.003	0.002	0.002	0.004	0.001	0.001	0.001	0.003	0.001
CIPLA	Lower (σ_{2j}^{L})	0.000	0.001	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.001	0.001
	Upper (σ_{2j}^{U})	0.001	0.002	0.001	0.000	0.000	0.002	0.001	0.000	0.002	0.002	0.002
BHEL	Lower (σ_{3j}^{L})	0.015	0.004	0.004	0.007	−0.001	0.013	−0.002	0.000	−0.002	0.002	0.003
	Upper (σ_{3j}^{U})	0.016	0.006	0.006	0.009	0.000	0.014	−0.001	0.000	−0.001	0.004	0.003
SBIN	Lower (σ_{4j}^{L})	0.005	0.005	0.003	0.005	0.003	0.003	0.000	0.001	−0.002	−0.001	0.000
	Upper (σ_{4j}^{U})	0.008	0.007	0.008	0.007	0.003	0.007	0.001	0.001	−0.001	0.003	0.001
DRREDDY	Lower (σ_{5j}^{L})	0.000	0.000	0.000	−0.001	0.000	0.002	0.000	0.001	0.000	−0.001	−0.001
	Upper (σ_{5j}^{U})	0.001	0.000	0.000	0.000	0.001	0.004	0.000	0.001	0.000	0.001	0.001
HDFCBANK	Lower (σ_{6j}^{L})	0.002	0.003	0.002	0.004	0.002	0.003	0.001	0.001	0.002	0.002	0.002
	Upper (σ_{6j}^{U})	0.005	0.005	0.006	0.007	0.002	0.005	0.003	0.002	0.002	0.002	0.002
HEROMOTOCO	Lower (σ_{7j}^{L})	−0.001	−0.002	−0.002	0.001	0.001	−0.002	0.000	0.000	−0.001	−0.003	0.001
	Upper (σ_{7j}^{U})	0.000	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.001	−0.001	0.002
INFY	Lower (σ_{8j}^{L})	−0.002	−0.001	0.001	−0.001	0.000	−0.001	−0.001	0.000	0.002	−0.001	0.000
	Upper (σ_{8j}^{U})	0.000	0.000	0.001	−0.001	0.001	0.002	0.000	0.001	0.006	0.000	0.001
SSLT	Lower (σ_{9j}^{L})	0.005	0.005	0.005	0.000	0.001	0.004	−0.002	−0.002	0.000	0.001	0.001
	Upper (σ_{9j}^{U})	0.010	0.009	0.012	0.007	0.001	0.008	0.000	0.000	0.001	0.002	0.003
ONGC	Lower (σ_{10j}^{L})	0.001	0.001	0.003	0.002	0.002	0.001	−0.001	0.000	−0.001	0.004	−0.001
	Upper (σ_{10j}^{U})	0.005	0.004	0.004	0.004	0.005	0.004	0.000	0.001	0.000	0.004	0.002
RELIANCE	Lower (σ_{11j}^{L})	0.001	0.002	0.001	0.003	0.002	0.001	0.000	0.000	0.000	−0.003	0.000
	Upper (σ_{11j}^{U})	0.004	0.005	0.005	0.003	0.002	0.004	0.001	0.001	0.001	0.002	0.001

Table 10 (continued)

Stocks		ICICIBANK	AXISBANK	BHARTIARTL	MARUTI	TCS	NTPC	BAJAJ-AUTO	COALINDIA
HDFC	Lower (σ_{1j}^{L})	0.000	−0.002	0.001	0.002	0.000	0.001	0.000	0.001
	Upper (σ_{1j}^{U})	0.002	0.001	0.002	0.003	0.000	0.001	0.001	0.002
CIPLA	Lower (σ_{2j}^{L})	0.000	−0.002	0.001	0.000	0.001	0.000	0.000	0.000
	Upper (σ_{2j}^{U})	0.000	0.000	0.001	0.002	0.001	0.001	0.001	0.000
BHEL	Lower (σ_{3j}^{L})	0.003	0.004	0.001	0.002	−0.001	0.001	0.000	0.002
	Upper (σ_{3j}^{U})	0.006	0.008	0.001	0.004	0.000	0.001	0.002	0.004
SBIN	Lower (σ_{4j}^{L})	0.000	0.000	0.001	0.005	−0.002	0.003	0.000	0.004
	Upper (σ_{4j}^{U})	0.006	0.007	0.003	0.009	−0.001	0.005	0.001	0.005
DRREDDY	Lower (σ_{5j}^{L})	0.000	−0.002	0.000	0.001	0.000	−0.001	0.000	0.000
	Upper (σ_{5j}^{U})	0.001	0.000	0.000	0.002	0.001	0.000	0.000	0.000
HDFCBANK	Lower (σ_{6j}^{L})	0.003	0.002	0.001	0.002	0.001	0.002	−0.001	0.002
	Upper (σ_{6j}^{U})	0.003	0.003	0.001	0.004	0.001	0.004	−0.001	0.003
HEROMOTOCO	Lower (σ_{7j}^{L})	0.001	0.000	0.001	0.001	−0.001	−0.001	0.003	0.000
	Upper (σ_{7j}^{U})	0.003	0.002	0.001	0.001	0.000	−0.001	0.003	0.000
INFY	Lower (σ_{8j}^{L})	−0.001	−0.003	0.000	−0.001	0.002	0.000	0.000	−0.002
	Upper (σ_{8j}^{U})	0.000	0.000	0.001	0.001	0.004	0.000	0.000	−0.001
SSLT	Lower (σ_{9j}^{L})	−0.001	0.001	0.002	0.003	0.001	0.004	−0.001	0.003
	Upper (σ_{9j}^{U})	0.003	0.004	0.002	0.007	0.001	0.007	0.002	0.005
ONGC	Lower (σ_{10j}^{L})	−0.003	−0.005	0.001	0.004	0.000	0.002	−0.003	0.001
	Upper (σ_{10j}^{U})	0.003	0.003	0.001	0.005	0.001	0.005	0.003	0.002
RELIANCE	Lower (σ_{11j}^{L})	−0.002	−0.001	0.001	0.002	0.000	0.002	0.000	0.002
	Upper (σ_{11j}^{U})	0.003	0.003	0.002	0.005	0.001	0.003	0.001	0.002

Table 10 (continued)

Stocks		HDFC	CIPLA	BHEL	SBIN	DRREDDY	HDFCBANK	HEROMOTOCO	INFY	SSLT	ONGC	RELIANCE
TATAPOWER	Lower (σ_{12j}^L)	0.001	0.000	0.015	0.005	0.000	0.002	−0.001	−0.002	0.005	0.001	0.001
	Upper (σ_{12j}^U)	0.003	0.001	0.016	0.008	0.001	0.005	0.000	0.000	0.010	0.005	0.004
HINDALCO	Lower (σ_{13j}^L)	0.001	0.001	0.004	0.005	0.000	0.003	−0.002	−0.001	0.005	0.001	0.002
	Upper (σ_{13j}^U)	0.003	0.002	0.006	0.007	0.000	0.005	0.001	0.000	0.009	0.004	0.005
TATASTEEL	Lower (σ_{14j}^L)	0.001	0.000	0.004	0.003	0.000	0.002	−0.002	0.001	0.005	0.003	0.001
	Upper (σ_{14j}^U)	0.003	0.001	0.006	0.008	0.000	0.006	0.001	0.001	0.012	0.004	0.005
LT	Lower (σ_{15j}^L)	0.002	0.000	0.007	0.005	−0.001	0.004	0.001	−0.001	0.000	0.002	0.003
	Upper (σ_{15j}^U)	0.002	0.000	0.009	0.007	0.000	0.007	0.001	−0.001	0.007	0.004	0.003
M&M	Lower (σ_{16j}^L)	0.001	0.000	−0.001	0.003	0.000	0.002	0.001	0.000	0.001	0.002	0.002
	Upper (σ_{16j}^U)	0.002	0.000	0.000	0.003	0.001	0.002	0.001	0.001	0.001	0.005	0.002
TATAMOTORS	Lower (σ_{17j}^L)	0.001	0.001	0.013	0.003	0.002	0.003	−0.002	−0.001	0.004	0.001	0.001
	Upper (σ_{17j}^U)	0.004	0.002	0.014	0.007	0.004	0.005	0.001	0.002	0.008	0.004	0.004
HINDUNILVR	Lower (σ_{18j}^L)	0.000	0.001	−0.002	0.000	0.000	0.001	0.000	−0.001	−0.002	−0.001	0.000
	Upper (σ_{18j}^U)	0.001	0.001	−0.001	0.001	0.000	0.003	0.000	0.000	0.000	0.000	0.001
ITC	Lower (σ_{19j}^L)	0.000	0.000	0.000	0.001	0.001	0.001	0.000	0.000	−0.002	0.000	0.000
	Upper (σ_{19j}^U)	0.001	0.000	0.000	0.001	0.001	0.002	0.000	0.001	0.000	0.001	0.001
WIPRO	Lower (σ_{20j}^L)	−0.001	0.000	−0.002	−0.002	0.000	0.002	−0.001	0.002	0.000	−0.001	0.000
	Upper (σ_{20j}^U)	0.001	0.002	−0.001	−0.001	0.000	0.002	0.001	0.006	0.001	0.000	0.001
SUNPHARMA	Lower (σ_{21j}^L)	0.002	0.001	0.002	−0.001	−0.001	0.002	−0.003	−0.001	0.001	0.004	−0.003
	Upper (σ_{21j}^U)	0.003	0.002	0.004	0.003	0.001	0.002	−0.001	0.000	0.002	0.004	0.002
GAIL	Lower (σ_{22j}^L)	0.000	0.001	0.003	0.000	−0.001	0.002	0.001	0.000	0.001	−0.001	0.000
	Upper (σ_{22j}^U)	0.001	0.002	0.003	0.001	0.001	0.002	0.002	0.001	0.003	0.002	0.001

Table 10 (continued)

Stocks		TATAPOWER	HINDALCO	TATASTEEL	LT	M&M	TATAMOTORS	HINDUNILVR	ITC	WIPRO	SUNPHARMA	GAIL
TATAPOWER	Lower (σ_{12j}^{L})	0.020	0.004	0.003	0.006	0.000	0.015	−0.002	−0.001	−0.001	−0.002	0.001
	Upper (σ_{12j}^{U})	0.025	0.008	0.009	0.006	0.002	0.020	0.001	0.003	0.002	0.004	0.001
HINDALCO	Lower (σ_{13j}^{L})	0.004	0.008	0.003	0.003	0.001	0.004	0.000	0.000	−0.001	−0.001	0.000
	Upper (σ_{13j}^{U})	0.008	0.012	0.011	0.005	0.001	0.008	0.001	0.001	0.001	0.003	0.002
TATASTEEL	Lower (σ_{14j}^{L})	0.003	0.003	0.005	0.002	0.001	0.003	−0.002	−0.001	0.000	0.002	0.000
	Upper (σ_{14j}^{U})	0.009	0.011	0.016	0.008	y002	0.010	0.001	0.000	0.002	0.004	0.001
LT	Lower (σ_{15j}^{L})	0.006	0.003	0.002	0.009	0.003	0.003	0.000	0.001	−0.002	0.000	0.002
	Upper (σ_{15j}^{U})	0.006	0.005	0.008	0.015	0.003	0.006	0.001	0.001	−0.002	0.003	0.003
M&M	Lower (σ_{16j}^{L})	0.000	0.001	0.001	0.003	0.004	0.000	0.000	0.000	−0.001	−0.001	0.000
	Upper (σ_{16j}^{U})	0.002	0.001	0.002	0.003	0.006	0.003	0.001	0.001	0.000	0.002	0.000
TATAMOTORS	Lower (σ_{17j}^{L})	0.015	0.004	0.003	0.003	0.000	0.018	−0.003	−0.001	0.000	−0.002	−0.002
	Upper (σ_{17j}^{U})	0.020	0.008	0.010	0.006	0.003	0.025	0.001	0.003	0.003	0.005	0.001
HINDUNILVR	Lower (σ_{18j}^{L})	−0.002	0.000	−0.002	0.000	0.000	−0.003	0.004	0.001	−0.001	0.001	0.000
	Upper (σ_{18j}^{U})	0.001	0.001	0.001	0.001	0.001	0.001	0.004	0.002	0.000	0.001	0.000
ITC	Lower (σ_{19j}^{L})	−0.001	0.000	−0.001	0.001	0.000	−0.001	0.001	0.003	0.000	0.001	0.000
	Upper (σ_{19j}^{U})	0.003	0.001	0.000	0.001	0.001	0.003	0.002	0.003	0.000	0.001	0.001
WIPRO	Lower (σ_{20j}^{L})	−0.001	−0.001	0.000	−0.002	−0.001	0.000	−0.001	0.000	0.007	−0.002	0.000
	Upper (σ_{20j}^{U})	0.002	0.001	0.002	−0.002	0.000	0.003	0.000	0.000	0.007	−0.002	0.001
SUNPHARMA	Lower (σ_{21j}^{L})	−0.002	−0.001	0.002	0.000	−0.001	−0.002	0.001	0.001	−0.002	0.022	−0.002
	Upper (σ_{21j}^{U})	0.004	0.003	0.004	0.003	0.002	0.005	0.001	0.001	−0.002	0.022	0.001
GAIL	Lower (σ_{22j}^{L})	0.001	0.000	0.000	0.002	0.000	−0.002	0.000	0.000	0.000	−0.002	0.003
	Upper (σ_{22j}^{U})	0.001	0.002	0.001	0.003	0.000	0.001	0.000	0.001	0.001	0.001	0.005

Table 10 (continued)

Stocks		ICICIBANK	AXISBANK	BHARTIARTL	MARUTI	TCS	NTPC	BAJAJ-AUTO	COALINDIA
TATAPOWER	Lower (σ_{12j}^L)	−0.002	−0.004	0.001	0.003	0.000	0.002	−0.001	0.005
	Upper (σ_{12j}^U)	0.003	0.005	0.004	0.006	0.002	0.006	−0.001	0.007
HINDALCO	Lower (σ_{13j}^L)	−0.001	−0.002	0.001	0.003	0.000	0.001	−0.001	0.002
	Upper (σ_{13j}^U)	0.004	0.003	0.002	0.007	0.000	0.004	−0.001	0.004
TATASTEEL	Lower (σ_{14j}^L)	−0.002	−0.001	0.001	0.003	0.000	0.002	−0.001	0.002
	Upper (σ_{14j}^U)	0.003	0.002	0.002	0.008	0.000	0.005	0.000	0.004
LT	Lower (σ_{15j}^L)	0.005	0.007	0.001	0.004	−0.002	0.002	0.002	0.003
	Upper (σ_{15j}^U)	0.005	0.008	0.002	0.007	−0.001	0.005	0.003	0.004
M&M	Lower (σ_{16j}^L)	−0.001	0.000	0.001	0.002	−0.001	0.001	0.001	0.001
	Upper (σ_{16j}^U)	0.002	0.001	0.002	0.004	−0.001	0.003	0.002	0.002
TATAMOTORS	Lower (σ_{17j}^L)	−0.002	−0.002	0.001	0.004	0.000	0.000	−0.002	0.003
	Upper (σ_{17j}^U)	0.003	0.001	0.003	0.007	0.002	0.003	−0.001	0.004
HINDUNILVR	Lower (σ_{18j}^L)	0.000	−0.001	−0.001	0.000	−0.001	0.000	0.000	0.000
	Upper (σ_{18j}^U)	0.000	0.000	0.001	0.002	0.000	0.002	0.001	0.001
ITC	Lower (σ_{19j}^L)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
	Upper (σ_{19j}^U)	0.001	0.001	0.002	0.002	0.000	0.001	0.001	0.002
WIPRO	Lower (σ_{20j}^L)	−0.002	−0.002	0.000	−0.002	0.002	0.000	−0.001	−0.001
	Upper (σ_{20j}^U)	0.000	−0.002	0.001	−0.001	0.003	0.000	0.001	−0.001
SUNPHARMA	Lower (σ_{21j}^L)	0.000	−0.001	−0.001	0.001	−0.002	0.002	−0.001	0.000
	Upper (σ_{21j}^U)	0.001	0.000	−0.001	0.005	−0.001	0.003	0.000	0.000
GAIL	Lower (σ_{22j}^L)	0.001	0.003	−0.001	−0.001	0.001	0.001	0.001	0.000
	Upper (σ_{22j}^U)	0.003	0.005	0.001	0.001	0.001	0.002	0.003	0.001

Table 10 (continued)

Stocks		HDFC	CIPLA	BHEL	SBIN	DRREDDY	HDFCBANK	HEROMOTOCO	INFY	SSLT	ONGC	RELIANCE
ICICIBANK	Lower (σ_{23j}^{L})	0.000	0.000	0.003	0.000	0.000	0.003	0.001	−0.001	−0.001	−0.003	−0.002
	Upper (σ_{23j}^{U})	0.002	0.000	0.006	0.006	0.001	0.003	0.003	0.000	0.003	0.003	0.003
AXISBANK	Lower (σ_{24j}^{L})	−0.002	−0.002	0.004	0.000	−0.002	0.002	0.000	−0.003	0.001	−0.005	−0.001
	Upper (σ_{24j}^{U})	0.001	0.000	0.008	0.007	0.000	0.003	0.002	0.000	0.004	0.003	0.003
BHARTIARTL	Lower (σ_{25j}^{L})	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.000	0.002	0.001	0.001
	Upper (σ_{25j}^{U})	0.002	0.001	0.001	0.003	0.000	0.001	0.001	0.001	0.002	0.001	0.002
MARUTI	Lower (σ_{26j}^{L})	0.002	0.000	0.002	0.005	0.001	0.002	0.001	−0.001	0.003	0.004	0.002
	Upper (σ_{26j}^{U})	0.003	0.002	0.004	0.009	0.002	0.004	0.001	0.001	0.007	0.005	0.005
TCS	Lower (σ_{27j}^{L})	0.000	0.001	−0.001	−0.002	0.000	0.001	−0.001	0.002	0.001	0.000	0.000
	Upper (σ_{27j}^{U})	0.000	0.001	0.000	−0.001	0.001	0.001	0.000	0.004	0.001	0.001	0.001
NTPC	Lower (σ_{28j}^{L})	0.001	0.000	0.001	0.003	−0.001	0.002	−0.001	0.000	0.004	0.002	0.002
	Upper (σ_{28j}^{U})	0.001	0.001	0.001	0.005	0.000	0.004	−0.001	0.000	0.007	0.005	0.003
BAJAJ-AUTO	Lower (σ_{29j}^{L})	0.000	0.000	0.000	0.000	0.000	−0.001	0.003	0.000	−0.001	−0.003	0.000
	Upper (σ_{29j}^{U})	0.001	0.001	0.002	0.001	0.000	−0.001	0.003	0.000	0.002	0.003	0.001
COALINDIA	Lower (σ_{30j}^{L})	0.001	0.000	0.002	0.004	0.000	0.002	0.000	−0.002	0.003	0.001	0.002
	Upper (σ_{30j}^{U})	0.002	0.000	0.004	0.005	0.000	0.003	0.000	−0.001	0.005	0.002	0.002

Table 10 (continued)

Stocks		TATAPOWER	HINDALCO	TATASTEEL	LT	M&M	TATAMOTORS	HINDUNILVR	ITC	WIPRO	SUNPHARMA	GAIL
ICICIBANK	Lower (σ^L_{23j})	−0.002	−0.001	−0.002	0.005	−0.001	−0.002	0.000	0.000	−0.002	0.000	0.001
	Upper (σ^U_{23j})	0.003	0.004	0.003	0.005	0.002	0.003	0.000	0.001	0.000	0.001	0.003
AXISBANK	Lower (σ^L_{24j})	−0.004	−0.002	−0.001	0.007	0.000	−0.002	−0.001	0.000	−0.002	−0.001	0.003
	Upper (σ^U_{24j})	0.005	0.003	0.002	0.008	0.001	0.001	0.000	0.001	−0.002	0.000	0.005
BHARTIARTL	Lower (σ^L_{25j})	0.001	0.001	0.001	0.001	0.001	0.001	−0.001	0.000	0.000	−0.001	−0.001
	Upper (σ^U_{25j})	0.004	0.002	0.002	0.002	0.002	0.003	0.001	0.002	0.001	−0.001	0.001
MARUTI	Lower (σ^L_{26j})	0.003	0.003	0.003	0.004	0.002	0.004	0.000	0.000	−0.002	0.001	−0.001
	Upper (σ^U_{26j})	0.006	0.007	0.008	0.007	0.004	0.007	0.002	0.002	−0.001	0.005	0.001
TCS	Lower (σ^L_{27j})	0.000	0.000	0.000	−0.002	−0.001	0.000	−0.001	0.000	0.002	−0.002	0.001
	Upper (σ^U_{27j})	0.002	0.000	0.000	−0.001	−0.001	0.002	0.000	0.000	0.003	−0.001	0.001
NTPC	Lower (σ^L_{28j})	0.002	0.001	0.002	0.002	0.001	0.000	0.000	0.000	0.000	0.002	0.001
	Upper (σ^U_{28j})	0.006	0.004	0.005	0.005	0.003	0.003	0.002	0.001	0.000	0.003	0.002
BAJAJ-AUTO	Lower (σ^L_{29j})	−0.001	−0.001	−0.001	0.002	0.001	−0.002	0.000	0.000	−0.001	−0.001	0.001
	Upper (σ^U_{29j})	−0.001	−0.001	0.000	0.003	0.002	−0.001	0.001	0.001	0.001	0.000	0.003
COALINDIA	Lower (σ^L_{30j})	0.005	0.002	0.002	0.003	0.001	0.003	0.000	0.001	−0.001	0.000	0.000
	Upper (σ^U_{30j})	0.007	0.004	0.004	0.004	0.002	0.004	0.001	0.002	−0.001	0.000	0.001

Table 10 (continued)

Stocks		ICICIBANK	AXISBANK	BHARTIARTL	MARUTI	TCS	NTPC	BAJAJ-AUTO	COALINDIA
ICICIBANK	Lower (σ_{23j}^L)	0.006	0.004	−0.003	−0.001	−0.001	0.000	0.001	−0.001
	Upper (σ_{23j}^U)	0.011	0.011	0.001	0.005	0.000	0.002	0.003	0.003
AXISBANK	Lower (σ_{24j}^L)	0.004	0.019	−0.003	−0.001	−0.003	0.001	0.002	−0.002
	Upper (σ_{24j}^U)	0.011	0.028	0.001	0.004	0.001	0.004	0.006	0.004
BHARTIARTL	Lower (σ_{25j}^L)	−0.003	−0.003	0.004	0.001	0.000	0.001	0.000	0.000
	Upper (σ_{25j}^U)	0.001	0.001	0.007	0.003	0.001	0.001	0.002	0.003
MARUTI	Lower (σ_{26j}^L)	−0.001	−0.001	0.001	0.007	−0.002	0.002	0.001	0.003
	Upper (σ_{26j}^U)	0.005	0.004	0.003	0.011	0.000	0.005	0.001	0.004
TCS	Lower (σ_{27j}^L)	−0.001	−0.003	0.000	−0.002	0.003	−0.001	−0.001	−0.001
	Upper (σ_{27j}^U)	0.000	0.001	0.001	0.000	0.004	0.000	0.000	−0.001
NTPC	Lower (σ_{28j}^L)	0.000	0.001	0.001	0.002	−0.001	0.004	0.000	0.003
	Upper (σ_{28j}^U)	0.002	0.004	0.001	0.005	0.000	0.007	0.001	0.004
BAJAJ-AUTO	Lower (σ_{29j}^L)	0.001	0.002	0.000	0.001	−0.001	0.000	0.005	0.000
	Upper (σ_{29j}^U)	0.003	0.006	0.002	0.001	0.000	0.001	0.008	0.000
COALINDIA	Lower (σ_{30j}^L)	−0.001	−0.002	0.000	0.003	−0.001	0.003	0.000	0.005
	Upper (σ_{30j}^U)	0.003	0.004	0.003	0.004	−0.001	0.004	0.000	0.006

Table 11 Bounds of fixed and variable transaction cost of 30 stocks in Rs

Scrip Name	Fixed transaction cost				Variable transaction cost			
	for buy		for sell		for buy		for sell	
	fb_j^L	fb_j^U	fs_j^L	fs_j^U	vb_j^L	vb_j^U	vs_j^L	vs_j^U
HDFC	20	30	7	10	0.667	0.700	0.350	0.480
CIPLA	20	30	7	10	0.600	0.677	0.560	0.760
BHEL	20	30	7	10	0.700	0.900	0.230	0.327
SBIN	20	30	7	10	0.333	0.600	0.130	0.140
DRREDDY	20	30	7	10	0.667	0.800	0.350	0.480
HDFCBANK	20	30	7	10	0.550	0.800	0.400	0.560
HEROMOTOCO	20	30	7	10	0.460	0.667	0.230	0.370
INFY	20	30	7	10	0.350	0.560	0.140	0.190
SSLT	20	30	7	10	0.080	0.180	0.030	0.060
ONGC	20	30	7	10	0.563	0.720	0.170	0.240
RELIANCE	20	30	7	10	0.667	0.700	0.350	0.480
TATAPOWER	20	30	7	10	0.600	0.677	0.200	0.226
HINDALCO	20	30	7	10	0.700	0.900	0.233	0.300
TATASTEEL	20	30	7	10	0.333	0.600	0.111	0.200
LT	20	30	7	10	0.667	0.800	0.222	0.267
M&M	20	30	7	10	0.550	0.800	0.183	0.267
TATAMOTORS	20	30	7	10	0.460	0.667	0.153	0.222
HINDUNILVR	20	30	7	10	0.350	0.560	0.117	0.187
ITC	20	30	7	10	0.080	0.180	0.027	0.060
WIPRO	20	30	7	10	0.563	0.720	0.188	0.240
SUNPHARMA	20	30	7	10	0.667	0.700	0.222	0.233
GAIL	20	30	7	10	0.600	0.677	0.200	0.226
ICICIBANK	20	30	7	10	0.700	0.900	0.233	0.300
AXISBANK	20	30	7	10	0.333	0.600	0.111	0.200
BHARTIARTL	20	30	7	10	0.667	0.800	0.222	0.267
MARUTI	20	30	7	10	0.550	0.800	0.183	0.267
TCS	20	30	7	10	0.460	0.667	0.153	0.222
NTPC	20	30	7	10	0.350	0.560	0.117	0.187
BAJAJ-AUTO	20	30	7	10	0.010	0.090	0.003	0.030
COALINDIA	20	30	7	10	0.480	0.700	0.160	0.233

investment. There are 7, 3, 2, 10, 7, 15, 17, 7, 5, 10, 6, 10, 9 and 6 units of shares should contain in respective selected stocks of the portfolio with the proportion of total investment 0.077, 0.092, 0.020, 0.016, 0.091, 0.111, 0.060, 0.039, 0.042, 0.053, 0.098, 0.040, 0.234 and 0.022. These are shown as x_j^* and w_j^* in columns of Table 13. The 1's in column of α_j^{b*} represent those stocks which will be purchased, and 0

Table 12 Currently holding stocks, and minimum and maximum unit of stocks bought and sold for the initial portfolio

Scrip Name	ξ_j	b_j^L	b_j^U	s_j^L	s_j^U	Scrip Name	ξ_j	b_j^L	b_j^U	s_j^L	s_j^U
HDFC	0	5	25	3	7	M&M	0	6	10	1	5
CIPLA	0	10	25	2	9	TATAMOTORS	0	3	16	1	7
BHEL	0	6	24	1	11	HINDUNILVR	0	7	18	4	9
SBIN	0	4	15	1	5	ITC	0	9	24	5	13
DRREDDY	0	8	20	3	10	WIPRO	0	5	10	2	6
HDFCBANK	0	6	30	5	12	SUNPHARMA	0	5	10	8	10
HEROMOTOCO	0	3	12	2	6	GAIL	0	10	35	6	30
INFY	0	7	22	4	9	ICICIBANK	0	6	17	1	16
SSLT	0	9	14	3	7	AXISBANK	0	4	16	4	16
ONGC	0	5	18	3	7	BHARTIARTL	0	8	28	2	19
RELIANCE	0	2	10	2	9	MARUTI	0	6	15	1	11
TATAPOWER	0	5	15	1	11	TCS	0	3	25	6	14
HINDALCO	0	10	25	6	14	NTPC	0	7	16	3	5
TATASTEEL	0	6	16	3	5	BAJAJ-AUTO	0	9	26	3	20
LT	0	4	26	2	14	COALINDIA	0	5	10	1	8

Table 13 The investment strategy for the initial investment

Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}	Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}
HDFC	7	0.077	1	0	7	0	M&M	7	0.091	1	0	7	0
CIPLA	0	0.000	0	0	0	0	TATAMOTORS	0	0.000	0	0	0	0
BHEL	0	0.000	0	0	0	0	HINDUNILVR	15	0.111	1	0	15	0
SBIN	0	0.000	0	0	0	0	ITC	17	0.060	1	0	17	0
DRREDDY	0	0.000	0	0	0	0	WIPRO	7	0.039	1	0	7	0
HDFCBANK	0	0.000	0	0	0	0	SUNPHARMA	5	0.042	1	0	5	0
HEROMOTOCO	3	0.092	1	0	3	0	GAIL	10	0.053	1	0	10	0
INFY	0	0.000	0	0	0	0	ICICIBANK	6	0.098	1	0	6	0
SSLT	0	0.000	0	0	0	0	AXISBANK	0	0.000	0	0	0	0
ONGC	0	0.000	0	0	0	0	BHARTIARTL	10	0.040	1	0	10	0
RELIANCE	2	0.020	1	0	2	0	MARUTI	0	0.000	0	0	0	0
TATAPOWER	0	0.000	0	0	0	0	TCS	9	0.234	1	0	9	0
HINDALCO	10	0.016	1	0	10	0	NTPC	0	0.000	0	0	0	0
TATASTEEL	0	0.000	0	0	0	0	BAJAJ-AUTO	0	0.000	0	0	0	0
LT	0	0.000	0	0	0	0	COALINDIA	6	0.022	1	0	6	0

Table 14 Current price and number of units of stocks holds in existing portfolio of 30 stocks

Scrip Name	Price (p_j)	ξ_j	Scrip Name	Price	ξ_j
HDFC	1095.60	7	M&M	1258.45	7
CIPLA	661.15	0	TATAMOTORS	526.55	0
BHEL	251.40	0	HINDUNILVR	730.00	15
SBIN	2695.00	0	ITC	353.15	17
DRREDDY	3172.75	0	WIPRO	561.40	7
HDFCBANK	905.20	0	SUNPHARMA	836.65	5
HEROMOTOCO	2928.80	3	GAIL	498.70	10
INFY	4034.80	0	ICICIBANK	1612.20	6
SSLT	255.10	0	AXISBANK	438.00	0
ONGC	403.60	0	BHARTIARTL	394.10	10
RELIANCE	990.55	2	MARUTI	3275.00	0
TATAPOWER	92.80	0	TCS	2588.00	9
HINDALCO	160.75	10	NTPC	146.85	0
TATASTEEL	487.20	0	BAJAJ-AUTO	2553.40	0
LT	1648.10	0	COALINDIA	359.05	6

represent otherwise. In column of α_j^{s*}, all values are zero i.e., no stocks should be sold as it is first time investment.

Next, we show how the rebalancing will be done for the time horizon $h = 1, 3, 6$ and 12 considering above obtained portfolio as current holding portfolio. For this propose, we consider minimum price on November 3, 2014 of each stock as current price for rebalancing. The number of units of stocks and current price of holding portfolio are given in Table 14.

Other input parameters such as variable and fixed transactions cost of each trading, and minimum and maximum limits of units of stock that can be bought or sold are same as those given in Tables 11 and 12. Also we keep an extra amount $v = \text{Rs.}\,0$ which has to be added on the amount available from current holding portfolio for rebalancing the portfolio. In addition to this, we assume that money consumed in total transaction cost should not exceed $d = \text{Rs.}\,1000$. Based on this information the rebalanced portfolio is obtained again by solving $\mathbf{IMV}_\lambda$ for the tolerance level of expected return of portfolio $\bar{\mu}_{fix} = [0.130, 0.060]$, and $\lambda = 0.6$. The efficient portfolio for different value of $h = 1, 3, 6$ and 12 are obtained and are given in Tables 15, 16, 17 and 18, respectively.

One may observe from Table 15 that if we rebalance the existing portfolio for a time horizon of one month i.e. $h = 1$, then the rebalanced portfolio contains the stocks of HDFC, HEROMOTOCO, HINDALCO, TATASTEEL, LT, M&M, TATAMOTORS, HINDUNILVR, ITC, WIPRO, SUNPHARMA, GAIL, ICICIBANK, BHARTIARTL, TCS and COALINDIA with number of units 1, 3, 4, 6, 4, 7, 3, 10, 17, 12, 5, 1, 3, 5, 13 and 6, respectively. This means that from the existing portfolio 6, 2, 6, 5, 9, 3 and 5 units of shares of HDFC, RELIANCE, HINDALCO, HINDUNILVR, GAIL, ICICIBANK and BHARTIARTL, respectively are sold out, while 6, 4, 3, 5,

Table 15 Optimal investment strategy at $h = 1$

Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}	Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}
HDFC	1	0.011	0	1	0	6	M&M	7	0.090	0	0	0	0
CIPLA	0	0.000	0	0	0	0	TATAMOTORS	3	0.016	1	0	3	0
BHEL	0	0.000	0	0	0	0	HINDUNILVR	10	0.075	0	1	0	5
SBIN	0	0.000	0	0	0	0	ITC	17	0.061	0	0	0	0
DRREDDY	0	0.000	0	0	0	0	WIPRO	12	0.069	1	0	5	0
HDFCBANK	0	0.000	0	0	0	0	SUNPHARMA	5	0.043	0	0	0	0
HEROMOTOCO	3	0.090	0	0	0	0	GAIL	1	0.005	0	1	0	9
INFY	0	0.000	0	0	0	0	ICICIBANK	3	0.049	0	1	0	3
SSLT	0	0.000	0	0	0	0	AXISBANK	0	0.000	0	0	0	0
ONGC	0	0.000	0	0	0	0	BHARTIARTL	5	0.020	0	1	0	5
RELIANCE	0	0.000	0	1	0	2	MARUTI	0	0.000	0	0	0	0
TATAPOWER	0	0.000	0	0	0	0	TCS	13	0.343	1	0	4	0
HINDALCO	4	0.007	0	1	0	6	NTPC	0	0.000	0	0	0	0
TATASTEEL	6	0.030	1	0	6	0	BAJAJ-AUTO	0	0.000	0	0	0	0
LT	4	0.067	1	0	4	0	COALINDIA	6	0.022	0	0	0	0
Expected return($\bar{\mu}$)	[0.014, 0.018]						Risk ($\bar{\gamma}$)	[0.000, 0.001]					

Table 16 Optimal investment strategy at $h = 3$

Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}	Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}
HDFC	1	0.011	0	1	0	6	M&M	7	0.090	0	0	0	0
CIPLA	10	0.067	1	0	10	0	TATAMOTORS	0	0.000	0	0	0	0
BHEL	0	0.000	0	0	0	0	HINDUNILVR	10	0.075	0	1	0	5
SBIN	0	0.000	0	0	0	0	ITC	11	0.040	0	1	0	6
DRREDDY	0	0.000	0	0	0	0	WIPRO	5	0.029	0	1	0	2
HDFCBANK	0	0.000	0	0	0	0	SUNPHARMA	10	0.085	1	0	5	0
HEROMOTOCO	3	0.090	0	0	0	0	GAIL	2	0.010	0	1	0	8
INFY	0	0.000	0	0	0	0	ICICIBANK	6	0.099	0	0	0	0
SSLT	9	0.023	1	0	9	0	AXISBANK	0	0.000	0	0	0	0
ONGC	0	0.000	0	0	0	0	BHARTIARTL	10	0.040	0	0	0	0
RELIANCE	0	0.000	0	1	0	2	MARUTI	0	0.000	0	0	0	0
TATAPOWER	0	0.000	0	0	0	0	TCS	9	0.238	0	0	0	0
HINDALCO	10	0.016	0	0	0	0	NTPC	0	0.000	0	0	0	0
TATASTEEL	9	0.045	1	0	9	0	BAJAJ-AUTO	0	0.000	0	0	0	0
LT	0	0.000	0	0	0	0	COALINDIA	11	0.040	1	0	5	0
Expected return($\bar{\mu}$)	[0.013, 0.016]						Risk ($\bar{\gamma}$)	[0.000, 0.001]					

Table 17 Optimal investment strategy at $h = 6$

Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}	Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}
HDFC	0	0.000	0	1	0	7	M&M	2	0.026	0	1	0	5
CIPLA	0	0.000	0	0	0	0	TATAMOTORS	0	0.000	0	0	0	0
BHEL	0	0.000	0	0	0	0	HINDUNILVR	22	0.164	1	0	7	0
SBIN	0	0.000	0	0	0	0	ITC	5	0.018	0	1	0	12
DRREDDY	8	0.259	1	0	8	0	WIPRO	3	0.017	0	1	0	4
HDFCBANK	0	0.000	0	0	0	0	SUNPHARMA	5	0.043	0	0	0	0
HEROMOTOCO	1	0.030	0	1	0	2	GAIL	4	0.020	0	1	0	6
INFY	0	0.000	0	0	0	0	ICICIBANK	3	0.049	0	1	0	3
SSLT	0	0.000	0	0	0	0	AXISBANK	0	0.000	0	0	0	0
ONGC	0	0.000	0	0	0	0	BHARTIARTL	10	0.040	0	0	0	0
RELIANCE	0	0.000	0	1	0	2	MARUTI	0	0.000	0	0	0	0
TATAPOWER	0	0.000	0	0	0	0	TCS	9	0.238	0	0	0	0
HINDALCO	0	0.000	0	1	0	10	NTPC	0	0.000	0	0	0	0
TATASTEEL	7	0.035	1	0	7	0	BAJAJ-AUTO	0	0.000	0	0	0	0
LT	0	0.000	0	0	0	0	COALINDIA	16	0.059	1	0	10	0
Expected return($\bar{\mu}$)	[0.015, 0.018]						Risk ($\bar{\gamma}$)	[0.000, 0.001]					

Table 18 Optimal investment strategy at $h = 12$

Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}	Scrip Name	x_j^*	w_j^*	α_j^{b*}	α_j^{s*}	y_j^{b*}	y_j^{s*}
HDFC	0	0.000	0	1	0	7	M&M	2	0.026	0	1	0	5
CIPLA	0	0.000	0	0	0	0	TATAMOTORS	0	0.000	0	0	0	0
BHEL	0	0.000	0	0	0	0	HINDUNILVR	8	0.060	0	1	0	7
SBIN	0	0.000	0	0	0	0	ITC	4	0.014	0	1	0	13
DRREDDY	8	0.259	1	0	8	0	WIPRO	1	0.006	0	1	0	6
HDFCBANK	0	0.000	0	0	0	0	SUNPHARMA	5	0.043	0	0	0	0
HEROMOTOCO	3	0.090	0	0	0	0	GAIL	4	0.020	0	1	0	6
INFY	0	0.000	0	0	0	0	ICICIBANK	3	0.049	0	1	0	3
SSLT	0	0.000	0	0	0	0	AXISBANK	0	0.000	0	0	0	0
ONGC	0	0.000	0	0	0	0	BHARTIARTL	21	0.084	1	0	11	0
RELIANCE	2	0.020	0	0	0	0	MARUTI	0	0.000	0	0	0	0
TATAPOWER	0	0.000	0	0	0	0	TCS	9	0.238	0	0	0	0
HINDALCO	0	0.000	0	1	0	10	NTPC	0	0.000	0	0	0	0
TATASTEEL	6	0.030	1	0	6	0	BAJAJ-AUTO	0	0.000	0	0	0	0
LT	0	0.000	0	0	0	0	COALINDIA	16	0.059	1	0	10	0
Expected return($\bar{\mu}$)	[0.014, 0.017]						Risk ($\bar{\gamma}$)	[0.000, 0.001]					

and 4 units of shares of TATASTEEL, LT, TATAMOTORS, WIPRO and TCS, respectively are bought in. Consequently, TATASTEEL, LT, and TATAMOTORS are newly bought stocks in rebalanced portfolio, while RELIANCE is complectly sold out from the existing portfolio. For the $h = 1$ optimal range of expected return and risk level of rebalanced portfolio are [0.014, 0.018] and [0.000, 0.001], respectively.

One may observe from Table 16 that if we rebalance the existing portfolio for a time horizon of thee month, i.e. $h = 3$, then the rebalanced portfolio contains the stocks of HDFC, CIPLA , HEROMOTOCO, SSLT, HINDALCO, TATASTEEL, M&M, HINDUNILVR, ITC, WIPRO, SUNPHARMA, GAIL, ICICIBANK, BHARTIARTL, TCS and COALINDIA with number of units 1, 10, 3, 9, 10, 9, 7, 10, 11, 5, 10, 2, 6, 10, 9 and 11, respectively. This means that from the existing portfolio 6, 2, 5, 6, 2 and 8 units of shares of HDFC, RELIANCE, HINDUNILVR, ITC, WIPRO, and GAIL, respectively are sold out, while 10, 9, 9, 5, and 5 units of shares of CIPLA, SSLT, TATASTEEL, SUNPHARMA and COALINDIA, respectively are bought in. Consequently, CIPLA, SSLT, and TATASTEEL are newly bought stocks in rebalanced portfolio, while RELIANCE is complectly sold out from the existing portfolio. For the $h = 3$ optimal range of expected return and risk level of rebalanced portfolio are [0.013, 0.016] and [0.000, 0.001], respectively. Similar type of observation can be drawn for Tables 17 and 18 for $h = 6$ and $h = 12$.

6 Conclusion

In this paper, we have developed a portfolio rebalancing model with interval parameters and interval transaction cost. Using our model one can find an optimal strategy for an initial investment. Furthermore, one can also frequently rebalance the existing portfolio by buying and selling of shares in order to get the desired return. It is hoped that results obtained in this paper will be useful to investor and portfolio analysts who can use interval data to obtain an efficient portfolio. Finally, one can develop a more complex model by considering liquidity, market impact cost, inflation, dividend etc.

Acknowledgments The authors would like to thank the referees for their comments and suggestions that led the paper into the current form.

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