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TECHNICAL NOTE

On the allocation of exclusive-use counters for airport check-in queues: static vs. dynamic policies

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Abstract In this paper we propose a static policy for the optimal allocation of a fixed number of exclusive-use check-in counters dedicated to a single flight. We first provide the motivation for considering the static policy by showing that the dynamic policy already available in the literature suffers from the curse of dimensionality. The objective is to minimize the (expected) total cost of waiting, counter operation, and passenger delay costs which we show to be convex in the number of counters allocated. In those cases where the passenger delay cost is difficult to estimate, we propose an alternative formulation and minimize the operating and waiting costs subject to a probabilistic service-level constraint. This constraint ensures that the probability of all passengers being cleared by the gate closing time exceeds a specific level. Finally, we provide a simple procedure for estimating the implied delay costs by exploiting the properties of the two optimization problems. Compared to the difficult-to-evaluate dynamic policy in other papers in the literature, the present static policy requires only a few function evaluations. This feature of the static policy

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makes it easy to find the optimal number of counters even when the number of booked passengers is in the hundreds.

Keywords Transportation · Queues · Airline check-in counters · Static policy · Stochastic models

1 Introduction

One of the key operations in any airport is the management of the limited number of check-in counters. In many of the airports around the world, and especially in airports in the Asia-Pacific region, check-in counters are managed by third-party service providers. These service providers handle the entire check-in operation by supplying all the resources required including the check-in counters, the appropriate number of check-in clerks and other personnel. Airlines demand a certain number of counters for each of their flights and, furthermore, they expect these counters to be at certain locations in the airport. The number of counters demanded by an airline depends on the capacity of the plane, the destination and other extraneous factors such as the image of the airline, the location and the number of counters its competitor uses, etc.

The service provider is contracted by the airport authority to maintain a certain service level in the check-in operation. Thus, it has to perform a balancing act in meeting: (1) the demands of the airlines in terms of the number of counters and other resources; (2) the service level standards set by the airport authority; and (3) the concerns about its own bottom line. The challenge is to complete the check-in process for all the arriving passengers within a specified time window subject to the above constraints. Currently, the allocation of counters to airlines is made using some simple rules-of-thumb based on past experience.

Takakuwa and Oyama [19] found that waiting at check-in counters makes up about 80 % of all the waiting times the passengers experience in an airport. For any passenger the journey begins at the check-in counters. So, an efficient check-in schedule should also aim to reduce congestion at the counters. A passenger who progresses through the check-in process will be highly satisfied and so will be happy to spend more time at the duty-free section. Consequently, the overall image of the airport will improve. Some authors even consider the level of service at the check-in to be a measure of tourism service quality (see Martín-Cejas [14]). Thus, it is clear that the check-in counter allocation problem studied in this paper is of strategic importance to airport operations. Yet, this topic has not received much attention in the literature. A search of the literature reveals only a few papers addressing this problem. A majority of these papers uses only simulation to analyze the problem.

To our knowledge, Parlar and Sharafali [16] were the first to demonstrate that this decision problem for exclusive use check-in counter systems is amenable to analytical treatment. In an exclusive use check-in counter system, each flight has a dedicated number of counters which stay open until at least half-an-hour before the scheduled departure of that flight. Examples of airports where such a system is in use are Terminal 1 and the budget terminal of Singapore Changi Airport, and the international terminal of Melbourne Tullamarine Airport in Australia. Modeling the arrival process as a death process, the authors of [16] derived the time-dependent transition probabilities for the underlying terminating queueing process. They used these probabilities in a stochastic dynamic programming model to analyze a periodic-review multi-counter single queue problem. Their approach yielded the optimal *dynamic* assignment of counters which minimized a suitable expected cost function. But, for problems with a large number of passengers, their approach required long computation times. Thus, their dynamic model suffers from the curse of dimensionality. A dynamic policy is also more complex as it requires periodic (or continuous) monitoring in order to decide in real time whether to open or close counters. Administratively, implementing such an open-and-close policy may be difficult and cumbersome resulting in higher overhead costs. Moreover, the counter clerks will have to alternate between periods of intense activity and of idleness. This is likely to have a demoralizing effect on them. Due to these and other administrative reasons, practitioners always prefer simple static policies (which advocate a fixed number of counters to be kept open throughout the operation time) over dynamic policies. For example, in the case of the airline overbooking problem, Barnhart et al. [3] highlight that in the airline industry 'few airlines have implemented such complex DP formulations because of the difficulties of providing adequate and accurate inputs.' Hence, in this research, we propose to investigate the static policy for the allocation of check-in counters to airlines.

The layout of the paper is as follows. In the next section, we provide a review of the literature. We then explain in some detail the rationale for this work in Section 3. In doing so, we also describe the exclusive use check-in counter queue model together with the assumptions and notation. In Section 4, we present our models. We first propose an unconstrained static optimization problem in Section 4.1. The objective function for this model comprises of the waiting cost, the service provisioning cost and the delay penalty cost. Of these costs, the delay penalty cost is very difficult to estimate. Instead one can use a probabilistic service level constraint. This static constrained optimization model is then presented in Section 4.2. An important use of this model is that it can also be employed to estimate the implied delay penalty cost. This aspect of the model is demonstrated in Section 4.3. For all these models, a closed form solution cannot be derived as the functions involved are transcendental in nature. So, we illustrate the usefulness of these models through numerical examples. We then compare the performance of the static model with that of the periodic review dynamic model of Parlar and Sharafali [16] in Section 5. Managerial insights are provided in Section 6. Finally, in Section 7, we provide some concluding remarks together with limitations and directions for future research.

2 Literature review

Parlar and Sharafali [16] highlight that simulation is the predominant tool used in the study of the airport check-in counter allocation problem. A few studies use pure mathematical models, while some other studies have used a combination of a mathematical model and simulation. We provide below an up-to-date review of the extant literature.

2.1 Pure mathematical programming approaches

Lee and Longton [13] appears to be the pioneering work that looked into modeling the airport check-in process. They employed queueing analysis for the problem and identified that the entire check-in operation is 'equivalent to combinations of, at most, four queueing processes of different types.' Later, in his book [12], Lee provides an account of this interesting experience and laments the futility of using $M/M/s$ queues to model the check-in process. Bruno and Genovese [5], Yan et al. [22, 23] pose this problem in a deterministic setting and model it as a binary integer program. The works are similar in spirit but differ in the objective function used. While Yan et al. [22, 23] aim to minimize the passengers' total walking distance, Bruno and Genovese [5] minimize the sum of the cost of providing service and the cost for passengers' waiting time. Using the demand placed by airlines for the check-in counters as input, Tang [20] proposes a network model to determine the minimum number of counters per day for common use check-in systems. Hsu et al. [10] consider four varieties of check-in facilities an airline can offer to the passengers, namely, (1) counter check-in, (2) self-service check-in, (3) online check-in and (4) barcode check-in. They then develop a model to allocate passengers dynamically to these facilities subject to maintaining a minimum waiting time for the passengers together with a constraint on the utilization of these facilities.

The system considered in all of the above works is common use check-in counter system. As stated above, Parlar and Sharafali [16] is the first work that addressed this issue for an exclusive use check-in counter system. In such a system, the arrivals occur at random from the finite number of passengers booked on that particular flight. As the arrivals will terminate once the last passenger arrives for check-in (or at time T the system close out time), the arrival process cannot be modeled as the traditional renewal process. Parlar and Sharafali [16] model the arrivals as events from a death process (see Bhat [4, p. 208]). Assuming the death rate to be non-stationary, they use stochastic dynamic programming to dynamically allocate counters to an airline. The objective function in their model is the sum of the cost of providing service, the cost for making customers wait and the cost of unfinished check-in at time T.

A related problem to the check-in counter allocation problem is the issue of workforce scheduling. A couple of papers in this direction are Cao et al. [6] and Stolletz [18]. For the Ottawa airport in Canada, Cao et al. [6] identify that the (passenger) agents' working schedule is the critical factor that impacted check-in system performance. The authors develop a linear programming model and a heuristic to determine alternative working schedules for the agents that minimized the total agent-hours subject to meeting the varying passenger load. Assuming non-stationary demand, Stolletz [18] uses a hierarchical workforce planning model. Incorporating employee preferences with flexible contracts, the author develops a binary linear program for the problem and illustrates the model through numerical studies.

2.2 Pure simulation approaches

As already mentioned, simulation is the predominant approach used by many researchers in this area. So, it is impossible to include every work in this short review. Hence, we mention below some noteworthy works. The work of Chun and Mak [9] stands out in the use of simulation for check-in counter management. Assuming a Poisson arrival stream at the counters and beta-distributed service times, they develop a comprehensive simulation-based decision support system for the Hong Kong International Airport to determine the number of counters to allocate to each departing flight. Joustra and van Dijk [11] develop a simulation toolbox to analyze: (1) common vs. exclusive use system; (2) capacity planning; (3) operational planning; and (4) personnel planning. They apply their tool box to analyze the system in Amsterdam Schiphol airport. Takakuwa and Oyama [19] simulate the passenger flow through the entire airport. As already pointed out, their analysis reveals that for any passenger, check-in accounts for 80 % of the total waiting time in the airport. With the objective to improve efficiency of check-in operations, Appelt et al. [2] used simulation and scenario analysis for the check-in procedure at the Buffalo Niagara International Airport.

2.3 Mixed approaches

In almost all the pure simulation-based works in the literature, the objective has been to compare alternatives rather than optimization. To our knowledge the work of van Dijk and van der Sluis [21] appears to be alone in combining simulation with integer programming to determine the optimal check-in service. Their work is also different in the aspect of modeling the arrival process. In all the simulation based papers, the focus is on common-use check-in systems. Such systems have continuous operation throughout the day (as they serve many departing flights through the day). So, the underlying processes do not terminate. In contrast, an exclusive use system is dedicated to a single flight. This system closes operation once all the booked passengers arrive and are checked in or at time T, whichever occurs earlier. Consequently, the simulation model for such a system (called ‘terminating simulation’) should also terminate once the above terminating event occurs. Van Dijk and van der Sluis [21] were the first to observe this phenomenon. So, they propose terminating simulation to analyze dedicated use check-in counter operations. As mentioned above, this phenomenon of the terminating nature was modeled by Parlar and Sharafali [16] through the use of a death-process based arrival process.

3 The rationale

In this section, we provide the rationale for this work. The prime reason for embarking on this research is that the only other analytical model in the literature fails to solve problems of realistic size. In order to explain this rationale, we introduce first the exclusive use check-in counter system together with some notation.

An exclusive-use check-in counter system (see Parlar and Sharafali [16]) is dedicated to a single flight. The service provider knows a priori the number of confirmed passengers, N, who will check in for that particular flight. Thus, the calling population size for this queueing system is finite. There is an extensive literature on finite population queueing models. In these models, any served customer usually goes back to the population and will return again to the queue after some random time. In such systems, the effective arrival rate will be the same at any time. But, in the check-in queue system a served customer leaves the system and will never return. So, the arrival rate to the check-in queue effectively decreases as the time to departure of the flight approaches. Consequently, this special finite population queue will never reach the steady state.

Parlar and Sharafali [16] assume that arrivals form a 'death process' (see Bhat [4, p. 208]) from the population of N travelers booked on the flight. An arrival at a check-in counter is a removal (or death) of that individual from this population. The random lifetime of a member in this population is assumed to follow an exponential distribution with mean λ^{-1}. Hence, for this process, if m passengers have already arrived by time t, the arrival rate at time t equals $\lambda(N-m)$.

The arriving passengers form a single line before the counters. The passenger at the head of the queue gets served by the next available counter. The authors assume that the instantaneous service rate at a counter is a function of both the number of passengers present and the number of counters open. This implies that if there were m arrivals and n service completions during $[0, t]$ resulting in $m-n$ passengers in the system (including the passengers undergoing service at the counters) at time t, then the instantaneous service rate at any counter is $c(m-n)\mu$, i.e., the probability that a departure occurs in $(t, t+\Delta t)$ is $c(m-n)\mu\Delta t + o(\Delta t)$.

Further, they assume a periodic-review model. In such a model, the service provider reviews the congestion in the system at equally spaced review epochs in order to decide whether additional counters need to be opened. The objective in their model is to minimize the total cost of operating this system over the period $[0, T]$. With $A(t)$ as the number of passengers that have arrived by time t, and $S(t)$ as the number of passengers that have been served by time t, the authors use the stochastic process $\{(A(t), S(t)) : t \geq 0\}$ for the analysis.

This process is clearly a Markov process due to the fact that the underlying distributions are exponential.

The highlight of that paper is the rather involved approach to obtain closed-form expressions for the transient transition probability functions,

$$P^{c}_{m,n}(i, j, t) = \Pr\{(A(t), S(t)) = (i, j) \mid (A(0), S(0)) = (m, n)\} \qquad (1)$$

of finding the system in state (i, j) at time t given that at time 0, the system was in state (m, n). As the state space could be very large for large N, their model suffers from the curse of dimensionality. A little reflection reveals that their periodic-review stochastic dynamic programming model requires the evaluation of $P^{c}_{m,n}(i, j, t)$ for all possible combinations of transitions. All these values have to be kept in RAM for the computation of the optimal dynamic policy. For an example problem with $N = 10$ passengers, the authors had to evaluate only 1,716 of these functions. But when N is set to 200 passengers, there will be 138,743,801 functions to account for. This requirement makes it extremely difficult to solve their model.

In order to understand this issue better, we ran some experiments of their periodic-review stochastic dynamic programming model. Table 1 presents the time taken to solve this model for various values of N on a Lenovo X200 PC running at 1.86 GHz clock speed with 3 GB of RAM. As the case corresponding to $N = 60$ itself took about 1,070.90 min (17.8 h) to finish, we did not consider values of N higher than 60.

To estimate the time to solve problems with $N > 60$, we used nonlinear regression to fit a power curve of the form $\tau(N) = aN^b$ to the data in Table 1. This gave us the best fit curve $\tau(N) = 7.759(10^{-9})N^{6.2797}$ as shown in Fig. 1 with a coefficient of determination of $r^2 = 0.99$.

The above results indicate that for a realistic passenger size of, say, $N = 250$, we would need $\tau(250) = 6163.57$ days (16.9 years) to optimally solve their model. This fact makes their model not suitable for implementation. So, for large N, alternative methods are required to find the optimal number of counters to allocate to flights. Obviously, one option before us is to use an approximate dynamic programming method (see Powell [17]) to solve the above stochastic dynamic programming model. The other option is to search for alternative policies that are simple and easily implementable. In this paper, we have chosen the second option because practitioners always prefer a simple to implement policy over a dynamic policy. The preference for such a policy

Table 1 Time taken to solve the dynamic programming model for various values of N, the number of passengers

N	τ, Time (in mins.)
10	0.16
20	1.94
30	14.52
40	86.53
50	399.96
60	1,070.90

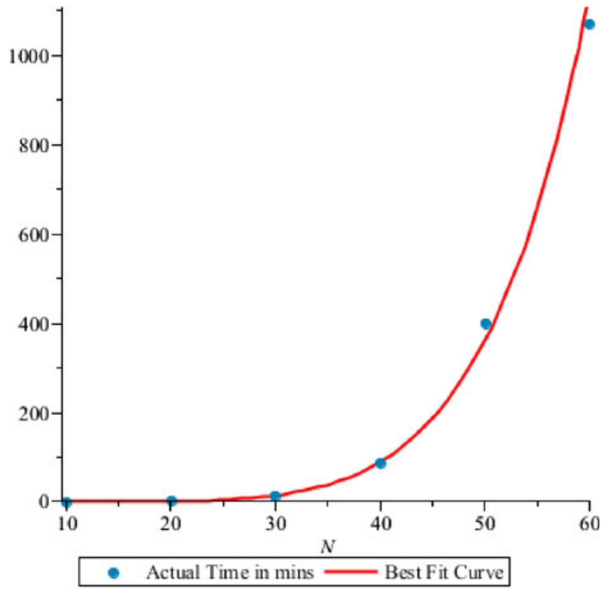

Fig. 1 Best fit curve $\tau(N) = 7.759(10^{-9})N^{6.2797}$ for the data on time taken to solve the dynamic programming model

would be even higher, if its performance were also as good as a policy based on approximate dynamic programming.

4 Static models for allocating check-in counters

We now introduce our static model for the optimal allocation of check-in counters to a single flight. The salient features together with some notation have already been explained in Section 3. The problem description is the same as in Parlar and Sharafali [16] except for the periodic review approach considered in that paper. We drop this assumption in our paper and replace it with the following:

- The airport service provider in our model is interested in the optimal static policy. In other words, for a particular flight, we have to find c, the optimal number of counters, to keep open throughout the period T.

The total cost function is comprised of, (1) the cost of operating the counters; (2) the cost for making passengers to wait; and (3) the penalty cost resulting from delayed passengers who have not yet been cleared at the closing time T (if the latter cost can be estimated with some degree of accuracy).

Before proceeding, for ease of reference, we give below in Table 2 the notation we use in this paper. The symbols are listed in alphabetical order.

The underlying stochastic process for the model in this paper is also $\{(A(t), S(t)) : t \geq 0\}$. This is a Markov process as already highlighted above.

Table 2 List of notation in alphabetical order

Symbol	Description
$A(t) = m$	Number of passengers that have arrived by time t equals m.
c	Number of check-in counters that are open (decision variable).
γ_0	Amortized fixed cost of the technology usage in managing congestion [$].
γ_1	The cost of making a passenger wait per unit time [$/passenger-time].
γ_2	The cost of operating a counter per unit time [$/counter-time].
γ_3	The penalty cost for each arrived passenger not cleared check-in by time T [$/passenger].
γ_4	Congestion monitoring and supervision cost [$/time].
h	Duration of supervision at every review epoch [time/review epoch].
K	The number of review epochs in the periodic review stochastic dynamic programming model.
λ^{-1}	The mean 'lifetime' of a passenger [time/passenger].
μ	The base service rate [passenger/time].
N	Number of passengers booked for the flight.
$P^c_{m,n}(i, j, t)$	Transient probability of finding the system in state $(A(t), S(t)) = (i, j)$ at time t given that the system was in state (m, n) at time 0 and c counters are open.
$S(t) = n$	Number of passengers that have been served by time t equals n.
T	Duration of time the check-in counter system will be open.
$\lfloor x \rfloor$; $\lceil x \rceil$	Floor of x; Ceiling of x.
$Y(t) = A(t) - S(t)$	Number of passengers in the system at time t.

Parlar and Sharafali [16] have shown that the transient transition probability functions defined in Eq. 1 are,

$$P^c_{m,n}(i, j, t) = \binom{N-m}{i-m} e^{-(N-i)\lambda t} \sum_{r=0}^{\max(i-m,m-n)} \binom{i-m}{r} [\alpha(c,t)]^{i-m-r} [\beta(c,t)]^r \times \binom{m-n}{j-n-r} e^{-(m-j+r)c\mu t} (1-e^{-c\mu t})^{j-n-r}, \quad t \geq 0, \tag{2}$$

for $c \geq 1, m \leq i \leq N, n \leq j \leq i$, where,

$$\alpha(c,t) \equiv \frac{\lambda}{\lambda - c\mu}(e^{-c\mu t} - e^{-\lambda t}), \tag{3}$$

$$\beta(c,t) \equiv (1 - e^{-\lambda t}) - \alpha(c,t). \tag{4}$$

The analysis in this paper depends on the convexity or otherwise of the functions $\alpha(c, t)$ and $\beta(c, t)$. These are first presented below as a lemma. We note that c, a decision variable denoting the number of counters to keep open, is naturally a discrete quantity. For analytical ease, we treat it as continuous and use calculus methods to prove the convexity of the above functions.

Lemma 1 *For a fixed value of t, the function $\alpha(c, t)$ is decreasing and convex in the decision variable c, and the function $\beta(c, t)$ is increasing and concave in c.*

Proof Please refer to Appendix A. □

Remark 1 Naturally, the above results apply only when $\lambda \neq c\mu$. In a practical problem where the parameters λ and μ are estimated from empirical data, it would be highly unlikely to encounter a case with $\lambda = c\mu$ since λ and μ would normally assume non-integer values. Nevertheless, this special case can also be examined since we can show that $\lim_{\lambda \to c\mu} \alpha(c, t) = c\mu t e^{-c\mu t}$. In this case, differentiating $\alpha(c, t)$, we find $\alpha'(c, t) = \mu t e^{-c\mu t}(1 - c\mu t)$ and $\alpha''(c, t) = 2\mu^2 t^2 e^{-c\mu t}\left(-1 + \frac{1}{2}c\mu t\right)$, implying that α is decreasing if $c\mu t > 1$ and convex if $c\mu t > 2$.

4.1 The unconstrained optimization problem

Let c be the number of counters allocated to a particular flight under consideration. Only these c counters will be open throughout the period $[0, T]$. Let N be the number of passengers with confirmed booking on this flight. Of these, some passengers might have checked in online. Let n be the number of such passengers, i.e., $S(0) = n$. Usually, at $t = 0$ some passengers might have already arrived and be waiting for the check-in counters to open. So, let $A(0) = m$ be the number of passengers who have arrived by time $t = 0$. Note that $A(0)$ includes those who have checked in online. Thus, the system actually opens with $m - n$ passengers waiting to check-in and $N - m$ passengers still to arrive.

Our objective is to find the optimal c so that the total cost of providing this check-in service is the least. To this end, we first derive explicit expressions for the cost terms.

4.1.1 The cost of waiting

If $Y(t)$ is the number in the system at time t, then $Y(t) = A(t) - S(t)$. Now, the total waiting time of all the passengers is given by $\int_0^T Y(t)\, dt$. So, $W(c)$, the total expected waiting time of all the passengers in $[0, T]$ is,

$$W(c) = E\left[\int_0^T Y(t)\, dt \mid A(0) = m, S(0) = n\right].$$

Hence, the total waiting cost of all the passengers in $[0, T]$ is $\gamma_1 W(c)$. We now have the following Lemma.

Lemma 2 *The total expected waiting time is*

$$W(c) = \left[\frac{\lambda(N - m)}{\lambda - c\mu} + (m - n)\right] \frac{(1 - e^{-c\mu T})}{c\mu} + \frac{(N - m)}{\lambda - c\mu}(e^{-\lambda T} - 1).$$

Also, $W(c)$ is decreasing and convex in c.

Proof Please refer to Appendix B. □

4.1.2 The cost of counter operation

As c counters are kept open throughout the interval $[0, T]$, the total number of hours of counter operation is $K(c) = cT$. Thus, the counter operation cost in $[0, T]$ is $\gamma_2 K(c)$. Since $K(c)$ is a linear function of c, it is also convex in c.

4.1.3 The cost of unfinished check-in at time T

It is expected that the system must clear check-in of all the arrived passengers by time T. If, due to certain reasons, some passengers are still waiting to clear check-in at T, then it might delay the take-off of that flight. Delaying the take-off of any flight will result in additional cost to the service provider. To calculate the cost of such unfinished work at time T, we use Eq. 11 to find $D(c)$, the expected number of passengers still to check-in at time T as,

$$D(c) = E[Y(T) \mid A(0) = m, S(0) = n] = (N - m)\alpha(c, T) + (m - n)e^{-c\mu T}.$$

Hence, the expected cost of unfinished check-in at time T is $\gamma_3 D(c)$. Using the same arguments as in Lemma 2, we again infer that $D(c)$ is also a decreasing convex function of c.

4.1.4 The total cost function

Now, combining all the three cost components, we obtain the total expected cost $G(c)$ during $[0, T]$ as $G(c) = \gamma_1 W(c) + \gamma_2 K(c) + \gamma_3 D(c)$. The unconstrained problem (UCP) can now be formulated as,

$$\text{UCP:} \quad \min_{c \in \mathcal{C}} G(c) = \gamma_1 W(c) + \gamma_2 K(c) + \gamma_3 D(c) \tag{5}$$

where $\mathcal{C} = \{c : c_{\min} \le c \le c_{\max}\}$ is the set of feasible values for the decision variable c. We call this version of the model the "unconstrained problem" since there are no explicit constraints on the decision variable other than the natural limits defined by $\mathcal{C}$.

Theorem 1 *The total expected cost $G(c)$ is convex in the decision variable c.*

Proof The proof is trivial since $W(c)$, $K(c)$ and $D(c)$ are all convex functions of c. □

Remark 2 We highlight that even if $K(c)$ were a nonlinear convex function, the general result of Theorem 1 would still be true.

Theorem 2 *Let c^0 be the solution of the equation*

$$-[\gamma_1 W'(c) + \gamma_3 D'(c)] = \gamma_2 T \tag{6}$$

If $c^0 \in \mathcal{C}$, then the optimal number of counters to allocate is given by $c^* = \lfloor c^0 \rfloor$ or $\lceil c^0 \rceil$; otherwise $c^* = c_{\min}$ or $c_{\max}$.

Proof Please refer to Appendix C. □

Example 1 Consider a flight with $N = 100$ passengers with confirmed booking. With the cost parameters as $(\gamma_1, \gamma_2, \gamma_3) = (40, 60, 80)$, the service provider is willing to allocate a maximum of 5 counters to this flight, i.e., $\mathcal{C} = [c_{\min}, c_{\max}] = [1, 5]$. Let the system operating time be $T = 1$ hour and the mean 'lifetime' of an arriving passenger be $\lambda^{-1} = (9.5)^{-1}$. The base service rate is $\mu = 6.3$. Thus, for all integer values of c, we have $\lambda \neq c\mu$. We also assume that at $t = 0$, seven passengers have already arrived and two have been cleared, i.e., $m = 7$ and $n = 2$.

With these parameter values, it is now easy to solve unconstrained optimization problem $\min_{c \in \mathcal{C}} G(c)$ to determine the optimal number of counters. The first order condition $G'(c) = 0$ yields $c^0 = 3.22 \in \mathcal{C}$. Since this value is not an integer, we examine the cost function at the two neighboring integer values of 3 and 4. We find $G(3) = 387.94$, and $G(4) = 395.87$. Thus, it is optimal to open $c^* = 3$ counters resulting in a minimum expected total cost of $G(c^*) = 387.94$.

4.2 The constrained optimization problem

In the unconstrained optimization problem formulated and solved above, cost is the primary concern. With the focus on cost alone, it is possible that the system might end up with more passengers not cleared through the system by time T. This in turn would create a chain reaction in delaying the take off of the plane or making the plane take off on time without some confirmed passengers. This event may happen if γ_3, the penalty cost for not clearing an arrived passenger by time T, is significantly underestimated. On the other hand, a higher value of γ_3 would imply more counters than necessary, thus resulting in higher cost. One way to deal with this dilemma is to consider a probabilistic service level constraint. This constraint will ensure that by closing time T the probability of any passenger being delayed is no more than a pre-specified level, say, $\theta = 0.01$.

This brings to attention the important issue of the estimation of the cost coefficients γ_1, γ_2 and γ_3. In reality, the coefficients γ_1 and γ_2 are somewhat easier to estimate, but it is very difficult to find estimates for γ_3. For example, as indicated in [16], European Organization for the Safety of Air Navigation (EOSAN)'s estimates for γ_1 range from €38 to €49 per hour. Similarly, Federal Aviation Administration estimates for personal travel is \$23.30, for business travel \$40.10 and for all purposes \$28.60 per hour (see, [16]). In our examples, we use $\gamma_1 = \$40$, which is very close to EOSAN's low-range and to the estimate recommended by Federal Aviation Administration for business class passengers. Aéroport International Strasbourg (as cited in [16]) provides estimates for γ_2 that range from €25 per hour (for a maintenance agent) to €70 per hour (for a project manager). We use an estimate of $\gamma_2 = \$60$ in our numerical examples.

Unfortunately, it is nearly impossible to estimate γ_3 to some degree of accuracy. A similar situation exists in the inventory control literature where the shortage penalty cost is difficult to estimate. The inventory control literature tackles this issue through the use of service level constraints. It is well established there that service level constraints help in estimating the imputed shortage cost (see Chopra and Meindl [8, pp. 350–351], and Nahmias [15, Ch. 5]). For example, Aardal et al. [1] consider a continuous review (r, Q) inventory system subject to a service level constraint and show how the shadow price of this constraint can be interpreted as the shortage cost corresponding to the service level guaranteed. A similar study was carried out by Çetinkaya and Parlar [7] for the economic order quantity model with planned backorders. They demonstrate how one can estimate the shortage cost implicitly through the use of service level constraints. They also show how more meaningful managerial insights can be gained through these approaches.

Motivated by these considerations, in this section we introduce a probabilistic service level constraint of the form $\Pr\{Y(T) \geq 1 \mid Y(0) = m - n\} \leq \theta$. Here, θ is the parameter chosen by the management. This constraint stipulates that the chances of finding more than one passenger with unfinished check-in at time T should not exceed θ. We note that, in practice, the management would be more comfortable to specify a θ rather than estimate γ_3.

Now, using Eq. 2, we have

$$\begin{aligned}\pi(c) &\equiv \Pr\{\text{No unfinished check-in at time } T \mid Y(0) = m - n\}\\ &= P^c_{m,n}(N, N, T).\end{aligned}$$

That is, $\pi(c)$ is the probability that at time $t = T$ all N passengers will be cleared given that at $t = 0$ the system opened with c counters and $m - n$ passengers. We can now derive that

$$\pi(c) = (1 - e^{-c\mu T})^{m-n}[\beta(c, T)]^{N-m}.$$

One would intuitively expect $\pi(c)$ to be an increasing function of c. This is indeed the case as shown in the next proposition.

Proposition 1 *The function $\pi(c)$ is increasing in the number of open counters c.*

Proof The proof follows from the fact that both $1 - e^{-c\mu T}$ and $\beta(c, T)$ are increasing and positive functions of c. □

Since $1 - \pi(c) = \Pr\{Y(T) \geq 1 \mid Y(0) = m - n\}$, the constraint $1 - \pi(c) \leq \theta$ requires that the chances of ending up with one or more waiting passengers at time T is very small, say θ, chosen by the management. Thus, the constrained optimization problem (CP) becomes,

$$\text{CP:} \quad \min_{c \in \mathcal{C}} F(c) = \gamma_1 W(c) + \gamma_2 K(c) \tag{7}$$

$$\text{s.t. } \pi(c) \geq 1 - \theta. \tag{8}$$

We illustrate the solution of the constrained problem with the following example.

Example 2 Consider again the same problem as in Example 1, but now assume that the unit cost of delayed passengers is not specified. Instead the management sets $\theta = 0.01$, i.e., it aspires to have a 0.99 probability of clearing all the passengers by the closing time T. We retain the values of the other parameters as in Example 1. First, solving the problem 7 without the constraint 8, the optimal value of c is again obtained as $c = 3$ but with a slightly lower cost $F(3) = 387.37$. However, at this value we find $\pi(3) = 0.98$ which does not satisfy the management's requirement of $\pi(c) \geq 0.99$.

Solving now the constrained problem CP in Eqs. 7 and 8, we obtain $c^* = 4$ at a higher cost of $F(4) = 393.53$ with $\pi(4) = 0.99$.

4.3 Calculation of the implied delay cost

In the last section, we developed a model with a service level constraint in lieu of γ_3. We also indicated that through this service level constraint we should be able to calculate the implied penalty cost. In this section, we provide a simple method for calculating this implied value of γ_3.

Let $\theta = \bar{\theta}$ be the specified service level parameter. Let $c^*(\bar{\theta})$ be the optimal solution to the constrained nonlinear programming problem CP given by Eqs. 7 and 8. It is possible that $c^*(\bar{\theta})$ may be different from c^* found in Section 4.1 for the original unconstrained problem UCP in Eq. 5. The natural question that now arises is, for what value(s) of γ_3, $c^* = c^*(\bar{\theta})$? To answer this question, we rename the expected cost function of the unconstrained problem (UCP) as $G(c, \gamma_3)$, where γ_3 is now unknown. As before, using calculus, the optimal c^* is the solution to the first order condition $\gamma_1 W'(c) + \gamma_2 T + \gamma_3 D'(c) = 0$. If we expect $c^* = c^*(\bar{\theta})$, then $\gamma_1 W'(c^*(\bar{\theta})) + \gamma_2 T + \gamma_3 D'(c^*(\bar{\theta})) = 0$. Solving for γ_3, we find the imputed penalty cost to be

$$\hat{\gamma}_3 = -\frac{\gamma_2 T + \gamma_1 W'(c^*(\bar{\theta}))}{D'(c^*(\bar{\theta}))}. \tag{9}$$

This value is actually the *upper bound* to the implied penalty cost. However, we recall that c^* is one or both of the integer neighbors of the non-integer optimal solution c^0 to UCP 5. So, there may be other values of $\hat{\gamma}_3$ which may also result in the same optimal solution $c^*(\bar{\theta})$ that is found in CP in Eqs. 7 and 8. Hence, we must also compute the *lower bound* for the implied cost $\hat{\gamma}_3$. This is obtained by solving either (1) $G(c^*(\bar{\theta})) = G(c^*(\bar{\theta}) - 1)$, or (2) $G(c^*(\bar{\theta})) = G(c^*(\bar{\theta}) + 1)$, depending on which side of c^0 the optimal integer solution is located. We illustrate this approach with the following example.

Example 3 Let $\theta = 0.01$, that is, the management wishes to have a 0.99 probability of clearing all the passengers by the time the system closes. Using the same parameters as in Examples 1 and 2, we will now estimate $\hat{\gamma}_3$. We recall that the optimal solution to the constrained problem CP in Eqs. 7 and 8 is $c^*(\bar{\theta}) = 4$ counters. So, the first order condition with unknown $\hat{\gamma}_3$ is $40W'(4) + 60 \cdot 1 + \hat{\gamma}_3 D'(4) = 0$, i.e., $\hat{\gamma}_3 = 12494.19$. This is the *upper bound* of the implied delay cost. To calculate the lower bound, we solve $G(3) = G(4)$ to find $\hat{\gamma}_3 = 2891.04$. Thus, for the given service level parameter $\theta = 0.01$, the implied delay cost γ_3 could be any value in the interval [2891.04, 12494.19]. The implication is that, for all $\gamma_3 \in [2891.04, 12494.19]$, the optimal integer solution to the original unconstrained problem UCP 5 is $c^* = 4$ counters.

If this range for the implied delay cost is deemed high or unrealistic by the management then the management may decide to open only three counters and be satisfied with a 0.98 probability of clearing all passengers.

5 Dynamic vs. static model—a comparison

Having established an-easy-to-use static model in the previous sections, we now embark on comparing its performance with that of the dynamic model that was proposed in Parlar and Sharafali [16]. We use numerical experiments to study this comparison.

As λ, the lifetime parameter of the passengers, is constant over the interval $[0, T]$ in the static model, we assume the same for the dynamic model too. Further, let $(T, K \mid c_{\min}, c_{\max}) = (1, 3 \mid 1, 5)$, and $(N \mid \lambda, \mu \mid \gamma_1, \gamma_2, \gamma_3) = (10 \mid 2.5, 5 \mid 40, 60, 100)$ be the *base values* for the numerical experiments with an initially empty system, i.e., $A(0) = S(0) = 0$. That is, a maximum of $c_{\max} = 5$ counters are available for the duration of $T = 1$ hour before the scheduled departure of the flight. At least $c_{\min} = 1$ counter will be open throughout T.

For the dynamic model, the number of review epochs is $K = 3$. This means that every 20 min a review will be made to decide whether to open an additional counter or not. Thus, for the periodic review dynamic programming model the decision epochs are $t_1 = 0$ h, $t_2 = \frac{1}{3}$ h and $t_3 = \frac{2}{3}$ h. However, for the static model, the decision is made at $t_1 = 0$ only.

We now recall, for easy reference, the periodic review stochastic dynamic programming model of Parlar and Sharafali [16]. The underlying stochastic process is $\{(A(t), S(t)), t \geq 0\}$. At time t_k the state of the system is (m, n). For the optimal policy, let $V_k(m, n)$ be the minimum expected cost-to-go from time t_k to the final time T. At time t_k, after observing the state (m, n), a decision is made to have $c_k \equiv c_k(m, n)$ counters open. This gives, for $k = 1, \ldots, K$,

$$V_k(m, n) = \min_{\substack{c_{\min} \leq c_k \leq c_{\max} \\ c_k(N,N)=0,\ V_k(N,N)=0}} \left[g_k(c_k) + \sum_{i=m}^{N} \sum_{j=n}^{i} P^c_{m,n}(i, j, t_{k+1} - t_k) V_{k+1}(i, j) \right]. \tag{10}$$

In the above, $g_k(c_k) = \gamma_1 \int_{t_k}^{t_{k+1}} \left[(N - m_k)\alpha(c_k, s) + (m_k - n_k)e^{-c_k \mu s} \right] ds + \gamma_2(t_{k+1} - t_k)c_k$ is the cost for the period (t_k, t_{k+1}), and the transition probability, $P^c_{m,n}(i, j, t_{k+1} - t_k)$, is calculated from Eq. 2.

Note that if all the passengers arrive before t_k and all of them get checked in by t_k, then the system will be closed at t_k. Consequently, $c_k(N, N) = 0$ and

$V_k(N, N) = 0$. Further, as per our assumption on the unfinished work at T, the boundary condition should be $V_{K+1}(m, n) = \gamma_3(m - n)$.

We ran numerical experiments by varying the parameter values around the base values mentioned above. Table 3 summarizes the results of these experiments. The base values are typeset in bold font. Hence $V_1(0, 0)$ is the optimal expected cost for the dynamic policy and $G(c^*)$ is optimal expected cost for the static policy. The percentage increase in costs if the static policy is implemented is given by $G(c^*)/V_1(0, 0) - 1$.

As expected, the total cost for the static policy is greater than that for the periodic review dynamic policy. We also note that the total costs increase with N. An interesting observation is that the percentage increases for $N = 30$ are significantly less than those for $N = 10$. For $N = 30$, the percentage increase in expected costs when the static policy is used varies between 14.62 and 20.35 %. At first sight, this may appear to be discouraging, but as we noted in Section 1, the periodic review dynamic policy requires extra efforts for book-keeping

Table 3 Comparison of the optimal cost $V_1(0, 0)$ of the dynamic policy in Parlar and Sharafali [16] with the optimal cost $G(c^*)$ of the static policy

N	λ	μ	γ_1	γ_2	γ_3	$V_1(0, 0)$	$G(c^*)$	$\frac{G(c^*)}{V_1(0,0)} - 1$ (%)	[Static $\succ$ Dynamic] iff $a_M >$
10	1.5	**5**	**40**	**60**	**100**	155.97	188.87	21.09	32.90
	2.5					150.23	182.97	21.79	32.74
	3.5					140.46	174.38	24.15	33.92
10	**2.5**	**5**	20	**60**	**100**	129.59	165.16	27.45	35.57
			40			150.23	182.97	21.79	32.74
			60			170.46	200.78	17.79	30.32
10	**2.5**	**5**	**40**	40	**100**	121.91	142.97	17.28	21.06
				60		150.23	182.97	21.79	32.74
				80		176.61	222.75	26.13	46.14
10	**2.5**	**5**	**40**	**60**	50	134.00	165.08	23.19	31.08
					100	150.23	182.97	21.79	32.74
					150	162.61	196.64	20.93	34.03
30	1.5	**5**	**40**	**60**	**100**	268.89	314.54	16.98	45.65
	2.5					259.49	301.37	16.14	41.88
	3.5					241.74	283.14	17.12	41.39
30	**2.5**	**5**	20	**60**	**100**	212.28	255.47	20.35	43.19
			40			259.49	301.37	16.14	41.88
			60			294.40	337.43	14.62	43.03
30	**2.5**	**5**	**40**	40	**100**	210.41	241.37	14.72	30.96
				60		259.49	301.37	16.14	41.88
				80		303.68	348.91	14.89	45.23
30	**2.5**	**5**	**40**	**60**	50	231.77	267.89	15.59	36.12
					100	259.49	301.37	16.14	41.88
					150	280.80	326.00	16.09	45.19

Here, the parameter values are varied around their base values (in bold) of $(N \mid \lambda, \mu \mid \gamma_1, \gamma_2, \gamma_3) = (10 \mid 2.5, 5 \mid 40, 60, 100)$. The last column indicates that if the monitoring cost a_M exceeds the numbers given, then the static policy dominates ($\succ$) the dynamic policy, i.e., the static policy has a lower cost and for monitoring congestion. In addition to technology requirements for monitoring, this will entail using a supervisor who will have to visit the counters at every review epoch, evaluate the congestion level at the counters and make a decision to open/close additional counters. Thus, for the dynamic policy, additional costs for manpower and technology usage need to be also included.

Let $\$\gamma_0$ be the amortized fixed cost per flight for technology usage (for example, Hamilton Airport in New Zealand[1] uses a required IRR of 9.5 % to recover the counter capital charge of \$410K over 15 years). Let h hours be the supervision time required for review and decision making by the supervisor. Hence, the additional cost for the periodic review dynamic policy can be modeled as $a_M = \gamma_0 + Kh\gamma_4$, where γ_4 is the supervision cost per unit time. We assume $\gamma_4 > \gamma_2$. Note that a_M is independent of the decision variable. Thus, for the dynamic policy the actual total cost is $a_M + V_1(0, 0)$. It then follows that the static policy proposed in this paper will be preferable to (i.e., dominate) the dynamic policy iff $G(c^*) < a_M + V_1(0, 0)$. (This is indicated in the last column of Table 3 by the label "[Static $\succ$ Dynamic] iff $a_M >$".) Since it is a simple matter of calculating the monitoring cost a_M, this comparison can be made relatively easily for each flight and the preferable policy (static or dynamic) can be determined when N is not too large, i.e., when N is less than 50 (as we have done in Table 3). Naturally, when $N > 50$, as discussed above, it would be impossible to evaluate the dynamic policy. To summarize, the static policy should be chosen: (1) when $N \le 50$ and $G(c^*) < a_M + V_1(0, 0)$; or, (2) when $N > 50$.

6 Managerial insights

The constant policy model we proposed in this paper is simple and easy to implement. Thus, managers can confidently make better informed decisions with regard to the allocation of counters to flights. The data required are easy to collect. The model is also flexible in the sense that it can be used for other purposes. For example, the manager can evaluate the performance of the optimal static allocation policy from the model against any standards stipulated by the airport authority. A key benchmark set by any airport authority is that, on average, the number in the system cannot exceed a given value, say η. This situation can easily be handled by introducing a constraint $\max_{t\in[0,T]} E[Y(t) \mid A(0) = m, S(0) = n] \le \eta$, where $E[Y(t) \mid A(0) = m, S(0) = n]$ is given by Eq. 11 in Appendix B.

Although the model is confined to exclusive-use counter operations, it has applicability to common-use counter operations as well. Common use counters are shared by several flights that are scheduled to depart during a specific

[1]Hamilton International Airport—Landing charges pricing methodology, March 2008. Downloaded from www.hamiltonairport.co.nz/file/downloads/pricing_11mar08.pdf on March 29, 2011.

interval of time. If one imagines this combined group of planes as a single plane, one can easily realize the applicability of our model to this situation. By exploiting this applicability, the service provider or the airlines can actually compare the results of this common use counters model to exclusive use models. They can then decide the appropriate system to use. A by-product of this work is the usefulness of our model in workforce planning. This is because the number of counters used determines the number of personnel required to manage the congestion at the counters.

The ground handlers in airports not only provide check-in service to airlines but also offer baggage, gate boarding, cargo and catering services. So, any delay at the check-in facility will have a cascading effect on the subsequent operations. We highlight that the model's output can help the service provider align her own other internal operations accordingly while also satisfying the external stakeholder requirements. Further, this simple-to-use model can also be used as a planning tool to make decisions on capacity and other infrastructure expansion.

7 Conclusion

In this paper we have proposed a static policy to manage the exclusive use airport check-in counter allocation problem. The model's objective is to minimize the total cost of operation of the counters while at the same time satisfying the requirements of the airport authority and airlines. The approach is simple and easy to implement unlike the dynamic programming-based methods. The model can easily accommodate any realistically sized problem with several hundred passengers. We have justified this by comparing the performances of the static and dynamic policies through numerical experiments.

One of the limitations of the model is the assumption that the system serves only one class of passengers. We argue that congestion at the counters is mainly due to the economy class passengers. Business and first class passengers enjoy priority service with dedicated counters. So, our model will be more than sufficient in that respect. One other feature not incorporated in the model is group arrivals. Again large group bookings are usually handled independently of the other ordinary passengers. It is our contention that groups like families can be accommodated easily in the model by using some approximations. The other limitation is the assumption of constant death rate λ for the arrival process. It is likely that λ may vary with time. The tacit assumption in our static model is that λ itself is the rate averaged over $[0, T]$. Such approximation is common in planning models, as in, e.g., the economic order quantity model where the deterministic annual demand is taken as the average of past demands.

Further research is possible in two directions. The current model assumes the operation time T to be given. The first extension could be to treat this also as a decision variable and determine the optimal duration for counter operation. In the current work we assume a single queue multiple counter system. One can relax this assumption and consider parallel queues. Although there is a loss of fairness in a parallel queuing system, we can cite many airports around the world where parallel queues are in use. We have also established that the static policy should be chosen over the dynamic policy when (1) the passenger size is below 50 and the cost of the static policy $G(c^*)$ is less than $a_M + V_1(0, 0)$, the total effective cost of the dynamic policy; or (2) the passenger size is above 50. So, for large planes, if the dynamic policy is better then we need to find faster algorithms to solve the dynamic model. We mention that research is underway by the authors for finding alternative stochastic dynamic programming methods to address the issue of determining the optimal solution for problems with large passenger sizes.

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Appendix

A Proof of Lemma 1

First, we define $\alpha'(c, t) \equiv d\alpha(c, t)/dc$, and note from Parlar and Sharafali [16, Lemma 2] that

$$\alpha'(c, t) = \frac{\lambda\mu}{(\lambda - c\mu)^2}\{e^{-c\mu t}[1 + t(-\lambda + c\mu)] - e^{-\lambda t}\} < 0.$$

Thus, $\alpha(c, t)$ is decreasing in c. Next, we similarly define $\alpha''(c, t) \equiv d^2\alpha(c, t)/dc^2$, and find

$$\alpha''(c, t) = -\frac{\lambda\mu^2}{(-\lambda + c\mu)^3}\left\{2e^{-c\mu t}\left[1 + t(-\lambda + c\mu) + \frac{1}{2}t^2(-\lambda + c\mu)^2\right] - 2e^{-\lambda t}\right\}$$

$$= \frac{2\lambda\mu^2 e^{-c\mu t}}{(-\lambda + c\mu)^3}\left\{e^{(-\lambda+c\mu)t} - \left[1 + t(-\lambda + c\mu) + \frac{1}{2}t^2(-\lambda + c\mu)^2\right]\right\}.$$

When $-\lambda + c\mu > 0$, it can be shown, using the properties of the exponential function and its Taylor expansion around zero that $e^{(-\lambda+c\mu)t} - [1 + t(-\lambda + c\mu) + \frac{1}{2}t^2(-\lambda + c\mu)^2] > 0$ and so in the above $\alpha''(c, t) > 0$. By the same argument, if $-\lambda + c\mu < 0$, then $e^{(-\lambda+c\mu)t} - \left[1 + t(-\lambda + c\mu) + \frac{1}{2}t^2(-\lambda + c\mu)^2\right] < 0$ implying again that $\alpha''(c, t) > 0$, thus proving the first part of the lemma. Moreover, since $\beta'(c, t) = -\alpha'(c, t) > 0$, and $\beta''(c, t) = -\alpha''(c, t) < 0$, the $\beta(c, t)$ function is increasing and concave in c. This proves the lemma. □

B Proof of Lemma 2

Parlar and Sharafali [16] have derived the probability generating function (p.g.f.) $\hat{\Pi}_{m,n}(u,t)$ of $Y(t)$ as,

$$\begin{aligned}\hat{\Pi}_{m,n}(u,t) &= E[u^{Y(t)} \mid A(0)=m, S(0)=n] \\ &= \{[1-\alpha(c,t)]+\alpha(c,t)u\}^{N-m}\left[(1-e^{-c\mu t})+e^{-c\mu t}u\right]^{m-n}.\end{aligned}$$

They further derive,

$$E[Y(t)\,|\,A(0)=m, S(0)=n] = \frac{d}{du}\hat{\Pi}_{m,n}(u,t)\bigg|_{u\to 1} = (N-m)\alpha(c,t)+(m-n)e^{-c\mu t}. \quad (11)$$

It then follows that the total expected wait of all passengers is,

$$W(c) = \int_0^T \left[(N-m)\alpha(c,t)+(m-n)e^{-c\mu t}\right] dt. \quad (12)$$

Performing the integration in Eq. 12, we obtain the explicit result:

$$W(c) = \left[\frac{\lambda(N-m)}{\lambda-c\mu}+(m-n)\right]\frac{(1-e^{-c\mu T})}{c\mu}+\frac{(N-m)}{\lambda-c\mu}(e^{-\lambda T}-1).$$

Differentiating $W(c)$ we have

$$W'(c) = \int_0^T \left[(N-m)\alpha'(c,t)-\mu t(m-n)e^{-c\mu t}\right] dt, \quad (13)$$

$$W''(c) = \int_0^T \left[(N-m)\alpha''(c,t)+(\mu t)^2(m-n)e^{-c\mu t}\right] dt. \quad (14)$$

Using Lemma 1, we see that $W(c)$ is decreasing in c since the integrand in Eq. 13 is negative for all $t \in [0,T]$. Thus $W'(c) < 0$. Moreover, using Lemma 1 again, as $\alpha''(c,t) > 0$ for all $t \in [0,T]$, and the second term in Eq. 14 is always positive, we have $W''(c) > 0$. □

C Proof of Theorem 2

Proof follows from Theorem 1. Since $G(c)$ is convex, c^0, the unique minimizer of $G(c)$ is the solution of the first order condition given by Eq. 6. We observe that the r.h.s. of Eq. 6 is a positive constant and that the l.h.s. is an increasing function of c. If $c^0 \in \mathcal{C}$, then c^* can be found by examining the cost function $G(c)$ at the nearest integer neighbor(s) of c^0. It then follows that $c^* = \lfloor c^0 \rfloor$ or $\lceil c^0 \rceil$. However, if $c^0 \notin \mathcal{C}$, then $G(c)$ must be a monotonic function of c. In this case the optimal solution c^* must be at one of the boundary points ($c_{\min}$, or $c_{\max}$) of the feasible region $\mathcal{C}$. □

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