OPSEARCH (Jan–Mar 2011) 48(1):73–85
DOI 10.1007/s12597-011-0036-3

TECHNICAL NOTE

Optimal ordering policies for Weibull distribution deterioration with associated salvage value under scenario of progressive credit periods

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Accepted: 4 September 2010 / Published online: 6 April 2011

Abstract The term "progressive credit periods" offered by the supplier to the retailer for settling the account is defined as follows: *If the retailer settles account by credit period M, then the supplier does not charge any interest. If the retailer pays after M but before N (N> M), then the supplier charges the retailer on an un–paid amount at the interest rate* Ic_1. *If the retailer settles after credit period N (N> M), then he will have to pay an interest rate* Ic_2 $(Ic_2 > Ic_1)$ *for the un–paid amount*. In this study, a mathematical model is developed when units in inventory deteriorate with respect to time and supplier offers two progressive credit periods. The salvage value is associated to deteriorated units. A flow–chart is given to find the optimal solution. The sensitivity analysis is carried out to analyze the effect of critical parameters on the optimal solution.

Keywords Weibull distribution to deterioration · Salvage value · Progressive credit periods

1 Introduction

The credit period is the most effective tool used by the supplier to encourage the retailer to buy more and attract new customers. It is also considered as the best

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option to price discounts. Brigham [1] defined credit period as "net 30", i.e. the supplier offers a retailer, the delay period of 30 days for settling the account.

Goyal et al. [2] explored the concept of progressive credit period and developed economic order quantity. Soni and Shah [3] developed a mathematical model when units in inventory are subject to constant deterioration under the scenario of progressive credit periods. Soni et al. [4] studied effect of inflation in above stated model. Levin et al.'s [5] quotation: "the presence of inventory has motivational effect on customer around it" is studied by Soni and Shah [6]. They developed a model in which demand is partially constant and partially dependent on the stock, and the supplier offers to retailer progressive credit period to settle the account.

The above stated model are derived under the assumption that deteriorated units in inventory is constant. However, items like fruits and vegetables, radioactive chemicals, medicines, blood components etc. deteriorate with time. Also it was assumed that the deterioration of units is complete loss to the retailer. It is observed that these deteriorated units can be made available at the reduced price. In this research article, an optimal ordering policy is established when units in inventory are subject to Weibull distribution deterioration and supplier suffers two progressive credit periods to the retailer to settle the account. A flow–chart is given to explore computational policy. The sensitivity analysis is carried out to study the variations in critical parameters on decision variable and objective function.

2 Assumptions and notations

The aforesaid model is developed using following assumptions:

- The inventory system deals with single item.
- The demand of R–units for an item is deterministic and known during the cycle time.
- Replenishment rate is infinite. Replenishment is instantaneous.
- Shortages are not allowed and lead–time is zero.
- The distribution of time for deterioration of the items is

$$\theta(t) = \alpha\beta t^{\beta-1} \tag{2.1}$$

  where $\alpha(0\leq\alpha<1)$ denotes scale parameter i. e. rate of deterioration denotes the shape parameter ($\beta\geq1$) and t ($t>0$) denotes the time to deterioration. The salvage value, $\gamma C(0\leq\gamma<1)$, is associated to the deteriorated units where C is the unit cost of an item.
- The deteriorated units can neither be repaired nor replaced during the cycle time.
- If the retailer pays by offered credit period M, then supplier does not charge any interest to the retailer. If the retailer pays after M and before second credit period N ($N>M$) then he can keep difference in unit sale price and unit cost in an interest bearing account at the interest rate of Ie per unit per annum. During [M, N], the supplier charges the retailer an interest rate of Ic_1 per unit per year. If the retailer pays after N, then supplier charges the retailer an interest rate of Ic_2 per unit per annum with $Ic_2>Ic_1$.

The following notations are used in the formulation of the model:

- R: the demand rate per annum,
- C: the unit purchase cost.
- γC: salvage value of deteriorated unit, $0 \leq \gamma < 1$
- P: selling price per unit. (P>C)
- h: the inventory holding cost per unit per year excluding interest charges.
- A: ordering cost per order.
- M: the first offered credit period in settling account by the supplier to the retailer without any extra charges.
- N: the second allowable credit period to settle the account with an interest charge of Ic_1 and M>N.
- Ic_1: the interest charged per $ in stock per year by the supplier when the retailer pays during [M, N].
- Ic_2: the interest charged per $ in stock per year the supplier when the retailer pays during [N, T]. Note that $Ic_2 > Ic_1$.
- Q: the procurement quantity (a decision variable).
- T: the replenishment cycle time (a decision variable).
- Q (t): the on–hand inventory level at any instant of time t, $0 \leq t \leq T$
- D(T): the number of units deteriorated during the cycle time T.
- K(T): the total inventory cost per time unit is the sum of: (a) ordering cost; OC, (b) inventory holding cost (excluding interest charges) IHC, (c) cost due to deterioration; DC, (d) interest charges; IC, for unsold items after the offered delay period M or N, minus (e) salvage value of deteriorated units; SV, and (f) interest earned from the sales revenue during the permissible delay period [0, M].

3 Mathematical model

The on–hand inventory depletes due to demand and deterioration of units. The instantaneous state of inventory at any instant of time t is governed by the differential equation.

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R, \quad 0 \leq t \leq \mathrm{T} \tag{3.1}$$

with boundary condition Q(0)=Q and Q(T)=0 and $\theta(t)$ is as given in (2.1). Taking series expansion and ignoring second and higher powers of α (assuming α to be very small), the solution of differential Eq. 3.1, using boundary condition Q (T)=0 is given by

$$Q(t) = R\left[T - t + \frac{\alpha T}{\beta + 1}\left(T^{\beta} - (1+\beta)t^{\beta}\right) + \frac{\alpha\beta t^{\beta+1}}{\beta + 1}\right], 0 \leq t \leq T. \tag{3.2}$$

Using Q (0)=Q, the procurement quantity is given by

$$Q = R\left[T + \frac{\alpha T^{\beta+1}}{\beta + 1}\right] \tag{3.3}$$

The number of units deteriorated; D (T) during one cycle is

$$\mathrm{D(T)} = \mathrm{Q} - \mathrm{RT} = \frac{\alpha R T^{\beta+1}}{\beta+1} \tag{3.4}$$

The various costs of total inventory cost of the system per cycle are as follows:

(a) Ordering cost;

$$\mathrm{OC} = \mathrm{A} \tag{3.5}$$

(b) Inventory holding cost;

$$\mathrm{IHC} = h\int_0^T Q(t)dt = hR\left[\frac{T^2}{2} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right] \tag{3.6}$$

(c) Cost due to deterioration of units;

$$\mathrm{CD} = \frac{CR\alpha T^{\beta+1}}{\beta+1} \tag{3.7}$$

(d) Salvage value of the deteriorated units;

$$\mathrm{SV} = \frac{\gamma CR\alpha T^{\beta+1}}{\beta+1} \tag{3.8}$$

The calculations of interest charged and interest earned depend on the length of cycle time; T and offered credit periods. There are three cases:

Case 1. $T \leq M$
Case 2. $M < T < N$
Case 3. $T \geq N$.

Next, we discuss computations of interest charged and interest earned in each case.

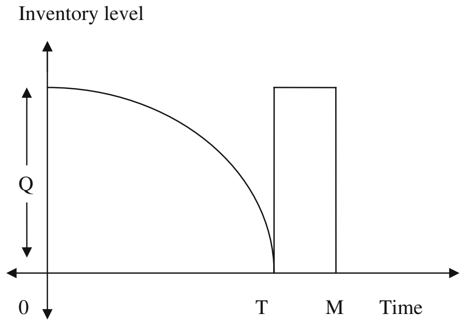
Case 1. $T \leq M$ i. e. cycle time ends before the first offered credit period. (Fig. 1)
Here, the retailer sells Q–units during [0, T] and is paying CQ to the supplier in full at time $M \geq T$. So interest charges are zero. i. e.

$$\mathrm{IC}_1 = 0 \tag{3.9}$$

During [0, T], the retailer sells unit at sale price; P per unit and deposit the revenue into an interest earning account at the rate of Ie per \$ per year. In the period [T, M], the retailer only deposits the total revenue into an account that earns Ie per \$ per year. Hence, interest earned during the cycle is

$$\mathrm{IE}_1 = PI_e\left[\int_0^T Rt\,dt + RT(M-T)\right] = PI_eRT\left(M - \frac{T}{2}\right) \tag{3.10}$$

Fig. 1 (T≤M)



Hence, the total cost of an inventory system per time unit is

$$\mathrm{K}_1(\mathrm{T}) = \frac{1}{T}(OC + IHC + CD + IC_1 - IE_1 - SV) \tag{3.11}$$

The optimum value of T=T_1 is the solution of non–linear equation $\frac{\partial K_1(T)}{\partial T} = 0$. The obtained T=$T_1$ minimizes the total cost because $\frac{\partial^2 K_1(T)}{\partial T^2} > 0$ for all T.

Case 2. M<T<N. (Fig. 2)

Here, interest earned, IE_2, during [0, M] is $IE_2 = PI_e \int_0^M Rtdt = \frac{PI_eRM^2}{2}$. The retailer has to pay for Q–units purchased at unit cost C \$ to the supplier up to time M, the retailer sells RM–units and has revenue PRM plus interest earned IE_2 to pay the supplier. Depending on the difference between the total purchase cost; CQ and the revenue; PRM+IE_2, two sub–cases may arise:

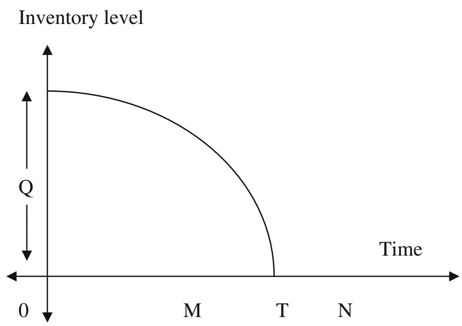
Sub–case 2.1 Let PRM+IE_2 $\geq$ CQ. i. e. the retailer has sufficient amount in his account to pay–off total purchase cost at M. Then interest charges,

$$IC_{2.1} = 0 \tag{3.12}$$

and interest earned,

$$IE_{2.1} = IE_2 = \frac{PI_eRM^2}{2} \tag{3.13}$$

Fig. 2 M<T<N



Therefore, the total cost of an inventory system per time unit is

$$K_{2.1}(T) = \frac{1}{T}(OC + IHC + CD + IC_{2.1} - IE_{2.1} - SV) \quad (3.14)$$

The optimal value of $T=T_{2.1}$ is a solution of non–linear equation $\frac{\partial K_{2.1}(T)}{\partial T} = 0$ and $T=T_{2.1}$ minimizes the total cost $K_{2.1}$ of an inventory system because $\frac{\partial^2 K_{2.1}(T)}{\partial T^2} > 0$ for all T.

Sub–Case 2.2: Let $PRM+IE_2<CQ$. Here, the retailer does not have sufficient money in his account to do payment at offered credit period; M the supplier charges retailer on the unpaid balance $U_1=CQ-(PRM+IE_2)$ at the interest rate Ic_1 at time M, therefore, interest charges; $IC_{2.2}$ per cycle is

$$IC_{2.2} = \frac{U_1{}^2 Ic_1}{2PR}\int_M^T Q(t)dt = \frac{U_1{}^2 Ic_1}{2P}\left(\frac{(T-M)^2}{2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left(T^{\beta+2} - M^{\beta+2}\right) - \frac{\alpha MT}{(\beta+1)}\left(T^{\beta} - M^{\beta}\right)\right) \quad (3.15)$$

and interest earned per cycle is

$$IE_{2.2} = IE_2 = \frac{PIe}{2}RM^2 \quad (3.16)$$

Therefore, the total cost of an inventory system per time unit is

$$K_{2.2}(T) = \frac{1}{T}(OC + IHC + CD + IC_{2.2} - IE_{2.2} - SV) \quad (3.17)$$

The optimum value of $T=T_{2.2}$ can be obtained by solving the non–linear equation $\frac{\partial K_{2.2}(T)}{\partial T} = 0$ which minimizes total cost $K_{2.2}$ because $\frac{\partial^2 K_{2.2}(T)}{\partial T^2} > 0$ for all T.

Case 3. $T \geq N$ i. e. supplier offers his retailer two progressive credit periods to settle the account before retailer's inventory reaches zero. (Fig. 3)

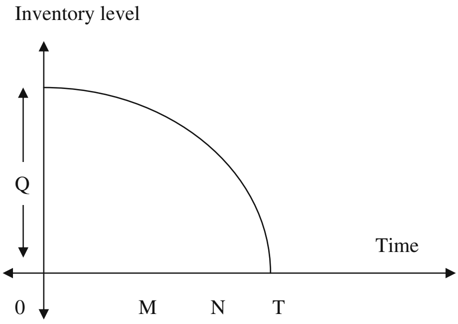
Based on the total purchase cost, CQ, total revenue $PRM+IE_2$ at M and total revenue $PRN+IE_2$ at N, three sub cases may arise.

Sub–case 3.1: Let $PRM+IE_2 \geq CQ$. This sub–case is same as sub–case 2.1 (Note: Here decision variable and objective function are designated by 3.1)

Sub–case 3.2: Let $PRM+IE_2<CQ$ and

$$PR(N-M) + \frac{PI_eR(N^2 - M^2)}{2} \geq CQ - (PRM + IE_2)$$

Fig. 3 T≥N



This sub–case is same as sub–case 2.2. (Note: Here decision variable and objective function is designated by 3.2.)

Sub–case 3.3: Let PRM+IE_2<CQ and $PR(N-M)+\frac{PI_eR(N^2-M^2)}{2}<CQ-(PRM+IE_2)$

Here, retailer does not have sufficient money in his account to pay–off total purchase cost at time N; he pays PRM+IE_2 at M and $PR(N-M)+\frac{PI_eR(N^2-M^2)}{2}$ at N. Hence, retailer will have to pay interest charges on unpaid balance; U_1=CQ – (PRM+IE_2) with interest rate Ic_1 during [M, N] and on unpaid balance; U_2=U_1 – $\left(PR(N-M)+\frac{PI_eR(N^2-M^2)}{2}\right)$ with interest rate Ic_2 during [N, T]. Hence, total interest payable per cycle is

$$IC_{3.3}=U_1Ic_1(N-M)+\frac{U_2{}^2}{PR}Ic_2\int_N^T Q(t)dt=U_1Ic_1(N-M)+\frac{U_2{}^2}{P}Ic_2\left\{\frac{(T-N)^2}{2}+\frac{\alpha\beta(T^{\beta+2}-N^{\beta+2})}{(\beta+1)(\beta+2)}+\frac{\alpha NT(N^\beta-T^\beta)}{\beta+1}\right\} \tag{3.18}$$

and interest earned

$$\mathrm{IE}_{3.3}=\mathrm{IE}_2=\frac{PI_eRM^2}{2} \tag{3.19}$$

Therefore, the total cost of an inventory system per time unit is

$$\mathrm{K}_{3.3}(\mathrm{T})=\frac{1}{T}(OC+IHC+CD+IC_{3.3}-IE_{3.3}-SV) \tag{3.20}$$

The necessary condition for $K_{3.3}$(T) to be minimum T= $T_{3.3}$ is $\frac{\partial K_{3.3}(T)}{\partial T}=0$ and sufficiency condition is $\frac{\partial^2K_{3.3}(T)}{\partial T^2}>0$ for all T.

In the next section, computation flow start is desired to search for the optimal solution.

4 Computational flow chart (Fig. 4)

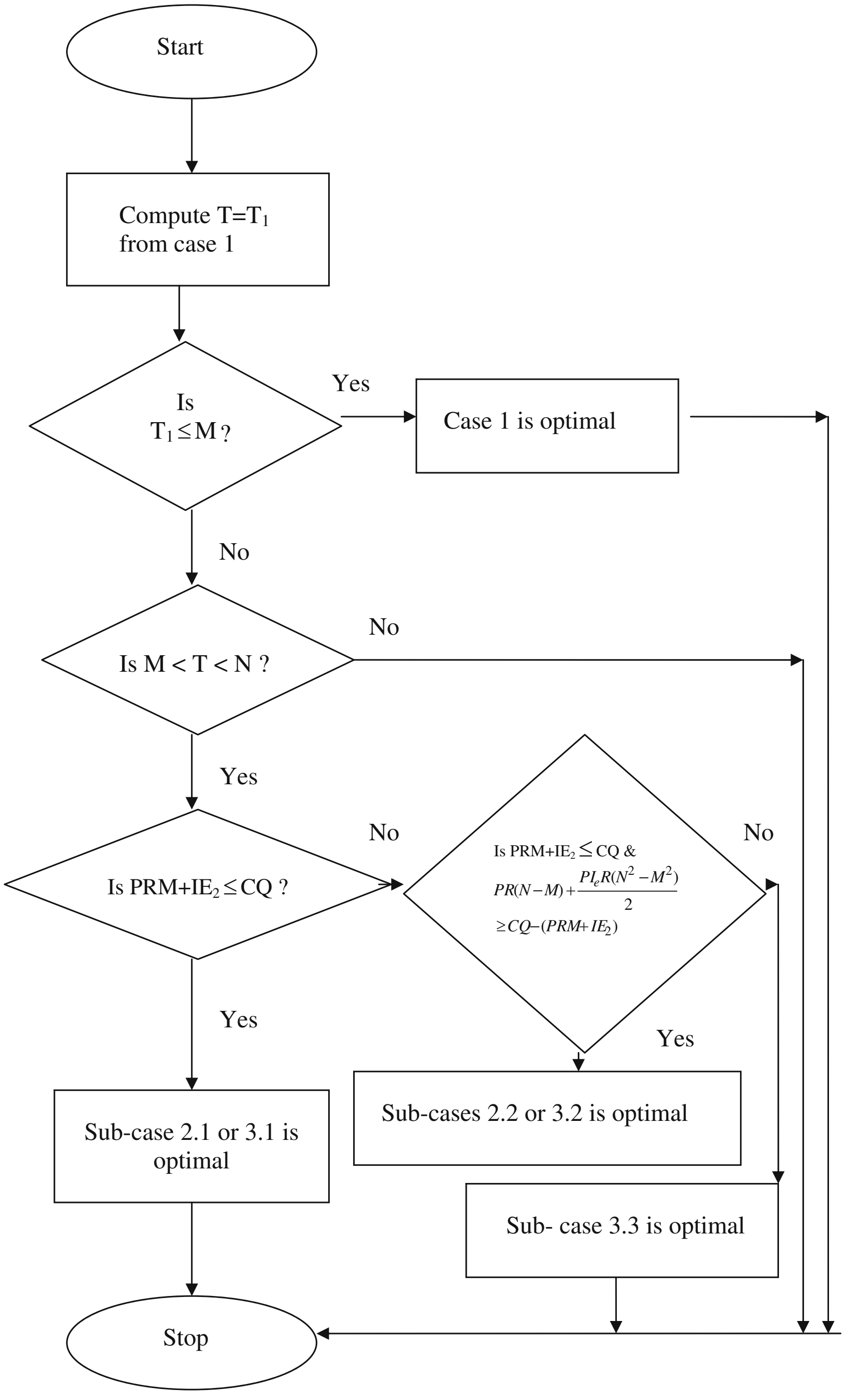


Fig. 4 Computational flow chart to search best optimal policy

5 Numerical example

Consider the following parametric values in appropriate units:

$$[A, C, h, P, R, I_e] = [200, 50, 5, 75, 1000, 0.12]$$

The effect of various parameters on decision variable and total cost of inventory system is exhibited in the following tables (Table 1):

Increase in first credit period; M decreases cycle time and total cost because retailer can earn more during offered credit period. Increase in second credit period; N increases cycle time significantly and decreases total cost of an inventory system (Table 2).

It is observed that more deterioration of units forces retailer to put frequent orders. Hence cycle time decreases and total cost of inventory system increases significantly (Table 3).

As shape parameter increases, cycle time increases and total cost of an inventory system decreases (Table 4).

Increase in salvage value of deteriorated units prolongs the cycle time and decreases total cost of inventory system significantly (Table 5).

The optimal cycle time and total cost of an inventory system are very sensitive to changes in offered progressive credit period and deterioration of units. Increase in

Table 1 Effect of two credit periods on decision policy (α=0.1, β=1.5, γ=0.2)

N M		28/365 (18%)	28/365 (20%)	28/365 (22%)
15/365	T	0.1400	0.1506	0.1587
(15%)	K	2010.90	1985.98	1982.11
20/365	T	0.1396	0.1500	0.1580
(16%)	K	1891.30	1859.46	1851.55
25/365	T	0.1395	0.1497	0.1575
(17%)	K	1783.11	1741.03	1726.82

Table 2 Effect of deterioration rate and first credit period on decision policy (N=35/365, Ic_2=22%, β= 1.5, γ=0.2)

α M		0.1	0.2	0.3
15/365	T	0.1587	0.1560	0.1531
(15%)	K	1982.11	2084.81	2184.54
20/365	T	0.1580	0.1552	0.1522
(16%)	K	1851.55	1953.20	2051.80
25/365	T	0.1575	0.1547	0.1516
(17%)	K	1726.82	1827.71	1925.52

Table 3 Effect of first credit period and shape parameter on decision policy (N=35/365, Ic_2=22%, α= 0.1, γ=0.2)

β M		1.5	2.5	3.5
15/365	T	0.1587	0.1609	0.1613
(15%)	K	1982.11	1888.82	1878.05
20/365	T	0.1580	0.1602	0.1606
(16%)	K	1851.55	1759.10	1748.47
25/365	T	0.1575	0.1597	0.1602
(17%)	K	1726.82	1634.97	1624.43

Table 4 Effect of first credit period and salvage value on decision policy (N=35/365, Ic_2=22%, α= 0.1, β=1.5)

γ M		0.2	0.4	0.6
15/365	T	0.1587	0.1593	0.1599
(15%)	K	1982.11	1956.74	1931.25
20/365	T	0.1580	0.1586	0.1592
(16%)	K	1851.55	1826.37	1801.04
25/365	T	0.1575	0.1581	0.1587
(17%)	K	1726.82	1701.74	1676.51

Table 5 Effect of second credit period and deterioration units on decision policy (M=20/365, Ic_1=16%, β=1.5, γ=0.2)

α N		0.1	0.2	0.3
28/365	T	0.1396	0.1377	0.1356
(18%)	K	1891.30	1976.05	2058.80
32/365	T	0.1500	0.1476	0.1450
(20%)	K	1859.46	1953.62	2055.25
35/365	T	0.1580	0.1552	0.1522
(32%)	K	1851.55	1953.20	2051.80

deterioration decreases cycle time and increases cycle time and decreases total cost. Increase in second credit period increases the cycle time and decreases total cost (Table 6).

Here, cycle time increases and total cost decreases significantly with increase in shape parameter and second offered credit period (Table 7).

Increase in salvage value of the deteriorated units increases cycle time and decreases total inventory cost (Table 8).

The model is very sensitive to deterioration of units in inventory and shape parameter. Increase in deterioration rate decreases cycle time and increases total cost. Increase in shape parameter increases cycle time and decreases total cost of an inventory system (Table 9).

Increase in salvage parameter for deteriorated units increases cycle time and decreases total inventory cost (Table 10).

No significant change is observed in decision variable and objective function for fixed shape parameter and increasing salvage parameter.

Table 6 Effect of second credit period and shape parameter on decision policy (M=20/365, Ic_1=16%, α=0.1, γ=0.2)

β N		1.5	2.5	3.5
28/365	T	0.1396	0.1413	0.1416
(18%)	K	1891.30	1813.43	1805.54
32/365	T	0.1500	0.1520	0.1523
(20%)	K	1859.46	1773.43	1764.05
35/365	T	0.1580	0.1602	0.1606
(22%)	K	1851.55	1759.10	1748.47

Table 7 Effect of second credit period and salvage parameter on decision policy (M=20/365, Ic_1=16%, α=0.1, β=1.5)

γ N		0.2	0.4	0.6
28/365	T	0.1396	0.1401	0.1405
(18%)	K	1891.30	1870.38	1849.36
32/365	T	0.1500	0.1505	0.1510
(20%)	K	1859.46	1836.17	1812.75
35/365	T	0.1580	0.1586	0.1592
(22%)	K	1851.55	1826.37	1801.04

Table 8 Effect of deterioration rate and shape parameter on decision policy (M=20/365, Ic_1=16%, N=35/365, Ic_1 =22%, γ=0.2)

β α		1.5	2.5	3.5
0.1	T	0.1580	0.1602	0.1606
	K	1851.55	1759.10	1748.47
0.2	T	0.1552	0.1597	0.1605
	K	1953.20	1771.16	1749.99
0.3	T	0.1552	0.1592	0.1604
	K	2051.80	1783.12	1751.51

Table 9 Effect of deterioration rate and salvage parameter on decision policy (M=20/365, Ic_1=16%, N=35/365, Ic_2 =22%, γ=0.2)

γ α		0.2	0.4	0.6
0.1	T	0.1580	0.1586	0.1592
	K	1851.55	1826.37	1801.04
0.2	T	0.1552	0.1564	0.1576
	K	1953.20	1904.00	1854.22
0.3	T	0.1552	0.1542	0.1561
	K	2051.80	1979.54	1906.49

Table 10 Effect of shape parameter and salvage parameter on decision policy (M=20/365, Ic_1=16%, N=35/365, Ic_2 =22%, α=0.2)

γ β		0.2	0.4	0.6
1.5	T	0.1580	0.1586	0.1592
	K	1851.55	1826.37	1801.04
2.5	T	0.1602	0.1603	0.1604
	K	1759.10	1756.16	1753.22
3.5	T	0.1606	0.1606	0.1606
	K	1748.47	1748.10	1747.73

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