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RESEARCH ARTICLE

Optical soliton solutions of stochastic Schrödinger–Hirota equation in birefringent fibers with spatiotemporal dispersion and parabolic law nonlinearity

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Abstract In this paper, the stochastic Schrödinger–Hirota equation with spatiotemporal dispersion and parabolic law nonlinearity is studied from the field of nonlinear optics, which is usually used to describe the optical soliton propagation in birefringent fibers. Firstly, the stochastic Schrödinger–Hirota equation is converted into nonlinear ordinary differential equation by using the traveling wave transformation. Secondly, the optical soliton solutions of the stochastic Schrödinger–Hirota equation are obtained by using the complete discriminant system method. Finally, three-dimensional, two-dimensional, and contour maps of optical soliton solutions of the stochastic Schrödinger–Hirota equation are drawn by using the Maple 2022 software. The optical soliton solutions obtained in this article can further help researchers understand the propagation of optical solitons in birefringent fibers.

Keywords Stochastic Schrödinger–Hirota equation · Optical soliton solution · Nonlinearity

Introduction

In recent years, the study of optical solitons in nonlinear partial differential equation (NLPDE) [1–25] has become a hot topic in the field of nonlinear science. The construction of optical soliton solutions of NLPDE can further explain scientific problems from the fields of physics, nonlinear optics and engineering technology. Therefore, many experts and scholars [26–36] have devoted themselves to the study of optical solitons in NLPDE. However, in the fields of natural science and engineering technology, the occurrence of random phenomena is inevitable [37, 38]. Therefore, the problem of considering the solution of NLPDE with random noise is a very difficult problem that has puzzled mathematicians and physicists [39–51]. Therefore, the main purpose of this paper is to construct optical soliton solutions for a class of stochastic NLPDE.

In this article, we consider the following the stochastic Schrödinger–Hirota equation in birefringent fibers with spatiotemporal dispersion and parabolic law nonlinearity [52]

$$\begin{cases} i\psi_t + a_1\psi_{xx} + b_1\psi_{xt} + (c_1|\psi|^2 + d_1|\phi|^2)\psi + (e_1|\psi|^4 + f_1|\psi|^2|\phi|^2 + g_1|\phi|^4)\psi + i[\alpha_1\psi_x + \lambda_1(|\psi|^2\phi)_x \\ +v_1(|\psi|^2)_x\psi + \theta_1|\psi|^2\psi_x + \gamma_1\psi_{xxx}] + \sigma_1(\psi - ib_1\psi_x)\frac{d\omega_1(t)}{dt} = 0, \\ i\phi_t + a_2\phi_{xx} + b_2\phi_{xt} + (c_2|\phi|^2 + d_2|\phi|^2)\phi + (e_2|\phi|^4 + f_2|\phi|^2|\psi|^2 + g_2|\psi|^4)\phi + i[\alpha_2\phi_x + \lambda_2(|\phi|^2\phi)_x \\ +v_2(|\phi|^2)_x\phi + \theta_2|\phi|^2\phi_x + \gamma_2\phi_{xxx}] + \sigma_2(\phi - ib_2\phi_x)\frac{d\omega_2(t)}{dt} = 0, \end{cases} \tag{1.1}$$

where $\psi(t,x)$ and $\phi(t,x)$ represent the soliton profiles. a_j and $b_j (j=1,2)$ stand for the coefficients of chromatic dispersions terms and spatio-temporal dispersion terms, respectively. c_j and $d_j (j=1,2)$represent the coefficients of self-phase modulation and cross-phase modulation terms, respectively. f_j, e_j and g_j $(j=1,2)$ stand for nonlinear terms. α_j and $\lambda_j (j=1,2)$ are the self-steepening terms. v_j and θ_j

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$(j = 1, 2)$ represent the nonlinear dispersion terms. $\frac{d\omega_j(t)}{dt}$ $(j = 1, 2)$ are the white noises. $\omega_j(t) (j = 1, 2)$ represents the standard Wiener processes. $\sigma_j (j = 1, 2)$ stands for the noises strength. In [52], Elsayed et al. obtained the optical soliton solutions of Eq. (1.1) using the Jacobi-elliptic expansion approach and the extended auxiliary equation algorithm, respectively. The main purpose of this article is to construct optical soliton solutions for equation Eq. (1.1) using the complete discriminant system method.

The rest of this article is organized as follows: In Sect. 2, Eq. (1.1) is transformed into the ODEs using traveling wave transformation. Moreover, the optical soliton solutions of stochastic Schrödinger–Hirota equation are obtained. In Sect. 3, a brief summary is presented.

Optical soliton solutions of Eq. (1.1)

Mathematical analysis

Firstly, let's consider the following transformation

$$\begin{aligned} &\psi(t,x) = \Psi(\xi)\mathbf{e}^{i[\eta_1(t,x)+\sigma_1\omega_1(t)-\sigma_1^2 t]}, \\ &\phi(t,x) = \Phi(\xi)\mathbf{e}^{i[\eta_2(t,x)+\sigma_2\omega_2(t)-\sigma_2^2 t]}, \\ &\xi = x - vt, \quad \eta_l(t,x) = -k_l x + \Omega_l t + \theta_l, l = 1, 2, \end{aligned} \tag{2.1}$$

where Ψ and Φ represent the amplitude portions of the solitons. $\eta_j(t,x)$, k_j, Ω_j and θ_j $(j = 1, 2)$ stand for the phase amplitude format, the frequencies of the solitons, the wave numbers and phase constants, respectively. v represents the soliton velocity. So, substituting Eq. (2.1) into Eq. (1.1) yields the real parts:

$$\begin{aligned} &(a_1 - b_1 v + 3k_1\gamma_1)\Psi''(\xi) - [a_1 k_1^2 - k_1\alpha_1 + \gamma_1 k_1^3 \\ &\quad + (\Omega_1 - \sigma_1^2)(1 - b_1 k_1)]\Psi(\xi) + d_1\Psi(\xi)\Phi^2(\xi) + (c_1 + k_1\lambda_1 \\ &\quad + k_1\theta_1)\Psi^3(\xi) + e_1\Psi^5(\xi) + f_1\Psi^3(\xi)\Phi^2(\xi) \\ &\quad + g_1\Psi(\xi)\Phi^4(\xi) = 0, \\ &(a_2 - b_2 v + 3k_2\gamma_2)\Phi''(\xi) - [a_2 k_2^2 - k_2\alpha_2 + \gamma_2 k_2^3 \\ &\quad + (\Omega_2 - \sigma_2^2)(1 - b_2 k_2)]\Phi(\xi) + d_2\Phi(\xi)\Psi^2(\xi) + (c_2 + k_2\lambda_2 \\ &\quad + k_2\theta_2)\Phi^3(\xi) + e_2\Phi^5(\xi) + f_2\Phi^3(\xi)\Psi^2(\xi) \\ &\quad + g_2\Phi(\xi)\Psi^4(\xi) = 0, \end{aligned} \tag{2.2}$$

and the imaginary parts:

$$\begin{aligned} &\gamma_1\Psi'''(\xi) - [2a_1 k_1 - b_1(\Omega_1 - \sigma_1^2) - \alpha_1 + 3\gamma_1 k_1^2 \\ &\quad + (1 - b_1 k_1)v]\Psi'(\xi) + (3\lambda_1 + 2\mu_1 + \theta_1)\Psi^2(\xi)\Psi'(\xi) = 0, \\ &\gamma_2\Phi'''(\xi) - [2a_2 k_2 - b_2(\Omega_2 - \sigma_2^2) - \alpha_2 + 3\gamma_2 k_2^2 \\ &\quad + (1 - b_2 k_2)v]\Phi'(\xi) + (3\lambda_2 + 2\mu_2 + \theta_2)\Phi^2(\xi)\Phi'(\xi) = 0. \end{aligned} \tag{2.3}$$

Next, let be the transformation $\Phi(\xi) = A\Psi(\xi)$, where A is a nonzero constant. Thus, Eqs. (2.2) and (2.3) can be convert into

$$\begin{aligned} &(a_1 - b_1 v + 3k_1\gamma_1)\Psi''(\xi) - [a_1 k_1^2 - k_1\alpha_1 + \gamma_1 k_1^3 \\ &\quad + (\Omega_1 - \sigma_1^2)(1 - b_1 k_1)]\Psi(\xi) + (A^2 d_1 + c_1 + k_1\lambda_1 \\ &\quad + k_1\theta_1)\Psi^3(\xi) + (e_1 + A^2 f_1 + A^4 g_1)\Psi^5(\xi) = 0, \\ &(a_2 - b_2 v + 3k_2\gamma_2)\Psi''(\xi) - [a_2 k_2^2 - k_2\alpha_2 + \gamma_2 k_2^3 \\ &\quad + (\Omega_2 - \sigma_2^2)(1 - b_2 k_2)]\Psi(\xi) + [d_2 + A^2(c_2 + k_2\lambda_2 \\ &\quad + k_2\theta_2)]\Psi^3(\xi) + (e_2 A^4 + f_2 A^2 + g_2)\Psi^5(\xi) = 0, \end{aligned} \tag{2.4}$$

and

$$\begin{aligned} &\gamma_1\Psi'''(\xi) - [2a_1 k_1 - b_1(\Omega_1 - \sigma_1^2) - \alpha_1 + 3\gamma_1 k_1^2 \\ &\quad + (1 - b_1 k_1)v]\Psi'(\xi) + (3\lambda_1 + 2\mu_1 + \theta_1)\Psi^2(\xi)\Psi'(\xi) = 0, \\ &\gamma_2\Psi'''(\xi) - [2a_2 k_2 - b_2(\Omega_2 - \sigma_2^2) - \alpha_2 + 3\gamma_2 k_2^2 \\ &\quad + (1 - b_2 k_2)v]\Phi'(\xi) + A^2(3\lambda_2 + 2\mu_2 + \theta_2)\Psi^2(\xi)\Psi'(\xi) = 0. \end{aligned} \tag{2.5}$$

Integrating Eq. (2.5) and assuming an integral constant of zero, we obtain

$$\begin{aligned} &\gamma_1\Psi''(\xi) - [2a_1 k_1 - b_1(\Omega_1 - \sigma_1^2) - \alpha_1 + 3\gamma_1 k_1^2 \\ &\quad + (1 - b_1 k_1)v]\Psi(\xi) + \frac{1}{3}(3\lambda_1 + 2\mu_1 + \theta_1)\Psi^3(\xi) = 0, \\ &\gamma_2\Psi''(\xi) - [2a_2 k_2 - b_2(\Omega_2 - \sigma_2^2) - \alpha_2 + 3\gamma_2 k_2^2 \\ &\quad + (1 - b_2 k_2)v]\Psi(\xi) + \frac{1}{3}A^2(3\lambda_2 + 2\mu_2 + \theta_2)\Psi^3(\xi) = 0. \end{aligned} \tag{2.6}$$

By the linearly independent principle on Eq. (2.6) yields

$$\begin{aligned} &\gamma_1 = \gamma_2 = 0, \quad \alpha_l = 2a_l k_l - b_l(\Omega_l - \sigma_l^2) + (1 - k_l b_l)v, \\ &3\lambda_l + 2\mu_l + \theta_l = 0, \quad l = 1, 2. \end{aligned} \tag{2.7}$$

Therefore, Eq. (2.3) are equivalent under following conditions

$$\begin{aligned} \frac{a_1 - b_1 v}{a_2 - b_2 v} &= \frac{a_1 k_1^2 - k_1\alpha_1 + \gamma_1 k_1^3 + (\Omega_1 - \sigma_1^2)(1 - b_1 k_1)}{a_2 k_2^2 - k_2\alpha_2 + \gamma_2 k_2^3 + (\Omega_2 - \sigma_2^2)(1 - b_2 k_2)} \\ &= \frac{A^2 d_1 + c_1 + k_1\lambda_1 + k_1\theta_1}{d_2 + A^2(c_2 + k_2\lambda_2 + k_2\theta_2)} = \frac{e_1 + A^2 f_1 + A^4 g_1}{e_2 A^4 + f_2 A^2 + g_2}. \end{aligned} \tag{2.8}$$

Consequently, the velocity of the soliton can be obtained from Eq. (2.8)

$$v = \frac{a_2[a_1k_1^2 - k_1\alpha_1 + (\Omega_1 - \sigma_1^2)(1 - b_1k_1)] - a_1[a_2k_2^2 - k_2\alpha_2 + (\Omega_2 - \sigma_2^2)(1 - b_2k_2)]}{b_2[a_1k_1^2 - k_1\alpha_1 + (\Omega_1 - \sigma_1^2)(1 - b_1k_1)] - b_1[a_2k_2^2 - k_2\alpha_2 + (\Omega_2 - \sigma_2^2)(1 - b_2k_2)]}, \tag{2.9}$$

and

$$A = [\frac{(a_2 - b_2v)(c_1 + k_1\lambda_1 + k_1\theta_1) - (a_1 - b_1v)d_2}{(a_1 - b_1v)(c_2 + k_2\lambda_2 + k_2\theta_2) - (a_2 - b_2v)d_1}]^{\frac{1}{2}},$$
$$e_1 = \frac{(a_1 - b_1v)(A^4e_2 + A^2f_2 + g_2) - A^2(a_2 - b_2v)(f_1 + A^2g_1)}{a_2 - b_2v}, \tag{2.10}$$

satisfying $(a_1 - b_1v)(c_2 + k_2\lambda_2 + k_2\theta_2) \neq (a_2 - b_2v)d_1$ and $(a_2 - b_2v)(c_1 + k_1\lambda_1 + k_1\theta_1) \neq (a_1 - b_1v)d_2$.

Thus, the first equation of Eq. (2.4) can be rewritten as

$$\Psi''(\xi) - \Theta_1\Psi^5(\xi) - \Theta_2\Psi^3(\xi) - \Theta_3\Psi(\xi) = 0, \tag{2.11}$$

where $\Theta_1 = \frac{e_1+A^2f_1+A^4g_1}{a_1-b_1v}$, $\Theta_2 = -\frac{A^2d_1+c_1+k_1\lambda_1+k_1\theta_1}{a_1-b_1v}$ and $\Theta_3 = -\frac{a_1k_1^2-k_1\alpha_1+(\Omega_1-\sigma_1^2)(1-b_1k_1)}{a_1-b_1v}$ satisfying $a_1 \neq b_1v$.

Multiplying both sides of Eq. (2.11) by $\Phi(\xi)$ and integrating once with respect to ξ, we have

$$(\Psi'(\xi))^2 = \frac{\Theta_1}{3}\Psi^6(\xi) + \frac{\Theta_2}{2}\Psi^4(\xi) + \Theta_3\Psi^2(\xi) + 2c, \tag{2.12}$$

where c is integral constant.

Setting $\Xi(\xi) = \Psi^2(\xi)$, then the integral expression of Eq. (2.12) is

$$\pm 2\sqrt{\varepsilon\frac{\Theta_3}{3}}(\xi - \xi_0) = \int \frac{d\Xi}{\sqrt{\varepsilon\Xi(\Xi^3 + P\Xi^2 + Q\Xi + D)}}, \tag{2.13}$$

where $P = \frac{3\Theta_2}{2\Theta_1}$, $Q = \frac{2\Theta_3}{\Theta_2}$ and $D = \frac{6c}{\Theta_1}$ satisfying $\Theta_1 \neq 0$.

Optical soliton solutions of Eq. (1.1)

Assume that $F(\Xi) = \Xi^3 + P\Xi^2 + Q\Xi + D$, $\Delta = -27(\frac{2P^3}{27} + D - \frac{PQ}{3})^2 - 4(Q - \frac{P^2}{3})^3$ and $\Gamma = Q - \frac{P^2}{3}$. Here, when $\Theta_3 > 0$, we take $\varepsilon = 1$. Conversely, when $\Theta_3 < 0$, we take $\varepsilon = -1$. Therefore, according to the complete discriminant system of third-order polynomials, we can obtain the optical soliton solution of Eq. (1.1).

(i) $\Delta = 0, \Gamma < 0, \varepsilon = 1$. Suppose that $F(\Xi) = \Xi^3 + P\Xi^2 + Q\Xi + D = (\Xi - \alpha)^2(\Xi - \beta)$, where α and β are the real constants satisfying $\alpha \neq \beta$. When $\alpha > \beta$ and $\Xi > \beta$, or when $\alpha < 0$ and $\Xi < 0$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\pm 2\sqrt{\frac{\Theta_3}{3}}(\xi - \xi_0) = \frac{1}{\alpha(\alpha - \beta)} \ln \frac{(\sqrt{\alpha(\Xi - \beta)} - \sqrt{\Xi(\alpha - \beta)})^2}{|\Xi - \alpha|}. \tag{2.14}$$

When $\alpha > \beta$ and $\Xi < 0$, or when $\alpha < \Xi$ and $\Xi < \beta$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\pm 2\sqrt{\frac{\Theta_3}{3}}(\xi - \xi_0) = \frac{1}{\alpha(\alpha - \beta)} \ln \frac{(\sqrt{-\alpha(\Xi - \beta)} - \sqrt{\Xi(\beta - \alpha)})^2}{|\Xi - \alpha|}. \tag{2.15}$$

When $\beta > \alpha > 0$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\pm 2\sqrt{\frac{\Theta_3}{3}}(\xi - \xi_0) = \frac{1}{\alpha(\beta - \alpha)} \arcsin \frac{\alpha(\Xi - \beta) + \Xi(\alpha - \beta)}{\beta|\Xi - \alpha|}. \tag{2.16}$$

(ii) $\varepsilon = -1$

When $\alpha > \beta$ and $\Xi > \beta$, or when $\alpha < 0$ and $\Xi < 0$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\pm 2\sqrt{-\frac{\Theta_3}{3}}(\xi - \xi_0) = \frac{1}{\alpha(\beta - \alpha)} \ln \frac{(\sqrt{\alpha(-\Xi + \beta)} - \sqrt{\Xi(\beta - \alpha)})^2}{|\Xi - \alpha|}. \tag{2.17}$$

When $\alpha > \beta$ and $\Xi < 0$, or when $\alpha < 0$ and $\Xi < \beta$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\pm 2\sqrt{-\frac{\Theta_3}{3}}(\xi - \xi_0) = \frac{1}{\alpha(\beta - \alpha)} \ln \frac{(\sqrt{-\alpha(-\Xi + \beta)} - \sqrt{\Xi(\alpha - \beta)})^2}{|\Xi - \alpha|}. \tag{2.18}$$

When $\beta > \alpha > 0$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\pm 2\sqrt{-\frac{\Theta_3}{3}}(\xi - \xi_0) = \frac{1}{\alpha(\alpha - \beta)} \arcsin \frac{(-\Xi + \beta)(\alpha - \gamma) + \Xi(\beta - \alpha)}{|\beta(\Xi - \alpha)|}. \tag{2.19}$$

Case II. If $\Delta = 0, \Gamma = 0$, then $F(\Xi) = (\Xi - \rho)^3$, where ρ is the real number.

(i) $\varepsilon = 1$ If $\Xi > \rho$ and $\Xi > 0$, or if $\Xi < \rho$ and $\Xi < 0$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\psi_1(t,x) = \pm \left[\frac{3\rho}{\rho^2\Theta_1(x - vt - \xi_0)^2 - 3} + \rho\right]^{\frac{1}{2}} \mathbf{e}^{i[\eta_1(t,x)+\sigma_1\omega_1(t)-\sigma_1^2 t]}. \tag{2.20}$$

(ii) $\varepsilon = -1$ If $\Xi > \rho$ and $\Xi < 0$, or if $\Xi < \rho$ and $\Xi > 0$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\psi_2(t,x) = \pm \left[\frac{3\rho}{-\rho^2\Theta_1(x - vt - \xi_0)^2 - 3} + \rho\right]^{\frac{1}{2}} \mathbf{e}^{i[\eta_1(t,x)+\sigma_1\omega_1(t)-\sigma_1^2 t]}. \tag{2.21}$$

Case III. If $\Delta > 0, \Gamma < 0, F(\Xi) = (\Xi - \theta_1)(\Xi - \theta_2)(\Xi - \theta_3)$, where θ_1, θ_2 and θ_3 are real numbers satisfying $0 < \theta_1 < \theta_2 < \theta_3$.

(i) $\varepsilon = 1$ When $\Xi > 0$ and $\Xi < \theta_3$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\psi_3(t,x) = \pm\sqrt{\frac{-\theta_1\theta_3\mathbf{sn}^2(\sqrt{-\theta_2(\theta_1-\theta_3)}\sqrt{\frac{\Xi_1}{3}}(x - vt - \xi_0), m)}{-\theta_3\mathbf{sn}^2(\sqrt{-\theta_2(\theta_1-\theta_3)}\sqrt{\frac{\Theta_1}{3}}(x - vt - \xi_0), m) - (\theta_1 - \theta_3)}}\mathbf{e}^{i[\eta_1(t,x)+\sigma_1\omega_1(t)-\sigma_1^2 t]}, \tag{2.22}$$

$$\psi_4(t,x) = \pm\sqrt{\frac{\theta_3(\theta_1-\theta_2)\mathbf{sn}^2(\sqrt{-\theta_2(\theta_1-\theta_3)}\sqrt{\frac{\Theta_1}{3}}(x - vt - \xi_0), m) - \theta_2(\theta_1 - \theta_3)}{(\theta_1-\theta_2)\mathbf{sn}^2(\sqrt{-\theta_2(\theta_1-\theta_3)}\sqrt{\frac{\Theta_1}{3}}(x - vt - \xi_0), m) - (\theta_1 - \theta_3)}}\mathbf{e}^{i[\eta_1(t,x)+\sigma_1\omega_1(t)-\sigma_1^2 t]}, \tag{2.23}$$

where $m^2 = \frac{\theta_3(\theta_1-\theta_2)}{\theta_2(\theta_1-\theta_3)}$.

(ii) $\varepsilon = -1$When $\theta_3 < \Xi < \theta_2$, we can obtain the solution of Eq. (1.1) from Eq. (2.13)

$$\psi_5(t,x) = \pm\sqrt{\frac{-\theta_1\theta_2\mathbf{sn}^2(\sqrt{-\theta_2(\theta_1-\theta_3)}\sqrt{\frac{\Theta_1}{3}}(x - vt - \xi_0), m) + \theta_1\theta_2}{-\theta_1\mathbf{sn}^2(\sqrt{-\theta_2(\theta_1-\theta_3)}\sqrt{\frac{\Theta_1}{3}}(x - vt - \xi_0), m) + \theta_3}}\mathbf{e}^{i[\eta_1(t,x)+\sigma_1\omega_1(t)-\sigma_1^2 t]}, \tag{2.24}$$

$$\psi_6(t,x) = \pm\sqrt{\frac{-\theta_2\theta_3}{(\theta_2-\theta_3)\mathbf{sn}^2(\sqrt{-\theta_2(\theta_1-\theta_3)}\sqrt{\frac{\Theta_1}{3}}(x - vt - \xi_0), m) - \theta_2}}\mathbf{e}^{i[\eta_1(t,x)+\sigma_1\omega_1(t)-\sigma_1^2 t]}, \tag{2.25}$$

where $m^2 = \frac{\theta_1(\theta_2-\theta_3)}{\theta_2(\theta_1-\theta_3)}$.

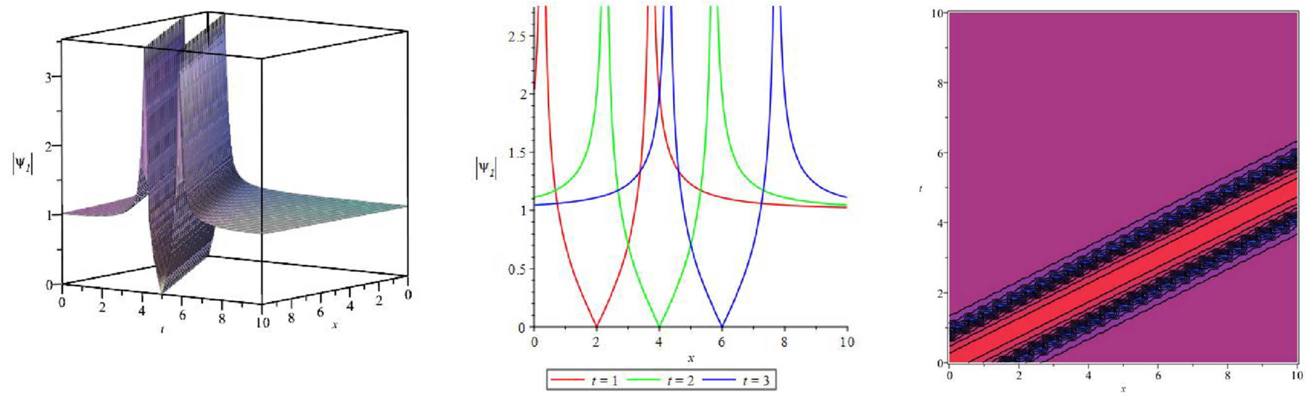
Case IV. If $\Delta < 0$, $F(\Xi) = (\Xi - \vartheta_1)[(\Xi - \vartheta_2)^2 + \vartheta_3^2]$, where ϑ_1, ϑ_2 and ϑ_3 are real numbers, and $\vartheta_1 > 0, \vartheta_2 > 0, \vartheta_3 > 0$. So, we can obtain the solution of Eq. (1.1) from Eq. (2.13), where the positive sign corresponds to $\varepsilon = 1$ and the symbol corresponds to $\varepsilon = -1$.

$$\psi_7(t,x) = \pm\sqrt{\frac{d_1\mathbf{cn}^2(\frac{\sqrt{\mp 6\vartheta_3 m_1\vartheta_1 c_1}}{3mm_1}(x - vt - \xi_0), m) + d_2}{d_3\mathbf{cn}^2(\frac{\sqrt{\mp 6\vartheta_3 m_1\vartheta_1 c_1}}{3mm_1}(x - vt - \xi_0), m) + d_4}}\;\mathbf{e}^{i[\eta_1(t,x)+\sigma_1\omega_1(t)-\sigma_1^2 t]}, \tag{2.26}$$

where $d_1 = \frac{1}{2}\vartheta_1(d_3 - d_4)$, $d_2 = \frac{1}{2}\vartheta_1(\vartheta_4 - d_3)$, $d_3 = \vartheta_1 - \vartheta_2 - \frac{\vartheta_3}{m_1}$, $d_4 = \vartheta_1 - \vartheta_2 - \vartheta_3 m_1$, $E = \frac{\vartheta_3^2-\vartheta_2(\vartheta_1-\vartheta_2)}{\vartheta_1\vartheta_3}$, $m_1 = E \pm \sqrt{E^2 + 1}$ and $m^2 = \frac{1}{1+m_1^2}$.

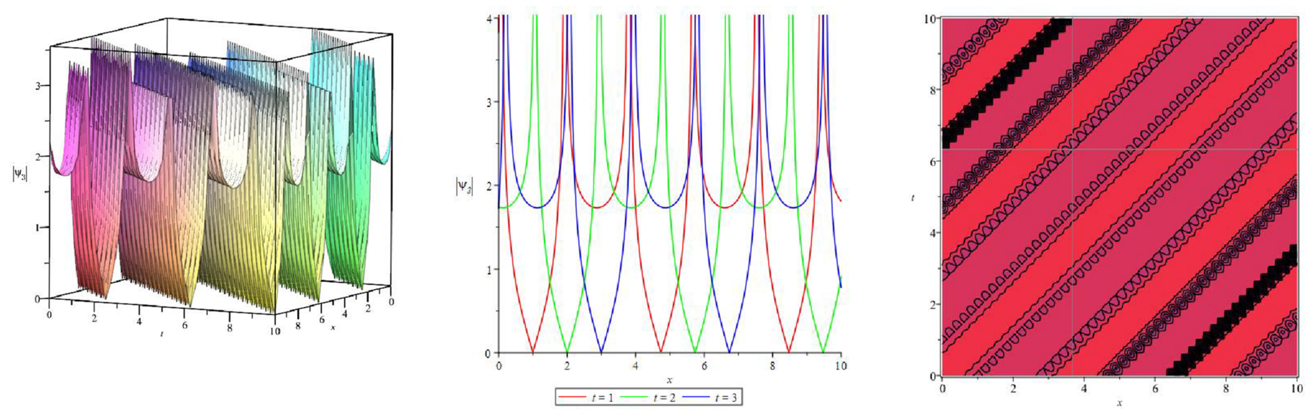
Numerical simulation

When $\Theta_1 = -1, \Theta_2 = 2, \Theta_3 = 3, \xi_0 = 0, v = 1, c = \frac{1}{6}$, we can plot three-dimensional diagram, two-dimensional diagram and contour plot of optical soliton solution $|\psi_1(t,x)|$ as shown in Fig. 1. When $\Theta_1 = 1, \Theta_2 = -4, \Theta_3 = \frac{11}{2}, \xi_0 = 0, v = 1, c = -1$, we can plot three-dimensional diagram, two-dimensional diagram and contour plot of optical soliton solution $|\psi_3(t,x)|$as shown in Fig. 2.



(a) 3D diagram (b) 2D diagram (c) Contour plot

Fig. 1 The graph of optical soliton solution $|\psi_1(t,x)|$ at $\Theta_1 = -1, \Theta_2 = 2, \Theta_3 = 3, \xi_0 = 0, v = 1, c = \frac{1}{6}$



(a) 3D diagram (b) 2D diagram (c) Contour plot

Fig. 2 The graph of optical soliton solution $|\psi_3(t,x)|$ at $\Theta_1 = 1, \Theta_2 = -4, \Theta_3 = \frac{11}{2}, \xi_0 = 0, v = 1, c = -1$

Conclusion

In the paper, the optical soliton solutions of stochastic Schrödinger–Hirota equation with spatiotemporal dispersion and parabolic law nonlinearity are investigated by using the complete discriminant system method. Moreover, we obtained seven sets of solutions and implicit function solutions, and we also draw three-dimensional diagram, two-dimensional diagram and contour plot of the solitons $|\psi_1(t,x)|$ and $|\psi_3(t,x)|$, which are meaningful for us to further understand the propagation of optical solitons. Compared with reference [52], our obtained solutions are more abundant.

Data availability No data were used for the research described in the article.

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