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RESEARCH ARTICLE

Parallel relay-assisted free-space optical communication using multi-pulse position modulation over the generalized turbulence channel model

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Abstract Relays play a pivotal role in free-space optical (FSO) communication system and thus has found tremendous applications mainly to enhance the signal quality and propagation distance coverage in the presence of atmospheric turbulence. Serial relaying or parallel relaying configurations are two major configurations used. In this paper, we analyze the performance of the parallel relay-assisted FSO influenced by the generalized Malaga (M) turbulence channel. In the first instance, the data modulation at the source and each relay is signaled using the multi-pulse position modulation. Next, closed-form expressions for the outage probability have been derived and later on used for the analysis of the influence of the threshold signal to noise ratio transmitter power as well as the number of relay nodes between the source and the destination on the overall outage probability. When direct transmission between source and destination is considered, the outage probability 3.965×10^{-7}, 5.783×10^{-6}, 7.156×10^{-3} for weak, moderate and strong turbulence, respectively. However, when a parallel relay configuration with three paths and two hops per path is considered, the outage probability improves to 6.23×10^{-20}, 1.93×10^{-16}, 3.96×10^{-7} for weak, moderate and strong turbulence, respectively, for a threshold signal-to-noise ratio of 10 dB. The results revealed that the use of parallel relay nodes is a clear performance improvement option in turbulence-affected FSO communication systems.

Keywords FSO · Parallel relays · Outage probability · MPPM · Turbulence

Introduction

Free-space optical communication systems have been considered and are now accepted as a viable solution for various applications chiefly due to advantages such as high security, high data rate, interference-free, ease of installation and high bandwidth among others [1, 2]. Important applications include broadband service delivery, earth to satellite links, inter-chip links among others [3]. Unlike radio frequency (RF) communication, FSO requires a direct line of sight between the transceivers. Hence, the operation of FSO communication suffers from the challenges of the atmospheric turbulence-induced scintillation, prevailing adverse weather conditions such as rain, snow and fog and the alignment between the transceivers [4]. The impact of these effects has been explained in various articles in the literature. Although some of the successful approaches toward reducing these effects are use of adaptive optics, aperture averaging, error control coding, etc., the use of relays in FSO has been one of the most effective ways to reduce the impact of these detrimental effects and thus improve coverage distance. As the coverage distance increases, the signal fading also increases, therefore increasing the number of short hops improves the signal integrity to some extend [5].

Various modulation schemes have been used in relay-aided FSO such as on-off keying (OOK) [6], binary phase-shift keying (BPSK) [7], differential phase-shift keying (DPSK), Polarization shift Keying (PolSK) and subcarrier modulation [8]. However, pulse position modulation has gained a remarkable ground in IM/DD FSO systems

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because its peak power is much larger than its average power, which condition is not satisfied by other pulse signaling techniques.

In order to create a PPM symbol constituting of L bits, a single pulse is placed into one of the $M = 2^L$ time slots. This is shown in Fig. 1. Because of orthogonality and lack of overlap between pulses, there is a remarkable improvement in PPM signaling bit error rate (BER). The decoding of the PPM symbols at the receiver is through determination of the slot containing the pulse and then performing inverse mapping across the transmitted bit sequence. High energy efficient PPM scheme requires a correspondingly huge bandwidth, and hence this has given way to multi-pulse position modulation (MPPM) scheme, in which more than one pulse is located within a symbol of M-time slots. Such a technique exhibit a good balance between the overall system throughput and link performance [9].

Due to the randomness of the FSO channel, different statistical atmospheric turbulence models such as K [10], gamma-gamma [11] and exponential Weibull [12] have been proposed and used for FSO performance analysis. However, Malaga (M) model represents a general model of atmospheric turbulence being able to characterize other less general previously proposed models [13]. The study conducted in [14] investigated the parallel relay FSO system influenced by the gamma-gamma fading model. A comparative analysis between PolSK and OOK was carried out based on the end-to-end outage probability concluded the superiority of PolSK over OOK. Using parallel all-optical relays over composite gamma-gamma channels, the performance analysis for a FSO communication system has been investigated in [15]. The multi-hop parallel free-space cooperative communication system employing BPSK modulation and decode-and-forward protocol has been analyzed under exponentiated Weibull (EW) fading channels [16]. Considering the atmospheric turbulence modeled

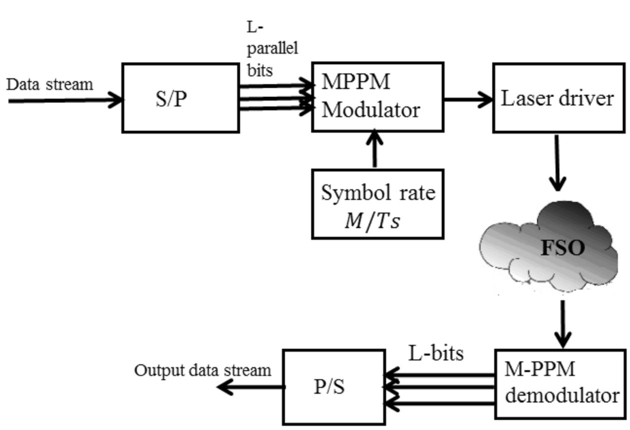

Fig. 1 Block diagram of MPPM free-space optical communication system

by gamma-gamma channel, and pointing errors, the research in [17] analyzed the performance of a parallel relay-assisted FSO transmission using decode-and-forward protocols. The study in [18] analyzed the multi-hop parallel relay FSO communication system using BPSK signaling over the gamma-gamma turbulence model. After considering the pointing error effect, it was shown that the relay-aided FSO system performance could be considerably enhanced using lower normalized beam width and jitter. In another study [19], spatial modulation has been used to improve the performance of a two-way relay cooperative communication system. The authors in [20] analyzed the BER performance of a multi-hop FSO system using M-ary PPM signaling under the log-normal channel. Their results showed that the MPPM performance could be improved by using a greater number of hops. Meanwhile, in [21], BER analysis of relay-assisted PSK with OFDM ROFSO system over Malaga distribution including pointing errors under various weather conditions has been carried out. Using BPPM over log-normal distribution, the optimal relay placement and diversity analysis issues in FSO communication have been investigated in [22]. The end-to-end outage probability for both the parallel and serial relay FSO system was analyzed over the Malaga channel in [23]. The MPPM signaling over the Malaga channel has been analyzed in [24].

It is evident in all the above-mentioned works, that although parallel and serial relay-assisted FSO has been analyzed over different atmospheric turbulence channels and using different modulation schemes, no work on MPPM parallel relay-assisted FSO influenced by Malaga channel has been reported in the literature. Motivated by the above analysis, in this paper, we focus on the parallel relay-aided FSO communication system employing MPPM signaling over Malaga channel. The advantage of the Malaga channel is that it is a more generic channel, which can be reduced to any of the previously proposed models by appropriate choice of the model parameters. The rest of the paper is organized as follows: in second section, the system and channel models are presented. In third section, the outage probability is analyzed followed by the results and discussion in fourth section. Finally, the paper is concluded in fifth section.

System and channel fading models

System model

The parallel relay-assisted FSO communication influenced by Malaga channel considered in this present study is illustrated in Fig. 2. The source node (S) is considered to transmit the same information to the destination node

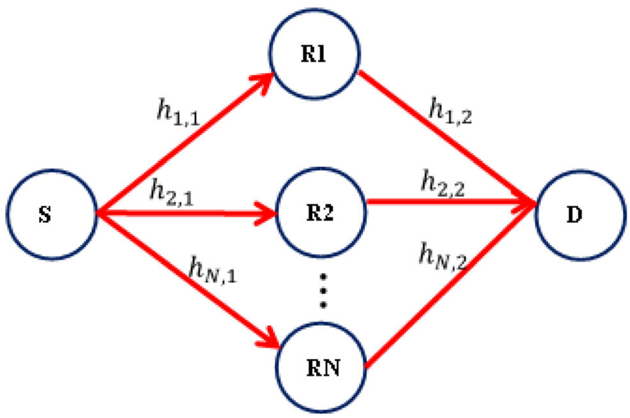

Fig. 2 System model

(*D*) indirectly, but via *N* parallel relay nodes of the M relay paths. We assume that the source node consists of *N* transmitters and the destination node has *N* receiver apertures. From the source node, the multiple copies of the information signal are transmitted via M hops per path.

In order to facilitate the description of the considered relay FSO system, we focus first on the point-to-point link. At the *j*th hop of the *i*th path, the received signal $y_{i,j}$ is expressed as [17];

$$y_{i,j} = Rh_{i,j}x_{i,j} + n_{i,j} \quad i \in \{1,2,\ldots N\}, j \in \{1,2,\ldots M\} \tag{1}$$

where R is the photodetector responsivity, $x_{i,j}$ is the signal transmitted in the *i*th path of the *j*th hop, $n_{i,j}$ is the additive white Gaussian noise of the *i*th path of the *j*th hop, M is the number of hops and finally $h_{i,j}$ is the channel irradiance of the *i*th path of the *j*th hop, which is expressed as:

$$h_{i,j} = h_{l_{i,j}} h_{s_{i,j}} h_{p_{i,j}} \tag{2}$$

where $h_{l_{i,j}}$ is the exponentiated path loss, $h_{s_{i,j}}$ is the laser beam attenuation due to atmospheric turbulence and $h_{p_{i,j}}$ is the attenuation due to misalignment between the FSO transmitter and receiver.

Various gases and particles in the atmosphere will absorb and scatter the propagating laser beam resulting in attenuation of the laser beam, which can be expressed the Beers–Lambert law as [25]:

$$h_{l_{i,j}} = e^{-\sigma L_{i,j}} \tag{3}$$

where σ is the attenuation coefficient and $L_{i,j}$ is the propagation distance of the *i*th path of the *j*th hop.

The atmospheric loss depends on the size and distribution of the scattering particles, which can be measured directly from the atmosphere. Hence the path loss of the atmosphere can be considered a constant over a long time. Without loss of generality, in this article $h_{l_{i,j}}$ has been considered as 1 dB [26].

We consider an IM/DD multi-pulse position modulation in which the transmitted data is modulated onto the intensity of the laser beam. Using the serial to parallel converter, the incoming serial bits are mapped into parallel stream and then converted to the MPPM symbol, where each symbol is positioned in any of the several M-time slots. The MPPM symbol is used to modulate the laser beam intensity. Finally, the laser beam is transmitted toward the receiver in free space by means of transmitting telescope. The receiving side consists of a receiving telescope followed by a photodetector which converts the optical signal to electrical signal. Because noise is added onto the signal as it propagates in the free-space channel, a low pass filter is used to remove such noise. Therefore, the received electrical signal is given as:

$$\begin{aligned} y(t) &= RP_R x(t) + n(t) \\ &= R\frac{M}{k}P_R \sum_{k=0}^{M-1} C_k rect\left(t - \frac{kT}{M}\right) + n(t) \end{aligned} \tag{4}$$

where $rect(t) = \begin{cases} 1, & \text{if } 0 \le t \le \dfrac{T}{M} \\ 0, & \text{otherwise} \end{cases}$. In Eq. (4), P_R is the average received optical power, x is the transmitted data, R is the photodetector responsivity and $n(t)$ is the additive white Gaussian noise with variance σ_n^2. Moreover, $C_k = 1$ for signal time slot and zero for non signal time slot. The average received optical signal can be expressed using the link range equation as [27];

$$P_R(h_{i,j}) = P_T \eta_T \eta_R G_T G_R \left(\frac{\lambda}{4\pi L_{i,j}}\right)^2 h_{i,j} \tag{5}$$

P_T is the transmitted optical power, η_T and η_R are the efficiencies of the transmitter and receiver optics, respectively. G_T, G_R are the transmitter and receiver telescope gains, respectively. λ is the operating wavelength, L is the transmission length and h is the channel state due to atmospheric turbulence. In this work, we assume that the transmitter and receiver telescope gains are equal and hence we have:

$$G_T = G_R = \left(\frac{\pi D}{\lambda}\right)^2 \tag{6}$$

D is the diameter of the receiving telescope. Next, we also make the assumption that $\eta_T = \eta_R = \eta$ and then substitute Eq. (6) into Eq. (5), we get:

$$P_R(h_{i,j}) = P_T \left(\frac{\eta A_r}{\lambda L_{i,j}}\right)^2 h_{i,j} \tag{7}$$

Equation (7) gives the received power at the photodiode where $A_r = \frac{\pi D^2}{4}$.

Channel model

The atmospheric turbulence is modeled using the generalized M distribution FSO model. It has been said that it is valid for all channel conditions from weak to strong and takes care of pointing errors also which happens between the transmitter and the receiver. In this model, the optical beam is considered as consisting of three components: (1) the line of sight component with power Ω, (2) the scattered component coupled to the line of sight with power $2\rho bo$, (3) the scattered component independent of the previous two components. The third component carries power of magnitude $2(1-\rho)bo$. Therefore, the total power of the scattered components is $2bo$. In this representation, the parameter ρ expresses the amount of coupling between the scattered and line of sight components. The probability distribution of the M distribution FSO turbulence is given by [13]:

$$f_{h_{i,j}}\left(h_{i,j}\right) = A\sum_{k=1}^{\beta} a_k h_{i,j}^{\frac{\alpha+k}{2}-1} K_{\alpha-k}\left(2\sqrt{\frac{\alpha\beta h_{i,j}}{\gamma\beta+\Omega'}}\right) \quad (8)$$

where

$$A = \frac{2\alpha^{\alpha/2}}{\gamma^{1+\frac{\alpha}{2}}\Gamma(\alpha)}\left(\frac{\gamma\beta}{\gamma\beta+\Omega'}\right)^{\beta+\frac{\alpha}{2}} \quad (9)$$

$$a_k = \binom{\beta-1}{k-1}\frac{(\gamma\beta+\Omega')^{1-\frac{k}{2}}}{(k-1)!}\left(\frac{\Omega'}{\Upsilon}\right)^{k-1}\left(\frac{\alpha}{\beta}\right)^{\frac{k}{2}} \quad (10)$$

In Eq. (8–10), $\Gamma(\cdot)$ is the gamma function, $K_v(\cdot)$ is the modified Bessel function of second kind and order v. Also α represents the effective number of large-scale scattering processes. Hence it is a positive number. β is a natural number that represents the amount of fading parameter. For simplicity, we have denoted $\gamma = 2(1-\rho)b_o$. Further, $\Omega' = \Omega + 2\rho b_o + 2\sqrt{2b_o\Omega\rho}\cos(\phi_A - \phi_B)$ is the average power of the coherent distribution. ϕ_A and ϕ_B are the deterministic phases of the LOS and coupled to LOS components, respectively.

Pointing errors

Pointing errors arises due to misalignment between the transmitter and the receiver. Hence its consideration is crucial for the determination of the FSO link performance. Pointing errors are hereby denoted by h_p, and its PDF is expressed as [28]:

$$f_{h_{p_{i,j}}} = \frac{g^2}{A_o^{g^2}}\left(h_{p_{i,j}}\right)^{g^2-1}, \quad 0 \le h_{p_{i,j}} \le A_o \quad (11)$$

In Eq. (11), $A_o = [\mathrm{erf}(v)]^2$ is the fraction of the collected optical power. $v = \sqrt{\frac{\pi}{2}}\cdot\frac{a}{\omega_z}$ in which a denotes the receiver telescope radius and ω_z is the laser beam width at a distance L. Furthermore, the equivalent beam width is given as $\omega_{zeq} = \left[\frac{\sqrt{\pi}\mathrm{erf}(v)\cdot\omega_z^2}{2ve^{-v^2}}\right]^{\frac{1}{2}}$. Finally, $g = \frac{\omega_{zeq}}{2\sigma_s}$ is the ratio between the effective beam width and the jitter standard deviation σ_s.

Combined channel fading model

The combined channel model for $h_{i,j}$ is given as:

$$f_{h_{i,j}}\left(h_{i,j}\right) = \int f_{h_{i,j}|h_{s_{i,j}}}\left(h_{i,j}|h_{s_{i,j}}\right).f_{h_{s_{i,j}}}\left(h_{s_{i,j}}\right)\mathrm{d}h_{s_{i,j}} \quad (12)$$

where $f_{h_{i,j}|h_{s_{i,j}}}\left(h_{i,j}|h_{s_{i,j}}\right)$ is the conditional probability for a given turbulence state $h_{s_{i,j}}$, defined as:

$$\begin{aligned} f_{h_{i,j}|h_{s_{i,j}}}\left(h_{i,j}|h_{s_{i,j}}\right) &= \frac{1}{h_{s_{i,j}}h_{l_{i,j}}} f_{I_p}\left(\frac{h_{i,j}}{h_{s_{i,j}}h_{l_{i,j}}}\right) \\ &= \frac{g^2}{A_o^{g^2}h_{s_{i,j}}h_{l_{i,j}}}\left(\frac{h_{i,j}}{h_{s_{i,j}}h_{l_{i,j}}}\right)^{g^2-1} \\ 0 \le h_{i,j} &\le A_o h_{s_{i,j}}h_{l_{i,j}} \end{aligned} \quad (13)$$

Thus, the PDF of $h_{i,j}$ can be simplified as:

$$f_{h_{i,j}}\left(h_{i,j}\right) = \frac{g^2 A}{\left(A_o h_{l_{i,j}}\right)^{g^2}} h_{i,j}^{g^2-1}\sum_{k=1}^{\beta} a_k \int_{\frac{h_{i,j}}{A_o h_{l_{i,j}}}}^{\infty} h_{i,j}^{\frac{\alpha+k}{2}-1-g^2} \times K_{\alpha-k}\left(2\sqrt{\frac{\alpha\beta h_{s_{i,j}}}{\mu\beta+\Omega'}}\right)\mathrm{d}h_{i,j} \quad (14)$$

The Bessel function can be expressed as a Meijer's G function using [29]. After simple manipulations, the final expression for the M channel with pointing errors can be expressed as:

$$f_{h_{i,j}}\left(h_{i,j}\right) = \frac{g^2 A}{2h_{i,j}}\sum_{k=1}^{\beta} a_k\left[\frac{1}{B}\right]^{\frac{\alpha+k}{2}} G_{1,3}^{3,0}\left[\frac{h_{i,j}}{BA_o h_{l_{i,j}}}\middle| \begin{matrix} 1+g^2 \\ g^2, \alpha, k\end{matrix}\right]. \quad (15)$$

$G_{p,q}^{m,n}[\cdot]$ is the Meijer's G function. Also $B = \alpha\beta/(\mu\beta+\Omega')$. The CDF of the M channel model can be expressed as [30]:

$$F_{h_{i,j}}\left(h_{i,j}\right) = \frac{g^2 A}{2}\sum_{k=1}^{\beta}\left(a_k\left[\frac{1}{B}\right]^{\frac{\alpha+k}{2}} G_{2,4}^{3,1}\left(\frac{h_{i,j}}{BA_o h_{l_{i,j}}}\middle| \begin{matrix} 1, 1+g^2 \\ g^2, \alpha, k, 0\end{matrix}\right)\right) \quad (16)$$

Further, for MPPM signaling, the electrical SNR as a function of the channel coefficient can be given as [24]:

$$\mathrm{SNR}\left(h_{i,j}\right) = RP_T\left(\frac{\eta A_r}{L\lambda}\right)^2\frac{\sqrt{M\mathrm{Log}_2 M}}{2\sigma_n}h_{i,j} \quad (17)$$

Outage probability analysis

The outage probability is defined as the probability that the end-to-end out signal to noise ratio is less than a specified threshold SNR_{th}. This SNR_{th} is the minimum value of the SNR above which a satisfactory quality of service is achieved [17]. Over a fading channel, the outage probability can be expressed as:

$$P_{out} = P\left(SNR\left(h_{i,j}\right) \le SNR_{th}\right) \quad (18)$$

By using Eqs. (16) and (17), the outage probability of the relay-assisted FSO system over Malaga channel can be obtained as:

$$\begin{aligned} P_{out\,i,j} &= \Pr\left(RP_T\left(\frac{\eta A_r}{L\lambda}\right)^2 \frac{\sqrt{M\mathrm{Log}_2 M}}{2\sigma_n} h_{i,j} \le SNR_{th}\right) \\ &= \Pr\left(h_{i,j} \le 2\sigma_n \left(\frac{\lambda L_{i,j}}{\eta A_r}\right)^2 \frac{SNR_{th}}{RP_T\sqrt{M\mathrm{Log}_2 M}}\right) \end{aligned} \quad (19)$$

$$P_{out,i} = \frac{Ag^2}{2}\sum_{k=1}^{\beta} a_k \left[\frac{1}{B}\right]^{\frac{\alpha+k}{2}} G_{2,4}^{3,1}\left[\frac{2\sigma_n}{BA_o h_{l_{i,j}}}\left(\frac{\lambda L_{i,j}}{\eta A_r}\right)^2 \frac{SNR_{th}}{RP_T\sqrt{M\mathrm{Log}_2 M}} \,\middle|\, \begin{matrix} 1, 1+g^2 \\ g^2, \alpha, k, 0 \end{matrix}\right] \quad (20)$$

In parallel relay transmission, the outage performance of each particular path depends on the outage probability of each hop in that concerned path. Therefore, the outage probability of the ith path can be expressed as:

$$P_{out,i} = 1 - \prod_{j=1}^{M}\left[1 - P_{out\,i,j}\right] \quad (21)$$

Therefore, the end-to-end outage probability can be derived as;

$$P_{out} = \prod_{i=1}^{N} P_{out\,i} = \prod_{i=1}^{N}\left\{1 - \prod_{j=1}^{M}\left[1 - P_{out\,i,j}\right]\right\} \quad (22)$$

The end-to-end outage probability for the MPPM-based FSO over the M channel can also be expressed as:

$$P_{out,MPPM} = \prod_{i=1}^{N}\left\{1 - \prod_{j=1}^{M}\left[1 - \frac{Ag^2}{2}\sum_{k=1}^{\beta} a_k\left[\frac{1}{B}\right]^{\frac{\alpha+k}{2}} G_{2,4}^{3,1}\left[\frac{2\sigma_n}{BA_o h_{l_{i,j}}}\left(\frac{\lambda L_{i,j}}{\eta A_r}\right)^2 \frac{SNR_{th}}{RP_T\sqrt{M\mathrm{Log}_2 M}} \,\middle|\, \begin{matrix} 1, 1+g^2 \\ g^2, \alpha, k, 0 \end{matrix}\right]\right]\right\} \quad (23)$$

Asymptotic analysis

We investigate the asymptotic performance of the parallel relay-assisted free-space optical communication system over the generalized atmospheric turbulence channel considering pointing errors also. This we do for a better insight into Eq. (23) for high values of the average SNR. Assuming that $p \le q$ and $z \to 0$ the Meijer's G function can be expressed in series form as Eq. (07.34.06.0006.01) [31]:

$$\begin{aligned} G_{p,q}^{m,n}\left[z \,\middle|\, \begin{matrix} a_1, \ldots a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots b_m, b_{m+1}, \ldots, b_q \end{matrix}\right] &= \sum_{k=1}^{m} \frac{\prod_{j=1, j\ne k}^{m} \Gamma\left(b_j - b_k\right) \prod_{j=1}^{n} \Gamma\left(1 - a_j + b_k\right)}{\prod_{j=n+1}^{p} \Gamma\left(a_j - b_k\right) \prod_{j=m+1}^{q} \Gamma\left(1 - b_j + b_k\right)} z^{b_k} \\ &\quad (1 + O(z)). \end{aligned} \quad (24)$$

Using the above result, Eq. (23) can be written as:

$$P_{out,MPPM} = \prod_{i=1}^{N}\left\{1 - \prod_{j=1}^{M}\left[1 - \frac{Ag^2}{2}\sum_{k=1}^{\beta} a_k\left[\frac{1}{B}\right]^{\frac{\alpha+k}{2}}\left(\sum_{k=1}^{3} \frac{\prod_{j=1, j\ne k}^{3}\Gamma\left(b_j - b_k\right)\Gamma(1 - a_1 + b_k)}{\Gamma(a_2 - b_k)\Gamma(1 - b_4 + b_k)} z^{b_k}(1 + O(z))\right)\right]\right\} \quad (25)$$

where $z = \frac{2\sigma_n}{BA_o h_{l_{i,j}}}\left(\frac{\lambda L_{i,j}}{\eta A_r}\right)^2 \frac{SNR_{th}}{RP_T\sqrt{M\mathrm{Log}_2 M}}$.

Results and discussion

In this section, we present the results for the outage probability performance of the parallel relay FSO system using the MPPM signaling over the M channel. In each relay path, the consecutive nodes are assumed equidistant along

Table 1 System parameters

Parameter	Symbol	Value
PD responsivity	R	0.5 A/W [14]
Wavelength	λ	1550 nm [10]
Attenuation coefficient	δ	0.1 dB/km [32]
Transmitter optics efficiency	η	0.8 [24]
Receiver optics efficiency	η	0.8 [24]
Link distance	H	1 km

the path from the source to the destination. Key system parameters used in the analysis are shown in Table 1.

Figure 3 presents the variation of the outage probability versus threshold SNR for the direct transmission and end-to-end transition via relays for the case of weak $(C_n^2 = 8.4 \times 10^{-15})$, moderate $(C_n^2 = 1.7 \times 10^{-14})$ and strong $\left(C_n^2 = 5 \times 10^{-14}\right)$ atmospheric turbulence conditions. Here we have considered three parallel relay paths and two hops per path. It is observed that there is a remarkable improvement in the outage probability when relays are used in parallel configuration than when only direct single path transmission without relays is considered. Obviously, better outage probability performance is observed in weak turbulence than in strong turbulence.

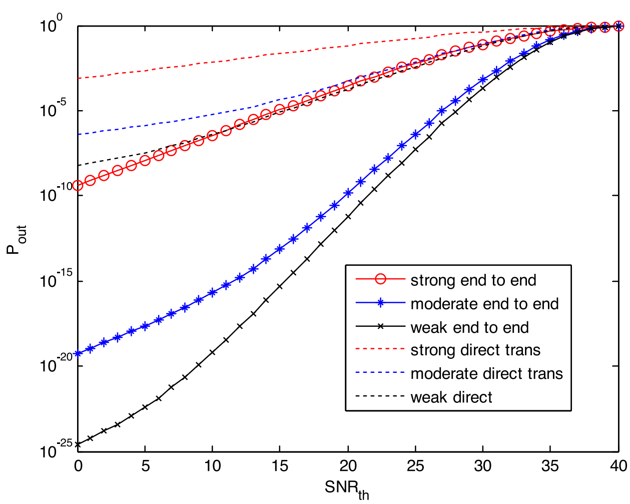

Fig. 3 Outage probability versus threshold SNR for the MPPM scheme

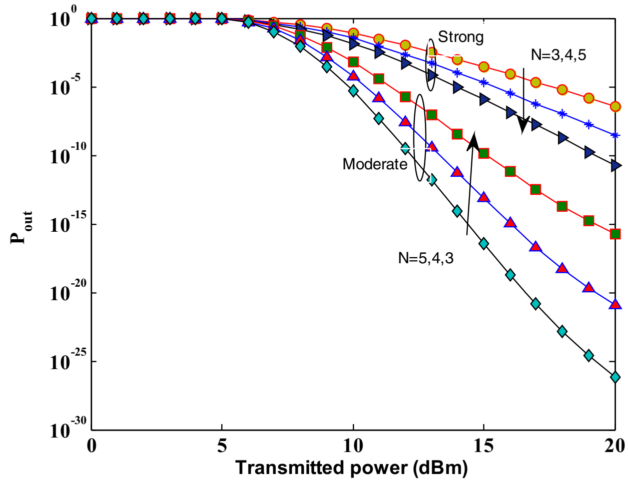

Fig. 4 Outage probability versus transmitted power

Moreover, the difference in the performance is more between the strong turbulence and moderate turbulence than between the moderate turbulence and weak turbulence. When $\mathrm{SNR_{th}} = 10\,\mathrm{dB}$, $P_{\mathrm{out}} = 6.23 \times 10^{-20}, 1.93 \times 10^{-16}, 3.96 \times 10^{-7}$ for weak, moderate and strong turbulence, respectively, when the transmission is through the parallel relays. The outage probability deteriorates to $P_{\mathrm{out}} = 3.965 \times 10^{-7}, 5.783 \times 10^{-6}, 7.156 \times 10^{-3}$ for weak, moderate and strong turbulence, respectively, when there is direct transmission between the source and the destination. These results are further summarized in Table 2.

The outage probability versus transmitted power for the MPPM FSO for direct and relay transmission systems in moderate and strong turbulence conditions is shown in Fig. 4. The outage probability is analyzed for 3, 4 and 5 relay paths with 2 relays in each path. From the figure, the outage probability decreases as the transmit power is increased from 0 dBm. Generally, for a given number of relay paths used, the outage probability is better in moderate turbulence than strong turbulence. For a fixed average transmit power, the outage probability will be lower if a greater number of relay nodes is used. For example, in moderate atmospheric turbulence, for a fixed transmit power of $10\,\mathrm{dBm}$, $P_{\mathrm{out}} = 6.47 \times 10^{-4}, 5.59 \times 10^{-5}$ and 4.84×10^{-6} for number of relay node = 3, 4 and 5, respectively. However, the outage probability is degraded

Table 2 Outage probability comparison for direct and relay-based transmission

Turbulence regime	Outage probability at $\mathrm{SNR_{th}} = 10\,\mathrm{dB}$	
	Direct transmission	Relay transmission (N = 3, M = 2)
Weak	3.96×10^{-7}	6.23×10^{-20}
moderate	5.78×10^{-6}	1.93×10^{-16}
Strong	7.15×10^{-3}	3.96×10^{-7}

Table 3 Comparison of the outage probability for different number of nodes and transmit power

Number of relay nodes (N)	Outage probability at $P_t = 10$ dBm		Outage probability at $P_t = 15$ dBm	
	Moderate turbulence	Strong turbulence	Moderate turbulence	Strong turbulence
3	6.47×10^{-4}	7.67×10^{-2}	1.41×10^{-10}	2.84×10^{-4}
4	5.59×10^{-5}	3.25×10^{-2}	7.35×10^{-14}	1.89×10^{-5}
5	4.84×10^{-6}	1.36×10^{-2}	3.83×10^{-17}	1.23×10^{-6}

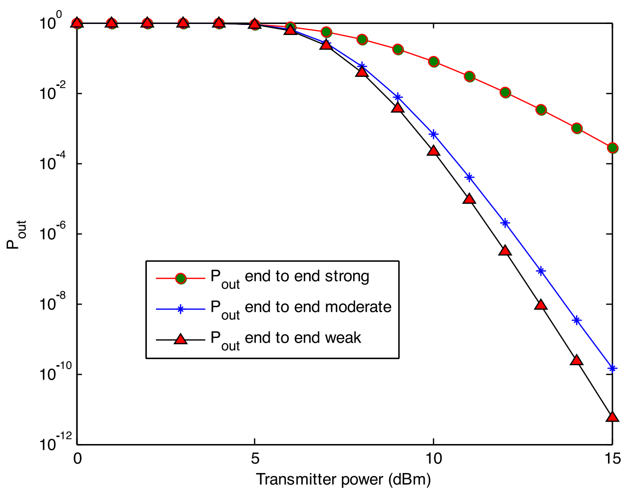

Fig. 5 End-to-end outage probability versus transmitter power in different turbulence strength

in strong turbulence. Further, by increasing the transmit power to 15 dBm, there is a remarkable improvement in the outage probability. These results are further clarified in Table 3.

Figure 5 is a plot of the end-to-end outage probability for the parallel relay-assisted FSO system based on MPPM over Malaga channel. We have considered the case of weak, moderate and strong atmospheric turbulence conditions. As noted earlier, increase in transmit power improves the outage probability, and the end-to-end outage probability is more favorable in weak turbulence than moderate or strong turbulence. For an applied transmit power of less than 5 dBm, there is no appreciable change in the end-to-end outage probability. However, when the transmit power is increased beyond 5 dBm, there is an abrupt reduction in the end-to-end outage probability across all turbulence regimes. For a fixed transmit power, the margin in the outage probability between the weak and moderate turbulence is small compared to the weak to strong turbulence margin. As an example, for an applied transmit power of 10 dBm, the end-to-end outage probability is $P_{\mathrm{out}} = 2.13 \times 10^{-4}$, 6.47×10^{-4} and 7.67×10^{-2} in weak, moderate and strong atmospheric turbulence, respectively.

The variation of the outage probability with transmit power for the case of weak turbulence and different number of relay nodes has been depicted in Fig. 6. Although there is a remarkable improvement in the outage probability when transmit power is increased, such an improvement is only noticed when the transmit power is at

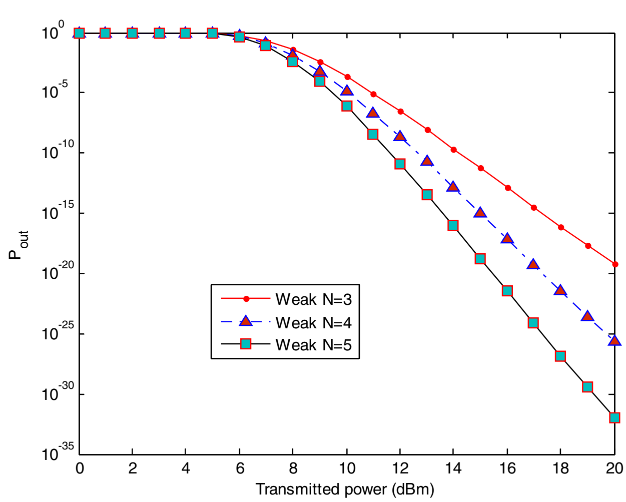

Fig. 6 Outage probability versus transmit power for different number of relay nodes with MPPM in weak turbulence

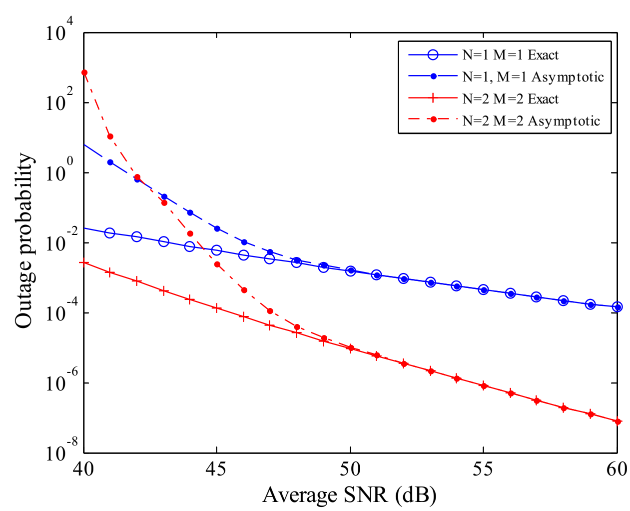

Fig. 7 Exact and asymptotic outage probability performance versus average electrical SNR (dB)

least 5 dBm. As the number of rely nodes per path is increased from 3 to 5, there is a corresponding improvement in the outage probability. As an example, when the transmitter power is 10 dBm, the outage probability is $P_{\text{out}} = 2.13 \times 10^{-4}, 1.27 \times 10^{-5}$ and 7.60×10^{-7} for $N = 3$, 4 and 5, respectively. However, when the transmitter power is increased to 15 dBm, the outage probability improves to $P_{\text{out}} = 5.79 \times 10^{-12}, 1.04 \times 10^{-15}$ and 1.87×10^{-19} for $N = 3$, 4 and 5, respectively. In general, these results indicate much more benefits of increasing the number of relay paths when the turbulence is adverse.

For a better insight into the performance of FSO at high SNR values, the exact and asymptotic outage probability versus average electrical SNR has been plotted in Fig. 7. We have considered a typical example for weak atmospheric turbulence scenarios for $N = 1$ $M = 1$ and $N = 2$ and $M = 2$. It can be observed that for all cases, regardless of the number of relays and relay paths, the for average SNR values above 45 dB, the asymptotic performance curve tends to meet with that of the exact outage probability curve.

Conclusion

In this article, the outage probability of the parallel relay-assisted FSO employing multi-pulse position signaling over the Malaga aggregated channel has been investigated. Considering atmospheric turbulence and misalignment errors, closed-form as well as asymptotic expressions for the outage probability has been derived and later on used for the analysis of the dependence of the outage probability on different factors such as the number of relay nodes, the transmitter power and the threshold signal to noise ratio. After comparing the outage performance using direct transmission, it has been observed that the use of parallel relay configuration with increased number of relay nodes results in a considerable improvement in the outage probability and hence enhances the overall performance of the FSO communication system.

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Declarations

Conflict of interest The authors declares that there are no conflicts of interest.

Code availability The code and data used is available upon request from the corresponding author.

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