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RESEARCH ARTICLE

Cubic–quartic optical soliton perturbation with Lakshmanan–Porsezian–Daniel model by sine-Gordon equation approach

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Abstract This paper recovers cubic–quartic optical solitons for perturbed Lakshmanan–Porsezian–Daniel model. This is for both with and without polarization. The sine-Gordon equation approach is the scheme adopted to retrieve the soliton solutions. A full spectrum of soliton solutions have emerged.

Keywords Solitons · Cubic–quartic · Maximum intensity

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Introduction

The most fundamental feature that sustains soliton formation and its stability is the existence of a delicate balance between chromatic dispersion (CD) and self-phase modulation (SPM). Once this balance is compromised, solitons fail to exist. Therefore, it is imperative to maintain the perfect harmony between the two key elements, CD and SPM. Occasionally, it may so happen that CD runs low. In such a situation, a new form of technology has been proposed. When CD was replaced by fourth-order dispersion (4OD), pure-quartic solitons were studied. However, for such solitons there is a limitation. No analytical closed-form soliton solution was available unless it is a stationary soliton. To overcome such shortcomings, cubic–quartic (CQ) solitons were proposed where, in addition to 4OD, third-order dispersion (3OD) effect was included. An abundance of results from CQ solitons have emerged and successfully floated across telecommunications industry, at least theoretically. Subsequently, the concept of CQ solitons, during such a crisis situation, was applied to a variety of other models that describe successful soliton transmission through optical fibers across trans-continental distances.

One of the models that describe soliton transmission through a variety of waveguides is perturbed Lakshmanan–Porsezian–Daniel (LPD) model, where the perturbation terms are all of Hamiltonian type [1–25]. This model first appeared during 1988 in the context of Heisenberg spin chain [18]. Later, it gained popularity in soliton studies and has been extensively studied in various contexts using a wide variety of mathematical methodologies. These include rogue waves, Darboux transform, collective variables, Adomian decomposition, semi-inverse variational principle, Jacobi's elliptic function expansion, exponential function expansion, trial equation approach, method of undetermined coefficients, Riccati equation approach and so on. The model was successfully studied in both contexts, namely polarization-preserving fibers as well as birefringent fibers. Today's paper will be addressing CQ solitons with perturbed LPD model, in both such forms of fibers, for the very first time. Sine-Gordon equation approach will be the integration scheme that is adopted here. This will reveal a spectrum of soliton solutions in both such fibers that are exhibited along with their respective existence criteria. The algorithm is illustrated and the extensive details of the mathematical phenomenon follow through.

Optical solitons

The retrieval of soliton solutions will be split into two subsections that deal with polarization-free fibers and when pulse polarization occurs. In both of the subsections, sine-Gordon equation approach will be employed to recover a wide spectrum of solitons. The details are exhibited in the next subsections.

Polarization-preserving fibers

The LPD equation in polarization-preserving fibers is:

$$\begin{aligned} iu_t + iau_{xxx} + bu_{xxxx} + c|u|^2u = \alpha(u_x)^2u^* \\ + \beta|u_x|^2u + \gamma|u|^2u_{xx} + \lambda u^2u^*_{xx} \\ + \delta|u|^4u \end{aligned} \tag{1}$$

where x and t that, respectively, stand for the spatial and temporal variables are independent variables. Linear temporal evolution is stood for the first term and $i = \sqrt{-1}$. The complex valued function $u(x,t)$ represents optical solitons in polarization-preserving fibers. a gives the coefficient of 3OD, while b is the coefficient of 4OD. Next, c stands for the coefficient of SPM and δ corresponds to two-photon absorption. Lastly, α, β, γ and λ are the coefficients of the nonlinear dispersion terms.

In presence of perturbation terms, the LPD equation for polarization-preserving fibers is:

$$\begin{aligned} iu_t + iau_{xxx} + bu_{xxxx} + c|u|^2u \\ = \alpha(u_x)^2u^* + \beta|u_x|^2u \\ + \gamma|u|^2u_{xx} + \lambda u^2u^*_{xx} + \delta|u|^4u \\ + i\left[\zeta\left(|u|^{2m}u\right)_x + \mu\left(|u|^{2m}\right)_x u \right. \\ \left. + \rho|u|^{2m}u_x\right] \end{aligned} \tag{2}$$

where ζ represents the self-steepening term, while μ and ρ are, respectively, the coefficients of the higher-order dispersion and nonlinear dispersion effects. Finally, full nonlinearity parameter is indicated by m.

To obtain the soliton solution, we set

$$u(x,t) = U(\xi)e^{i\varphi}, \quad \xi = x - vt, \quad \varphi(x,t) = -\kappa x + \omega t + \theta_0 \tag{3}$$

where the speed is denoted by v, while the frequency, wave number and phase center are stood for by κ, ω and θ_0, respectively. Also, the soliton amplitude and its phase component are, respectively, represented by $U(\xi)$ and $\varphi(x,t)$.

Plugging Eq. (3) into Eq. (2) yields the real equation

$$\begin{aligned} bU^{(iv)} + \left(3a\kappa - 6b\kappa^2\right)U'' - (\lambda+\gamma)U^2U'' \\ - (\alpha+\beta)(U')^2U + \left(c + \alpha\kappa^2 + \gamma\kappa^2 + \lambda\kappa^2 - \beta\kappa^2\right)U^3 \\ + \left(b\kappa^4 - a\kappa^3 - \omega\right)U \\ - \delta U^5 - (\kappa\zeta + \kappa\rho)U^{2m+1} = 0 \end{aligned} \tag{4}$$

and the imaginary equation

$$\begin{aligned} (a - 4b\kappa)U''' + \left(4b\kappa^3 - 3a\kappa^2 - v\right)U' \\ + 2\kappa(\gamma + \alpha - \lambda)U'U^2 \\ - (\rho + \zeta + 2m\mu + 2m\zeta)U'U^{2m} = 0. \end{aligned} \tag{5}$$

Equations (4) and (5) are reduced to

$$\begin{aligned} bU^{(iv)} + 6b\kappa^2U'' - \left(3b\kappa^4 + \omega\right)U \\ + \left(c - 4\gamma\kappa^2 + 2\kappa\mu + 2\kappa\zeta\right)U^3 \\ - \delta U^5 = 0 \end{aligned} \tag{6}$$

with

$$m = 1, \tag{7}$$

$$a = 4b\kappa, \tag{8}$$

$$\alpha = -2\gamma, \tag{9}$$

$$\rho = -2\mu - 3\zeta, \tag{10}$$

$$\lambda = -\gamma, \tag{11}$$

$$\beta = 2\gamma, \tag{12}$$

$$v = -8b\kappa^3. \tag{13}$$

Equation (6) can be integrated in order to designate the soliton profile, while Eq. (13) yields the soliton speed and Eqs. (7)–(12) gives the constraints.

The sine-Gordon equation method admits that the solution form of Eq. (6) is:

$$U(\xi)=\sum_{i=1}^{N}\cos^{i-1}(V(\xi))[B_i\sin(V(\xi)) + A_i\cos(V(\xi))]+A_0 \tag{14}$$

where A_i and B_i $(0\le i\le N)$ are constants, N represents the balance number and $V(\xi)$ ensures

$$V'(\xi)=\sin(V(\xi)) \tag{15}$$

with

$$\begin{aligned}&\sin(V(\xi))=\ \mathrm{sech}\ (\xi)\ \text{or}\ \sin(V(\xi))=\mathrm{i\ csh}\ (\xi),\\ &\cos(V(\xi))=\tanh(\xi),\quad \text{or}\ \cos(V(\xi))=\coth(\xi).\end{aligned} \tag{16}$$

By virtue of the balancing principle applied in Eq. (6), Eq. (14) becomes

$$U(\xi)=B_1\sin(V(\xi))+A_1\cos(V(\xi))+A_0. \tag{17}$$

Putting Eq. (17) with Eq. (15) into Eq. (6) leads to

Case-1

$$\begin{aligned}\delta&=\frac{3(4\gamma\kappa^2-c-2\kappa\mu-2\kappa\zeta)^2}{2b(3\kappa^2-10)^2},\quad \omega\\ &=-b\left(3\kappa^4+12\kappa^2-16\right),\quad A_0=0,\\ A_1&=\pm\sqrt{-\frac{4b(3\kappa^2-10)}{c-4\gamma\kappa^2+2\kappa(\mu+\zeta)}},\quad B_1=0.\end{aligned} \tag{18}$$

Plugging Eq. (18) with Eq. (16) into Eq. (17) leads to the dark soliton

$$\begin{aligned}u(x,t)=\pm&\sqrt{-\frac{4b(3\kappa^2-10)}{c-4\gamma\kappa^2+2\kappa(\mu+\zeta)}}\\ &\tanh\left(x+8b\kappa^3t\right)e^{i\left(-\kappa x-b\left(3\kappa^4+12\kappa^2-16\right)t+\theta_0\right)}\end{aligned} \tag{19}$$

and the singular soliton

$$\begin{aligned}u(x,t)=\pm&\sqrt{-\frac{4b(3\kappa^2-10)}{c-4\gamma\kappa^2+2\kappa(\mu+\zeta)}}\\ &\coth\left(x+8b\kappa^3t\right)e^{i\left(-\kappa x-b\left(3\kappa^4+12\kappa^2-16\right)t+\theta_0\right)}.\end{aligned} \tag{20}$$

These solitons are valid for

$$b\left(3\kappa^2-10\right)\left(c-4\gamma\kappa^2+2\kappa(\mu+\zeta)\right) <0. \tag{21}$$

Case-2

$$\begin{aligned}\delta&=\frac{3(4\gamma\kappa^2-c-2\kappa\mu-2\kappa\zeta)^2}{2b(3\kappa^2+5)^2},\quad \omega\\ &=-b\left(3\kappa^4-6\kappa^2-1\right),\quad A_0=0,\\ A_1&=0,\quad B_1=\pm\sqrt{\frac{4b(3\kappa^2+5)}{c-4\gamma\kappa^2+2\kappa(\mu+\zeta)}}.\end{aligned} \tag{22}$$

Inserting Eq. (22) with Eq. (16) into Eq. (17) causes to the bright soliton

$$\begin{aligned}u(x,t)=\pm&\sqrt{\frac{4b(3\kappa^2+5)}{c-4\gamma\kappa^2+2\kappa(\mu+\zeta)}}\\ &\mathrm{sech}\ \left(x+8b\kappa^3t\right)e^{i\left(-\kappa x-b\left(3\kappa^4-6\kappa^2-1\right)t+\theta_0\right)}\end{aligned} \tag{23}$$

and the singular soliton

$$\begin{aligned}u(x,t)=\pm&\sqrt{-\frac{4b(3\kappa^2+5)}{c-4\gamma\kappa^2+2\kappa(\mu+\zeta)}}\\ &\mathrm{csch}\ \left(x+8b\kappa^3t\right)e^{i\left(-\kappa x-b\left(3\kappa^4-6\kappa^2-1\right)t+\theta_0\right)}.\end{aligned} \tag{24}$$

The bright soliton will exist provided the following constraint condition remains valid:

$$b\left(3\kappa^2+5\right)\left(c-4\gamma\kappa^2+2\kappa(\mu+\zeta)\right)>0. \tag{25}$$

while the singular soliton will exist provided the condition (26) holds:

$$b\left(3\kappa^2+5\right)\left(c-4\gamma\kappa^2+2\kappa(\mu+\zeta)\right)<0. \tag{26}$$

Case-3

$$\begin{aligned}\delta&=\frac{6(4\gamma\kappa^2-c-2\kappa\mu-2\kappa\zeta)^2}{b(6\kappa^2-5)^2},\quad \omega\\ &=-b\left(3\kappa^4+3\kappa^2-1\right),\quad A_0=0,\\ A_1&=\pm\sqrt{-\frac{b(6\kappa^2-5)}{2(c-4\gamma\kappa^2+2\kappa(\mu+\zeta))}},\quad B_1\\ &=\pm\sqrt{\frac{b(6\kappa^2-5)}{2(c-4\gamma\kappa^2+2\kappa(\mu+\zeta))}}.\end{aligned} \tag{27}$$

Inserting Eq. (27) with Eq. (16) into Eq. (17) yields the combo singular soliton

$$u(x,t) = \left\{ \begin{array}{l} \pm\sqrt{-\dfrac{b(6\kappa^2-5)}{2(c-4\gamma\kappa^2+2\kappa(\mu+\zeta))}}\coth(x+8b\kappa^3 t) \\ \pm\sqrt{-\dfrac{b(6\kappa^2-5)}{2(c-4\gamma\kappa^2+2\kappa(\mu+\zeta))}}\,\text{csch}\,(x+8b\kappa^3 t) \end{array} \right\} e^{i\left(-\kappa x-b\left(3\kappa^4+3\kappa^2-1\right)t+\theta_0\right)}. \tag{28}$$

The combo singular soliton is valid for

$$b\left(6\kappa^2-5\right)\left(c-4\gamma\kappa^2+2\kappa(\mu+\zeta)\right)<0. \tag{29}$$

Birefringent fibers

The coupled system derived from the equation (2) for birefringent fibers without four-wave mixing is:

$$\begin{aligned} &iq_t+ia_1q_{xxx}+b_1q_{xxxx}+\left(c_1|q|^2+d_1|r|^2\right)q \\ &=\left\{\alpha_1(q_x)^2+\beta_1(r_x)^2\right\}q^* \\ &+\left(\gamma_1|q_x|^2+\lambda_1|r_x|^2\right)q+\left(\delta_1|q|^2+\zeta_1|r|^2\right)q_{xx} \\ &+\left(\mu_1q^2+\rho_1r^2\right)q^*_{xx} \\ &+\left(f_1|q|^4+g_1|q|^2|r|^2+h_1|r|^4\right)q \\ &+i\Big[\eta_1\left(|q|^2q\right)_x+\vartheta_1\left(|r|^2r\right)_x \\ &+\left\{\theta_1\left(|q|^2\right)_x+\epsilon_1\left(|r|^2\right)_x\right\}q \\ &+\left(\tau_1|q|^2+\sigma_1|r|^2\right)q_x\Big], \end{aligned} \tag{30}$$

$$\begin{aligned} &ir_t+ia_2r_{xxx}+b_2r_{xxxx}+\left(c_2|r|^2+d_2|q|^2\right)r \\ &=\left\{\alpha_2(r_x)^2+\beta_2(q_x)^2\right\}r^* \\ &+\left(\gamma_2|r_x|^2+\lambda_2|q_x|^2\right)r+\left(\delta_2|r|^2+\zeta_2|q|^2\right)r_{xx} \\ &+\left(\mu_2r^2+\rho_2q^2\right)r^*_{xx} \\ &+\left(f_2|r|^4+g_2|r|^2|q|^2+h_2|q|^4\right)r \\ &+i\Big[\eta_2\left(|r|^2r\right)_x+\vartheta_2\left(|q|^2q\right)_x \\ &+\left\{\theta_2\left(|r|^2\right)_x+\epsilon_2\left(|q|^2\right)_x\right\}r \\ &+\left(\tau_2|r|^2+\sigma_2|q|^2\right)r_x\Big] \end{aligned} \tag{31}$$

where the complex valued functions $q(x,t)$ and $r(x,t)$ account for optical solitons in birefringent fibers. For $l=1,2$, a_l are the coefficients of 3OD, while b_l represent the coefficients of 4OD. Then, c_l and f_l stands for the coefficients of SPM, while d_l, g_l and h_l account for the coefficients of cross-phase modulation. Lastly, α_l, β_l, γ_l, λ_l, δ_l, ζ_l, μ_l, ρ_l, η_l, ϑ_l, θ_l, ε_l, τ_l and σ_l are the coefficients of the nonlinear dispersion terms.

To look for soliton solution to the governing model, the solution hypothesis picked is:

$$q(x,t)=U_1(\xi)e^{i\varphi},\quad r(x,t)=U_2(\xi)e^{i\varphi},\quad \xi=x-vt,\quad \varphi(x,t)=-\kappa x+\omega t+\theta_0. \tag{32}$$

Inserting Eq. (32) into Eqs. (30) and (31) yields the real equation

$$\begin{aligned} &b_lU_l^{(iv)}+\left(3\kappa a_l-6\kappa^2b_l\right)U_l'' \\ &-(\rho_l+\zeta_l)U_{\bar{l}}^2U_l'' \\ &-(\delta_l+\mu_l)U_l^2U_l'' \\ &-(\alpha_l+\gamma_l)\left(U_l'\right)^2U_l \\ &-(\beta_l+\lambda_l)U_l\left(U_{\bar{l}}'\right)^2+\left(\kappa^2\beta_l-\kappa^2\lambda_l\right. \\ &\left.+\kappa^2\zeta_l+\kappa^2\rho_l+d_l-\kappa\sigma_l\right)U_lU_{\bar{l}}^2 \\ &+\left(\kappa^4b_l-\kappa^3a_l-\omega\right)U_l \\ &+\left(c_l+\kappa^2\delta_l+\kappa^2\alpha_l+\kappa^2\mu_l\right. \\ &\left.-\kappa^2\gamma_l-\kappa\eta_l-\kappa\tau_l\right)U_l^3-\kappa\vartheta_lU_{\bar{l}}^3-h_lU_lU_{\bar{l}}^4 \\ &-g_lU_l^3U_{\bar{l}}^2-f_lU_l^5=0 \end{aligned} \tag{33}$$

and the imaginary equation

$$\begin{aligned} &(a_l-4\kappa b_l)U_l'''+\left(4\kappa^3b_l-3\kappa^2a_l-v\right)U_l' \\ &+(2\kappa\zeta_l-2\kappa\rho_l-\sigma_l)U_l'U_{\bar{l}}^2 \\ &+(2\kappa\alpha_l+2\kappa\delta_l-2\kappa\mu_l-3\eta_l-\tau_l-2\theta_l)U_l'U_l^2 \\ &+(2\kappa\beta_l-2\epsilon_l)U_lU_{\bar{l}}'U_{\bar{l}}-3\vartheta_lU_{\bar{l}}'U_{\bar{l}}^2=0 \end{aligned} \tag{34}$$

where $l=1,2$ and $\bar{l}=3-l$. Eqs. (33) and (34) reduce to

$$\begin{aligned} &b_lU_l^{(iv)}+6\kappa^2b_lU_l''-\left(3\kappa^4b_l+\omega\right)U_l \\ &+\left(d_l+c_l-4(\delta_l+\zeta_l)\kappa^2\right. \\ &\left.+2\kappa(\epsilon_l+\theta_l+\eta_l+\vartheta_l)\right)U_l^3 \\ &-(h_l+g_l+f_l)U_l^5=0 \end{aligned} \tag{35}$$

with

$$U_{\bar{l}}=U_l, \tag{36}$$

$$a_l=4\kappa b_l, \tag{37}$$

$$\alpha_l=-\beta_l-2\delta_l-2\zeta_l+\frac{\tau_l+\sigma_l+2\epsilon_l+2\theta_l+3\eta_l+3\vartheta_l}{2\kappa}, \tag{38}$$

$$\gamma_l=-\lambda_l+2\delta_l+2\zeta_l-\frac{\tau_l+\sigma_l+2\epsilon_l+2\theta_l+3\eta_l+3\vartheta_l}{2\kappa}, \tag{39}$$

$$\mu_l=-\rho_l-\delta_l-\zeta_l, \tag{40}$$

$$v = -8\kappa^3 b_l. \tag{41}$$

Utilizing the balancing principle in Eq. (35), Eq. (14) changes to

$$U_l(\xi) = B_1 \sin(V_l(\xi)) + A_1 \cos(V_l(\xi)) + A_0. \tag{42}$$

Substituting Eq. (42) with Eq. (15) into Eq. (35) leads to

Case-1

$$\omega = -b_l\left(3\kappa^4 + 12\kappa^2 - 16\right), \quad A_0 = 0, \quad B_1 = 0,$$

$$A_1 = \pm\sqrt{-\frac{4b_l(3\kappa^2 - 10)}{c_l + d_l - 4\kappa^2(\delta_l + \zeta_l) + 2\kappa(\epsilon_l + \eta_l + \theta_l + \vartheta_l)}},$$

$$\begin{aligned} f_l = &-\frac{1}{2b_l(3\kappa^2 - 10)^2}\left(-3c_l^2 - 3d_l^2 - 12\kappa^2\vartheta_l^2 - 6c_l d_l\right.\\ &- 48\kappa^4\delta_l^2 - 48\kappa^4\zeta_l^2 - 12\kappa^2\epsilon_l^2\\ &- 12\kappa^2\eta_l^2 - 12\kappa^2\theta_l^2 + 200b_l g_l + 200b_l h_l + 18\kappa^4 b_l g_l\\ &+ 18\kappa^4 b_l h_l - 120\kappa^2 b_l g_l - 120\kappa^2 b_l h_l\\ &- 12\kappa c_l\vartheta_l - 12\kappa d_l\epsilon_l - 12\kappa d_l\eta_l - 12\kappa d_l\theta_l - 12\kappa d_l\vartheta_l\\ &- 96\kappa^4\delta_l\zeta_l + 48\kappa^3\delta_l\epsilon_l + 48\kappa^3\delta_l\eta_l\\ &+ 48\kappa^3\delta_l\theta_l + 48\kappa^3\delta_l\vartheta_l + 48\kappa^3\epsilon_l\zeta_l\\ &+ 48\kappa^3\eta_l\zeta_l + 48\kappa^3\theta_l\zeta_l + 48\kappa^3\zeta_l\vartheta_l + 24\kappa^2 c_l\delta_l + 24\kappa^2 c_l\zeta_l\\ &+ 24\kappa^2 d_l\delta_l + 24\kappa^2 d_l\zeta_l - 24\kappa^2\epsilon_l\eta_l - 24\kappa^2\epsilon_l\theta_l\\ &- 24\kappa^2\epsilon_l\vartheta_l - 24\kappa^2\eta_l\theta_l\\ &- 24\kappa^2\eta_l\vartheta_l - 24\kappa^2\theta_l\vartheta_l\\ &\left.- 12\kappa c_l\epsilon_l - 12\kappa c_l\eta_l - 12\kappa c_l\theta_l\right). \end{aligned} \tag{43}$$

Plugging Eq. (43) with Eq. (16) into Eq. (42) leads to the dark solitons

$$\begin{aligned} q(x,t) &= \pm\sqrt{-\frac{4b_1(3\kappa^2 - 10)}{c_1 + d_1 - 4\kappa^2(\delta_1 + \zeta_1) + 2\kappa(\epsilon_1 + \eta_1 + \theta_1 + \vartheta_1)}}\\ &\tanh\left(x + 8b_1\kappa^3 t\right)e^{i\left(-\kappa x - b_1\left(3\kappa^4 + 12\kappa^2 - 16\right)t + \theta_0\right)}, \end{aligned} \tag{44}$$

$$\begin{aligned} r(x,t) &= \pm\sqrt{-\frac{4b_2(3\kappa^2 - 10)}{c_2 + d_2 - 4\kappa^2(\delta_2 + \zeta_2) + 2\kappa(\epsilon_2 + \eta_2 + \theta_2 + \vartheta_2)}}\\ &\tanh\left(x + 8b_2\kappa^3 t\right)e^{i\left(-\kappa x - b_2\left(3\kappa^4 + 12\kappa^2 - 16\right)t + \theta_0\right)} \end{aligned} \tag{45}$$

and the singular solitons

$$\begin{aligned} q(x,t) &= \pm\sqrt{-\frac{4b_1(3\kappa^2 - 10)}{c_1 + d_1 - 4\kappa^2(\delta_1 + \zeta_1) + 2\kappa(\epsilon_1 + \eta_1 + \theta_1 + \vartheta_1)}}\\ &\coth\left(x + 8b_1\kappa^3 t\right)e^{i\left(-\kappa x - b_1\left(3\kappa^4 + 12\kappa^2 - 16\right)t + \theta_0\right)}, \end{aligned} \tag{46}$$

$$\begin{aligned} r(x,t) &= \pm\sqrt{-\frac{4b_2(3\kappa^2 - 10)}{c_2 + d_2 - 4\kappa^2(\delta_2 + \zeta_2) + 2\kappa(\epsilon_2 + \eta_2 + \theta_2 + \vartheta_2)}}\\ &\tanh\left(x + 8b_2\kappa^3 t\right)e^{i\left(-\kappa x - b_2\left(3\kappa^4 + 12\kappa^2 - 16\right)t + \theta_0\right)}. \end{aligned} \tag{47}$$

These solitons are valid for

$$b_l\left(3\kappa^2 - 10\right)\left(c_l + d_l - 4\kappa^2(\delta_l + \zeta_l) + 2\kappa(\epsilon_l + \eta_l + \theta_l + \vartheta_l)\right) < 0. \tag{48}$$

Case-2

$$\omega = -b_l\left(3\kappa^4 - 6\kappa^2 - 1\right), \quad A_0 = 0, \quad A_1 = 0,$$

$$B_1 = \pm\sqrt{\frac{4b_l(3\kappa^2 + 5)}{c_l + d_l - 4\kappa^2(\delta_l + \zeta_l) + 2\kappa(\epsilon_l + \eta_l + \theta_l + \vartheta_l)}},$$

$$\begin{aligned} f_l = &-\frac{1}{2b_l(3\kappa^2 + 5)^2}\left(-3c_l^2 - 3d_l^2 - 12\kappa^2\vartheta_l^2 - 6c_l d_l\right.\\ &- 48\kappa^4\delta_l^2 - 48\kappa^4\zeta_l^2 - 12\kappa^2\epsilon_l^2\\ &- 12\kappa^2\eta_l^2 - 12\kappa^2\theta_l^2 + 50b_l g_l + 50b_l h_l + 18\kappa^4 b_l g_l\\ &+ 18\kappa^4 b_l h_l + 60\kappa^2 b_l g_l + 60\kappa^2 b_l h_l\\ &- 12\kappa c_l\vartheta_l - 12\kappa d_l\epsilon_l - 12\kappa d_l\eta_l - 12\kappa d_l\theta_l - 12\kappa d_l\vartheta_l\\ &- 96\kappa^4\delta_l\zeta_l + 48\kappa^3\delta_l\epsilon_l + 48\kappa^3\delta_l\eta_l\\ &+ 48\kappa^3\delta_l\theta_l + 48\kappa^3\delta_l\vartheta_l + 48\kappa^3\epsilon_l\zeta_l\\ &+ 48\kappa^3\eta_l\zeta_l + 48\kappa^3\theta_l\zeta_l + 48\kappa^3\zeta_l\vartheta_l + 24\kappa^2 c_l\delta_l\\ &+ 24\kappa^2 c_l\zeta_l + 24\kappa^2 d_l\delta_l + 24\kappa^2 d_l\zeta_l - 24\kappa^2\epsilon_l\eta_l\\ &- 24\kappa^2\epsilon_l\theta_l - 24\kappa^2\epsilon_l\vartheta_l - 24\kappa^2\eta_l\theta_l\\ &- 24\kappa^2\eta_l\vartheta_l - 24\kappa^2\theta_l\vartheta_l\\ &\left.- 12\kappa c_l\epsilon_l - 12\kappa c_l\eta_l - 12\kappa c_l\theta_l\right). \end{aligned} \tag{49}$$

Inserting Eq. (49) with Eq. (16) into Eq. (42) causes to the bright solitons

$$\begin{aligned} q(x,t) &= \pm\sqrt{\frac{4b_1(3\kappa^2 + 5)}{c_1 + d_1 - 4\kappa^2(\delta_1 + \zeta_1) + 2\kappa(\epsilon_1 + \eta_1 + \theta_1 + \vartheta_1)}}\\ &\text{sech}\left(x + 8b_1\kappa^3 t\right)e^{i\left(-\kappa x - b_1\left(3\kappa^4 - 6\kappa^2 - 1\right)t + \theta_0\right)}, \end{aligned} \tag{50}$$

$$r(x,t) = \pm\sqrt{\frac{4b_2(3\kappa^2+5)}{c_2+d_2-4\kappa^2(\delta_2+\zeta_2)+2\kappa(\epsilon_2+\eta_2+\theta_2+\vartheta_2)}} \operatorname{sech}\left(x+8b_2\kappa^3 t\right)e^{i\left(-\kappa x-b_2\left(3\kappa^4-6\kappa^2-1\right)t+\theta_0\right)} \tag{51}$$

and the singular solitons

$$q(x,t) = \pm\sqrt{-\frac{4b_1(3\kappa^2+5)}{c_1+d_1-4\kappa^2(\delta_1+\zeta_1)+2\kappa(\epsilon_1+\eta_1+\theta_1+\vartheta_1)}} \operatorname{csch}\left(x+8b_1\kappa^3 t\right)e^{i\left(-\kappa x-b_1\left(3\kappa^4-6\kappa^2-1\right)t+\theta_0\right)}, \tag{52}$$

$$r(x,t) = \pm\sqrt{-\frac{4b_2(3\kappa^2+5)}{c_2+d_2-4\kappa^2(\delta_2+\zeta_2)+2\kappa(\epsilon_2+\eta_2+\theta_2+\vartheta_2)}} \operatorname{csch}\left(x+8b_2\kappa^3 t\right)e^{i\left(-\kappa x-b_2\left(3\kappa^4-6\kappa^2-1\right)t+\theta_0\right)}. \tag{53}$$

The bright solitons are valid for

$$b_l\left(c_l+d_l-4\kappa^2(\delta_l+\zeta_l)+2\kappa(\epsilon_l+\eta_l+\theta_l+\vartheta_l)\right) > 0 \tag{54}$$

while the singular solitons are valid for

$$b_l\left(c_l+d_l-4\kappa^2(\delta_l+\zeta_l)+2\kappa(\epsilon_l+\eta_l+\theta_l+\vartheta_l)\right) < 0. \tag{55}$$

Case-3

$$\begin{aligned}
&\omega=-b_l\left(3\kappa^4+3\kappa^2-1\right),\ A_0=0,\\
&A_1=\pm\sqrt{-\frac{b_l(6\kappa^2-5)}{2(c_l+d_l-4\kappa^2(\delta_l+\zeta_l)+2\kappa(\epsilon_l+\eta_l+\theta_l+\vartheta_l))}},\\
&B_1=\pm\sqrt{\frac{b_l(6\kappa^2-5)}{2(c_l+d_l-4\kappa^2(\delta_l+\zeta_l)+2\kappa(\epsilon_l+\eta_l+\theta_l+\vartheta_l))}},\\
&f_l=-\frac{1}{b_l(6\kappa^2-5)^2}(-6{c_l}^2-6{d_l}^2-24\kappa^2{\vartheta_l}^2-12c_ld_l\\
&-96\kappa^4{\delta_l}^2-96\kappa^4{\zeta_l}^2-24\kappa^2{\epsilon_l}^2\\
&-24\kappa^2{\eta_l}^2-24\kappa^2{\theta_l}^2+25b_lg_l+25b_lh_l+36\kappa^4b_lg_l\\
&+36\kappa^4b_lh_l-60\kappa^2b_lg_l-60\kappa^2b_lh_l\\
&-24\kappa c_l\vartheta_l-24\kappa d_l\epsilon_l-24\kappa d_l\eta_l-24\kappa d_l\theta_l-24\kappa d_l\vartheta_l\\
&-192\kappa^4\delta_l\zeta_l+96\kappa^3\delta_l\epsilon_l+96\kappa^3\delta_l\eta_l\\
&+96\kappa^3\delta_l\theta_l+96\kappa^3\delta_l\vartheta_l+96\kappa^3\epsilon_l\zeta_l\\
&+96\kappa^3\eta_l\zeta_l+96\kappa^3\theta_l\zeta_l+96\kappa^3\zeta_l\vartheta_l+48\kappa^2c_l\delta_l\\
&+48\kappa^2c_l\zeta_l+48\kappa^2d_l\delta_l+48\kappa^2d_l\zeta_l-48\kappa^2\epsilon_l\eta_l\\
&-48\kappa^2\epsilon_l\theta_l-48\kappa^2\epsilon_l\vartheta_l-48\kappa^2\eta_l\theta_l\\
&-48\kappa^2\eta_l\vartheta_l-48\kappa^2\theta_l\vartheta_l\\
&-24\kappa c_l\epsilon_l-24\kappa c_l\eta_l-24\kappa c_l\theta_l).
\end{aligned} \tag{56}$$

Substituting Eq. (56) with Eq. (16) into Eq. (42) yields the combo singular solitons

$$q(x,t)=\left\{\begin{array}{l}\pm\sqrt{-\frac{b_1(6\kappa^2-5)}{2(c_1+d_1-4\kappa^2(\delta_1+\zeta_1)+2\kappa(\epsilon_1+\eta_1+\theta_1+\vartheta_1))}}\coth(x+8b_1\kappa^3t)\\ \pm\sqrt{-\frac{b_1(6\kappa^2-5)}{2(c_1+d_1-4\kappa^2(\delta_1+\zeta_1)+2\kappa(\epsilon_1+\eta_1+\theta_1+\vartheta_1))}}\operatorname{csch}\left(x+8b_1\kappa^3t\right)\end{array}\right\} e^{i\left(-\kappa x-b_1\left(3\kappa^4+3\kappa^2-1\right)t+\theta_0\right)}, \tag{57}$$

$$r(x,t)=\left\{\begin{array}{l}\pm\sqrt{-\frac{b_2(6\kappa^2-5)}{2(c_2+d_2-4\kappa^2(\delta_2+\zeta_2)+2\kappa(\epsilon_2+\eta_2+\theta_2+\vartheta_2))}}\coth(x+8b_2\kappa^3t)\\ \pm\sqrt{-\frac{b_2(6\kappa^2-5)}{2(c_2+d_2-4\kappa^2(\delta_2+\zeta_2)+2\kappa(\epsilon_2+\eta_2+\theta_2+\vartheta_2))}}\operatorname{csch}\left(x+8b_2\kappa^3t\right)\end{array}\right\} e^{i\left(-\kappa x-b_2\left(3\kappa^4+3\kappa^2-1\right)t+\theta_0\right)}. \tag{58}$$

The combo singular solitons are valid for

$$b_l(6\kappa^2 - 5)\left(c_l + d_l - 4\kappa^2(\delta_l + \zeta_l) + 2\kappa(\epsilon_l + \eta_l + \theta_l + \vartheta_l)\right) < 0. \tag{59}$$

Conclusions

This paper secures CQ solitons with perturbed LPD model for both polarization-preserving fibers and birefringent fibers. The sine-Gordon equation approach has made this retrieval of the complete spectrum of soliton solutions possible. The results thus form a new list in the data bank of nonlinear evolution equations and its soliton solutions. The list opens up a floodgate of opportunities with CQ solitons for LPD model. This would include locating the conservation laws and identifying the respective conservative quantities. One would also need to address the perturbation theory to form the adiabatic dynamics of soliton perturbation. Next, CQ solitons with Bragg gratings in presence of dispersive reflectivity for LPD model is a possibility. Additional integration methods, such as semi-inverse variational principle or Lie symmetry, would give a new perspectives to the model. A further extension is to address such dynamics with DWDM topology would be absolutely necessary. The results of such studies would gradually and sequentially be reported.

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Compliance with ethical standards

Conflict of interest The authors also declare that there is no conflict of interest.

References

1. R.T. Alqahtani, M.M. Babatin, A. Biswas, Bright optical solitons for Lakshmanan–Porsezian–Daniel model by semi-inverse variational principle. Optik **154**, 109–114 (2018)
2. A.A. Al Qarni, A. Ebaid, A.A. Alshaery, H.O. Bakodah, A. Biswas, S. Khan, M. Ekici, Q. Zhou, S.P. Moshokoa, M.R. Belic, Optical solitons for Lakshmanan–Porsezian–Daniel model by Riccati equation approach. Optik **182**, 922–929 (2019)
3. A. Al Qarni, M.A. Banaja, H.O. Bakodah, A.A. Alshaery, Q. Zhou, A. Biswas, S.P. Moshokoa, M.R. Belic, Bright optical solitons for Lakshmanan–Porsezian–Daniel model with spatio-temporal dispersion by improved Adomian decomposition method. Optik **181**, 891–897 (2019)
4. A.A. Al Qarni, A.A. Alshaery, H.O. Bakodah, Optical solitons for the Lakshmanan–Porsezian–Daniel model by collective variable method. Results Opt. **1**, 100017 (2020)
5. S. Arshed, A. Biswas, F.B. Majid, Q. Zhou, S.P. Moshokoa, M. Belic, Optical solitons in birefringent fibers for Lakshmanan–Porsezian–Daniel model using $\exp(-\phi(\xi))$-expansion method. Optik **170**, 555–560 (2018)
6. A. Bansal, A. Biswas, H. Triki, Q. Zhou, S.P. Moshokoa, M. Belic, Optical solitons and group invariant solutions to Lakshmanan-Porsezian–Daniel model in optical fibers and PCF. Optik **160**, 86–91 (2018)
7. A. Biswas, Y. Yildirim, E. Yasar, R.T. Alqahtani, Optical solitons for Lakshmanan–Porsezian–Daniel model with dual-dispersion by trial equation method. Optik **168**, 432–439 (2018)
8. A. Biswas, A.H. Kara, R.T. Alqahtani, M.Z. Ullah, H. Triki, M. Belic, Conservation laws for optical solitons of Lakshmanan–Porsezian–Daniel model. Proc. Rom. Acad. Ser. A **19**(1), 39–44 (2018)
9. A. Biswas, M. Ekici, A. Sonmezoglu, H. Triki, F.B. Majid, Q. Zhou, S.P. Moshokoa, M. Mirzazadeh, M. Belic, Optical solitons with Lakshmanan–Porsezian–Daniel model using a couple of integration schemes. Optik **158**, 705–711 (2018)
10. A. Biswas, Y. Yildirim, E. Yasar, Q. Zhou, S.P. Moshokoa, M. Belic, Optical solitons for Lakshmanan–Porsezian–Daniel model by modified simple equation method. Optik **160**, 24–32 (2018)
11. A. Biswas, M. Ekici, A. Sonmezoglu, M.M. Babatin, Optical solitons with differential group delay and dual dispersion for Lakshmanan–Porsezian–Daniel model by extended trial function method. Optik **170**, 512–519 (2018)
12. M. Ekici, Optical solitons in birefringent fibers for Lakshmanan–Porsezian–Daniel model by extended Jacobi's elliptic function expansion scheme. Optik **172**, 651–656 (2018)
13. M.M.A. El-Sheikh, H.M. Ahmed, A.H. Arnous, W.B. Rabie, A. Biswas, A.S. Alshomrani, M. Ekici, Q. Zhou, M.R. Belic, Optical solitons in birefringent fibers with Lakshmanan–Porsezian–Daniel model by modified simple equation. Optik **192**, 162899 (2019)
14. J.V. Guzman, R.T. Alqahtani, Q. Zhou, M.F. Mahmood, S.P. Moshokoa, M.Z. Ullah, A. Biswas, M. Belic, Optical solitons for Lakshmanan–Porsezian–Daniel model with spatio-temporal dispersion using the method of undetermined coefficients. Optik **144**, 115–123 (2017)
15. J.V. Guzman, A. Biswas, M.F. Mahmood, Q. Zhou, S.P. Moshokoa, M. Belic, Optical solitons with polarization-mode dispersion for Lakshmanan–Porsezian–Daniel model by the method of undetermined coefficients. Optik **171**, 114–119 (2018)
16. M.B. Hubert, G. Betchewe, M. Justin, S.Y. Doka, K.T. Crepin, A. Biswas, Q. Zhou, A.S. Alshomrani, M. Ekici, S.P. Moshokoa, M. Belic, Optical solitons with Lakshmanan–Porsezian–Daniel model by modified extended direct algebraic method. Optik **162**, 228–236 (2018)
17. A.J.M. Jawad, M.J. Abu-AlShaeer, A. Biswas, Q. Zhou, S. Moshokoa, M. Belic, Optical solitons to Lakshmanan–Porsezian–Daniel model for three nonlinear forms. Optik **160**, 197–202 (2018)
18. M. Lakshmanan, K. Porsezian, M. Daniel, Effect of discreteness on the continuum limit of the Heisenberg spin chain. Phys. Lett. A **133**(9), 483–488 (1988)
19. A. Nirmeh, New analytical method based on Riccati equation for finding soliton solutions of nonlinear Lakshmanan–Porsezian–Daniel (LPD) equation. J. New Res. Math. **4**(16), 93–100 (2019)
20. V.N. Serkin, T.L. Belyaeva, Optimal control for soliton breathers of the Lakshmanan–Porsezian–Daniel, Hirota, and cmKdV models. Optik **175**, 17–27 (2018)
21. Y. Yang, T. Suzuki, X. Cheng, Darboux transformations and exact solutions for the integrable nonlocal Lakshmanan–Porsezian–Daniel equation. Appl. Math. Lett. **99**, 105998 (2020)

22. Y. Ye, C. Hou, D. Cheng, S. Chen, Rogue wave solutions of the vector Lakshmanan–Porsezian–Daniel equation. Phys. Lett. A. **384**(11), 126226 (2020)
23. H. Yepez-Martinez, J.F. Gomez-Aguilar, M-derivative applied to the soliton solutions for the Lakshmanan–Porsezian–Daniel equation with dual-dispersion for optical fibers. Opt. Quantu. Electron. **51**, 31 (2019)
24. Y. Yildirim, Optical soliton molecules of Lakshmanan–Porsezian–Daniel model in birefringent fibers by trial equation technique. Optik **203**, 162690 (2020)
25. E.M.E. Zayed, R.M.A. Shohib, M.M. El-Horbaty, A. Biswas, M. Ekici, A.S. Alshomrani, F.B. Majid, Q. Zhou, M.R. Belic, Optical solitons in birefringent fibers with Lakshmanan–Porsezian–Daniel model by the aid of a few insightful algorithms. Optik **200**, 163281 (2020)