

Simultaneous self mode locked and self-Q-switched lasers with active medium functioning as saturable absorber

Devasis Haldar¹ 

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Abstract It is hereby reported through this paper an observation of simultaneously self-mode-locking and self-Q-switching in fibre lasers, where there is no need for bulk active or passive cavity components like shutters or dye to restore phase relationship of oscillating modes but uses just a single piece of fibre to produce ultrashort pulses of high peak power. The design, performance as well as the principle of operation are discussed.

Keywords Self-mode-locking · Self-Q-switching · Saturable absorber · Kerr effect · Erbium-doped fiber

Introduction

Fluorescence in lasers comes from absorption of light at one wavelength to get excited to higher level and again radiate back to lower (ground) level through spontaneous emission. The emitted radiation is of longer wavelength. Laser action starts from tiny amount of fluorescence in the laser resonator mode.

Fluorescence linewidth comes from plotting gain curve of laser against frequency and is narrow when transition is between narrow levels. It makes it easy to achieve population inversion. In rare-earth-doped silica, such as the erbium (Er^{3+})-doped fibre, the fluorescence linewidth is broad [1, 2] and that permits to use them for design of tunable sources and broadband amplifiers. However in any case, the broadened laser cavity in rare-earth-doped singlemode optical fibres may support oscillations in many

modes simultaneously and hence are ideal for use in ultrashort pulses. A rare variety of tunable light sources with ultrashort pulses (say femtosecond pulses) using different mode locking techniques are coming up for future optical communication systems, sensors and other applications [3–6].

However what lacked in these cases is simultaneous self-mode-locking and self-Q-switching in fibre lasers with just a single piece of fibre to produce ultrashort pulses of high peak power. They are all complex and many often riddled with bulk components. In this paper an observation of simultaneously self-mode-locking and self-Q-switching in fibre lasers, with just a single piece of fibre to produce ultrashort pulses of high peak power, is reported. The additional advantage and newness of the present system is that there is no requirement of bulk active or passive components like shutter or dye for restoring the phase relationship of oscillating modes.

The design, performance as well as the operating method of our fibre laser are discussed in the paper.

Historical achievements

Sustained pulsation in the light output from various laser sources have been reported much early [7, 8]. If we can achieve nonuniform excitation of the laser medium, we can have the repetitive pulsing in say, GaAs laser. Lee and Roldan [9] achieved repetitive Q-switched light pulses in GaAs lasers. With split or tandem double-section stripe geometry, the non-uniform excitation has been achieved in controlled manner. These leads to self-induced sustain pulsation which could be best described as repetitive Q-switching. Here one section of the diode is pumped with a current below the threshold, while the other section is

✉ Devasis Haldar
devasishaldar62@gmail.com

¹ ECE Department, SRM University, Modinagar, India

pumped several times above the threshold. While one section of the laser diode is used for amplification, the other section is used as absorber.

Generation of ultrashort pulses using a cw-pumped Ti:Al₂O₃ has been reported [10, 11]. Since no active or passive element was introduced into the cavity for producing short pulses, this laser was regarded as self-mode locked. The underlying phenomenon is referred to the optical Kerr effect. The optical Kerr effect varies spatially along with the laser beam profile due to uneven intensity distribution and makes an amplified intense short pulse in solid state gain medium. The pulse experience a phase retardation [12].

Interaction between saturated population inversion and the optical signal were used to describe self-mode-locking and sustained self-pulsing in a unidirectional ring-fibre laser. [13]. In [14] self Q-switching and self-mode locking was demonstrated in a ring laser. In [15] erbium doped fibre lasers were formed with more than one fibre and results were explained using ion pair interaction model. In [16], self pulsing is demonstrated in good cavity and bad cavity configuration.

However, none of these have simplicity of linear single piece of fibre laser.

As will be seen, for the current fibre lasers and the self-mode-lock and self-Q-switch therein, the explanations for the operation are based to the discussion given above.

Basic principles

The end-pumping configuration for fibre lasers gives an exponential distribution of pump power along the fibre cavity, which can be expressed as

$$\pm P_p(z) = P(o) \exp(\Gamma_p(-\sigma_{pa}N_1) - \alpha)z \tag{1}$$

where P(z) is the pump power function of axis z along fibre. P(o) is the launched pump power at z = 0 fibre end, Γ_p is the confinement factor to account the doped region within core, σ_{pa} denotes the absorption cross section at pump frequency ($\nu_p = c/\lambda_p$), N_1 is the population density (number of ions per unit volume) of Er³⁺ in the ground state ⁴I_{15/2}- considered same as lower level of transition for three level system. The term of $\Gamma_p(-\sigma_{pa}N_1)$ takes into account the fact that the doped region within core provides gain for the entire fibre mode. α is the absorption of the lasers medium at the pump wavelength. The transmission loss, which is small compared with the parameter α , has been omitted in the Eq. (1). The \pm sign in the LHS of the same equation indicates the direction of propagation of pump wave (positive for the forward direction and negative for the backward direction).

In accordance with the Eq. (1), the population inversion will also have an exponential distribution along fibre,

except for the part close to z = 0 where the activate ions may all be excited and the population inversion is then unified. This is in agreement with the fact that the laser medium pump rate is proportional to local pump power. For the fibre lasers of three level systems, the part of the rare earth doped far from the pump end, the pump doesn't have enough energy to create population inversion. This part now plays role of absorber rather than gain medium, i.e. due to absence of population inversion the signal won't be amplified, it will rather be absorbed. This is because lower transition level is same as the ground level and absorption here take a start from the ground level. The absorption band of this absorber and that of the lasing transition are the same. Thus at higher intensity as excited states are populated due to absorption of signal by unexcited ions, the absorption decreases. Thus the absorber possesses the characteristics of saturability.

The fibre will also have a radial distribution of rare-Earth (Er³⁺) ions. The concentration of ions will be larger at the centre compare to edges. Intensity of beam will be higher at the centre of the beam.

Optical Kerr effect produces a variation of refractive index as described by the formula $n = n_0 + n_2I$, where n_0 and n_2 are the linear and non-linear components of the refractive index and I is the intensity of radiation. Since n_2 is positive in most materials, the refractive index becomes larger in areas where intensity is higher—here at the centre of the beam. This creates a focusing density profile which potentially leads to collapse of a beam on itself.

Thus optical Kerr effect will play a key role in self focusing the beam and causing high peak intensity pulses with smaller beam size. These peak intensity pulses will be effective in extracting energy from the laser gain medium and convert it to useful power output and its extraction efficiency will in any case be higher than that of the cw beam.

Spotsizes, which has a physical meaning of energy concentration degree to the fibre centre, is function of wavelength (square root of wavelength) [17]. Thus being of longer wavelength, the radial intensity distribution of the lasing light is not identical to the intensity distribution of the pump light. There won't be any population inversion at the edges of the core which will therefore take part in the working role of a saturable absorber.

The high power pulses are produced on account of optical Kerr effect and as the absorber part transmits the peak of fluctuation, these initial fluctuation grow into a narrow pulse 'bouncing' to and fro within the cavity. By such operation phase relationship of oscillating modes are continuously restored producing a period train of mode locked pulses.

Therefore it is the optical Kerr effect along with the above mentioned conditions combined within fibre lasers

of three-level system explains the all fibre self-mode-locking and self-Q-switching laser.

The irradiance I' as a function of time t due to mode locking can be written by the standard relation [18]:

$$I'(t) = E_0^2 [\sin^2(N\varphi/2) / \sin^2(\varphi/2)] \quad (2)$$

where $\varphi = \delta\omega t$; $\delta\omega$ = angular frequency separation between the modes = $\pi c/L$ (c is the velocity of light and L the optical pathlength between the mirrors).

General experimental investigation

Investigations are done on a number of Er^{3+} -doped fibre lasers with different dopant concentration levels and different index-profiles all at lasing wavelength of $1.55 \mu\text{m}$. The energy levels considered were of three level system ${}^4I_{11/2}$ to ${}^4I_{15/2}$ with ${}^4I_{11/2}$ having almost zero population, used as pump band, ${}^4I_{13/2}$ as metastable band and ${}^4I_{15/2}$ as ground state band. Rolls of erbium doped fibre with a diameter of $9.5 \mu\text{m}$ were developed initially for laboratory application primarily to set up and study an Erbium

doped fibre amplifier. It was found to be providing self pulsing phenomenon when excited with an argon ion pumped dye laser operating at 650 nm . Argon-ion pumped dye laser was used as cw pumping source. A schematic diagram of the experimental set up is drawn in Fig. 1.

The experimental system for this study comprised an end-pumping configuration. Both ends of the fibre were abutted to mirrors to form the laser cavity. Pump beam was launched into the active fibre with the help of microscope objective. The output mirror had a reflectivity of 50% at the lasing wavelength. The experimental diagram also shows a schematic diagram of a variable directional coupler where power in the coupled port can be varied using a tapping screw. One of its arm is focussed into the cavity mirror. In the other arm a laser beam of $1.53 \mu\text{m}$ wavelength, from a GaAlAs semiconductor diode is focussed after being modulated by a current source at 100 kHz . The isolator protects the diode from being exposed to fibre laser light coupled from the other arm.

Light from this arm (the one where light from semiconductor diode is launched) is made to couple to the

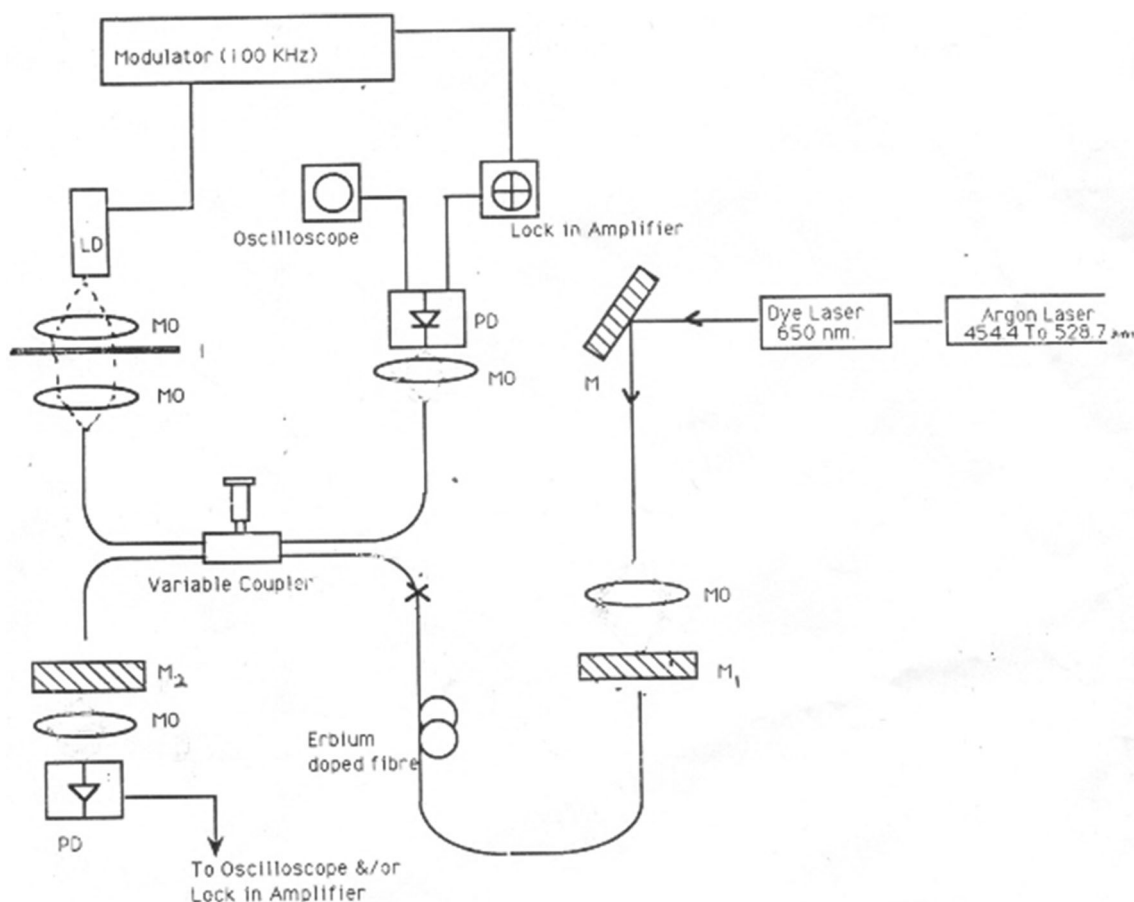


Fig. 1 The experimental set up. LD semiconductor laser diode at $1.53 \mu\text{m}$, PD photodetector, I isolator, M, M_1 and M_2 are mirrors, MO microscope objective

coupled port in the other arm, by suitably adjusting the tapping screw of the variable directional coupler. These modulated injected photons, having the same wavelength as the lasing light would get guided in the long single mode waveguide of the fibre laser cavity in the other arm and would undergo either gain or loss depending upon how they are incident upon and how the polarisation of the incident photon matches or mismatches with that of the lasing medium.

The laser output was detected using a Ge-detector with a 1 GHz bandwidth. A 1 GHz bandwidth oscilloscope was used to show the laser performance in the time domain. A lock-in-amplifier is added for phase sensitive detection.

A strain of self-mode-locked pulses have been obtained in almost all the tested three level Er^{3+} doped fibre level system. Sometimes deviation existed from self-locking; however a repetitive kind of pulsing has been found to impose over the cw background. The pulse time interval Δt has been equal to $2L/c$ where c is the velocity of light and L is the length of the cavity. This is the round trip time of lasing light within the cavity. Figure 2 shows a typical oscilloscope picture of self-mode-locked pulses from an Er^{3+} -doped fibre laser. Two fibre samples were chosen which gave simultaneously self-mode-locked and self-Q-switched pulses. What these two fibres have in common is a low dopant concentration, namely 78 ppm for the sample 1 and 66 ppm for the sample 2, respectively. In the experiments accomplished, no self-Q-switched pulses were observed in those Er^{3+} -doped fibre lasers with dopant level higher than 240 ppm.

On reducing the cavity fibre length of the two chosen samples stated above, the self-Q-switched phenomenon started disappearing and only cw self-mode-locking pulses survived.

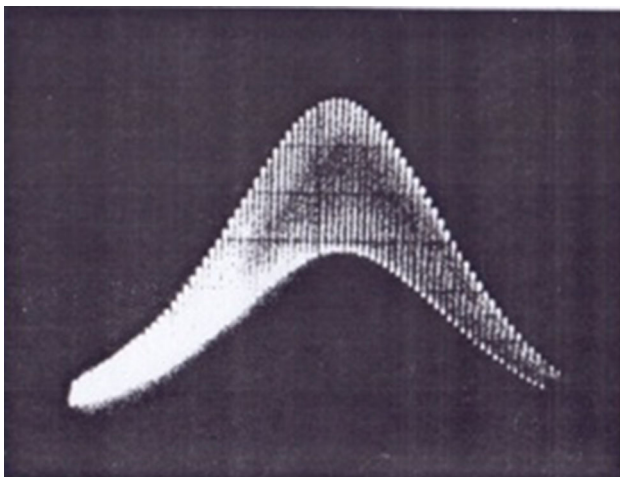


Fig. 2 Self mode locked pulses

When two types of fibres, one initially gives self-Q-switching while the other does not, were spliced together to form a laser cavity, it was found that the new laser did not give self-Q-switched pulses, no matter which part of cavity fibre is located near the pump source.

Thus one can find a length greater than length L_0 and up to a maximum length of L where one can realize the Q-switched pulses. We will see that value of L_0 in our experimental set up is 0.95 m in one case and 2.95 m in another case. The maximum length in both cases remained at 19.5 m. Later a qualitative explanation for this phenomenon is given on basis of population difference of the saturable gain medium varying along length of fibre. In the discussion part, a theoretical model is presented to evaluate the point at which Q-switched pulses have started to build up.

Simultaneously self-mode-locking and self-Q-switching operations

The sample 1 Er^{3+} -doped fibre, demonstrating simultaneously self-mode-locking and self-Q-switching operation was fabricated using the MCVD method. It must be mentioned again that the erbium doped fibre was initially grown not basically for self-mode-locking or self-Q-switching, but for purpose of laboratory general study on erbium doped fibre amplifier. However they started showing such phenomenon when pumped with argon pumped dye laser operating at $650 \mu\text{m}$. The host medium for the dopant was $\text{GeO}_2/\text{SiO}_2$ glass.

The dopant concentration, at 78 ppm, was relatively low to allow the laser cavity to be formed from various fibre lengths between 0.5 to 19.5 m. The tailor-made refractive index profile of the fibre cross-section is shown in Fig. 3. The cutoff wavelength of the fibre was $1.2 \mu\text{m}$, ensuring a single-mode operation at the lasing wavelength.

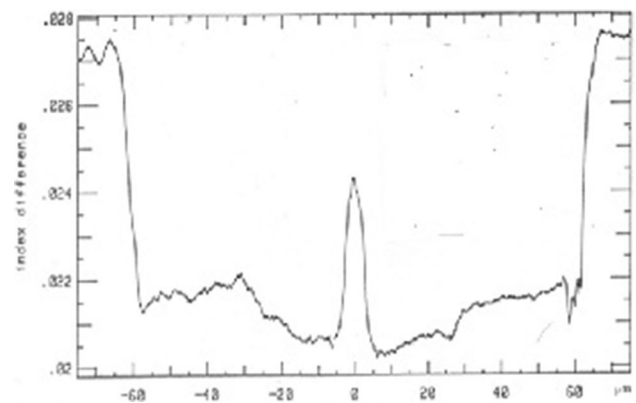


Fig. 3 The refractive index profile of-the fibre cross-section

Table below briefly give performance of sample 1 and sample 2.

	Concentration of Er ³⁺ (in ppm)	Length for self-mode-lock (m)	Length for self-Q-switched (m)
Sample 1	78	0.5–0.95	0.95–19.5
Sample 2	66	2.9–19.5	

The performance of the laser formed using sample 1 is explained as the follows:

1. Refer to the table above. For working of the fibre laser as self-mode-locked and self-Q-switched simultaneously, it was imperative that pump power exceed 18 mW at 660 nm. This value of pump power may be regarded as a threshold value of pump power. Also, noteworthy point is that self-Q-switching occurred for longer cavity lengths. When only self-mode-locked pulses were observed, it was in combination with a cw which look similar to noise.
2. The self-Q-switched pulses occurred immediately above threshold pump power. Normal cw operation of the laser which occurred at smaller fibre length, does not occur above 0.95 m without Q-switched pulsing. A typical measurement of these Q-switched pulses is shown in Fig. 4, where time base was 20 μ s per division.
3. The pump power and the fibre length (as stated in the table) were varied and different self-Q-switched pulses with repetition frequency ranging from 1.5 to 95 kHz were obtained. Also the peak power of the pulses was an increasing function of the pump power. Both the repetition frequency and the pulse peak power, for the laser with 19.5 m long fibre are measured as a function of pump rate. The widths of the pulses were 5 and 2 μ s for the lowest and highest pumping rate respectively.

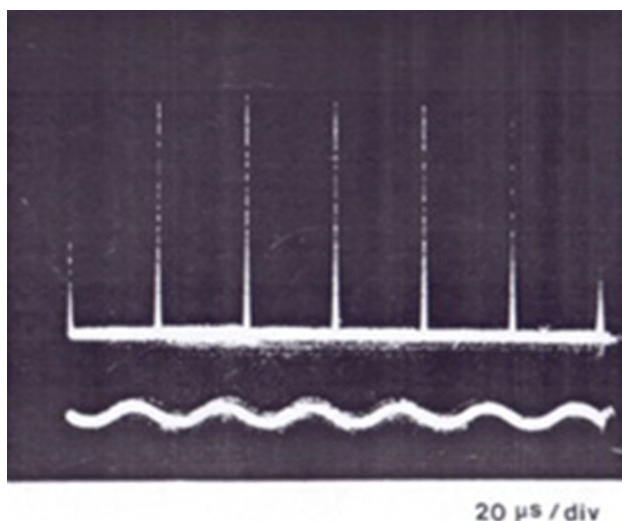


Fig. 4 Self Q-switched pulses

The best performance was measured with a cavity formed by fibre length of 4 m, in which a peak power of 530 mW for the Q-switched pulses, with 7.5 kHz repetition rate and 1.2 μ s width, has been achieved at a pump level of 60 mW absorbed power.

4. Self-mode-locked pulses were obtained within the envelope of the pulses. The repetition rate of the self-mode-locked pulses were as decided by the cavity length. A typical pulse train with 24 MHz repetition rate is shown in Fig. 5. A pulse width of 450 ps has been measured but this measurement width was limited by response of detector as well as of the oscilloscope electronics.

The sample 2 fibre, which also gives simultaneously self-mode-locked and self-Q-switched pulses, has an erbium dopant concentration of 66 ppm within GeO₂/SiO₂ glass host. The lasing action occurred for fibre length of 2.4 to 30 m. Simultaneously self-mode-locking and self-Q-switching operations occurred for the fibre length range from 2.9 to 19.5 m with a similar characteristics to the laser constructed using the sample 1. For fibre lengths shorter than 2.9 m or longer than 19.5 m, no self-Q-switching could be observed. The picture in Fig. 6 shows the self-Q-switched pulses from the lasers with a cavity fibre length of 10 m.

Normal relaxation oscillation pattern can be observed from a laser with a fibre length of 2.9 m. As mentioned before, when the laser is operating in its cw mode, self-mode-locked pulses were still existing as shown in Fig. 2. It was also noticed that the simultaneously self-mode-locking and self-Q-switching of operations sample 2 could sometimes be influenced by fibre environments such as pressing or twisting the fibre and made the self-Q-switched pulses extinguished.

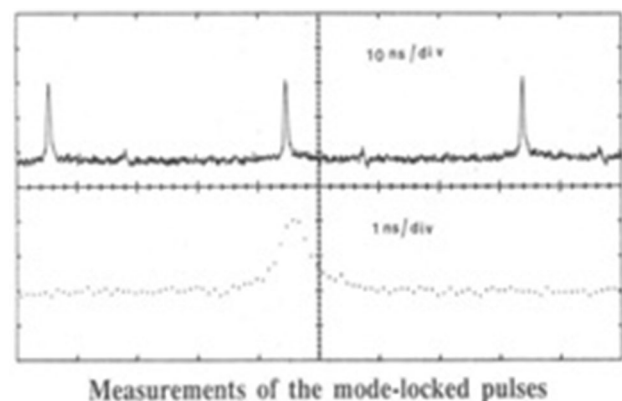


Fig. 5 Measurement of the mode locked pulses

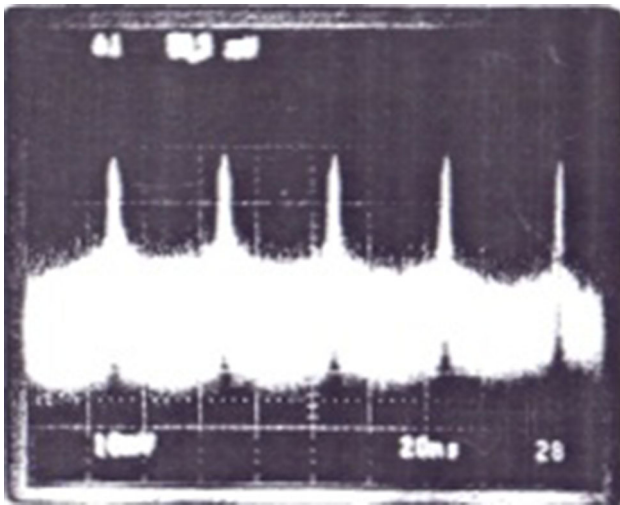


Fig. 6 Self Q-switched pulses

Discussion

Considering only longitudinal nonuniform excitation, the rate equation for the gain medium can be written as

$$dN_g(z,t)/dt = R_p(z,t) - N_g(z,t)/\tau_g - K_g N_g(z,t) n(z,t) \tag{3}$$

and the cavity photon linear intensity $n(z,t)$ equation is given by

$$dn(z,t)/dt = [K_g N_g(z,t) - 1/\tau_c] n(z,t) \tag{4}$$

In these equations, R_p represents the pump rate. For cw pumping, R_p has a steady exponential distribution related to Eq. (1). $N_g(z,t)$ is population difference of the saturable gain medium and K_g stands for coupling coefficient for the same. τ_g is the population recovery rate for the gain medium. τ_c stands for the photon cavity lifetime caused by cavity losses. Here loss due to absorption is not included. In Eq. (4) the rate of spontaneous emission into the lasing mode (R_{sp}) is neglected. For steady state condition, which can give an expression for the output power, the left hand side of above two equations must be equal to zero. For small $n(z,t)$, $dn(z,t)/dt$ is positive and this implies N_g must exceed threshold value N_{gth} . One can find this threshold from Eq. (3) for steady state case when number of photons is zero.

The steady state photon density (n_s), on the other hand, can be written as:

$$n_s = \tau_c R_p - R_{pth} \tag{5}$$

where R_{pth} is given by (N_{gth}/τ_g) , the threshold pump rate.

Noteworthy point is that $N_g(z,t)$ in the above equations might also be negative for a three-level laser system. If we define L_o to be the point, at which the population difference is equal to zero, just at junction of self-Q-switching, i.e.

$$N_g(z,t) = \begin{cases} > 0 & 0 < z < L_o \\ = 0 & z = L_o \\ < 0 & L_o < z < L \end{cases} \tag{6}$$

where L is assumed to be the fibre length. It is the part of fibre with negative $N_g(z,t)$ that plays a role of saturable absorber.

It has been theoretically shown in references [7] and [9] that the rate equations with a form similar to Eqs. (3)–(6) have pulsed as well as cw solutions. The pulsed solution occurs for the phase plane where the region is basically unstable. The cw solution can be attributed to the stable region.

There were however two important observations noticeable:

(1) The self-mode-locking was also found independently of fibre length (2) the self-Q-switching is influenced by application of mechanical pressure or by twisting the cavity fibre. These observations points to the fact that apart from longitudinal nonuniform excitation, it is the radial nonuniform excitation which is also contributing to appearance of saturable absorbing characteristics. However it is not certain which one is dominating.

However it is sure that self-Q-switching is a matter of relatively low dopant concentration with longer fibre length.

If we consider the longitudinal nonuniform excitation alone, all the parameters in Eqs. (3) and (4) affect the self-Q-switching operation, or there can be so called second threshold conditions as described in Ref. [19]. Therefore, fibre length, output mirror reflectivity (related to τ_c) and dopant concentration are all critical for obtaining self-Q-switched pulses.

If we want to include the saturable absorber part, the Eq. (4) for the cavity photon linear intensity $n(z,t)$, will be rewritten in the following form:

$$dn(z,t)/dt = [K_g N_g(z,t) - K_a N_a - 1/\tau_c] n(z,t) \tag{7}$$

with new rate equation for the saturable absorber:

$$dN_a(t)/dt = -[N_a(t) - N_{a0}]/\tau_a - K_a N_a(t) n(t) \tag{8}$$

Here N_a and K_a are the population difference and coupling coefficient of saturable absorbing medium, N_{a0} is the initial unsaturated value of N_a and τ_a is the population recovery rate for the saturable absorber. Under the assumption that recovery time τ_a for saturation absorber is very less compared to Q switch pulsewidth τ_p [19], the steady state solution ($dN_a/dt = 0$) leads to

$$N_a(t) \sim N_{a0}/[1 + (K_a \tau_a) n(t)] \text{ and thus Eq. (7) becomes:}$$

$$dn(z,t)/dt = [K_g N_{g0}(z,t) - K_a N_{a0} - 1/\tau_c] n(z,t) \tag{9}$$

N_{g0} is the laser inversion at the point the Q-switched pulse just starts to build up. The quantity in bracket is the initial growth rate for photon number.

To explore the exact conditions for simultaneous mode locking and Q-switching a number of laser diode pumped Nd³⁺-doped fibre lasers of a four level system have also been investigated. The lasing wavelength was at 1.08 μm for transition from level $^4\text{F}_{3/2}$ to level $^4\text{I}_{11/2}$. Here no self-mode locking or self-Q-switching were seen. The reason is attributed to empty lower level in a four level system. Therefore there did not occur any negative value of Ng(z,t). Hence one can conclude that the phenomenon of self pulsing is a consequence of active medium when it itself function as a saturable absorber.

It is believed that the self-mode-locking performance of fibre lasers is affected by fibre dispersion around the lasing wavelength. The W-type index profile shown in Fig. 3 benefits the self-mode-locking performance which makes the results described above much better than using other fibres. On the other hand, when self-Q-switching occur simultaneously with self-mode-locking, a more stable operation can be obtained due to higher photon intensity.

On electronics level, the self-Q-switched fibre laser is always accompanied with frequency jitter. This makes measurement of width of mode-locked pulse quite complex. The results reported above are deteriorated by the frequency jitter and limited by the measurement instruments.

Using acousto-optic deflector as an active Q-switch in the cavity formed by sample 1 fibre provides stronger Q-switched pulses, which are far more stable. Here the performance of mode locking remain unaffected. Thus one can combine active Q-switch and passive mode locking for design and fabrication of a pulsed laser source where Q-switching frequency can be controlled.

There lies two alternative to go in for high energy mode locked pulses in fibre lasers. (1) To increase the fibre cavity length to the extent it reduces repetition rate. Clearly, here we will get very large length. (2) To go to achieve mode locked fibre laser in state of Q-switching. We have discussed both the alternates here achievable by means of all fibre techniques.

The application of pulsed laser source is multiple—for various equipment, for communication, for various instruments, for OTDR etc.

Simplest configuration is obviously the first advantage of the all-fibre self-mode-locked and self-Q-switched laser over all other active and passive methods. The behaviour of traditional passive Q-switching or mode-locking using saturable-absorber dye solutions depends critically on the saturation properties of the gain medium and saturable absorber, and these can deteriorate with time or with repeated laser shots. However as a solid state material, all-fibre self-mode-locked and self-Q-switched laser does not suffer from these facts. Furthermore, the all-fibre self-

mode-locked and self-Q-switched lasers are compatible with existing fibre optic components and systems.

Conclusions

Self-pulsing in laser sources is inherently a phenomenon in rare-earth-doped singlemode fibre lasers. We have found that the longitudinal and radial nonuniform excitation of the active fibre can be made use of to form a saturable absorber, and just one piece of rare-earth-doped fibre can be used to generate a laser cavity which can provide simultaneously self-mode-locked and self-Q-switched pulses. Thus one can well achieve a new type of practical pulsed fibre whose main features include simplicity of device which can be easily pumped with just LD, easy to connect to fibres and its components, having high peak power and extremely narrow pulsing width.

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