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RESEARCH ARTICLE

Waveguiding effect on optical spatial solitons in centrosymmetric photorefractive materials

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Abstract Photorefractive solitons have been studied in a waveguide that is made of centrosymmetric material. The dynamical equations pertaining to characteristics of solitons have been derived under paraxial ray and Wentzel–Kramers–Brillouin (WKB) approximations. It has been predicted that the planar waveguide structure enhances self-focusing effect and reduces the threshold power requirement for soliton formation. The waveguide that is embedded in the photorefractive crystal leads to the trapping of low power solitary wave which otherwise would not have formed spatial solitons at this low power in this material. The minimum requirement of power for self-trapping in the material decreases with the increase in the value of waveguide co-efficient. The existence of bistable states has also been predicted.

Keywords Spatial solitons · Bistable state · Centrosymmetric material

Introduction

Optical soliton propagation has been an intensively researched topic during last three decades due to its implications in optical communications and signal processing [1–20]. Optical solitons are electromagnetic waves which are localized in space, or time or both [1–3].They can be created in nonlinear media and have been observed in plasmas [6], optical fibres [7], photorefractive media [8, 9], bulk nonlinear media [10, 11] etc. Photorefractive spatial solitons possesses several distinctive properties for which they become very attractive. For example, they can be formed using very low power which is of the order of a few microwatts. Photorefractive materials exhibit saturating nonlinearity, thereby (2 + 1)D spatial solitons do not suffer collapse in these materials. Photorefractive spatial solitons can be created at the telecommunication wavelength thus enhancing their real life applications. These solitons could possess fast response time which is a few microsecond. Due to above special features, optical spatial solitons in photorefractive media draw special attention in the soliton community [6–16]. They find important applications in optical switching, beam steering, optical interconnects, parallel computing, reconfigurable optical circuits etc. [17–19, 22–24].

The steady state photorefractive solitons were first theoretically predicted by Segev et al. [8] in 1992. Next year these solitons were detected experimentally by Duree et al. [9]. Till date three different types of steady state photorefractive solitons have been theoretically predicted and experimentally detected. These are screening solitons (SS) [8, 9, 12–14], photovoltaic (PV) solitons [15, 16] and screening photovoltaic (SP) solitons [17–20]. These solitons have been found to exist in different varieties such as bright, dark and grey, scalar as well as vector configurations.

Most of the earlier experimental as well as theoretical works on photorefractive solitons employed non centrosymmetric photorefractive (NCSPR) materials. However, Segev et al. [21] predicted that centro-symmetric photorefractive (CSPR) materials may also support spatial

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solitons. Del Re et al. [22] confirmed the prediction by observing them in CSPR media, in particular, (1 + 1)D and (2 + 1)D steady state photorefractive spatial solitons in centrosymmetric paraelectric KLTN crystals have been observed [22, 23]. These screening solitons are fundamental to both in the understanding of 2D solitons and in applications. In a recent review their application potential has been elegantly highlighted [24].

During last two decades, through tremendous progress [8, 9, 12–25] has been made in the understanding of propagation characteristics of spatial solitons in bulk photorefractive media, not much progress have been reported on the properties of solitons in photorefractive waveguides. In fact, baring sole exception of Ref. [26], serious attempt is yet to be made to examine and reveal properties of these solitons in photorefractive waveguides. Unlike temporal solitons, the requirement of threshold power for the creation of spatial solitons is very large [3]. Therefore, creation of spatial solitons in laboratory is more difficult in comparison to the creation of temporal solitons [3]. However, above difficulty may be overcome easily if spatial solitons are created in a waveguide. The waveguide, due to its light guiding property, will counter balance the self-defocusing induced beam divergence. The self-defocusing of an optical beam can be completely or partially eliminated due to the wave guiding effect of the waveguide. Hence, in a waveguide, the threshold power required for the soliton formation would be substantially lower in comparison to the threshold power required in bulk media. This may be helpful in creating spatial solitons at relative ease. Therefore, formation of spatial solitons in a photorefractive waveguide needs further investigation. Thus, in this paper, using WKB approximation, we investigate the propagation characteristics of bright spatial solitons in a biased planar photorefractive waveguide that is made of centrosymmetric material. This paper has been organized as follows. The mathematical model for the propagational characteristics of bright optical spatial solitons in a planar waveguide has been developed in "Mathematical formulation" section. In addition, the results have been also presented in this section. "Conclusion" section contains a brief conclusion of our investigations.

Mathematical formulation

The waveguide considered in the present paper is a planar waveguide, made of centro-symmetric materials such as paraelectric potassium lithium tantalate niobate (*KLTN*) or potassium tantalate niobate $KTa_{0.65}Nb_{0.35}O_3(KTN)$. The waveguide in this material may be fabricated using selectively diffused construction in which a dopant is diffused into a selected layer of the bulk material to change the refractive index of the layer [32]. A channel waveguide can be easily created by changing the refractive index by selective diffused construction. Note that in this method a waveguide is created in a single material by changing the refractive index selectively, thereby creating the core and cladding.

We consider a polarized optical beam which is propagating in *z*-direction through a biased waveguide that is made of centro-symmetric photorefractive (CSPR) crystal. The waveguide is placed in such a way that the optical *c*-axis of the CSPR crystal lies along the *x*-axis. A bias field is also applied along this direction. The beam is considered to be polarized in *x*- direction, and is allowed to diffract along that direction only. The governing equation for spatial evolution of the electric field $\vec{E}(x,z)$ associated with the propagating optical beam can be written as

$$\nabla^2\vec{E}(x,z) + (k_0 n)^2\vec{E}(x,z) - bx^2\vec{E}(x,z) = 0, \tag{1}$$

where $k_0 = 2\pi/\lambda_0$, λ_0 is the free space wavelength of the incident beam, n is the perturbed refractive index of the crystal [13, 22] which is related to its unperturbed value n_0 by the relation $n^2 = n_0^2 - n_0^4 g_{eff}\varepsilon_0^2(\varepsilon_r - 1)^2 E_{sc}^2$, b is the waveguide parameter. The positive value of b determines the strength of the waveguide and the value of this parameter is decided during fabrication. Application of bias field does not change the value of b. The parameter g_{eff} represents the effective quadratic electro-optic coefficient, ε_0 and ε_r being the permittivity of the free space and relative permittivity of the CSPR crystal. E_{sc} represents the steady state space charge field developed in the crystal due to the electro-optic effect. Above expression of refractive index signifies that unlike non-centrosymmetric materials, the same centrosymmetric material cannot be made focusing or defocusing type by changing the direction of external field. The expression for the space charge field in the CSPR waveguide can be obtained from the charge transport model of Kukhtarev et al. [27] which includes rate, current, Poisson equation together with Gauss' law. The detail derivation of steady state space charge field can be found elsewhere [24] which can be written as:

$$E_{sc} = \frac{I_\infty + I_d}{I + I_d} E_0, \tag{2}$$

where I_d represents the so called dark irradiance, E_0 is the external bias field and $I(x,z)$ represents the power density profile of the optical beam, $I_\infty = I(x \to \pm\infty)$. The electric field $\vec{E}(x,z)$ of the beam envelope is assumed to be slowly varying, so that it can be written as

$$\vec{E}(x,z) = \hat{x}\varphi(x,z)e^{ikz}, \tag{3}$$

where $\varphi(x,z)$ is the slowly varying wave envelope propagating through the PR media and k is the wave number in the crystal, related to the corresponding quantity k_0 in free space by $k = k_0 n_0$. Substituting the anstanz (3) into the wave Eq. (1), and employing slowly varying envelope approximation, we obtain

$$i\frac{\partial\varphi(x,z)}{\partial z} + \frac{1}{2k_0n_0}\frac{\partial^2\varphi(x,z)}{\partial x^2} - \frac{1}{2}k_0n_0^3g_{eff}\varepsilon_0^2(\varepsilon_r - 1)^2E_{sc}^2\varphi(x,z) - bx^2\varphi = 0. \tag{4}$$

Inserting the expression of E_{sc} from Eq. (2) in (4), we obtain

$$i\frac{\partial U(\xi,s)}{\partial\xi} + \frac{1}{2}\frac{\partial^2U(\xi,s)}{\partial s^2} - \beta\left(\frac{\rho+1}{1+|U|^2}\right)^2 U - \delta s^2 U = 0, \tag{5}$$

where $\rho = I_\infty/I_d$, $I(x,z) = (n_0/2\eta_0)|\varphi(x,y)|^2$, $\varphi(x,y) = (2\eta_0 I_d/n_0)^{1/2}U(\xi,s)$, $\xi = z/(kx_0^2)$, $s = x/x_0$, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$, $\beta = \frac{(k_0x_0)^2n_0^4g_{eff}}{2}E_0$ and $\delta = bk_0x_0^4n_0$, x_0 being an arbitrary spatial width taken for scaling. The nonlinear contribution to the refractive index of the PR crystal due to charge drift comes from the third term in Eq (5). Equation (5) is a nonintergrable equation and cannot be solved exactly. However, there are several approximate methods which can be used to solve above equation. For example, approximation method like Segev's method [8, 17, 18], paraxial method of Akhmanov [7, 28, 29], Variational method of Anderson [30] or moment method of Vlasov [31] can be used to solve Eq. (5). Each of these methods have their advantage and disadvantage. However, since, paraxial ray approximation method is very simple and gives very illustrative physical picture, in this communication, we employ the paraxial method of Akhmanov [26, 27] and restrict our study to investigations on bright solitons only, hence, $\rho = 0$. In order to solve Eq. (5), we assume that the slowly varying beam envelope $U(\xi,s)$ can be taken as:

$$U(\xi,s) = U_0(\xi,s)e^{-i\Omega(\xi,s)}, \tag{6}$$

where $U(\xi,s)$ and $\Omega(\xi,s)$ represents the beam envelope and its phase, respectively; $U_0(\xi,s)$ is a pure real quantity. Substituting the ansatz (6) in the evolution Eq. (5), we obtain following equation:

$$\left(i\frac{\partial U_0}{\partial\xi} + U_0\frac{\partial\Omega}{\partial\xi}\right) + \frac{1}{2}\left\{\frac{\partial^2U_0}{\partial s^2} - 2i\frac{\partial U_0}{\partial s}\frac{\partial\Omega}{\partial s} - iU_0\frac{\partial^2\Omega}{\partial s^2} - U_0\left(\frac{\partial\Omega}{\partial s}\right)^2\right\} - \beta\frac{1}{(1+U_0^2)^2}U_0 - \delta s^2U_0 = 0. \tag{7}$$

Equating real and imaginary parts of above equation, we get following two equations:

$$\frac{\partial U_0}{\partial\xi} - \frac{\partial U_0}{\partial s}\frac{\partial\Omega}{\partial s} - \frac{1}{2}U_0\frac{\partial^2\Omega}{\partial s^2} = 0, \tag{8}$$

and

$$U_0\frac{\partial\Omega}{\partial\xi} + \frac{1}{2}\frac{\partial^2U_0}{\partial s^2} - \frac{1}{2}U_0\left(\frac{\partial\Omega}{\partial s}\right)^2 - \beta\Phi_1(\xi,s)U_0 - \delta s^2U_0 = 0, \tag{9}$$

where $\Phi_1(\xi,s) = \frac{1}{(1+|U_0|^2)^2}$.

In Eq. (9), $\Phi_1(\xi,s)$ account for the nonlinearity induced in the PR material due to the space charge induced refractive index change by the external bias field. The last term in Eq. (9) is due to the planar waveguide structure of the PR material. The last two terms control the diffraction of the beam, leading to its shape preserving propagation. We look for a self-similar spatial soliton solution for which the electromagnetic field energy is confined in the central region of the beam. Of the many possible solutions, the Gaussian solution gives very good analytical results, comparable to those found from pure numerical simulations. Hence, the amplitude and phase of solitons are taken in the following form:

$$U_0(\xi,s) = \frac{U_{00}}{\sqrt{(\xi)}}e^{-s^2/2r^2f^2(\xi)}, \tag{10}$$

$$\Omega(\xi,s) = \frac{s^2}{2}\Gamma(\xi) + \Psi(\xi), \tag{11}$$

$$\Gamma(\xi) = -\frac{1}{f(\xi)}\frac{df(\xi)}{d\xi}, \tag{12}$$

where U_{00} is the normalized peak power of the soliton, r is a positive constant and $f(\xi)$ is the variable beam width parameter such that the product $rf(\xi)$ gives the spatial width of the soliton. Without any loss of generality, we assume at $\xi = 0, f = 1$. In addition, we further assume that the soliton forming beam is nondiverging when it enters the photorefractive crystal i.e.,$\frac{df}{d\xi} = 0$ at $\xi = 0$. The nonlinear contributions Φ_1 to the refractive index are expanded in Taylor series, from which under first order approximation we obtain:

$$\Phi_1(\xi,s) = \frac{1}{\left(1+\frac{U_{00}^2}{f}\right)^2} + s^2\frac{2U_{00}^2/(r^2f^3)}{\left(1+\frac{U_{00}^2}{f}\right)^3}. \tag{13}$$

Substitution of Eqs. (10)–(13) in Eq. (9) results in an equation which contains different powers of s^2, equating the coefficients of various powers of s^2 on both sides of this equation, we obtain the evolution equation for the beam width parameter $f(\xi)$ as

$$\frac{d^2f(\xi)}{d\xi^2} = \frac{1}{r^4f^3(\xi)} - 4\beta\frac{\frac{P_0}{r^2f^2(\xi)}}{\left(1+\frac{P_0}{f(\xi)}\right)^3} - 2\delta f(\xi). \tag{14}$$

Above equation describes the evolution of the beam width parameter of the spatial soliton of normalized power $P_0 = (U_{00}^2)$ in the photorefractive waveguide. Depending on the value of power and the relative strength of the system parameters β, the optical solitary wave may diverge, compress or get self-trapped. A trapped solitary wave i.e., a spatial soliton, is achieved when the beam width remains invariant so that the left-hand side of Eq. (14) is zero. Therefore, setting the left hand side of above equation equal to zero, we get following equation which gives a relationship between threshold power P_{ot} and bandwidth r for stationary propagation:

$$\frac{1}{r^4} = 2\delta + \frac{4\beta P_{ot}}{r^2(1+P_{0t})^3}. \tag{15}$$

Equation (15) establishes a relationship between beam width (r), threshold power P_{ot} and waveguide parameter δ. This equation is also known as the existence equation of optical solitons that is propagating through the photorefractive waveguide.

Examining above equation one can get an idea about the type of solitons that is permissible through the waveguide, it also gives an idea about the threshold power requirement for stable soliton formation. Above equation possesses four roots of the beam width parameter r. A careful examination of these roots reveals that two of these roots are complex, one is real and positive, while the remaining one is real and negative. The beam width parameter r has to be real positive, hence, the real positive root is identified as the width of the soliton. In order to have an idea about the requirement of threshold power for soliton of different beam width, we have demonstrated the variation of r with threshold power P_{ot} for different waveguide parameter in Fig. 1. Form Fig. 1, it is amply clear that for a given spatial width of the soliton, there exists two different threshold power, one at low power while the other at high power. This is is a signature of the existence of bistable solitons. In the low-power regime, the width of the soliton state is not much affected with the variation of power. In addition, it should be pointed out that as a consequences of bistable behavior, a soliton with a specific spatial width can be formed at two different threshold powers and these may be identified as P_{ot1} and P_{0t2}. However, in the high-power region, it is clear from the figure that the width of the spatial soliton increases with an increase in the value of the waveguide parameter δ. To this end, we now examine the behavior of spatial solitons at different peak power in absence of any waveguidig effect. In order to do that the variation of normalized beam width parameter with

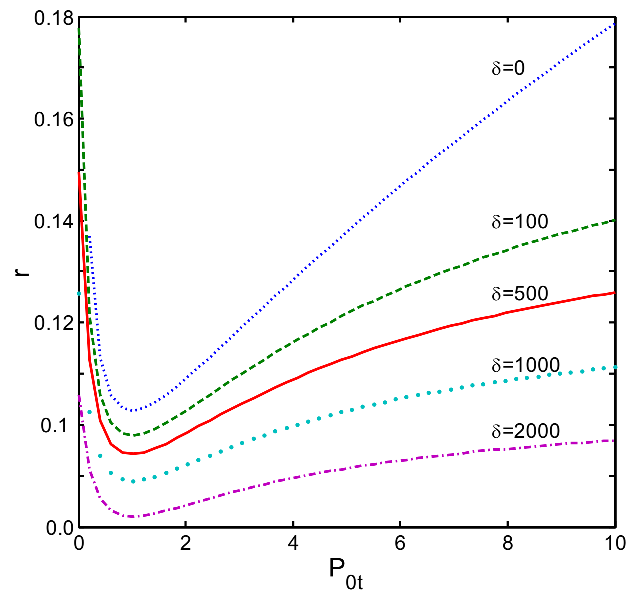

Fig. 1 Variation of equilibrium spatial beam width r with threshold power P_{0t} of solitons; $\beta = 157.9$

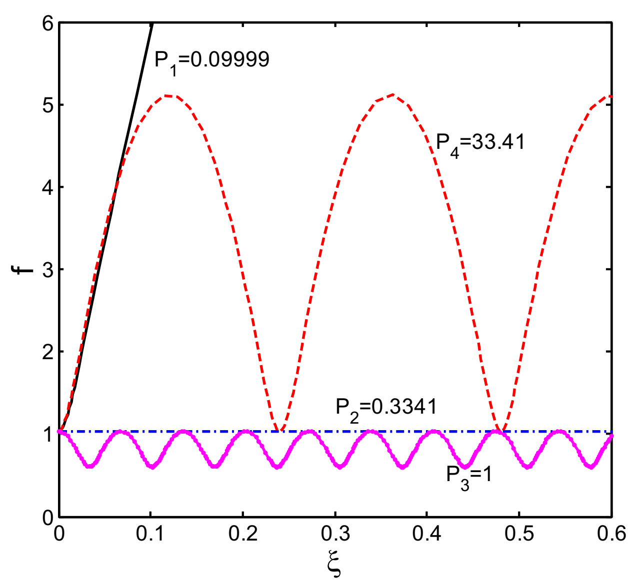

Fig. 2 Variation of variable beam width parameter $f(\xi)$ with normalized distance of propagation ξ at four different soliton peak power. For all curves $\beta = 157.9$; $f(\xi) = 1$ and $\frac{df}{d\xi} = 0$ at $\xi = 0$

distance of propagation has been demonstrated in Fig. 2. For a lucid illustration of the beam dynamics inside the photorefractive waveguide, we have selected four different power regimes and depicted the variation of beam width at these powers. Particularly, we have chosen,$P_1(= 0.099) < P_{ot1}$, $P_2(= 0.3341) = P_{ot1}$, $P_{ot1} < P_3(= 1) < P_{ot2}$, $P_4(= 33.41) > P_{ot2}$. It is evident that at P_1, the beam width diverges to a very large value since the soliton peak power

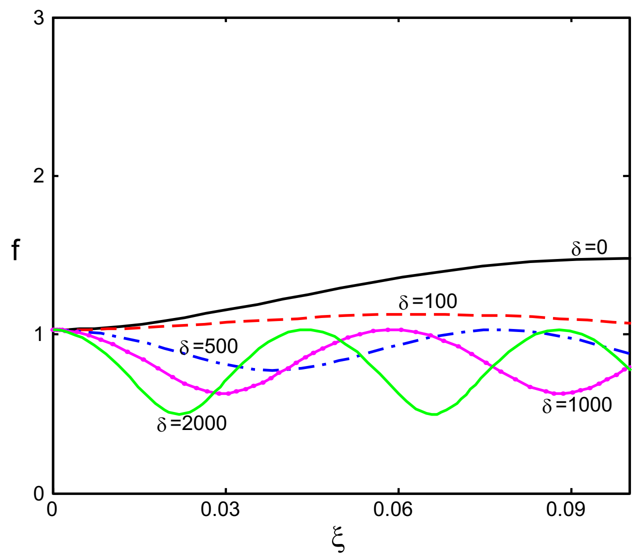

Fig. 3 Variation of variable beam width parameter $f(\xi)$ with normalized distance of propagation ξ for five different values of the wave guide parameter. $\beta = 157.9$, $P_0 = 0.06947$ and $r = 0.1489$; $f(\xi) = 1$ and $\frac{df}{d\xi} = 0$ at $\xi = 0$

is less than the threshold power. At threshold power $P = P_{ot1}$, the spatial soliton propagates maintaining its shape unchanged, which has been manifested by constant beam width $f = 1$.The beam width of a spatial soliton oscillates with oscillation amplitude less than unity when the soliton peak power lies between two threshold values P_{ot1} and P_{ot2}.

At this stage it would be appropriate to investigate the effect of waveguding on the propagation characteristics of the soliton. Particularly, it would be appropriate to examine whether a soliton with peak power less than threshold power would propagate as a stable entity or diverge. Therefore, we have demonstrated the beam dynamics at threshold peak power in Fig. 3. It is evident from the figure that the spatial soliton diverges in absence of waveguiding when the peak power is less than the threshold power. This has been demonstrated by the thick solid curve in Fig. 3. However, the same soliton propagates as a stable entity in presence of finite waveguiding. For sufficiently large $\delta(>500)$, the behavior of f is oscillatory, with an amplitude always less than one, indicating trapping of the soliton even at peak powers less than the threshold power. The higher the value of δ, the smaller the power required to trap the soliton in the waveguide.

Conclusion

In conclusion, we have examined the possibility of optical spatial soliton propagation at low power in optical waveguide that is embedded in a centrosymmetric photorefractive material. The waveguide structure augments the self-focusing effect of the photorefractive material, consequently the requirement of minimum threshold power for self-trapped propagation reduces. Solitons with lower peak power can propagate as a stable entity due to the waveguiding effect which otherwise would have diverged. The larger the waveguide co-efficient, the lower is the minimum requirement of threshold power of the beam that can be self-trapped. We have identified four different power regimes in which the soliton beam width parameter possesses distinct behavior.

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