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RESEARCH ARTICLE

# Some aspects of wave and quantum approaches at description of movement of twisted light

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**Abstract** The existence of twisted light may be inferred from modern quantum concepts and experimental data. These waves possess energy, impulse and angular momentum. However, the Maxwell's four-dimensional theory of electromagnetism does not imply the existence of waves with these properties. This article develops a model generalizing the theory of electromagnetism in such a way that it would be possible to obtain equations of twisted electromagnetic waves. Generalization is implemented by introduction of a space-time with a more complex structure compared to the four-dimensional space-time. Such spaces include a seven-dimensional space-time, which allows to describe not only translational, but also rotational motion of bodies. A model developed by the author provides the following results: 1) generalization of the theory of electromagnetism in which it is possible to obtain equations of twisted light waves, 2) solution describing interference of light waves oppositely twisted, 3) the formula relating the energy, impulse and angular momentum of electromagnetic wave, 4) justification of a new phenomenon - redshift due to electromagnetic waves screwing.

**Keywords** Maxwell's theory of electromagnetism · Seven-dimensional space-time · Twisted light · Angular momentum · Interference · Redshift

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## Introduction

Any light wave can carry not only energy $E = \hbar\omega$ and an impulse $p = k\hbar$, but an angular momentum, as well. The angular momentum of light consists of two components: a spin angular momentum and an orbital angular momentum. The spin angular momentum $L_s = l\hbar$ is related to wave polarization, and the orbital angular momentum $L_o = n\hbar$ is related to dependence of a wave phase from direction, where $l$, $n$ are some whole quantum numbers. An electromagnetic wave is characterized by two directions: a wave vector **k** and an electric-field vector **E** constantly being mutually perpendicular. As a result, another degree of freedom occurs, i. e. rotation of the electric-field vector around the wave vector. With such rotation change of the electric-field vector shows as a phenomena of polarization. In a common plane light wave $n = 0$ all wavefronts follow each other. Therefore, if we take a wave cross-section, the wave phase will be equal in each point of the cross-section (Fig. 1).

The work by Allen L. et al. [1], published in 1992, proposed schemes for formation and detection of twisted light, and was the first to suggest that the electromagnetic wave had the orbital angular momentum apart from the spin moment. In the twisted light the wavefront resembles a spiral directed towards the side of wave propagation with the phase changing in each cross-sectional point depending on the direction. For instance, at $n = \pm 1$ (Fig. 1b, d) the wave phase changes by $2\pi$ for one turn of the wavefront, and at $n = \pm 2$ (Fig. 1a, e) for a half-turn.

Straight twisted light beams were experimentally generated by [2] in 1995. Twisted electromagnetic waves were formed technically with a specific prism of variable thickness. Difference in thickness of the prism makes it possible for the wave phase, which has passed through the thicker prism layer, to lag from the wave phase, which has passed

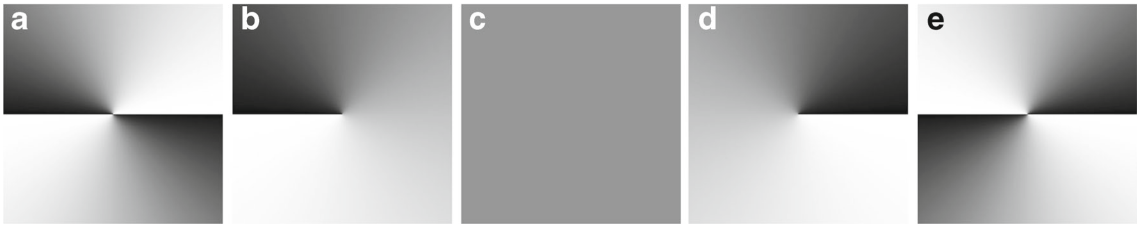


**Fig. 1** A phase profile of a light ray, with quantum numbers: **a**) $n = -2$, **b**) $n = -1$, **c**) $n = 0$, **d**) $n = +1$, **e**) $n = +2$. The wave phase equal to zero is colored black, the wave phase equal to $2\pi$ is colored white

through the less thick prism layer. Thus, at the output different wave sections have a different phase depending on the direction (Fig. 2a), while the amplitude attains the distribution Fig. 2b. The advanced methods for generating twisted light allow to simultaneously form several beams differently twisted. The work [3] used a pitch-fork hologram for these purposes.

An experiment described in the work [4] proves that the light wave possess not only the impulse, but the orbital angular momentum, as well. The authors describe an experiment, where a microparticle was suspended in the focus of a laser beam. When having absorbed light, the microparticle started turning. Thereby, the turning direction depended on twisting direction of the laser light.

According to the work [5], it was possible to generate twisted x-ray waves with photon energy equal to 99 eV due to developed technologies for formation and registration of the twisted light.

The twisted light has various applications today, including the quantum information theory, microcomputer control and astrophysical researches [6]. For instance, in 2010 B. Thide with the colleagues issued the work [7], describing a method allowing determination of rotation features of black holes based on the analysis of an angular momentum of the light passing near an accretion disc.

The simplified equation of the twisted light wave propagating along the axis $Ox$, with the phase depending on time $t$, component $x$ and the wavefront rotation angle $\varphi$ (Fig. 2a), is of the following form:

$$E = E_0 \cdot \exp\left(-i\left(\omega t - kx - n\varphi\right)\right).$$

However this equation does not flows from the Maxwell's four-dimensional theory of electromagnetism, suggesting that the standard theory of electromagnetic fields shall be corrected.

## Choice of description method for twisted waves

Therefore, the purpose of this work is to build a model of electromagnetism, naturally involving a possibility to describe twisted electromagnetic waves, and to investigate the properties of the built model.

Since the electromagnetic wave in a three-dimensional space has additional degrees of freedom, we may suppose that to describe movement of the twisted electromagnetic waves it is reasonable to use a space-time with a more complex structure, rather than the four-dimensional space-time.

The works by mathematicians B. Riemann, G. Weyl, E. Cartan, J. Schouten and others show that spaces may be

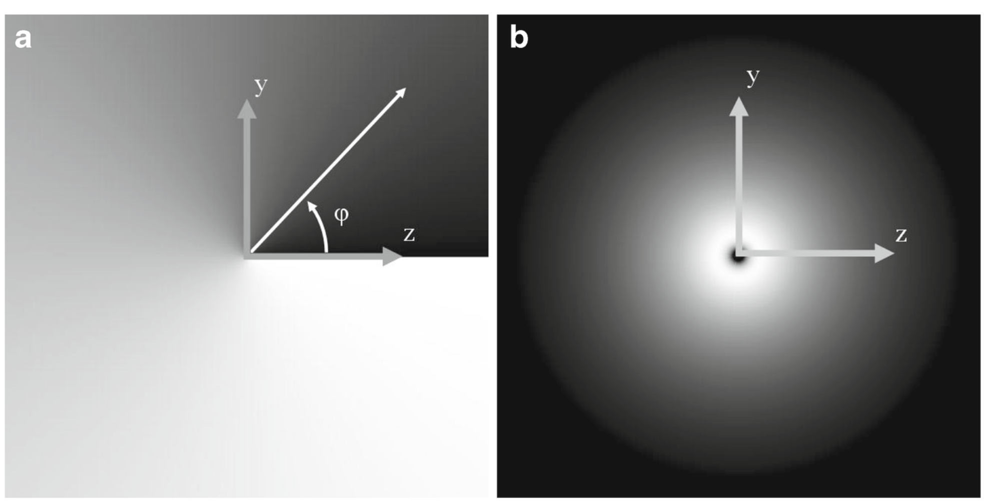


**Fig. 2** **a**) Phase profile of the light beam with $n = +1$ depending on the turning angle of the wavefront $\varphi$. In the figure, the wave phase equal to zero is colored black, the wave phase equal to $2\pi$ is colored white. **b**) The amplitude of the twisted light beam; maximum amplitude is colored white, while the minimum one is colored black

characterized by curvature, as well as by torsion and nonmetricity. For instance, to describe the observed phenomena the modern cosmology uses the Riemann-Cartan space with curvature and torsion or a more common affine-metric space with curvature, torsion and nonmetricity, particularly the Weyl-Cartan space with the nonmetricity of Weyl type [8–17]. However all attempts of G. Weyl to use the concepts of scale conversion with parallel translation to create a geometrical theory of electromagnetism failed that makes us to decline the concepts of development of a generalized theory of electromagnetism with the use of spaces with torsion and nonmetricity.

Another example of the space-time accounting for additional degrees of freedom is a model of the seven-dimensional space-time. As the works [18–21] show, to explain the dynamics not only of translational motion, but of rotational motion, as well, of bodies in gravitational fields, one may use the seven-dimensional space-time which includes, apart from time and three spatial components, three components aligning the body in the space $x^4 = \varphi$, $x^5 = \psi$, $x^6 = \theta$, the so called Euler angles. The equations of the geodesic seven-dimensional space-time allow to develop not only equations for translational motion, but equations describing rotational motion of gyroscopes [21], as well.

The metric tensor takes the following form for the inane seven-dimensional space-time [18, 21]:

$$\begin{aligned} g_{00} &= -g_{\alpha\alpha} = 1, \\ g_{45} &= g_{54} = -\frac{J_\omega \cos(\theta)}{m}, \\ g_{44} &= g_{55} = g_{66} = -\frac{J_\omega}{m}, \end{aligned} \tag{1}$$

where $J_\omega$- the angular momentum of a test body relating to the rotation axis, $m$ - the weight of the test body, $\alpha = 1, 2, 3$. The paradox of notation of the metric tensor [22, 23], which, regardless of four-dimensional metric tensors of the general relativity, depends on the parameters of the test body (inertia momentum to weight ratio), makes it possible to suppose that the space is not an absolute value and the gravitational field depends on the test body inserted into it. However it is important that deflection of geodesic lines, along which the test body move, is the only method available today to detect gravitation. That is why we may suggest about presence or lack of gravitation only in the view of the moving test body. As a result, it is no wonder that the metric tensor describing the gravitational field will depend on the parameters of the test body. This concept makes to reconsider the notion of relativity: not only motion becomes relative, but the space-time itself depends on the viewed test body.

As was shown in the works [18, 23], the use of seven-dimensional space-time for description of rotational motion of bodies proved to be successful. This includes motion of gyroscopes in gravitational fields, description of motion of the galactic disc related to stellar rotation, as well as change of rotational speed of the bodies with the change of the gravitational potential. From the above it can be concluded that the model of the seven-dimensional space-time shall be used to develop the generalized theory of electromagnetism.

## Derivation of twisted waves equations

We shall generalize the Maxwell's four-dimensional theory of electromagnetism by the space-time of seven dimensions to describe the twisted electromagnetic waves. Interaction of particles is described by a field of force with properties, regardless of the classical theory, characterized by a 7-vector $A_i$. Further it will be called a7-potential with the components being functions of the coordinates, time and angles of orientation [18]. Three spatial components of the 7-potential $A^k$ form a three-dimensional vector, called a field rotational potential, a time component will be called a scalar potential $A^0{=}\Phi$, and three orientating components of the 7-potential will form the field rotational potential. The index of the 7-potential $A^k$ will be lowered by the metric tensor (1)

$A_i = g_{ik} A^k$.

The Lagrange function for a charged body in the electromagnetic field takes the following form for the seven-dimensional space-time:

$$L = -mc^2\sqrt{1 - \frac{V^2}{c^2} - \frac{J_\omega \omega^2}{mc^2}} - q\Phi + \frac{q}{c}(A_i V^i) + \frac{q}{c}(A_j \omega^j). \tag{2}$$

Therefore, it is important that due to uniformity and isotropy of the space it may be supposed that the rotational part of the potential of the electromagnetic field depends only on the angle coordinates, while the dimensional part depends on the linear coordinates.

The Lagrange equations determine the equations of the charge motion in the given electromagnetic space:

$$\frac{d}{dt}\frac{\partial L}{\partial u^i} - \frac{\partial L}{\partial x^i} = 0, \tag{3}$$

where $L$ is defined by the formula (2). The velocity derivative is a generalized impulse of the body $p_i = \partial L / \partial u^i$. Then the motion equation will be as follows:

$$\frac{d}{dt}\left(p_k + \frac{q}{c}A_k\right) = \frac{q}{c}\partial_k\left(-c\Phi + (A_n u^n)\right), \tag{4}$$

where $k, n = 1, 2, 3, 4, 5, 6$. Following notation of the total differential of the 7-potential and transformation, the following will be derived:

$$\frac{dp_k}{dt} = -\frac{q}{c}\frac{\partial A_k}{\partial t} - q\partial_k\Phi + \frac{q}{c}\left(u^n \varepsilon_{knl}\varepsilon_{sdh} g^{ls} g^{dm} g^{kf} \partial_m A_f\right), \tag{5}$$

where $\varepsilon_{knl}$ - symbols similar to the Levi-Civita symbols [24], with the coefficients running the values from 1 through 6. Since dimensional and angular coordinates may not be multiplied in the considered space, the Levi-Civita symbols will have the following structure:

$$\varepsilon_{ikl} = \left\{ \begin{array}{l} -1\ \ (1,2,3);\ (2,3,1);\ (3,1,2);\ (4,5,6);\ (5,6,4);\ (6,4,5) \\ +1\ \ (3,2,1);\ (1,3,2);\ (2,1,3);\ (6,5,4);\ (4,6,5);\ (5,4,6) \\ 0 \end{array} \right\} \tag{6}$$

In the left-hand parts of the (5) there is a time derivative of the body impulse. Therefore, in the right-hand parts of the equations there is a force impacting the body. By analogy with the theory of electromagnetism, the force impacting the body will be divided into two parts: one depending on the velocity and another not depending.

The force of the first type is the electric field:

$$E_k = -\frac{1}{c}\frac{\partial A_k}{\partial t} - \partial_k \Phi. \tag{7}$$

The force of the second type is the magnetic field:

$$H_r = \varepsilon_{rdh} g^{dm} g^{hf} \partial_m A_f. \tag{8}$$

By combining the parametric (7) and (8) the first Maxwell equation modified for the seven-dimensional case may be developed:

$$\varepsilon_{rdh} g^{dm} g^{hf} \partial_m E_f = -\frac{1}{c}\frac{\partial}{\partial t} H_r. \tag{9}$$

By multiplying (8) by a seven-dimensional nabla operator $\nabla = \partial_k dx^k$ the second modified Maxwell equation may be established:

$$g^{kh} \partial_k H_h = 0. \tag{10}$$

The product of the charge density $\varepsilon$ by the seven-dimensional velocity vector $u^k$ [18] will be called a current density 7-vector:

$$j^k = \varepsilon u^k. \tag{11}$$

Three spatial components thereof form three-dimensional current density, while the rotational components form charge rotational density.

A seven-dimensional tensor of the electromagnetic field will be introduced:

$$F_{hk} = \partial_h A_k - \partial_k A_h. \tag{12}$$

By locating the field equation by the principle of least action the following equation may be developed:

$$\partial_k F^{ik} = -\frac{4\pi}{c} j^i, \tag{13}$$

which is the second pair of the Maxwell equations noted in the seven-dimensional form. By substituting different values $i$ and tensor components (12) the following equations will be established:

$$g^{km} \partial_k E_m = 4\pi \varepsilon, \tag{14}$$

$$\varepsilon_{klm} g^{lh} g^{mf} \partial_h H_f = \frac{1}{c}\frac{\partial E_k}{\partial t} + \frac{4\pi}{c} j_k, \tag{15}$$

The (10), (9) alongside with the (14), (15) define the electromagnetic field in the seven-dimensional space. Further these equations will be called the Maxwell seven-dimensional equations.

To establish equations of electromagnetic wave the Maxwell seven-dimensional equations in the inane space-time will be considered, i. e., $\varepsilon = 0$ $j_k = 0$. Let us take a seven-dimensional rotation of the (9):

$$\varepsilon_{eru} g^{rb} g^{us} \partial_b \varepsilon_{sdh} g^{dm} g^{hf} \partial_m E_f = -\frac{1}{c}\frac{\partial}{\partial t} \varepsilon_{eru} g^{rb} g^{us} \partial_b H_s.$$

By using the (15) a wave equation of the following type may be noted:

$$\varepsilon_{eru} \varepsilon_{sdh} g^{rb} g^{us} g^{dm} g^{hf} \partial_b \partial_m E_f = -\frac{1}{c^2}\frac{\partial^2 E_e}{\partial t^2}. \tag{16}$$

The product $\varepsilon_{eru}\varepsilon^{mfu}$ is the true sixth-rank tensor [25], which may expressed as a combination of product of components of an identity tensor. A proposal that the functions of time, dimensional and rotational coordinates are components of the electrical induction vector $E = E^2(t, x, \varphi)$ allows to significantly simplify the (16) as follows:

$$\left(\partial_1^2 + \frac{1}{R_{in}^2}\partial_4^2\right) E + \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E = 0. \tag{17}$$

where $R_{in}$ - a radius of inertia. The (17) in case of motion of motion of the wave along the axis $Ox$ and rotation around this axis with the phase turn by angle $\varphi$, will be solved as follows:

$$E = A \cdot \exp\left(-i\left(\omega t - \frac{2\pi}{\lambda}x - \frac{2\pi}{\mu}\varphi + \sigma\right)\right), \tag{18}$$

where $\omega$ - a cyclical oscillation frequency; $\lambda = cT$ - the wave length; $\mu = \Omega T$ - will be called a wave rotation angle, it is the angle by which the wave phase rotates per one time period; $\Omega$ - an angular velocity of the wave phase rotation; $\sigma$ - an initial oscillation phase. Therefore the developed (18) is the equation of the twisted electromagnetic wave with the phase changing depending on the time, coordinate and rotation angle.

## Twisted waves interference

Let us take two twisted electromagnetic waves $E'$ and $E''$ with equal amplitudes, equal wavelengths and equal in

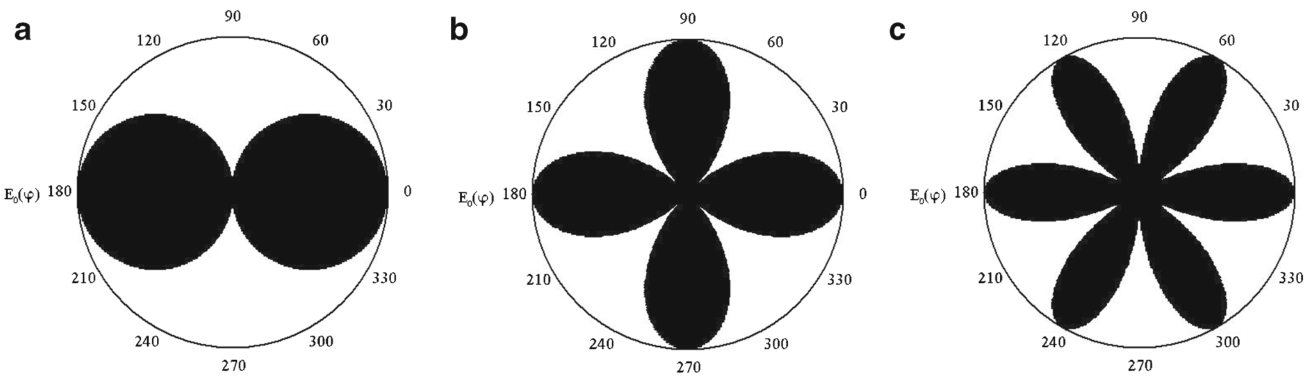


**Fig. 3** At interference of two inversely twisted waves the amplitude will depend on direction as follows: **a**) with the rotation angle $\mu = 2\pi$, **b**) with the rotation angle $\mu = \pi$, **c**) with the rotation angle $\mu = 2\pi/3$

modulus rotation angles $\mu_1 = -\mu_2 = \mu$ are distributed along the axis $Ox$:

$$E' = A \cdot \exp\left(-i\left(\omega t - \frac{2\pi}{\lambda}x - \frac{2\pi}{\mu}\varphi + \sigma_1\right)\right), \tag{19}$$

$$E'' = A \cdot \exp\left(-i\left(\omega t - \frac{2\pi}{\lambda}x + \frac{2\pi}{\mu}\varphi + \sigma_2\right)\right). \tag{20}$$

Interference of the waves (19) and (20) results in the wave:

$$E = E' + E''.$$

The initial oscillation phases may be represented as $\sigma_1 = \eta + \chi$ and $\sigma_2 = \eta - \chi$. Then as a result of the wave interference we shall get a rotating similarity of the standing wave:

$$E = 2A\cos\left(\frac{2\pi}{\mu}\varphi - \chi\right)\exp\left(-i\left(\omega t - \frac{2\pi}{\lambda}x + \eta\right)\right). \tag{21}$$

From (21) it is seen that if we take:

$$E_0(\varphi) = 2A\cos\left(\frac{2\pi}{\mu}\varphi - \chi\right)$$

as amplitude of the resulting wave, then the (21) reduces to an equation of a standard progressive wave with amplitude depending on the rotation angle $\varphi$:

$$E = E_0(\varphi)\exp\left(-i\left(\omega t - \frac{2\pi}{\lambda}x + \eta\right)\right).$$

Figure 3 shows the profiles of amplitudes.

## Angular momentum of twisted wave

By substituting (18) to (17) we develop the formula connecting wavelength, oscillation frequency and rotating velocity:

$$\frac{1}{\lambda^2} + \frac{1}{\mu^2 R_i^2} - \frac{\nu^2}{c^2} = 0.$$

By expressing the wave frequency we obtain dependability of the frequency on the wavelength and rotation angle in the gravitational field:

$$\nu^2 = \frac{c^2}{\lambda^2} + \frac{c^2}{R_i^2\mu^2}. \tag{22}$$

As the work [18] shows, the Lagrange function for a free body, in the metric tensor (1), is of the following form:

$$L = -mc^2\sqrt{1 - \frac{V^2}{c^2} - R_i^2\frac{\omega^2}{c^2}},$$

where $m$ - the weight of the body, $V$ - the translational velocity of the body, $\omega$ - the angular velocity of rotation of the body.

The impulse and angular momentum of a solid body are traditionally considered as a vector with components equal to derivatives of the Lagrange function by the corresponding velocity components. The impulse of the solid body is equal to:

$$p^k = \frac{mV^k}{\sqrt{1 - \frac{V^2}{c^2} - R_i^2\frac{\omega^2}{c^2}}},$$

while the angular momentum of the solid body is equal to:

$$p_\omega^n = \frac{J_\omega \omega^n}{\sqrt{1 - \frac{V^2}{c^2} - R_i^2\frac{\omega^2}{c^2}}},$$

where $J_\omega = m \cdot R_i^2$ - the inertia momentum of the body. The energy of the body will be calculated according to the formula:

$$E = \frac{mc^2}{\sqrt{1 - \frac{V^2}{c^2} - R_i^2\frac{\omega^2}{c^2}}}.$$

The total relativistic energy, impulse and angular momentum may be combined as the seven-dimensional impulse:

$$P_7 = \left(\frac{E}{c}, p^k, \frac{p_\omega^n}{R_i^2}\right).$$

To develop a formula connecting the energy, impulse and angular momentum, we shall find the squared 7-impulse of the solid body. Since the squared 7-velocity is equal to the squared velocity of light, then the connection between the energy, impulse and angular momentum will be equal to:

$$E^2 = m^2c^4 + p^2c^2 + \frac{p_\omega^2 c^2}{R_i^2}.$$

To describe a photon the connection between the energy, impulse and angular momentum takes the following form:

$$E^2 = p^2c^2 + \frac{p_\omega^2 c^2}{R_i^2}. \tag{23}$$

By substituting the energy of photon to $E = h\nu$ and using the formula (22) we obtain:

$$p^2 + \frac{p_\omega^2}{R_i^2} = \frac{h^2}{\lambda^2} + \frac{h^2}{R_i^2\mu^2}.$$

The first summand of the right part of the equation is the squared impulse of photon:

$$p = \frac{h}{\lambda}. \tag{24}$$

The second summand is the squared angular momentum of photon:

$$p_\omega = \frac{h}{\mu}. \tag{25}$$

Let us compare the angular moment of photon derived from the formula (25) with the orbital angular momentum of impulse $L_o = n\hbar$, derived from the quantum theory. From the formulae it is seen that quantum number $n$ is the relation of $2\pi$ to the rotation angle of wave $\mu$:

$$n = \frac{2\pi}{\mu}.$$

The formula allows to explain the physical meaning of quantum number $n$. This value is reciprocal of the part of shift of the wave phase for the period of time, that may be presented as a relation:

$$n = \frac{\omega}{\Omega}, \tag{26}$$

where $\omega$ - the cyclic frequency of electromagnetic wave, $\Omega$ - the angular velocity of shift of the wave phase.

Let us consider propagation of the electromagnetic wave through a passive torsional device, for instance, through the pitch-fork hologram. According to the energy-conservation law the energy of a non-twisted electromagnetic wave:

$$E_1 = \frac{hc}{\lambda_1}$$

prior to the propagation through the pitch-fork hologram and the energy of the twisted electromagnetic wave:

$$E_2 = \sqrt{\frac{h^2c^2}{\lambda_2^2} + \frac{h^2c^2}{R_i^2\mu_2^2}}$$

following the propagation will be equal: $E_1 = E_2$. Therefore, the reduced energy-conservation law may be noted as follows:

$$\frac{1}{\lambda_1^2} = \frac{1}{\lambda_2^2} + \frac{1}{R_i^2\mu_2^2}. \tag{27}$$

As follows from the developed (27), there is a new phenomenon of redshift at twisting light. This phenomenon is that with twisting of electromagnetic waves by the passive device the twisted electromagnetic wavelength will be greater compared to the non-twisted electromagnetic wavelength.

## Conclusion

Let us number the main results of this work obtained within the framework of generalization of the Maxwell theory of electromagnetism by seven-dimensional space-time:

1) The equations of twisted light waves within the wave model have been developed. In particular, the phenomenon of oppositely twisted light waves has been shown, which, like any phenomenon of interference, is resulting from the wave nature of light with the quantitative conformities depending on the wavelength $\lambda$ and the rotation angle $\mu$.
2) From the point of view of wave equations the physical meaning of the quantum number $n$has been explained, and the formula of the orbital angular momentum of impulse (25) for the electromagnetic wave has been developed.
3) The formula of connection of the energy, impulse and orbital angular momentum for the electromagnetic wave (23) has been obtained.
4) Based on the energy-conservation law, the new phenomenon of redshift at twisting of electromagnetic wave has been described.

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