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Facies Analysis, Markov Model and Linking of Sub-environments in the Early Permian Barakar Coal Measures of Godavari Gondwana Basin of Southeastern India

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ABSTRACT

The early Permian Barakar coal measures Formation of Gondwana Supergroup in the Godavari basin comprise siliciclastic fluvial facies of passive continental margin. The lithofacies relationship and their sequences are analyzed quantitatively using embedded Markov Chain analysis along with improved binomial probability method. Summarized lithofacies sequence show the following order: (CGSD= Matrix supported conglomerate, gritty to pebbly / coarse-grained tabular and trough cross bedded sandstones)→(MGSD= Medium grained trough and planar cross bedded sandstones)→(FGSD= Fine-grained parallel and ripple drift cross-laminated sandstones)→(SH= Shale/inter-bedded sandstone-shale and gray shale)→(C=coal/ shaly coal and carbonaceous shale)→(CGSD= Matrix supported conglomerate, gritty to pebbly/coarse-grained planar and trough bedded sandstones). These repetitive cycles accumulated in a low sinuosity, high gradient braided streams which became moderately sinuous at places. The lithofacies relationships showing fining upward tendency suggest a progressive upward decline in current competency. The sequence is considered as the classical example of a sand dominated braided river. The lateral changes in lithofacies association indicate shifting of sub-environments from stream channel, overbank levee and peat forming back swamp to flood plains of fluvial system. It is suggested that the channel wandering along with differential intra-basinal tectonism and subsidence controlled cyclic repetition during early Permian sedimentation in the Godavari basin.

INTRODUCTION

Facies analysis is a key aspect in most of the geological and geophysical studies leading to sedimentary basin analysis. In the coal bearing strata, an accumulation of diverse lithological intervals (e.g. sandstone, shale or siltstone, and coal) is a result of differential subsidence of basin floor (Maejima et al., 2008) due to varying sedimentation rate (Miall, 2013). Such information is usually obtained from the surface stratigraphic sections and the examination of core logs of the area. Additionally, a liberal interpretation of Walther's Law (Middleton, 1973) also might allow reconstruction of the lateral facies mosaic (Doveton, 1994; Parks, et al., 2000). Gradual transition from one facies to another implies that the corresponding sub environments were adjacent laterally and vertically. Subsequently the relationships between lithofacies are analyzed and a depositional model (i.e. model cycle) is structured with some confidence level. The randomness of the occurrences may be tested using Chi-square test.

Markov Chain represents a most appropriate and convenient method to model and interpret lithofacies relationship (Khan and Tewari, 2007; Hota, et al., 2012; Maria et al., 2014). It is a statistical technique that allows modeling stratigraphic sequences using transition probability from discrete state to the next (Davis, 2002). Such dependence suggests that in the sedimentary processes at a specific time the facies distribution have memory which is useful in environmental interpretation that could be demonstrated by analyzing adjacent sections (Soto et al., 2014). In fact, the Markovian analysis allows evaluating the state of change in terms of its relative probability of occurrence. The lithologies are not only repeated vertically, but partially depend from one up on another. Hence a sedimentary sequence can be described as a series of rock or bed which overlay or underlay one another with a predictable probability pattern (Davis, 2002). Nevertheless, this kind of studies is considered useful in defining the genetic relationships among the lithofacies and so as to corresponding paleo-environments within an environmental spectrum.

The purpose of the present study is to analyze lithofacies from early Permian coal bearing Barakar Formation of Godavari-Gondwana basin, southeastern India to develop a Markov chain algorithm. In view of the limited availability of outcrop sections, the deep core logs of the area are utilized for lithofacies analysis. The present analysis would help evaluate statistically cyclic characters of lithofacies and their stationarity through space and time *viz-a-viz* sub-environments of fluvial system.

GEOGRAPHICAL AND GEOLOGICAL SETTING

In the Indian subcontinent, the Gondwana basins occur mainly as outliers within elongated depressions on the Precambrian/Archaean shield. These basins are *en echelon* in three principal belts, namely Koel-Damodar, Son-Mahanadi and Godavari basins and the Gondwana Supergroup span in age from late Carboniferous/ early Permian to early Cretaceous, with an aggregate thickness of 6000-7000 m. The Godavari-Gondwana basin of south-eastern India, trending in a northwest-southeast direction (Fig. 1) for a distance of about 500 km from north of Chandrapur (19º5'N: 79º17' E) in Maharashtra to south of Aswarapet (17º15' N: 80º00' E) in Andhra Pradesh. It contains a thick pile of mainly fluviatile sediments underlain by thin glacial beds with a cumulative thickness of about 6000-7000 m; marine deposits are also intercalated at places in the basal parts. The late Carboniferous/ early Permian to lower Cretaceous stratigraphic record of the Gondwana basin is illustrated in Fig. 1. The lowermost Gondwana lithounit comprises 200-370 m thick Talchir Formation and consists of diamictite, conglomerate, pebble beds and light green sandstone and shale. These are succeeded conformably by the coal-bearing Barakar Formation which ranges in thickness from 750-350 m covering an area of about 600 sq. km. However, in the southeastern extremity of the PGG basin, the thickness of Barakar Formation is reduced to about 100m. Unlike in the Koel-Damodar Gondwana basin where coal seams are thick and laterally persistent, they are thin, laterally

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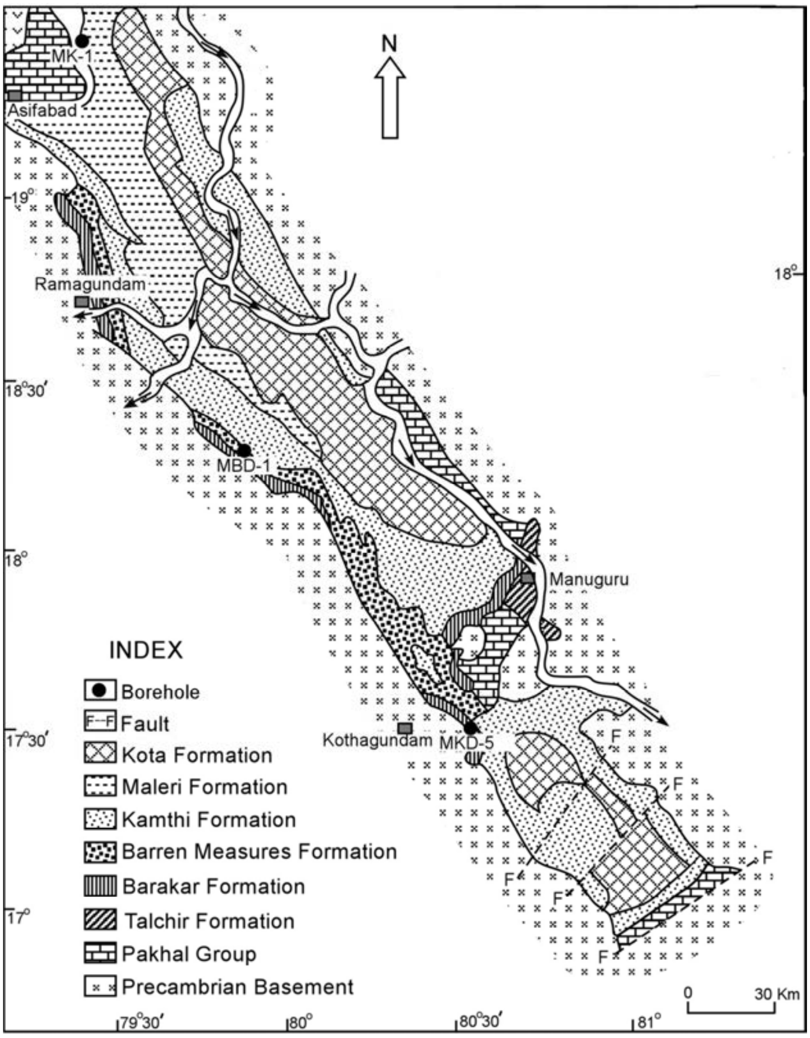

Fig.1. Geological setup of Godavari Gondwana basin showing location of studied profiles.

discontinuous and fewer in number here, and hence demarcation of Barakar overlying Barren Measures Formation is often difficult in this area.The Barakar Formation of Godavari basin is divisible into the Lower and the Upper on lithological grounds. The Lower member is 100m to 250 m thick and is characterized by the pebbly to gritty and coarse grained sandstone with lenses of conglomerate with few bands of siltstone/shale and coal seam with absence of any workable coal seam. The upper member attains a maximum thickness about 500 m and is characterized by fining upward coal-bearing cyclothems containing 2 to 12 coal seams (Murty and Rao, 1996). With the usage of satellite images, supported by deep borehole data, it is now possible to construct a detailed lithostratigraphic succession of the Permian stratigraphy in the Godavari-Gondwana basin (Table 1).

MATERIALS AND METHOD

Due to scanty nature and paucity of continuous exposures, the succession of lithofacies has been studied from about 1583.50 m cored rock sequence in the boreholes MK-001 (Kaghaznagar sub basin), MBD-001 (Mulung sub-basin) and MKD-005 (Kothagadum sub-basin) of Godavari-Gondwana basin. Individual lithofacies are identified on the basis of gross lithology, and internal sedimentary structures and boundary condition following Miall (1996) and Sengupta (2012). The lithologies observed from the cored sequences as well as on surface outcrops are condensed into five facies states and are analyzed by cumulating the data for the entire Gondwana basin. Figure 2 is a graphic representation of the parts of studied core logs showing vertical sequence of the Barakar lithofacies. Summarized lithofacies of the Barakar Formation along with their sedimentary domain is given in Fig.3. These lithofacies are:

- Facies CGSD: matrix supported conglomerate, gritty to pebbly / coarse-grained planar and trough cross bedded sandstones (*Gm, Gt-Gp*).
- Facies MGSD: medium grained trough and planar cross bedded sandstones (*Sm and St-Sp*).
- Facies FGSD: fine-grained parallel and ripple drift cross-laminated sandstones (*Sr-Fl and Fm*).
- Facies SH: shale/interbedded sandstone-shale and gray shale) (*Fl*).
- Facies COAL: coal/shaly coal and carbonaceous shale (C).

Lithofacies Interpretation

***Lithofacies CGSD** (Gm, Gt-Gp)*: This lithofacies assemblage is developed in the basal part of the Barakar Formation(4 to 10 % by volume) and occurs as elongated channel-like geometry with erosional base lying unconformably on the irregular Archaean basement (Fig.4A). Individual occurrences are 1 to 5 m thick and 5 to 50 m wide, and essentially massive enclosing polymodal, sub-angular to sub- rounded pebbles, cobbles and boulders. These clasts comprising

Table 1. Litho-stratigraphy of Pranhita-Godavari Gondwana (PGG) valley basin, Andhra Pradesh (modified after Raja Rao, 1982)

Age	Formation	Lithology
Lower to Middle Triassic	Kamthi	Compact ferruginous sandstone often cross-bedded with subordinate siltstone and clay bands
Upper Permian	Raniganj	Thick monotonous sequence of medium-coarse grained cross-bedded sandstone. Occasionally contain pebble beds.
Middle Permian	Barren Measures	Mostly cross-bedded medium to coarse grained sandstones, micaceous siltstone and associated with thick variegated shale.
Lower Permian	Barakar	*Upper*: Medium-very coarse cross-bedded sandstone with subordinate shale's and siltstone with few workable coal seams.
		Lower: Conglomeratic pebbly very coarse-coarse sandstone associated with thin impersistent coal seams.
Permo-Carboniferous	Talchir	Base typified by a tillite with unsorted and stratified rocks with outsized clasts succeeded by green cross-bedded sandstone
Proterozoic-Archaean		Pakhal phyllites and Sullavai quartzite

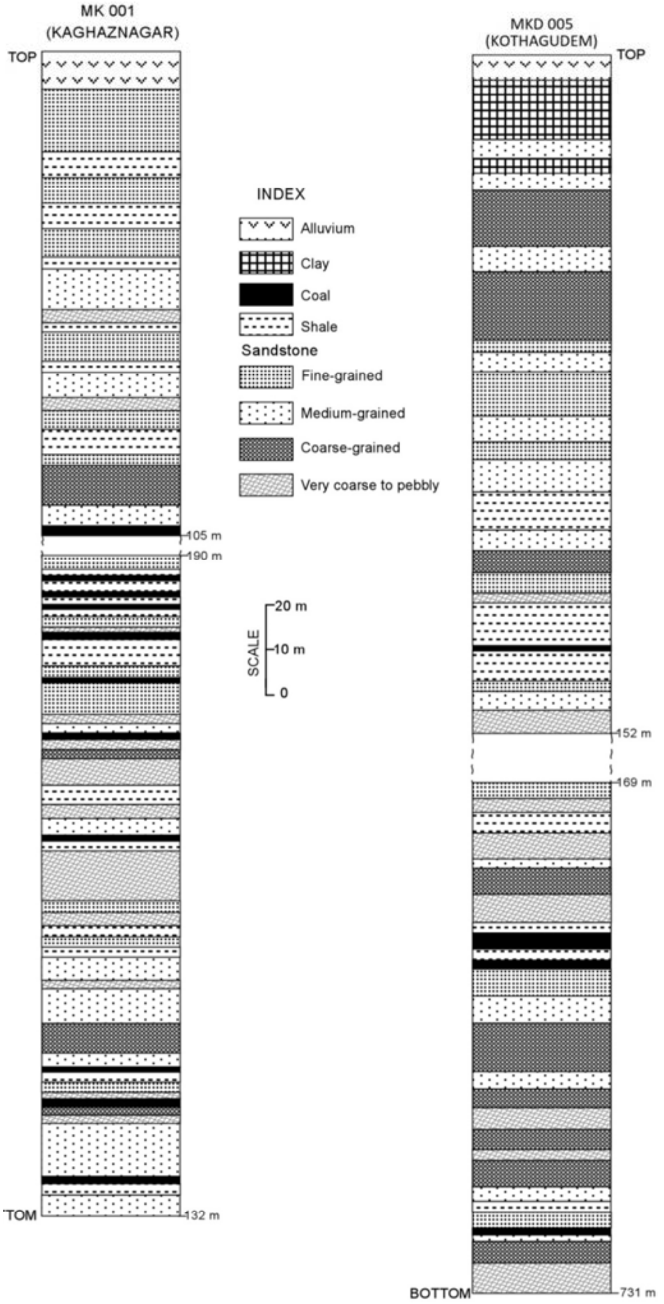

Fig.2. Parts of Barakar Formation reproduced from the bore log profiles of MK-001 from Kaghaznagar sub basin (northeast) and MKD-005 from Kothagadum sub basin (southwest)

igneous and metamorphic rocks are embedded in sandy or silty matrix with ferruginous and argillaceous cement. This lithofacies is nearly identical to gravels and conglomerates in the proximal portions of pro-glacial outwash streams (Rust, 1978) and to modern humid region alluvial fans of Japan, Taiwan (Saito and Oguchi, 2005), ancient humid region alluvial sediments (Bowman, 2018). Similar lithofacies has been interpreted as debris flow, channel-lag or longitudinal braid bars of low sinuous streams elsewhere in other Gondwana basins (Casshyap and Tewari, 1984; Sengupta, 2012). The lithofacies assemblage should represent an active bed-loaded system such as pseudo-plastic debris flow (Miall, 2010).

Matrix supported conglomerate sub-lithofacies is developed above the clast supported conglomerate lithofacies. Clasts are poorly to moderately sorted, angular to sub-rounded, embedded in clayey to sandy matrix. Frequent lateral shifting and vertical aggradations of channel bars in gravelly braided streams may have deposited these lithofacies (Casshyap and Tewari,1984).The matrix supported conglomerate sub-lithofacies is suggestive of occasional development of normal grading in a strong cohesive force (McCabe and Eyles, 1988). This lithofacies has been also attributed to fan accumulation in response to tectonic uplift along shoulders of graben/half graben or rift system (Miall, 1981) and may also resembles the outer/ distal alluvial fan facies of McGowan and Groat (1971).

The trough and planar cross-bedded conglomerate lithofacies (Gt-Gp) contains polymictic conglomerates with grey to dark grey matrix (Fig. 4 B). Facies *Gp* represents progradational down-stream deposition of gravels as transverse or longitudinal bars during flood peaks in low-sinuosity channels (Malaza et al., 2013; Tewari and Khan, 2017). Textural variations within the foresets of cross-bedding shown as alternating gravel and coarse sand dominated layers may be caused by variations in sorting due to fluctuating hydraulic conditions and gravel-clasts over-passing. The erosional base of facies *Gt* represents channel scour that was formed by avulsion at relatively high-water stage or bar dissection during a falling–water stage (Miall,1996). The presence of trough cross-bedding suggest a migration of mega bed-forms, where their lee-faces are the likely sites of avalanching and grain fall (Collinson and Thompson, 1982).

To sum up, the gravely facies association suggest deposition on low –relief longitudinal bars (sheet- bars of Miall, 2013) in the proximal (upper) reaches of braided streams. As the bars migrated, tabular foresets were added to the slip-faces of bars and horizontally stratified or poorly bedded sediments later accumulated on the tops of bars. As a result, cross-bedded sets are always overlain by horizontally stratified sandstones or pebble conglomerates. Collinson (2002) interpreted this facies as channelized debris flow.

Lithofacies MGSD (Sm, St-Sp): Sandstones are the dominant lithofacies in the Barakar sequence constituting about 70-80 % by volume; thickness of sandstone bodies ranges from about 6 to 12 m. Generally, they overlie the conglomerate and / or occur as a basal part of the fining-upward sequence and grade laterally and vertically into medium and fine-grained sandstones.

Very coarse to coarse grained sandstone occurs as small and large channel shaped and commonly form multistory bodies with concave upward and scoured base with nearly flat top. These sandstones show development of successive sets of large scale trough cross-beds with frequent planar foresets (Fig. 5 A, B). Such cross bedded cosets are attributed to down current migration of sand dune and sand waves in shallow water streams (Sengupta, 2012; Miall, 2013). The multistory and multilateral sandstone sequences indicate lateral shifting of sub-channels within low sinuosity streams and are perhaps formed when rate of migration within the aggrading channel belt is large enough to cause superposition of channel bars, before the channel belt is abandoned. Thin and lens like shale between two successive channel sandstone bodies in a vertical sequence represents suspension on top of channel bar during low flow (Collinson, 1970; Rust, 1972)

Facies *Sm* consists of sandstone that lacks any observable internal sedimentary structures, whereas, facies *St-Sp* consists of tabular and planar cross-beddings showing textural variations within. Some of facies *Sp* have angular foresets, others having asymptotic foresets. The base of this facies is typically flat. The sandstone belonging to this facies is lithic sandstone, brownish gray to dark gray, with granules concentrated in some foreset. As in the case of facies *Sp*, this facies shows textural variations, where coarse sands and granules tend to concentrate in foreset. It generally has an erosional base. Massive sandy beds of facies *Sm* perhaps formed in response to depositional processes (Yeganeh et al., 2012) or by post-depositional deformation (Allen, 1986). In the present interpretation, deformation is considered as irrelevant based on the absence of its indicators in any bed associated

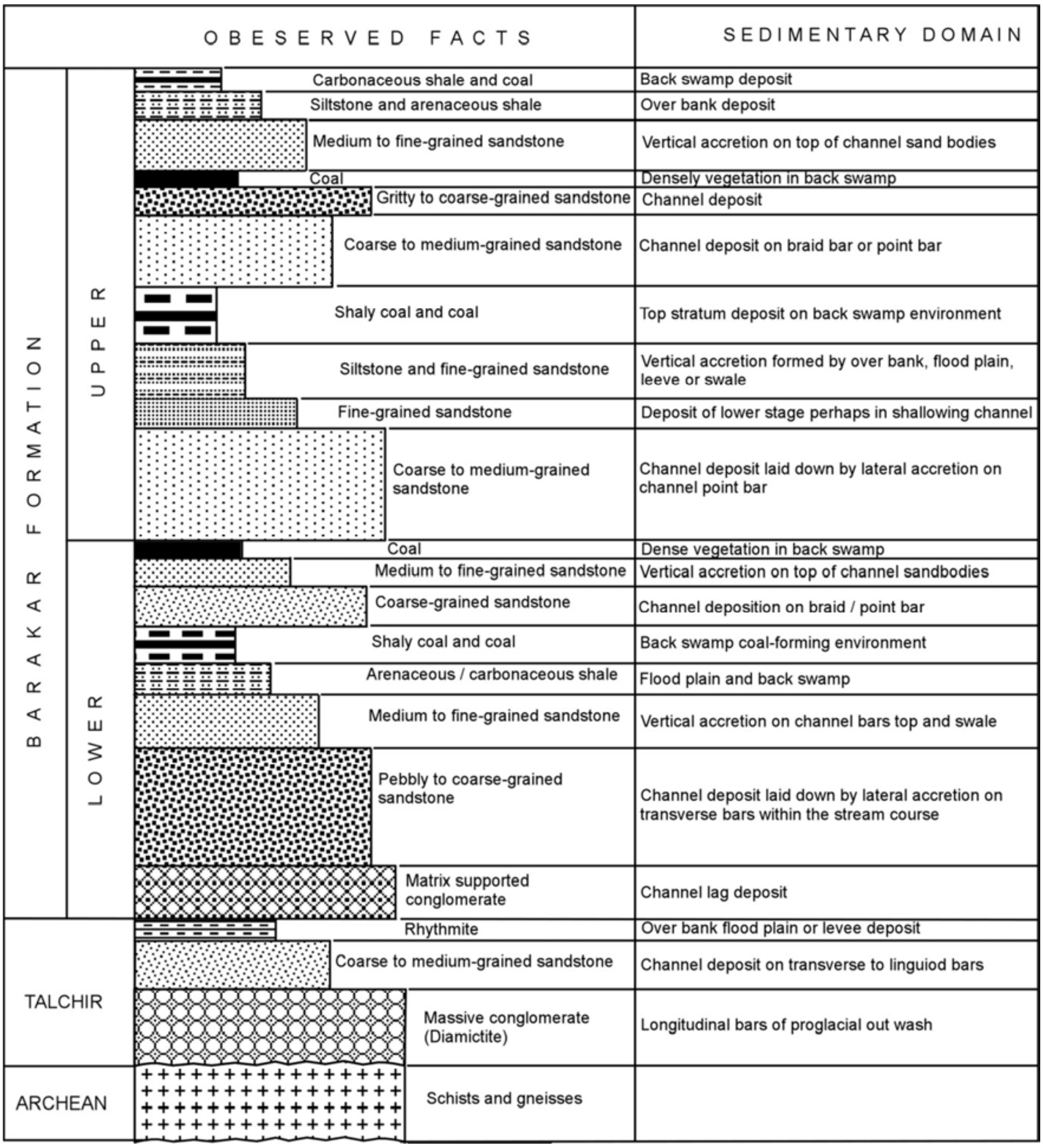

Fig.3. Summarized lithofacies order of Barakar Formation and corresponding sedimentary domain

with facies *Sm*. Accordingly, this facies is interpreted as resulting from transport and deposition of sediments by short-lived mass flow.

Facies *St*, which is characterized by the presence of trough cross-bedding and it association with facies *Sp* and *Sm*, most likely developed by the migration of 3-D dunes that stacked up to generate bar forms in channel under the conditions of the upper part of the lower flow regime (Sengupta, 2012; Miall, 2013).The presence of planar cross-bedding and the thickness of the sets of cross-beds of facies Sp suggest that it might have been formed by migration of 2-D dunes or bars while Miall (2010) interpreted it as transverse bars formed under lower flow regime. Their lee faces were the likely sites of avalanching of coarse grained sands and granules. Textural variations, where the coarser sands and granules tend to concentrate in foreset, were formed because sand is typically sorted by a process of ripple migration up to the stoss side of the dune or bars.

Lithofacies FGSD: Fine sandstone (*Sh-Sr*), Siltstone (*Fm*) and Shale (*SH*).

Thinly bedded, ripple laminated, fine grained sandstone (*Sr*) and laminated shale (*SH*) lithofacies mostly occur interbedded with each other; constitutes about 5-12 % of total bulk strata. They are as thick as 1m and 10 m and locally occur independently resting above sandy lithofacies with gradational contact.

The lithofacies (*Sh-Sr*) consists of fine to very fine grained sandstone with a variety of ripple, parallel lamination and cross-laminations (Fig. 6 A, B). Ripple cross-lamination is locally well displayed due to intermixing of carbonaceous material in fine grained sandstone and siltstone. The stratification appears as change in grain size and by the presence of very thin fine grained siliciclastic rock laminae. The base of facies *(Sh-Sr)* is flat and irregular, but not erosional. This lithofacies correspond to deposition on the upper parts of sandy bars and in abandoned flood plains of low sinuous streams during periods of decreasing discharge. The thin and lenticular occurrence of associated shale facies are suggestive of rapid shifting of channel bars.

The lithofacies (*Fm*) *and* (*SH*) consists of massive siltstone and shale lacking any distinct sedimentary structures though faint parallel-lamination is visible in places. This lithofacies is fine grained, medium bedded (10 -50 cm thick) and commonly lenticular (5- 30 m wide). Parallel lamination suggests that the sediments were deposited in

Fig.4. Massive conglomerate lithofacies lying above Archean basement in the lower part of the Barakar Formation (A); Trough cross bedded pebbly sandstone/ conglomerate lithofacies (B)

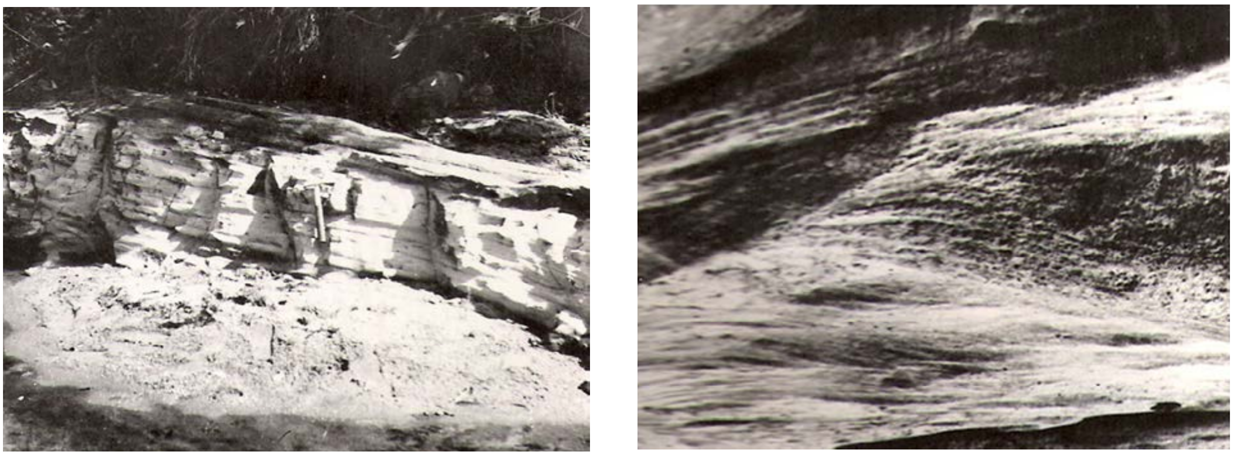

Fig.5. Planar (A) and Trough (B) cross bedded coarse to medium grained lithofacies of Barakar Formation

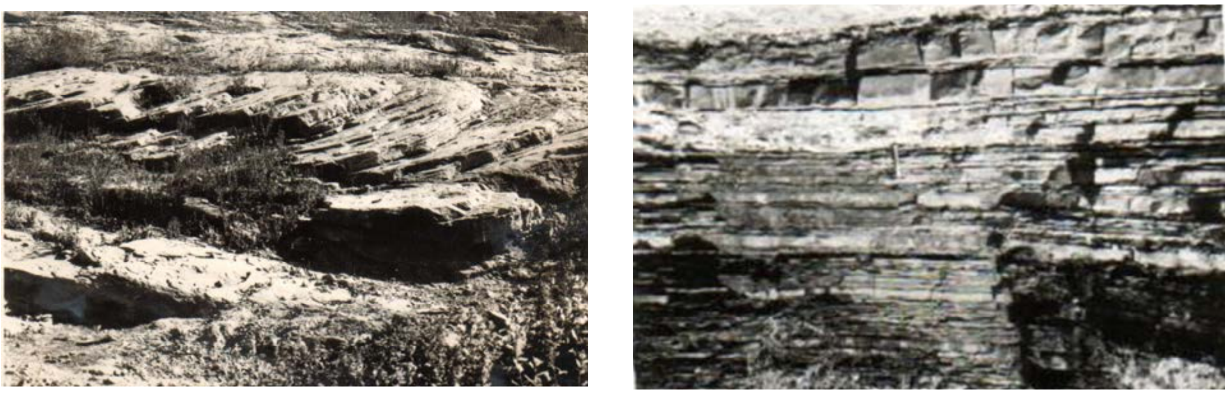

Fig.6. Ripple cross laminated fine grained sandstone lithofacies (A); Interbedded sequence of fine grained sandstone/ siltstone and laminated shale (B)

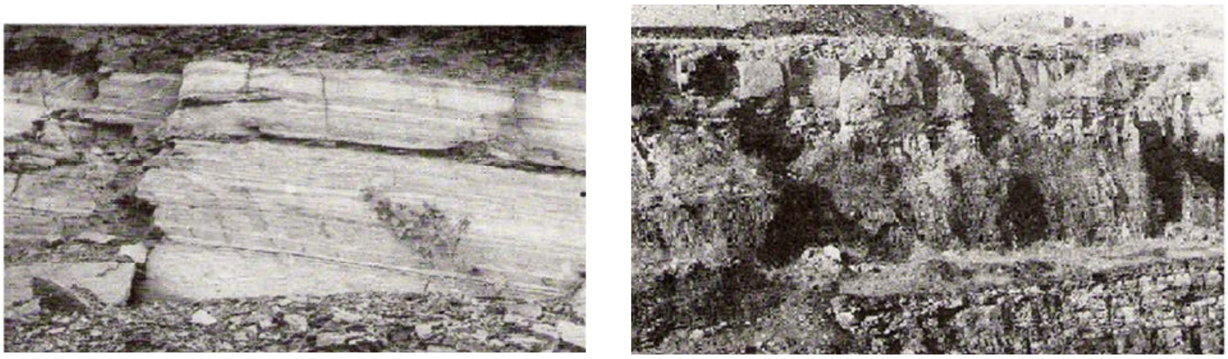

Fig.7. Thinly laminated fine grained sandstone interbedded with laminated shale (A); and vertical quarry face in an open cut mine showing laminated siltstone/shale in between coal seams (B)

moderately drained floodplain environment. Facies (*Fm*) *and* (*SH*) may represent deposition from suspension and from weak traction currents in overbank areas where sedimentation rates would be highest, or deposition in topographically low parts of the flood plains (Miall, 1996; Malaza, 2013). Rust (1978) interpreted that this facies is formed where water energy is sufficiently low to allow settling of suspended fine silt on top of channel bar. The dominant sedimentary process in such areas is suspension fallout accompanied by periodic input of current transported sands. Thick and persistent beds of interbedded fine sandstone, siltstone and shale are attributed to deposition in back swamp/flood basin surrounded by floodplain where the flow remained restricted and sluggish. Sandy bands within it probably were introduced during periodic floods.

Lithofacies (C)**:** This lithofacies is generally developed at the top of the fining upward sequence and constitutes less than 5 % of the strata; thickness varies from a few centimeters to a few meters and tends to lenses out laterally within tens of meters. It is marked by two thick and persistent coal seams and few thin and impersistent coal seams. These coal seams are usually wedged between channel sandstone bodies or may have carbonaceous shale or shale below and channel sandstone above. The coal seams include thin to thick layers of carbonaceous shale/mudstone, siltstone and fine grained sandstone, which were probably introduced during periodic floods as crevasse-splay and represent a relatively short-lived peat forming back swamp during that time. This type of association in coal seams with its top and bottom rock association is observed in rapidly shifting sandy braided river environment (Hazzeldine and Anderson, 1980).

Markov Chain Analysis

Markov chains are mathematical models of probabilistic processes, which generate random sequences of outcomes to certain probability. Detailed treatments can be found in Davis (2002) and examples applied to Sedimentology in Doveton (1994), Tewari et al., (2009), Khan and Tewari (2007), Hota et al., (2012), and Tita and Djomeni (2016). The basic premise of a Markov process is that if the outcomes of all the first *n* events of a series of events are known, then the probabilities of outcomes in the t^{th} experiments are also known. The t^{th} step transition probability is given by $p_{ij}(t) = P_r[ft=S_j/f_{t-1} = S_i]$, where p_{ij} is the transition probability (P_r) of an event *i* to *j* in which the outcome function *f* takes a value S_j at a time 's' that depends only on the directly outcome function '*f*' at the time *t*-1 having value S_i. If the set of all possible outcomes is finite, it is a finite stochastic process.

Markov chains describe Markov process if:

i) There is a finite set of outcomes depending on the outcomes before the t^{th} set, *i.e.*$P_r[ft=S_j/f_{t-1}=S_i)$ ^$p] = P_r[ft=S_j/f_{t-1}=S_i]$. This condition is called Markov property.

ii) The probability $p_{\mathbf{t+1}}$ (0) that outcome 0 will occur at trial t is known if we know what outcome occurred on trial *t*-1 i.e. $p_{ij}(t) = P_r[f_t = S_j/f_{t-1} = S_i]$

iii) The dependence of $p_{\mathbf{t+1}}$ (0) on the previous outcome is independent of 't' i.e. it is the same for trail 2 as for 100.

Embedded Markov chains (EMC) are models that do not allow self-to-self transition *i.e.* having zero entries in the dominant diagonal. Markov process can be one-dimensional or act in higher dimensions and can be conditioned on boundary conditions or not (Elfeki and Dekker, 2005). In this study, emphasis is on unconditioned, one-dimensional process with single-step transitions analogous to the expression of fluvial dynamics (Tewari et al., 2009; Hota et al., 2012; Khan and Tewari, 2013). More complex Markov chain models can be applied to use the information gained by our approach to predict facies outside the known outcrop (Parks et al., 2000; Elfeki and Dekker, 2005).

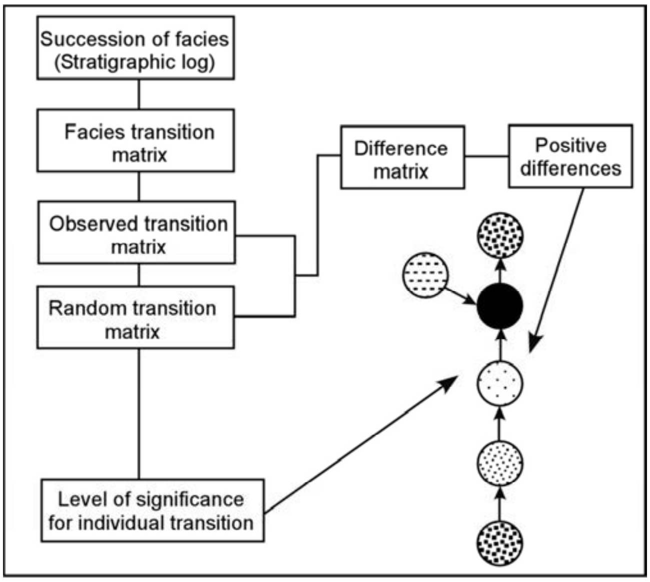

Fig.8. Flow diagram of embedded Markov Chain (modified after Walker, 1984).

Figure 8 illustrates the summary of Markov chain model used in this study. The basic objective is to derive the non-random, sequential components by subtracting the portion which can be explained by random model. To facilitate comparison between, this is best done using transition probabilities calculated from the tally matrix of observed transition. The difference matrix is then used to rank the transition on increasing non-random strength which can often be presumed to have originated from process relationship and their influence upon the succession of features concerned. Among the two methods (Embedded Markov Model and Regular Markov Model) of structuring data from actual lithologic successions, the "Embedded Markov Model" (Krumbein and Dacey, 1969) is applied in the present study to structure data matrix of lithofacies transitions.

The application of embedded Markov chain (EMC) involves following six steps:

(1) Tabulation of the transition (transition frequency matrix,(f_{ij}) from which transition probability matrix (p_{ij}) is computed by dividing each element of the transition frequency matrix by the corresponding row total *i.e.* $p_{ij} = f_{ij}/n_{i+}$;

(2) Computation of independent trails (probability) or random probability matrix $r_{ij}= n_{i+} / n_{++}-n_{i+}$ where n_{i+} is row total and n_{++} total transition in the system;

(3) Deducing difference matrix ($d_{ij}=p_{ij}-r_{ij}$);

(4) Constructing an expected frequency matrix $e_{ij} = r_{ij} .n_{i+}$

(5) Test for randomness and significance;

(6) Construction of a facies relationship diagram (FRD) for the positive values of the difference matrix or the resulting values of any significant test used.

Binomial Probability Analysis

Harper (1984) suggested that the transition probabilities greater than random must be tested for statistical significance considering a transition as significant if we cannot rule out the null hypothesis of its randomness. Thus, an objective procedure is to choose a given significance level and, for each transition between two facies, compute the probability of having at least the observed number of successions in *N* trails. This corresponds to the binomial probability (BP). Considering the null hypothesis that a transition occurs at random, it can be rejected if the binomial probability is greater than or equal to the chosen level of significance. Xu and McCarthy (1998) suggested that to reduce the risk of null hypothesis rejection, the level of significance must be chosen considerable low. Harper (1984) proposed improvement methods of facies analysis using binomial probability of at least n_{obs} succession in N_{trails}, and is given by

$$\Sigma_{n=n_{obs}}^{n=N} C\ (N, n)\ p^{n}\, q^{N-n} \qquad (1)$$

Where C (N, *n*) = the number of possible combinations of N objects taken *n* at a time, and is given by

$$C\ (N, n) = N!\ /\ (N, n)!\ n! \qquad (2)$$

p = the probability of success on a single trial. $q = 1-p$. N = the total number of upward transitions of any facies into all other facies (i.e. row totals). n = the number of upward transitions of any facies into any other facies. q = the probability of failure on a single trial.

Binomial probability of facies transition having positive values between observed and independent trail probability were chosen at significance level between 0.05 to 0.10 in the present study then marked the significant transitions *i.e.,* significant level 5-10%,and when BP 0.05-0.10 the transition is not significant. In other words reject the null hypothesis if the probability computed in step (2) is greater than or equal to the level of significance chosen, otherwise, do not reject the null hypothesis.

Test of Significance: Non parametric Chi-square test as described by Billingsley (1961) has been applied to ascertain whether the given sequence has a Markovian "memory" or no "memory". The test statistics is

$$\chi^{2} = \Sigma_{i=1}^{n}\ \Sigma_{j=1}^{n}\ (f_{ij} - f_{i}e_{ij})^{2}\ /\ f_{i}\,e_{ij} \qquad (3)$$

Such a test would have $(m\text{-}1)^2 - m$ degree of freedom for an original m X m tally matrix; f_{ij}= transition count matrix; f_i = frequency distribution of states; e_{ij} = independent trails matrix. Following the convention of Fisher, acceptance of the null hypothesis will be assumed at the 5% confidence level and rejection of the null hypothesis at the 1% confidence level.

Some statistical references (Griffiths, 1967, p.351) suggested that a chi-square test is valid only when the minimum expected frequency in any cell exceeds 5. However recent studies (Miller, 1983) show that this requirement may be relaxed so that expected cell frequencies may be in fact less than 5. So this criterion was ignored for the validity of the chi-square test, in the present study.

Markov Model, Cyclicity and Depositional Environments

Table 2 lists various Markov matrices including the transition-count, transition-probability, independent probability and difference matrix. Binomial probability for all differences with level of significance of 0.05 and 0.10 is shown in Table 3. The facies relationship diagram (FRD) for the entire area (Fig. 9) suggests that cycles are fining upward asymmetric type and strongly influenced by a Markovian mechanism. An ideal Barakar cycle consists of conglomerate or coarse grained sandstone at the base is succeeded by medium- and fine- grained sandstones, shale and coal seam at the top, which suggests a progressive decline in current competency from lower to upper sequence.Each complete cycle begin with basal conglomerate or coarse grained sandstone and terminating with coal seam, suggest the deposition within channel system and its subsequent abandonment and burial under peat backswamp. The repetitive sub-sequences are the result of lateral migration of the sub-environment, and relatively rapid subsidence to allow the accumulation and deposition of a sub-sequence. Casshyap and Tewari (1984) interpreted coal bearing finning upward Barakar cycles as channel wandering whereas thicker cycles with laterally extensive coal seams on top are attributed to differential subsidence of basin floor (Maejima et al., 2008). It is more likely that an extensive coal seam thus formed show, in different parts of the basin, a "floor" of variable lithology – sandstone (channel deposit), shale and carbonaceous shale and siltstone (over-bank deposit).

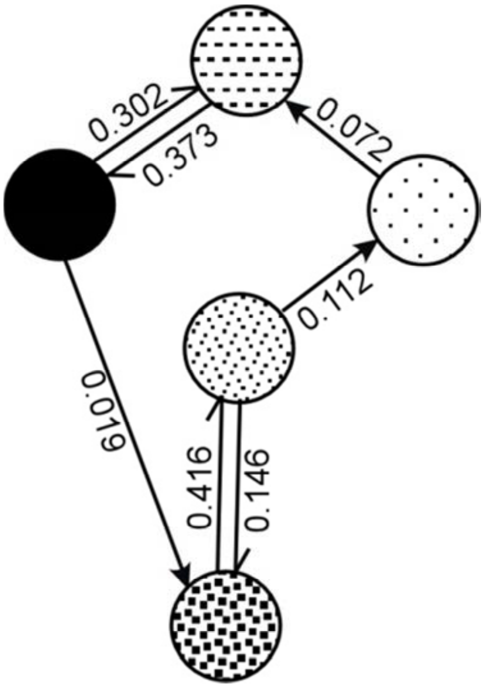

Fig.9. Facies relationship diagram (FRD) for entire area based on positive values of the difference matrix (Table 8) showing frequently upward lithofacies transition in the Barakar Formation, Godavari-Gondwana basin.

The chi-square statistics for the Barakar succession using five lithofacies types in the entire study area at an appropriate degree of freedom at 95% level of confidence is listed in Table 2. However, as pointed out by Walker (1984) and Harper (1984), in subtracting the random from observed probabilities one does not know whether a given difference is significant or not. To overcome this aspect of lithofacies analysis, Harper (1984) proposed improvement method of facies sequence analysis using binomial probability.

The onset of the Barakar sedimentation in the Godavari valley basin began commonly with erosional and irregular surface which were subsequently overlain by crudely stratified conglomerate to pebbly, stratified planar and trough cross bedded sandstone lithofacies (CGSD). These depositories representing high strength channel lag,and the channeled form of some conglomerates of the facies *Gm* is due to flows passively occupying the preexisting alluvial topography, including the channels (Miall, 1996). The graded bedding facies Gm indicates deposition from single current as the energy and flow strength diminished. The nature of its erosional base suggests that the facies was deposited following a flood that eroded the strata underneath. The presence of a matrix possibly resulted from post infiltration of open-framework gravel by sand (De Celles et al., 1991). The occurrences of trough cross bedding suggests a migration of mega bed-forms, where lee faces are the likely sites of avalanching and grain fall (Collinson et al., 2006). The probability of transition from conglomeratic facies (CGSD) to medium grained sandstone (MGSD) ranges from 61 to 74 % and corresponding binomial probability

Table 2. Cumulative Markov matrices of lithofacies transition in the Barakar Formation, Pranhita Godavari Gondwana basin

Lithofacies	CGSD	MGSD	FGSD	SH	C
A. Transition Count Matrix (f_{ij})					
CGSD	0	138	37	13	09
MGSD	106	0	64	42	13
FGSD	30	41	0	45	14
SH	30	25	16	0	97
C	38	13	10	71	0
B.Transition Probability Matrix (p_{ij})					
CGSD	0	0.700	0.187	0.066	0.046
MGSD	0.471	0	0.284	0.186	0.057
FGSD	0.230	0.315	0	0.346	0.107
SH	0.178	0.148	0.095	0	0.577
C.Random Probability matrix (r_{ij})					
CGSD	0	0.343	0.198	0.256	0.201
MGSD	0.314	0	0.207	0.268	0.210
FGSD	0.273	0.311	0	0.232	0.193
SH	0.288	0.329	0.190	0	0.161
C	0.273	0.312	0.181	0.233	0
D. Expected Frequency Matrix (e_{ij})					
CGSD	0	67.57	39.00	50.43	39.60
MGSD	70.65	0	46.57	60.30	47.25
FGSD	35.49	40.43	0	30.16	23.79
SH	48.38	55.27	31.92	0	34.42
C	36.06	41.18	23.89	30.75	0
E. Difference matrix (d_{ij})					
CGSD	0	+0.357	- 0.011	-0.191	-0.155
MGSD	+0.157	0	+0.077	-0.082	-0.153
FGSD	-0.043	+0.004	0	+0.114	-0.076
SH	-0.110	-0.181	-0.095	0	+0.384
C	+0.015	-0.214	-1.060	+0.304	0
F. Chi-square value matrix ($\chi 2$)					
CGSD	0	73.41	1.02	26.43	23.64
MGSD	17.68	0	6.52	5.55	23.39
FGSD	0.85	0.05	0	7.50	4.03
SH	6.98	16.57	7.92	0	113.77
C	0.11	19.33	8.07	52.69	0

Test of Significant

Computed value of $\chi 2$	Degree of freedom	Limiting value of χ^2 at 99% significance
415.51	15	37.7

$2.227\ e^{-18}$ which is definitely significant at 0.05 levels suggesting that due to lateral migration of the stream, the channel lag and proximal channel deposits were covered by distal channel sediments in braided stream for the lower part and was perhaps deposited vertically in sinuous stream for the upper part. The presence of planar and trough cross bedding in the facies suggest that it might be formed by migration of 2-D and 3-D dunes or bars. Their lee faces were the likely sites of avalanching of coarse- to medium grained sands. A noteworthy feature in lithofacies transition is a two way transition between (CGSD) and (MGSD), implying that interbedding of the two lithofacies and corresponding sub environments has a greater probability of occurrence in the observed data than would be expected if the lithofacies were interbedded randomly. There are 21-33% chance of (MGSD) is successively overlain by (FGSD) with binomial probabilities between 0.0001-0.0389 is less than to the level of significance chosen (0.05) indicating that the discontinuous beds of fine clastic correspond to deposition by vertical accretion on the top of channel sand bars during lower flow conditions and periods of reduced discharge. The massive nature of FGSD may be interpreted as due to migration of either low-amplitude bed-forms or deposition under the plane bed conditions of the upper flow regimen (Miall, 2013; Boggs, 2017). Alternatively, the fine grained sandstones may derive from high-energy sheet floods that spilled over from the channels into a lower-energy environment during discharges that were too voluminous to be confined in the main fluvial channel system (Ghazi and Mountney, 2009). Similarly there are 30-45% chances of shale to transit from fine grained sandstone (FGSD). These are well in agreement with the occasional preservation of fine clastic facies in the braided river complex. The shale facies (SH) may be formed under two different conditions; during flood stage and in shallow water (Miall, 1996) or in upper plane bed condition at the transition from subcritical to supercritical flows. Occurrence of parallel laminated shale suggests that the sediments were deposited in flood plain environments with frequent variation of energy condition, thus resulting in grain size variation vertically or laterally. The occasional occurrence of carbonaceous shale indicates a moderate growth of vegetation in and around the basin. Shale/mudstone show well and high (55-60%) preference to occur before coal with significant binomial probability between $1.338\ e^{-7}$ and $4.9830\ e^{-16}$ at a significant level of 0.05 indicate that coal has been deposited after abandonment of sedimentation cycles in swampy low lying areas favorable for the deposition of coal. In most of the cases coal is succeeded by coarse grained sandstone facies (CGSD) with erosional contact characterizing asymmetrical cycles and rapid to and fro shifting of channel during deposition. The thick coal seam indicates a long persistent, slowly subsiding, moderately drained, and densely vegetated back swamp, and long period of stable time, whereas, thin coal seams contain abundant carbonaceous mudstone that indicates a short lived flooding during that period (Tewari and Khan, 2015). Due to lateral migration of the stream channel the peat swamp deposits were buried under channel deposits with initiation of new cycle as coal (C) show 30 % probability of transition and binomial probability 0.0711 on area level which is significant at the 0.10 level to coarse grained sandstone or the transition between lithofacies C and lithofacies CGSD is rare, if the transition occur randomly. Rapid and frequent lateral shift of channel course, a common phenomenon in modern river basin may favorably explain the development of fining upward cycles.

The lateral migration of the sub-environments may produce the characteristics subsequences of lithofacies, provided that regional subsidence is sufficient to allow the deposits to accumulate and bury them before the constituent lithologies are eroded. And the accumulation of thick pile of lower Gondwana was possibly due to

Table 3. Cumulative Binomial Probability (BP) of lithofacies transition having positive difference values between transition probability (p_{ij}) and random probability (r_{ij}) in Pranhita Godavari Gondwana valley basin

Lithofacies transition	*p*	N	*n*	Probability*
CGSD→MGSD	0.343	197	138	0.0006
MGSD→CGSD	0.314	225	106	0.0001
MGSD→FGSD	0.207	225	64	0.0013
FGSD→SH	0.232	130	30	0.0452
SH→C	0.193	168	97	$3.7732e^{-7}$
C→SH	0.233	132	71	$2.9384e^{-14}$
C→CGSD	0.273	132	38	0.0711
FGSD→MGSD	0.273	130	30	0.0452

p= Transition Probability for random sequence (r_{ij}).
N=Total number of *i* in a row of transition count matrix (f_{ij}).
n= Total number of transition from $i \rightarrow j$ in a transition matrix.
*= BP < 0.10 is significant.

subsidence of the Godavari basin along NW-SE axis. The Markov algorithm in the Barakar succession is, however, shown to be non-stationary when examined over the area as a whole suggesting that the sub-environments presumably changed through time and space on a regional scale (Table 10) as was observed in case of Talchir Gondwana basin (Hota and Maejima, 2004). It is due to lateral variation in lithofacies types and differential rate of subsidence of channel, overbank and flood plain complexes.

Overall, the early Permian Barakar Formation of Godavari basin of southeastern India resembles to those of the South Saskatchewan river in Canada, a classical example of a sand-bed braided river (Smith, et al., 2006). Because of the relatively large size of the river channels (sand bodies up to 15 m, recorded in the bore log) that the paleo-rivers reaching the study area likely had relatively large catchment areas. Therefore, vast areas of Godavari basin were drained by river with relatively large fluvial systems running from south and southeast to north and northwest (Casshyap and Tewari, 1984). Dispensation of channel scours suggests that the channel shifting was a conspicuous feature and westward avulsion of the streams may caused the overstepping of the Barakar Formation over the older formations.

The above interpretation suggest deposition under channelized condition in moderately sinuous stream in an alluvial fan to fluvial plain setting (Fig.9). Consequently, thegradient of the stream is reduced to form comparatively fine-grained and small scale sequences with several horizons of thin to thick coal seams. The peat forming swamps were very short lived and most probably were moderate to well drained, which is indicated by the presence of repeated sequence of coarse to fine grained clastic with intervening thin seam. The thick coal seam indicates along persistent, slowly subsiding, moderately drained, and densely vegetated back swamp. The thick coal seams probably result from the combined interaction of various factors, like localized aggradations of channels, slow and steady subsidence of the basin area, abundant rainfall to grow luxuriant vegetation (Tewari and Khan, 2015). Carbonaceous shale is commonly observed as a part of facies progression capping floodplain succession in the lower reaches of a fluvial stream system. Gradually, this situation changed to more peneplained condition to deposit thick coal material (seams) in a long moderately drained to poorly drained and densely vegetated peat forming back-swamps along with sinuous streams. The claystone/ shale and siltstone were deposited in natural levee or flood plain basin. But the occurrence of carbonaceous shale/mudstone indicates more stagnant condition in back swamp and abandoned channel conditions with spares vegetation (Fig. 9). The coal-bearing Barakar cycles are statistically stationary through space in three coal sub basins across the Godavari basin suggesting that coal cycles so recorded should represent lateral association of various sub-environments in the similar manner and characterize fluvial system following Walther's Law of facies. The local versus regional changes may be attributed to broad regional variations in depositional environment that are not significant at the local scale.

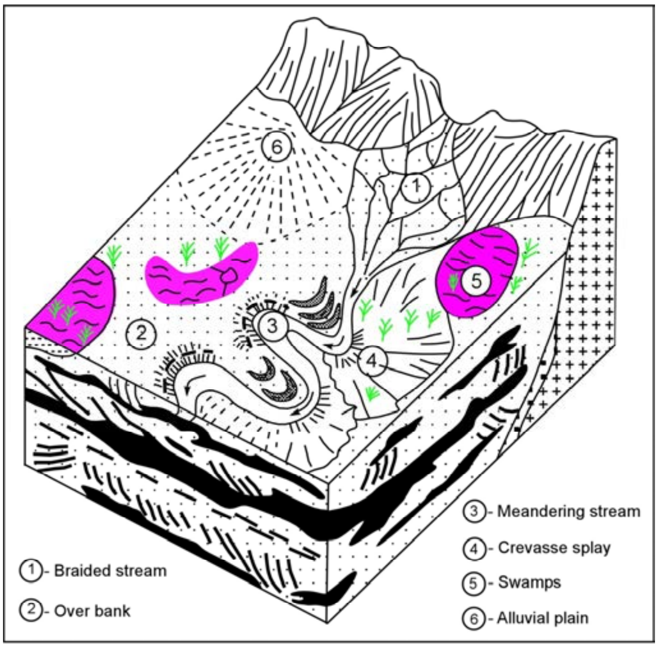

Fig.10. Block diagram showing braided to moderately sinuous streams model for the Barakar Formation of Pranhita-Godavari-Gondwana basin.

CONCLUSIONS

Markov chain analysis and binomial probability method in subsurface boreholes profiles brought out definite lithofacies relationship and in corresponding facies areas of early Permian Barakar Formation of Godavari basin. The five lithofacies recognized in the Barakar Formation indicate asymmetrical cyclic pattern through space and time. Each complete cycle exhibits upward decrease in grain size due to decline of flow intensity. The various lithofacies are interpreted as in-channel, overbank levees, and flood plain areas of low to moderately sinuous streams. Each lithofacies of fining upward cycles can be linked with different sub-environments of braided channel fluvial system and their organization may be ascribed to lateral migration of streams in response to differential subsidence. Intra basinal differential subsidence might have been responsible for the rejuvenation and consequent initiation of a successive cycle. Rapid and frequent lateral shift of channel course, a common characteristic of river channel may favorably explain the development of asymmetrical fining upward cycles.

The coal bearing Barakar cycles, charactering fluvial system following Walther's Law of Facies, and represents lateral association of various sub-environments in the similar manner. It is suggested that these Barakar cycles are autocyclic in nature: the sediment distributive mechanisms; the lateral migration of streams triggered by intra basinal differential subsidence or in other words the local versus regional changes may be attributed to broad regional variations in depositional environment that are not significant at the local scale.

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