

Assessment of Reservoir Rock Properties from Rock Physics Modeling and Petrophysical Analysis of Borehole Logging Data to Lessen Uncertainty in Formation Characterization in Ratana Gas Field, Northern Potwar, Pakistan

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ABSTRACT

Petrophysical evaluation and rock physics analysis are the important tools to relate the reservoir properties like porosity, permeability, pore fluids with seismic parameters. Nevertheless, the uncertainties always exist in the quantification of elastic and seismic parameters estimated through wireline logs and rock physics analysis. A workflow based on statistical relationships of rock physics and logs derived elastic and seismic parameters with porosity and the percentage error exist between them is given. The statistical linear regressions are developed for early Eocene Chorgali Formation between various petrophysically factors determined from borehole logging of well Ratana – 03 drilled in tectonically disturbed zone and the seismic and elastic parameters estimated through rock physics modeling. The rock physics constraints such as seismic velocities, effective density and elastic moduli calculated from Gassmann fluid substitution analysis are in harmony and close agreement to those estimated from borehole logs. The percentage errors between well logs and rock physics computed saturated bulk modulus (K_{sat}), effective density (ρ_{eff}), compressional and shear wave velocities (V_p and V_s) are 1.31%, 4.23 %, 5.25% and 4.01% respectively. The permeability of reservoir intervals show fairly strong linear relationship with the porosity, indicating that the reservoir interval of the Chorgali Formation is permeable and porous thus having large potential of hydrocarbon accumulation and production.

INTRODUCTION

The prime objective of formation evaluation is the accurate measurements of reservoir constraints such as clay content, level of water saturation, porosity, hydrocarbon saturation permeability, seismic velocities, effective density etc. (Zamanek et al., 1970; Hussain et al., 2017a). Most commonly it is presumed that on specific scale limits the reservoir rock interval is linear, isotropic and homogeneous. While, on the contrary, several spatial and vertical diversities in the reservoir rocks exist on different scales and porous rock-fluid interactions result in different petrophysical quantities (Honarpour et al., 1985; Alemu et al., 2013). Fluid replacement modeling is a significant component of rock physics studies, which provides easy ways to discriminate fluid nature and its quantity in reservoirs rock. Numerous theoretical methods of fluid replacement modeling and empirical relations (Gassmann, 1951; Duffy and Mindlin, 1957; Wyllie et al., 1958; Krief et al., 1990) have been presented to analyze the interaction between rock units and pore filled fluids in saturated rock unit and to examine the fluid saturation. These practiced models describe the association

between the modulus of the fluid filled rock (K_{sat}), the bulk modulus of the dry rock skeleton (K_{fram}), the rock-forming minerals (matrix) (K_{min}) and to the bulk modulus of the reservoir fluid (K_f). But, most of these approaches are based on some hypothesis and therefore can be practiced under specific conditions. Krief et al. (1990) presented a model that directly compute the saturated rock modulus from bulk modulus of rock grains without taking into account the modulus of dry rock. This model is almost based on empirical formulas. In their rock physics model, Duffy and Mindlin (1957), presented complex relationships to find saturated velocities based on porosity, lithology, effective pressure and mechanical compaction. They also proposed the array of identical spheres as a face centered cube to compute the Poisson's ratio and Young's modulus that are further used to estimate seismic velocities. Wyllie et al. (1956) method is based on seismic array theory and only works well when wavelength of seismic wave is smaller as compared grain and pore sizes. Properties of dry rock modulus are also not taken into account by this model and under estimate the P wave velocity in vuggy or secondary porosity reservoirs. Fluid replacement model presented by Gassmann (1951) is commonly and more frequently used in rock physics modeling as it estimates the saturated rock modulus as a function of bulk modulus of dry rock, rock forming matrix, pore fluids and reservoir porosity. Gassmann model is fairly simple and has clear physical meanings of fluid replacement equations. At production and filed development stages, it estimates the fluid replacement effects on seismic and elastic properties more precisely (Ahmed et al., 2017).

Appropriate rock physics models (RPM) must require to quantify and monitor the variation in reservoir parameters (Nguyen and Nam, 2011; Saxena et al., 2013) and it also help to achieve the feasibility study for time-lapse seismic monitoring by displaying the variations in seismic signature due to production and fluid injection associated parameters at in-situ conditions (Kazemeini et al., 2010). The effect of reservoir fluids saturations, changes in elastic properties of the reservoir at different field development stages and stress fluctuations are analyzed through the rock physics based forward modeling (Li, 2009; Mavko et al., 2009). The seismic velocities, effective density and elastic moduli are the essential properties that are estimated from the numerical analysis of geophysical well logs such as sonic transit time log and density log. Similarly, fluid substitution model (Gassmann, 1951) is also used to anticipate the bulk modulus of pore filled with fluid rock unit and effective density that is further used to compute seismic velocities at in-situ conditions.

In the present study, a statistical work flow has been developed between wireline logs and rock physics modeling derived seismic and

elastic parameters to define the uncertainty and percentage error exist between them. P and S wave velocities, effective density, bulk modulus of saturated rock estimated from well logs at reservoir zone of well Ratana – 03 and calculated by applying Gassmann’s model at the same interval are correlated and plotted against reservoir porosity. Linear relationships are established between numerous reservoir properties to assess percentage error and uncertainty in the estimation of several quantities.

GEOLOGY AND SUBSURFACE STRUCTURAL ANALYSIS OF THE STUDY AREA

The Potwar basin is the western part of the Upper Indus basin which is situated in the northern Pakistan and is located near the foothills of the Himalayas (Kazmi and Jan, 1997). The geographic position of the study area lies between latitude 32° – 34° N and longitude 70° – 74° E. This is an oil and gas producing basin that formed when the Indian and the Eurasian plates collided with each other. The Potwar marine facies have great potential of hydrocarbon that almost accounts for 48% of the world known petroleum and is still a very good prospect for oil and gas exploration wells and drilling and exploration activities (Riva, 1983). This onshore basin is surrounded on the west by river Indus, on the north by Parachinar-Muree fault, on the east by Jehlum fault and on the south by Surghar and Salt Ranges (Siddiqui et al., 1998). Satellite image of Pakistan highlighting study area (Ratana field) has been shown in Fig.1. The interpreted subsurface structures by using seismic reflection data and 2D depth map are shown in Fig. 2a and 2b respectively. The depth conversion of seismic data is carried by using seismic velocities estimated during velocity analysis by constant stack velocity methods. Initially, the root mean square (RMS) velocities are converted in interval velocities by using Dix formula (1955) and then interval velocities are further transformed into average velocities as shown in Fig. 2c. The conversion of RMS velocities (V_{rms}) into interval velocities (V_{int}) is carried out by using the the following equation:

$$V_{int,n}^2 = \frac{V_{rms,n}^2 T_n - V_{rms,n-1}^2 T_{n-1}}{T_n - T_{n-1}} \quad (1)$$

here, n denotes the number of velocity time pairs at a particular common depth point (CDP).

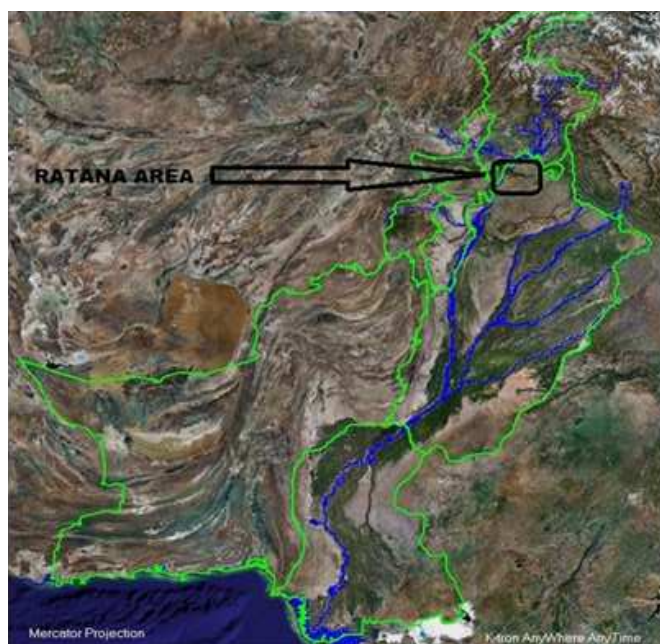


Fig. 1. Satellite image of Pakistan showing the location of the Ratana area.

Average velocities (V_{ave}) are used for time to depth conversion and are derived from the V_{int} by the equation:

$$V_{ave_i} = \frac{\sum_{i=1}^n V_{int_i} (T_i - T_{i-1})}{T_i} \quad (2)$$

These velocity functions (V_{rms} , V_{int} and V_{ave}) are plotted in the Fig. 2c. The main structural features of the area are pop up anticline, salt cored anticline, the Soan Syncline, and the Salt Range Thrust. These 2D depth images are prepared by using seismic velocities. Figure 2a and 2b demonstrates the subsurface structures mainly comprising of thrust faults with alternating anticlines and synclines.

DATA SOURCES AND METHODS TO EXTRACT THE PETROPHYSICAL AND ROCK PHYSICS PARAMETERS

In this study, we have used the seismic reflection data as well as the complete set of wireline logs such as spectral gamma ray (SGR), bulk density (RHOB), sonic transit time for compressional wave (DT), electro log deep (LLD), self-potential (SP) and neutron-porosity (NPHI). By interpreting the seismic data we have map the reservoir rock qualitatively and petrophysical study of logs data has been done for quantitative formation evaluation. Logs derived parameters like porosity, mineralogical composition, pore fluids saturation, seismic velocities and elastic moduli are further used in rock physics modeling (RPM). To measure these uncertainties between logs derived parameters and calculated via RPM, statistical regression analysis is also performed. Crossplots among different petrophysical parameters were also prepared to verify log calculated values against those derived from rock physics modeling. The complete mathematical workflow used in the current work to compute the logs and rock physics parameters is given below.

Logs Derived Parameters

Logs derived parameters (seismic velocities, density, porosity, shale volume etc.) are important ingredients of RPM. The compressional wave velocity (V_p) is calculated by taking the reciprocal of sonic interval transient time (Δt). Since shear wave log is not available, therefore Castagna’s well-known formula (Castagna et al., 1985) is used to find S wave velocity (V_s). The reservoir density is computed via density log (RHOB).

Evaluation of porosity from the well logs data is an important tool that allows a better characterization of the reservoirs under study in their technical and economical contents (Azzam and Shazly, 2012). Wyllie et al. (1956) gave a velocity-porosity relation used to find reservoir porosity by using transient time measured by sonic tool (Δt), transient time of interstitial fluids (Δt_L) and transient time rocks matrix (Δt_{ma}). The Wyllie’s time average equation is given below.

$$\Delta t = \phi \Delta t_L + (1 - \phi) \Delta t_{ma} \quad (1)$$

After calculating V_p , V_s and reservoir density (ρ_{log}), the bulk modulus of saturated rock from wireline logs is estimated as

$$K_{sat} = \rho_{log} (V_p^2 - 4/3V_s^2) \quad (2)$$

The quality of reservoir also depends on the amount of clay present in it. Therefore quantifying the shale volume (V_{sh}) is also very important (Ahmed et al., 2017; Hussain et al., 2017b). By estimating gamma ray index (I_{GR}) from SGR log, shale volume is estimated with the help of different mathematical formulas (Larionov, 1969; Stieber, 1970; Clavier et al., 1977). The water saturation within reservoir pores is estimated from resistivity log by using Archie’s equation (1942).

Rock Physics Parameters

In this section, the complete quantitative workflow used to extract the rock physics parameters (P and S wave velocities, effective density

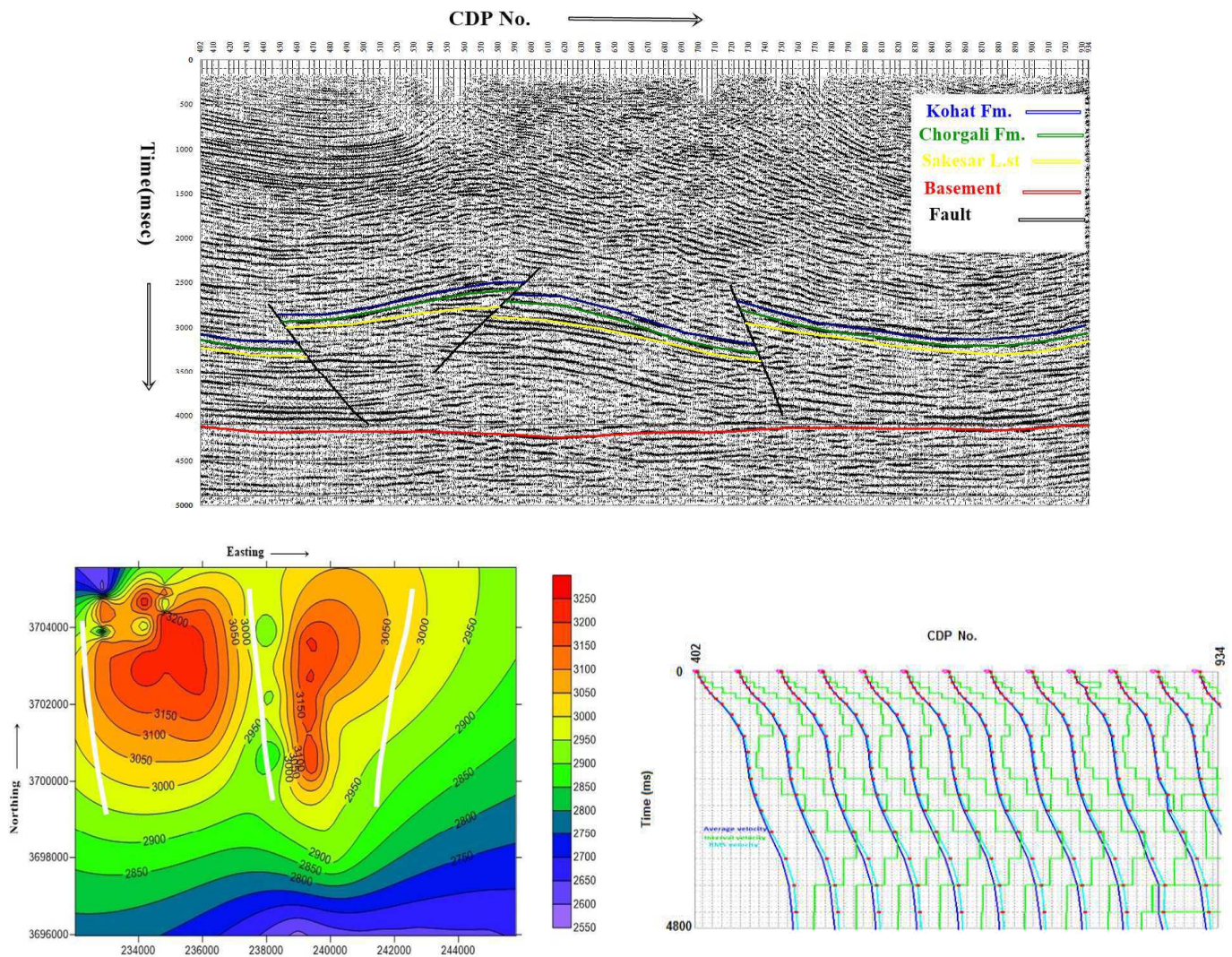


Fig. 2. (a) Interpreted seismic section (NP86-09) describing the subsurface structures and faults. Four formations: Kohat Formation, Chorgali Formation (reservoir), Sakesar Limestone and Basement rock are mapped. The study area is a thrust bounded and east-west trending pop-up structure. The subsurface structures are showing complete disharmony with due to sub thrust play. **(b)** 2D depth contour map showing the alternate anticlines and synclines at Chorgali level are presented. **(c)** Seismically derived velocities V_{avg} (average velocity in blue color), V_{rms} (root mean square velocity in sky blue color with red nodes) and V_{int} (interval velocity with green) are displayed at different common depth points (CDPs) of seismic line NP86-09.

and saturated rock bulk modulus) by applying rock physics modeling are discussed. Bulk modulus of reservoir rock is a function of dry rock modulus (K_{frame}), reservoir fluids modulus (K_{fl}), modulus of rock forming matrix (K_{matrix}) and reservoir porosity (ϕ) as presented by Gassmann's relation (Gassmann, 1951) and is given below.

$$K_{sat} = K_{frame} + \frac{\left(\frac{1 - K_{frame}}{K_{matrix}}\right)^2}{\left(\frac{\phi}{K_{fl}} + \frac{(1 - \phi)}{K_{matrix}} + \frac{K_{frame}}{K_{matrix}^2}\right)} \quad (3)$$

The input constraints required for Gassmann's algorithm are calculated by lab analysis of core samples or from wireline logs by using well known mathematical relations (Khalid and Ahmed 2016). All the functions of Eq. 2 by applying different formulas such as for K_{fl} by using Wood's relation (1941), K_{frame} discussed by (Zhu and McMechan, 1990), and K_{matrix} by VRH average method have been computed (Voigt, 1910; Reuss, 1929; Hill, 1952).

The shear modulus of saturated rock (μ_{sat}) is not affected by pore fluid can be determined by using density (ρ_{log}) and shear wave velocity log as given below.

$$\mu_{sat} = \rho_{log} V_s^2 \quad (4)$$

The principal objective of RPM is to compute the seismic velocities (P and S) and effective density at borehole (in-situ) conditions like temperature, pressure, mineral ingredients, brine salinity, reservoir porosity and pore filled fluids type (brine, oil or gas) and saturation level. Compressional (V_p) and shear (V_s) wave velocity can be computed using known moduli (bulk and shear modulus) and effective density as

$$V_p = \left(\frac{K_{sat} + 4\mu_{sat}/3}{\rho_{eff}}\right)^{1/2} \quad (5)$$

$$V_s = \left(\frac{\mu_{sat}}{\rho_{eff}}\right)^{1/2} \quad (6)$$

Whereas the effective density (ρ_{eff}) of saturated rock as a function

mineral density (ρ_{min}) and fluid density (ρ_f) is estimated by using equation as

$$\rho_{eff} = (1 - \phi) \rho_{min} + \phi \rho_f \quad (7)$$

After calculating the seismic and elastic parameters such as V_p , V_s , ρ_{eff} and K_{sat} from wireline logs and rock physics modeling by using above described quantitative workflow (from Eqs.1–7), their crossplots are made against reservoir porosity. The results of RPM and wireline logs are related with each other. Statistical analysis is also performed to quantify the error and uncertainty between RPM and logs derived values. The input constraints used for rock physics analysis and Gassmann’s fluid substitution modeling are given in the Table 1.

The percentage error is measured by using the following equation:

$$\%Error = \frac{X_2 - X_1}{X_1} \times 100 \quad (8)$$

Here the X_1 and X_2 are the logs and rock physics derived values. But the greater value is always in the place X_2 , either it is derived from RPM or wireline logs so as to express percentage error. The percentage error found in P and S wave velocities, effective density, saturated bulk modulus computed from rock physics with respect to these parameters are derived from borehole logs. These two final outputs from both methods (logs and rock physics) are then cross plotted against porosity and the percentage error corresponding to each sample to reduce the uncertainty in evaluation of reservoir rock in Ratana gas field.

RESULTS

The results obtained by applying above described quantitative work flow is elaborated. This section is mainly categorized into two parts. In first part, the results of petrophysical studies are described and all the parameters required for rock physics modeling are extracted. While in the second part, the relationships between reservoir porosity and physical parameters such as saturated rock bulk modulus, effective density, P and S wave velocity etc. derived from wireline logs and from rock physics modeling are made and the error (%) between wireline logs and rock physics parameters is also presented.

Table 1. Input parameters used in rock physics modeling and Gassmann fluid substitution are given. These properties are derived by using the mathematical formulas given in Batzle and Wang (1992) and Mavko et al. (2009)

Parameters	Symbols	Numerical values	Units
Bulk modulus of calcite	$K_{calcite}$	70.2	GPa
Bulk modulus of clay	K_{clay}	21.00	GPa
Density of calcite	$\rho_{calcite}$	2.71	gm/cm ³
Density of clay	ρ_{clay}	2.58	gm/cm ³
Calcite percentage	V_{cal}	84.00	%
Clay percentage	V_{clay}	16.00	%
Bulk modulus of matrix	K_{matrix}	56.00	GPa
Bulk modulus of dry rock	K_{frame}	26.55	GPa
In-situ density of water	ρ_w	0.960	gm/cm ³
In-situ density of brine	ρ_{brine}	1.178	gm/cm ³
V_p in water at in-situ condition	V_{p-w}	1582.5	m/s
V_p in brine at in-situ condition	$V_{p-brine}$	1729	m/s
Bulk modulus of brine	K_{brine}	3.524	GPa
Specific gravity of gas	SG	0.6	
Gas constant	R	8.314	
Bulk modulus of gas	K_{gas}	0.1205	GPa
Bulk modulus of rock fluid	K_{fl}	0.233	GPa

Petrophysical Evaluation

Petrophysical characteristics of the Early Eocene Chorgali Formation of Ratana gas field have been assessed through the analysis of wire line logging records of an exploratory well Ratana 3. The detailed petrophysical study of the of reservoir interval (4780–4840 m) of Chorgali Formation by using a complete set wire line logs such as SGR, DT, RHOB, LLD etc. is shown in Fig. 3. Volume of shale (V_{sh}) is one of the most important petrophysical parameter, required to define reservoir quality as well as reservoir character. Shale volume is calculated to estimate shale contents in the reservoir. The analysis reveals that the Chorgali Formation mainly consists of carbonate minerals (about 80 % dolomite), however some clayey minerals (about 4 20%) are also present. The spectral gamma ray (SGR) curve shows small values (near about 30 %) in the reservoir zone (Fig. 3) indicating the presence of small radioactive minerals. Hydrocarbon saturation

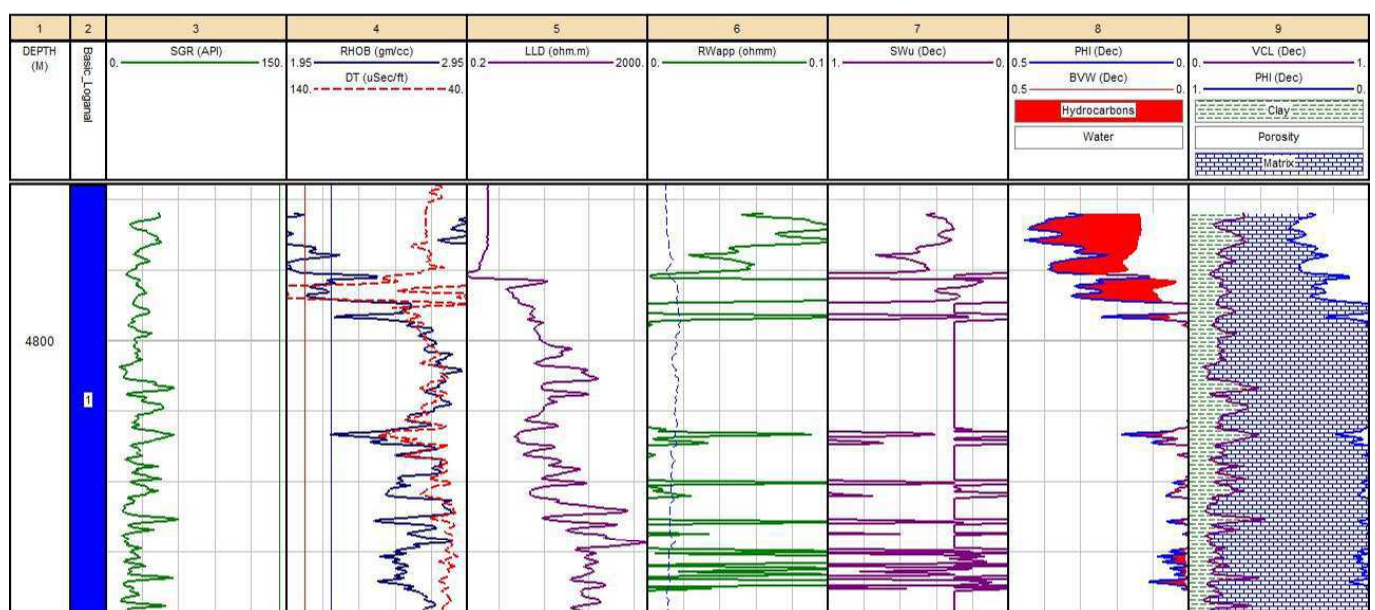


Fig. 3. Petrophysical evaluation of Chorgali Formation in Ratana – 03 well. Various input log curves such as spectral gamma ray (SGR), density (RHOB), sonic (DT), electro log deep (LLD) and derived log curves including water resistivity (RWapp), water saturation, porosity (PHI), bulk volume of water, volume of clay (V_{cl}) and matrix are displayed at Chorgali Formation zone.

affected by resistivity and conductivity logs. The water saturation curve demonstrates that the fluids saturation in the reservoir (from 4780–4800 m) is about 0.31 % of gas and 0.69 % of water. The average reservoir porosity is approximately 20 %. The zone (red colour) with high porosity and resistivity with low water saturation has been marked as region containing hydrocarbon.

Relations between Porosity and Rock Physics/Well Logs derived Parameters and Error (%) Analysis

It is worth mentioning that the methodology adopted in this work is the integration of the rock physics modeling and petrophysical analysis. The main advantage of this integration is to link between elastic and petrophysical properties of the rock-fluid composite, which reduced the uncertainty in prediction reservoir properties.

The physical properties of Chorgali Formation such as effective bulk modulus, effective density, P and S wave velocities computed by rock physics analysis at bore well conditions (temperature, pressure, fluid saturation, reservoir lithology etc.) are calibrated with well logs (sonic and density). Rock physics modeling is applied in the reservoir intervals by considering the uniform distribution fluids within the pores.

The substantial relationships between porosity and seismic parameters exist in the Chorgali Formation. The logs derived porosity has inverse relations with all other seismic constraints. The relationship

of compressional wave velocity derived from sonic log and RPM with porosity is shown in the Fig. 4a. Primary wave velocity (V_p) results have a good correlations with reservoir porosity. The P-wave velocities measured from sonic transit log are in very close association with those computed by RPM with small % age of error (is 4 5.25 %) as demonstrated in Fig. 4b. The porosity-velocity relationship is determined using a linear regression with regression coefficient $R^2 \sim 0.66$. The P wave velocity predicted from the sonic and RPM in the reservoir zone varies from 3000–5500 m/s.

The shear wave velocity is derived from the compressional wave velocity using Castagana’s relation (1985), while for rock physics modeling Eq. 6 is used. The logs and RPM V_s as a function of porosity (ϕ) are plotted in Fig. 5a. S wave velocities computed via logs and RPM have strong correlation and are inversely related to the porosity. Shear wave velocities have high regression coefficient (R^2 4 0.69) as compared to compressional wave velocity but with small percentage error (4 4.01 %) between V_s calculated from logs and rock physics model (Fig. 5b).

Figures 6a and 6b present the statistical analysis of logs and RPM derived effective density as a function of reservoir porosity. The logs density is derived from the density log (RHOB) and the RPM density has computed by using Eq. 7. The effective density and porosity have good correlation with regression coefficient 4 0.769 (Fig. 6a). The

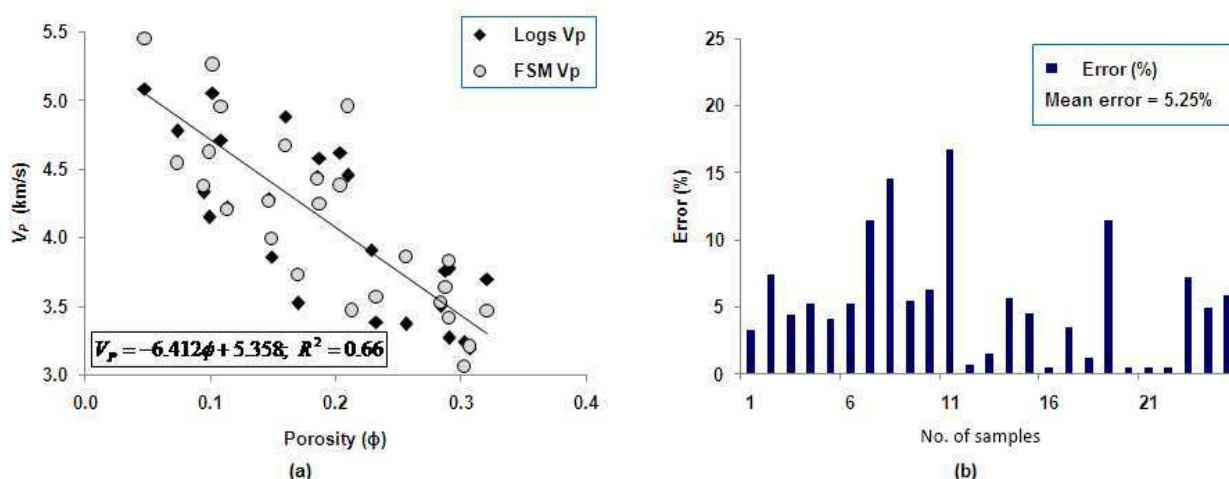


Fig. 4. (a) Relationship between logs and fluid substitution model (FSM) derived P-wave velocity (V_p) and porosity (ϕ) with their regression coefficient (R^2 4 0.66) value is shown. (b) Percentage error between P-wave (V_p) values of well logs and FSM is given. Velocities derived from both methods are very close to each other with mean error (5.25 %).

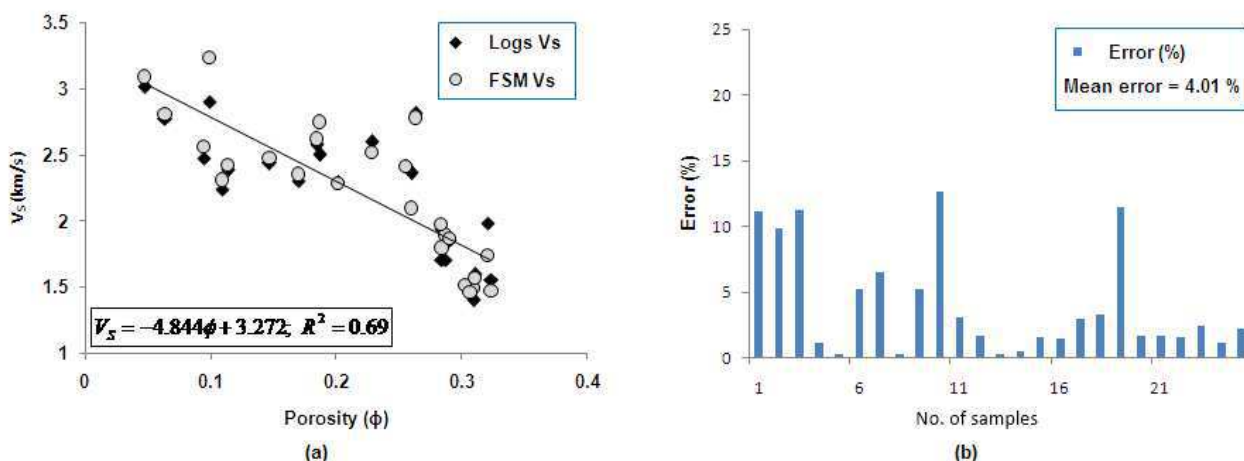


Fig. 5. (a) Statistical relation between well logs and fluid substitution model estimated S-wave velocity (V_s) and porosity (ϕ). Both porosity and shear wave velocity have high regression coefficient (R^2 4 0.69). The percentage error (4.01) between S-wave velocities (V_s) values of show the validity of Gassmann’s equation to predict seismic velocities in the study area.

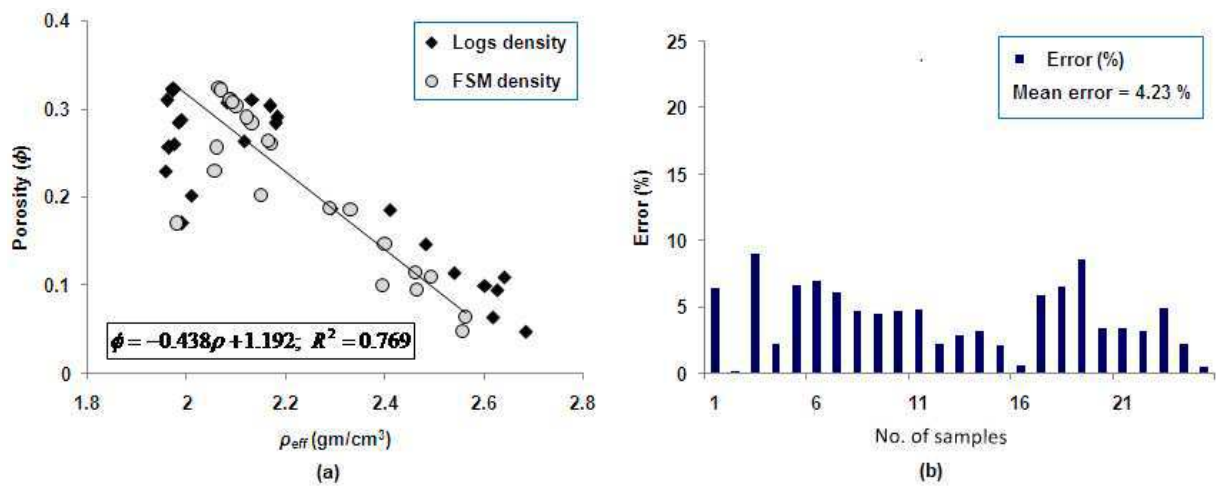


Fig. 6. (a) Effective density computed for Chorgali interval is plotted versus reservoir porosity (ϕ). (b) Mean error (4.23 %) value present between effective density values (ρ_{eff}) from density log and FSM are also portrayed.

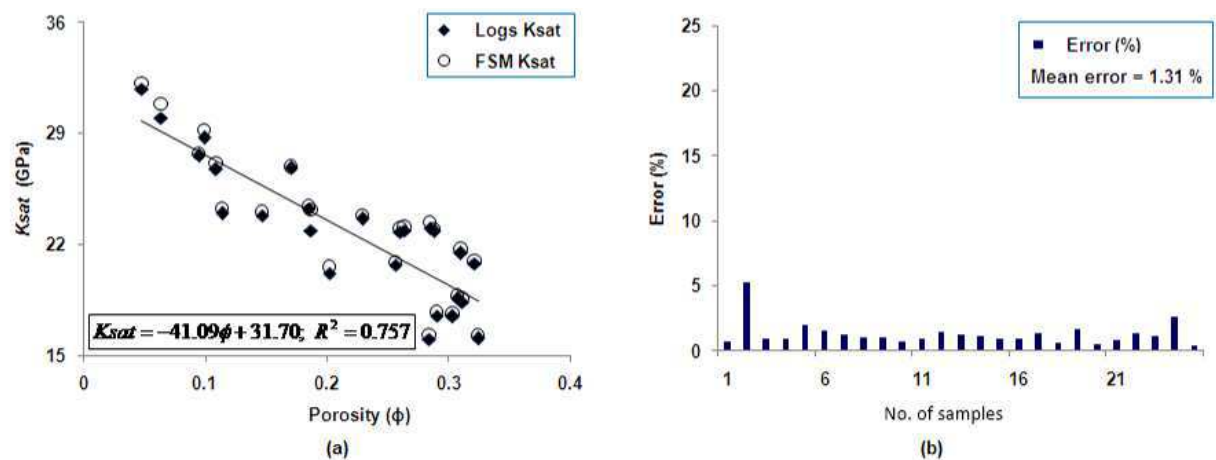


Fig. 7. (a) Sonic log and FSM obtained saturated bulk modulus (K_{sat}) are plotted against porosity (ϕ) and have high regression coefficient (R^2 0.757). (b) Mean percentage error value between bulk moduli is also displayed. K_{sat} derived by both methods shows very close agreement with each other.

percentage error analysis between logs and RPM density is given in the Fig. 6b.

The modulus of incompressibility calculated from borehole logs by using Eq. (2) is linked with K_{sat} derived from RPM by using Eq. (3) as a function of porosity (ϕ) values for the reservoir zone of well Ratana 3 (Fig. 7a). The inverse relation between K_{sat} and ϕ exists in the well Ratana 3 with robust correlation coefficient (R^2 0.757) is found. The percentage error analysis between derived logs and RPM has also been presented in the Fig. 7b. Both logs and RPM derived values show good correlation (Fig. 7b) with small percentage of error (1.31 %).

Relation between Porosity and Permeability

Porosity and permeability are the two essential parameters that describe the reservoir quality and to estimate the hydrocarbon reserves. Therefore it is important to analyze the variation trend of porosity and permeability in the reservoir zone. In Figure 8, the porosity and permeability along with regression coefficient (R^2 0.66) are plotted against each other. A linear regression is established to define the association between porosity and permeability.

CONCLUSION

The statistical relationships of rock physics and well logs derived parameters with porosity for early Eocene Chorgali Formation are

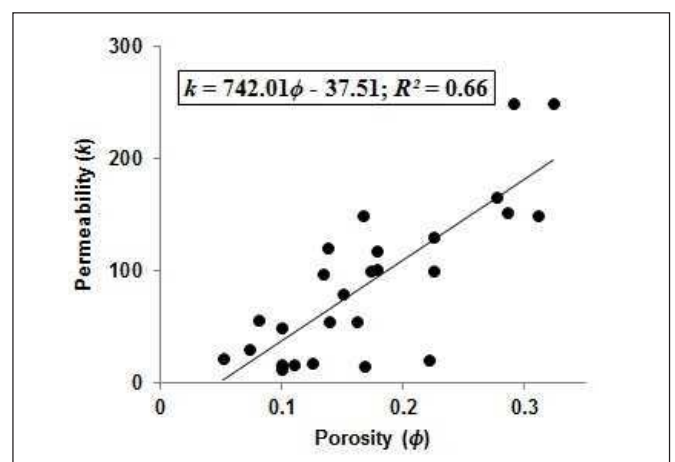


Fig. 8. Porosity and permeability crossplot at reservoir zone. Permeability increases with increase in porosity.

developed. The P and S wave velocities, effective density and saturated rock modulus derived by using Gassmann Fluid replacement algorithms show close agreement with the logs derived velocities, effective density and bulk modulus. The percentage of errors between logs and rock physics derived saturated bulk modulus (K_{sat}), effective

density (ρ_{eff}), P wave velocity (V_p) and S wave velocity (V_s) varies and lies between 1.31 – 5.25 %. The highest percentage of error (5.25 %) exists between compressional wave velocities derived from both methods. While K_{sat} measured by both algorithms shows less error (1.31 %) because the dry rock modulus is calculated by reversing the Gassmann equation. The statistical analysis shows that Gassmann fluid replacement model gives very accurate results and hence very practical for reservoir evaluation and can be used for field development and reservoir monitoring.

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