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ORIGINAL RESEARCH

Existence and Exponential Stability of Anti-periodic Solutions of High-order Hopfield Neural Networks with Delays on Time Scales

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Abstract On time scales, a class of delayed high-order Hopfield neural networks are considered. We establish some sufficient conditions on the existence and exponential stability of anti-periodic solutions for the following Hopfield neural networks with time-varying and distributed delays

$$\begin{aligned} x_i^{\Delta}(t) = & -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_j\left(x_j(t-\gamma_{ij}(t))\right) \\ & + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t) \int_0^{\infty} k_{ij}(\theta) g_j(x_j(t-\theta))\Delta\theta \int_0^{\infty} k_{il}(\theta) g_l(x_l(t-\theta))\Delta\theta \\ & + \quad I_i(t), \quad i = 1, 2, \ldots, n \end{aligned}$$

on time scales. Finally, an example is given to show the effectiveness of the proposed method and results.

Keywords Time scales · Time-varying delays · Distributed delays · Anti-periodic solution · High-order Hopfield neural networks

Introduction

Hopfield neural networks have been extensively studied and developed in recent years, and there has been considerable attention in the literature on Hopfield neural networks with time delays (see, e.g., [1–7]). Since high-order Hopfield neural networks (HHNNs) have a stronger approximation property, faster convergence rate, greater storage capacity, and higher fault tolerance than lower-order Hopfield neural networks, the study of high-order Hopfield neural networks has recently gained a lot of attention. Moreover, there have been extensive results on

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the problem of the existence and stability of equilibrium points, periodic solutions and almost periodic solutions of high-order Hopfield neural networks in the literature. We refer the reader to [8–14] and the references cited therein. Continuous-time and discrete-time Hopfield-type neural networks have been applied to model identification, optimization, etc. Recently, in order to unify the study of continuous-time and discrete-time Hopfield-type neural networks, Ref. [15] has introduced and studied the existence and exponential stability of periodic solution to the following high-order Hopfield neural network with delays on time scales

$$\begin{aligned} x_i^{\Delta}(t) = &-c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_j\left(x_j(t-\gamma_{ij}(t))\right) \\ &+ \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t) \int_0^{\infty} k_{ij}(\theta) g_j(x_j(t-\theta))\Delta\theta \int_0^{\infty} k_{il}(\theta) g_l(x_l(t-\theta))\Delta\theta \\ &+ \ I_i(t), \quad i = 1, 2, \ldots, n, \quad t \in (0, \infty)_{\mathbb{T}}, \end{aligned} \tag{1}$$

where $\mathbb{T}$ is an ω-periodic time scale which has the subspace topology inherited from the standard topology on $\mathbb{R}$, n corresponds to the number of units in a neural network, $x_i(t)$ corresponds to the state vector of the ith unit at the time t, $c_i(t)$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs, $a_{ij}(t)$ and $b_{ijl}(t)$ are the first- and second-order connection weights of the neural network, $\gamma_{ij}(t) \geq 0$ corresponds to the time delay required in processing and transmitting a signal from the jth unit to the ith unit at time t, k_{ij} is the kernel, and $I_i(t)$ denotes the external inputs at time t, f_j and g_j are the activation functions of signal transmission.

However, very few results are available on the existence and exponential stability of anti-periodic solutions for neural networks, while the existence of anti-periodic solutions plays an important role in characterizing the behavior of nonlinear differential equations (see [16–19]).

Motivated by the above, the main aim of this paper is to study the existence and exponential stability of anti-periodic solutions of (1). Without loss of enerality, we assume that $0 \in \mathbb{T}$. For each interval J of $\mathbb{R}$, we denote $J_{\mathbb{T}} = J \cap \mathbb{T}$.

The system (1) is supplemented with initial values given by

$$x_i(s) = \phi_i(s), \quad s \in (-\infty, 0]_{\mathbb{T}}, \quad i = 1, 2, \ldots, n,$$

where $\phi_i(\cdot)$ denote continuous functions defined on $(-\infty, 0]_{\mathbb{T}}$.

Throughout this paper, we assume that

(H_1) $c_i(t) \in C(\mathbb{T}, \mathbb{R}^+)$, $a_{ij}, b_{ijl}, I_i \in C(\mathbb{T}, \mathbb{R})$, $c_i(t+\omega) = c_i(t)$, $a_{ij}(t+\omega) = a_{ij}(t)$, $b_{ijl}(t+\omega) = -b_{ijl}(t)$, $I_i(t+\omega) = -I_i(t), \gamma_{ij} \in C(\mathbb{T}, \mathbb{T})$, $\gamma_{ij}(t+\omega) = \gamma_{ij}(t)$, i, j,
$l = 1, 2, \ldots, n$.

(H_2) Functions $f_j, g_j \in C(\mathbb{R}, \mathbb{R})$, $f_j(u) = -f_j(-u)$, $f_j(0) = 0$, $g_j(u) = g_j(-u)$, and there exist constants $L_j, H_j, N_j > 0$ such that $|f_j(u_1) - f_j(u_2)| \leq L_j|u_1 - u_2|$, $|g_j(u_1) - g_j(u_2)| \leq H_j|u_1 - u_2|$, $|g_j(u_1)| \leq N_j$, for all $u_1, u_2 \in \mathbb{R}$, $j = 1, 2, \ldots, n$.

(H_3) The delay kernels $k_{ij} : [0, \infty)_{\mathbb{T}} \to \mathbb{R}^+$ are real-valued piecewise continuous, and there exists an $\alpha_0 > 0$ such that functions

$$K_{ij}(\alpha) = \sum_{m=1}^{\infty} e_{\alpha}(0, m\omega) \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)\Delta\theta,$$

are right-dense continuous for $\alpha \in [0, \alpha_0)_{\mathbb{T}}$ and $K_{ij}(0) = 1, \quad i, j = 1, 2, \ldots, n.$

Preliminaries

In this section, we shall first recall some basic definitions and lemmas which are used in what follows.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \qquad \rho(t) = \sup\{s \in \mathbb{T} : s < t\} \qquad \text{and} \qquad \mu(t) = \sigma(t) - t.$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}_k = \mathbb{T}$.

A function $f : \mathbb{T} \to \mathbb{R}$ is right-dense continuous provided it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense point and each left-dense point, then f is said to be continuous function on $\mathbb{T}$.

For $y : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative of $y(t)$, $y^{\Delta}(t)$, to be the number (if it exists) with the property that for a given $\varepsilon > 0$, there exists a neighborhood U of t such that

$$\left|[y(\sigma(t)) - y(s)] - y^{\Delta}(t)[\sigma(t) - s]\right| < \varepsilon|\sigma(t) - s|$$

for all $s \in U$.

If y is continuous, then y is right-dense continuous, and if y is delta differentiable at t, then y is continuous at t.

Let y be right-dense continuous. If $Y^{\Delta}(t) = y(t)$, then we define the delta integral by

$$\int_a^t y(s)\Delta s = Y(t) - Y(a).$$

Definition 1 [20,21] If $a \in \mathbb{T}$, $\sup \mathbb{T} = \infty$, and f is right-dense continuous on $[a, \infty)$, then we define the improper integral by

$$\int_a^{\infty} f(t)\Delta t = \lim_{b \to \infty} \int_a^b f(t)\Delta t$$

provided this limit exists, and we say that the improper integral converges in this case. If this limit does not exist, then we say that the improper integral diverges.

Definition 2 [22] For each $t \in \mathbb{T}$, let N be a neighborhood of t, then, for $V \in C_{rd}[\mathbb{T} \times \mathbb{R}^n, \mathbb{R}^+]$, define $D^+V^{\Delta}(t, x(t))$ to mean that, given $\varepsilon > 0$, there exists a right neighborhood $N_{\varepsilon} \subset N$ of t such that

$$\frac{[V(\sigma(t), x(\sigma(t))) - V(s, x(\sigma(t))) - \mu(t, s) f(t, x(t))]}{\mu(t, s)} < D^+V^{\Delta}(t, x(t)) + \varepsilon$$

for each $s \in N_{\varepsilon}$, $s > t$, where $\mu(t, s) \equiv \sigma(t) - s$. If t is rd and $V(t, x(t))$ is continuous at t, this reduces to

$$D^+V^{\Delta}(t, x(t)) = \frac{V(\sigma(t), x(\sigma(t))) - V(t, x(\sigma(t)))}{\sigma(t) - t}.$$

Definition 3 [23] We say that a time scale $\mathbb{T}$ is periodic if there exists $p > 0$ such that if $t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the smallest positive p is called the period of the time scale.

Definition 4 [23] Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with period p. We say that the function $f : \mathbb{T} \to \mathbb{R}$ is periodic with period ω if there exists a natural number n such that $\omega = np, \quad f(t+\omega) = f(t)$ for all $t \in \mathbb{T}$ and ω is the smallest number such that $f(t+\omega) = f(t)$.

If $\mathbb{T} = \mathbb{R}$, we say that f is periodic with period $\omega > 0$ if ω is the smallest positive number such that $f(t+\omega) = f(t)$ for all $t \in \mathbb{T}$.

A function $r : \mathbb{T} \to \mathbb{R}$ is called regressive if

$$1 + \mu(t)r(t) \neq 0$$

for all $t \in \mathbb{T}^k$.

If r is regressive function, then the generalized exponential function e_r is defined by

$$e_r(t,s) = \exp\left\{\int_s^t \xi_{\mu(\tau)}(r(\tau))\Delta\tau\right\}, \quad \text{for } s,t \in \mathbb{T},$$

with the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\operatorname{Log}(1+hz)}{h} & \text{if } h \neq 0, \\ z & \text{if } h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, we define

$$p \oplus q := p + q + \mu pq, \qquad \ominus p := -\frac{p}{1+\mu p}, \qquad p \ominus q := p \oplus (\ominus q).$$

Then the generalized exponential function has the following properties.

Lemma 1 [20] *Assume that $p, q : \mathbb{T} \to \mathbb{R}$ are two regressive functions. Then*

(i) $e_0(t,s) \equiv 1$ *and* $e_p(t,t) \equiv 1$*;*
(ii) $e_p(\sigma(t),s) = (1+\mu(t)p(t))e_p(t,s)$*;*
(iii) $e_p(t,\sigma(s)) = \frac{e_p(t,s)}{1+\mu(s)p(s)}$*;*
(iv) $\frac{1}{e_p(t,s)} = e_{\ominus p}(t,s)$*;*
(v) $e_p(t,s) = \frac{1}{e_p(s,t)} = e_{\ominus p}(s,t)$*;*
(vi) $e_p(t,s)e_p(s,r) = e_p(t,r)$*;*
(vii) $e_p(t,s)e_q(t,s) = e_{p\oplus q}(t,s)$*;*
(viii) $\frac{e_p(t,s)}{e_q(t,s)} = e_{p\ominus q}(t,s)$.

Lemma 2 [20] *Assume $f, g : \mathbb{T} \to \mathbb{R}$ are delta differentiable at $t \in \mathbb{T}^k$. Then*

$$(fg)^\Delta(t) = f^\Delta(t)g(t) + f(\sigma(t))g^\Delta(t) = f(t)g^\Delta(t) + f^\Delta(t)g(\sigma(t)).$$

Lemma 3 (Chain Rule [20]) *Assumed that $v : \mathbb{T} \to \mathbb{R}$ is strictly increasing and $\tilde{\mathbb{T}} := v(\mathbb{T})$ is a time scale. Let $\omega : \tilde{\mathbb{T}} \to \mathbb{R}$. If $v^\Delta(t)$ and $\omega^{\tilde{\Delta}}(v(t))$ exist for $t \in \mathbb{T}^k$, then*

$$(\omega \circ v)^\Delta = (\omega^{\tilde{\Delta}} \circ v)v^\Delta.$$

Lemma 4 *Let $\mathbb{T}$ be an ω-periodic time scale. Then $\sigma(t+\omega) = \sigma(t) + \omega$ for all $t \in \mathbb{T}$.*

Proof By using the definition of forward jump operator, we have $\sigma(t)+\omega \geq t+\omega$, then $\sigma(t)+\omega \geq \sigma(t+\omega)$, now we claim that $\sigma(t)+\omega = \sigma(t+\omega)$. If it is not true, we assume that $\sigma(t+\omega) = t_1^* < \sigma(t)+\omega$. From the definition of infimum(inf), we know that there exists a $t_2^* \in \mathbb{T}, \quad t_2^* > t+\omega$, such that

$$t_2^* < t_1^* + \frac{\sigma(t)+\omega-t_1^*}{2} = \frac{\sigma(t)+\omega+t_1^*}{2} < \sigma(t)+\omega. \tag{2}$$

From (2), we obtain $t_2^* - \omega < \sigma(t)$, on the other hand, since $t_2^* > t+\omega, \quad t_2^* - \omega \geq \sigma(t)$, which is a contradiction. The proof of Lemma 4 is complete.

From Lemma 4, we obtain the following Lemma.

Lemma 5 *Let $\mathbb{T}$ be a ω-periodic time scale. Then $\mu(t)$ is a ω-periodic function.*

Proof

$$\mu(t+\omega) = \sigma(t+\omega) - t - \omega = \sigma(t)+\omega-t-\omega = \sigma(t)-t = \mu(t).$$

Definition 5 The anti-periodic solution $x^*(t)$ of system (1) is said to be exponentially stable if there exists a positive constant λ such that for every $\delta \in \mathbb{T}$, there exists $N = N(\delta) \geq 1$ such that every solution of (1) satisfies

$$|x_i(t) - x_i^*(t)| \leq N||\phi - \phi^*||_1 e_{\ominus\lambda}(t,\delta), \quad t \in (0,\infty)_{\mathbb{T}},$$

where

$$||\phi-\phi^*||_1 = \max_{1\leq i\leq n} \max_{\delta\in(-\infty,0]_{\mathbb{T}}} |\phi_i(\delta) - \phi_i^*(\delta)|.$$

For the sake of convenience, we introduce the following notations:

$$c_i^+ = \max_{t\in[0,\omega]_{\mathbb{T}}} c_i(t), \quad c_i^- = \min_{t\in[0,\omega]_{\mathbb{T}}} c_i(t), \quad \gamma = \max_{1\leq i,j\leq n}\{\gamma_{ij}^+\}, \gamma_{ij}^+ = \max_{t\in[0,\omega]_{\mathbb{T}}} |\gamma_{ij}(t)|,$$

$$b_{ijl}^+ = \max_{t\in[0,\omega]_{\mathbb{T}}} |b_{ijl}(t)|, \quad a_{ij}^+ = \max_{t\in[0,\omega]_{\mathbb{T}}} |a_{ij}(t)|, \quad I_i^+ = \max_{t\in[0,\omega]_{\mathbb{T}}} |I_i(t)|, \quad I = \max_{1\leq i\leq n} I_i^+.$$

Main results

Lemma 6 *Assume that (H_1)–(H_3) hold. Suppose further that*

(H_4) *there exists a constant $\eta > 0$ such that*

$$-c_i^- + \sum_{j=1}^n a_{ij}^+ L_j + 2\sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ H_j N_l < -\eta < 0,$$

and $\bar{x}(t) = (\bar{x}_1(t), \bar{x}_2(t), \ldots, \bar{x}_n(t))^T$ is a solution of (2) with initial conditions

$$\bar{x}_i(s) = \bar{\phi}_i(s), \quad |\bar{\phi}_i(s)| < \frac{I + \sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ N_j N_l}{\eta}, \quad s \in (-\infty,0]_{\mathbb{T}}, \quad i = 1,2,\ldots,n. \tag{3}$$

Then

$$|\bar{x}_i(t)| < \frac{I + \sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ N_j N_l}{\eta}, \quad t \in (0,\infty)_{\mathbb{T}}, \quad i = 1,2,\ldots,n. \tag{4}$$

Proof Assume that (4) does not hold. Then, there exists $i \in \{1, 2, \ldots, n\}$ and the first time $t_0 \in (0, \infty)_{\mathbb{T}}$ such that

$$|\bar{x}_i(t_0)| = \frac{I + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l}{\eta}, \quad |\bar{x}_i(t)| < \frac{I + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l}{\eta}, \quad t \in (-\infty, t_0)_{\mathbb{T}},$$

$$|\bar{x}_j(t)| < \frac{I + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l}{\eta}, \quad j \neq i, \quad t \in (-\infty, t_0)_{\mathbb{T}}, \quad j = 1, 2, \ldots, n.$$

Calculating the upper left derivative of $|\bar{x}_i(t_0)|$, together with $(H_1)-(H_4)$, we have

$$\begin{aligned}
0 &\le D^{+}|\bar{x}_i(t_0)|^{\Delta} \\
&\le -c_i^{-}\,|\bar{x}_i(t_0)| + \sum_{j=1}^{n} a_{ij}^{+} L_j \left|\bar{x}_i(t_0 - \gamma_{ij}(t_0))\right| \\
&\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} \Bigg[\int_0^\infty k_{ij}(\theta)(g_j(x_j(t_0-\theta)) - g_j(0))\Delta\theta \int_0^\infty k_{il}(\theta) g_l(x_l(t_0-\theta))\Delta\theta \\
&\quad + \int_0^\infty k_{ij}(\theta) g_j(0)\Delta\theta \int_0^\infty k_{il}(\theta)(g_l(x_l(t_0-\theta)) - g_l(0))\Delta\theta \\
&\quad + \int_0^\infty k_{ij}(\theta) g_j(0)\Delta\theta \int_0^\infty k_{il}(\theta) g_l(0)\Delta\theta\Bigg] + I_i^{+} \\
&\le \left[-c_i^{-} + \sum_{j=1}^{n} a_{ij}^{+} L_j + 2\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} H_j N_l\right]\left(\frac{I + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l}{\eta}\right) \\
&\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l + I_i^{+} \\
&< -\eta\left(\frac{I + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l}{\eta}\right) + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l + I_i^{+} \\
&< 0,
\end{aligned}$$

which is a contradiction and hence (4) holds. The proof of Lemma 6 is complete.

Lemma 7 *Let $(H_1)-(H_4)$ hold. Suppose that $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ is the solution of (1) with initial value (3). Let $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ be an arbitrary solution of (1) with initial value $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T$. Then there exists $\lambda > 0$, such that for every $\delta \in (-\infty, 0]_{\mathbb{T}}$, there exist $N = N(\delta) \ge 1$ such that $x(t)$ satisfies*

$$|x_i(t) - x_i^*(t)| \le N||\phi - \phi^*||_1 e_{\ominus\lambda}(t, \delta), \quad t \in (0, \infty)_{\mathbb{T}}, \quad i = 1, 2, \ldots, n.$$

Proof Set $z(t) = x(t) - x^*(t)$, then from (1), we have

$$\begin{aligned} z_i^{\Delta}(t) = &-c_i(t)z_i(t) + \sum_{j=1}^{n} a_{ij}(t)\left[f_j(x_j(t-\gamma_{ij}(t))) - f_j\left(x_j^*(t-\gamma_{ij}(t))\right)\right] \\ &+ \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)\left[\int_0^{\infty} k_{ij}(\theta)g_j(x_j(t-\theta))\Delta\theta \int_0^{\infty} k_{il}(\theta)g_l(x_l(t-\theta))\Delta\theta\right. \\ &\left.- \int_0^{\infty} k_{ij}(\theta)g_j\left(x_j^*(t-\theta)\right)\Delta\theta \int_0^{\infty} k_{il}(\theta)g_l\left(x_l^*(t-\theta)\right)\Delta\theta\right], \end{aligned} \tag{5}$$

where $t \in (0,\infty)_{\mathbb{T}}, \quad i = 1, 2, \ldots, n$, and the initial values are

$$z_i(s) = \phi_i(s) - \phi_i^*(s) = \psi_i(s), \quad s \in (-\infty, 0]_{\mathbb{T}}, \quad i = 1, 2, \ldots, n.$$

It is obvious that there exist a $k_0 \in \mathbb{N}$ such that $\gamma \le k_0\omega$. Let $A_i(\xi)$ be defined by

$$\begin{aligned} A_i(\xi) = c_i^- - \xi - \sum_{j=1}^{n} a_{ij}^+ L_j e_{\xi}(0, -k_0\omega) \\ -2\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^+ N_j H_l \sum_{m=1}^{\infty} e_{\xi}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)\Delta\theta, \end{aligned}$$

where $\xi \in [0, +\infty)$, $i = 1, 2, \ldots, n$. It is clear that

$$A_i(0) = c_i^- - \sum_{j=1}^{n} a_{ji}^+ L_j - 2\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^+ N_j H_l > 0, \quad i = 1, 2, \ldots, n.$$

Since $A_i, i = 1, 2, \ldots, n$ are continuous on $[0,\infty)$ and $A_i(\xi) \to -\infty$, as $\xi \to +\infty$, there exists $\xi_i > 0$ such that $A_i(\xi_i) = 0$ and $A_i(\xi) > 0$, for $\xi \in (0, \xi_i)$. By choosing $\lambda = \min\{\min_{1\le i\le n}\{\xi_i\}, \frac{\alpha_0}{2}\}$, we obtain

$$\begin{aligned} A_i(\lambda) = c_i^- - \lambda - \sum_{j=1}^{n} a_{ij}^+ L_j e_{\lambda}(0, -k_0\omega) \\ -2\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^+ N_j H_l \sum_{m=1}^{\infty} e_{\lambda}(0, -m\omega) \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)\Delta\theta \ge 0, \end{aligned}$$

where $i = 1, 2, \ldots, n$. Consider the following Lypunov function

$$G_i(t) = e_{\lambda}(t, \delta)|z_i(t)|, \quad \delta \in (-\infty, 0]_{\mathbb{T}}, \quad t \in \mathbb{T}, \quad i = 1, 2, \ldots, n. \tag{6}$$

Using (5) and (6) while calculating the upper right derivative of $G_i(t)$, we obtain

$$
\begin{aligned}
D^+\left(G_i^{\Delta}(t)\right) &\le \lambda e_\lambda(t,\delta)|z_i(t)| \\
&\quad + \quad e_\lambda(\sigma(t),\delta)\mathrm{sign} z_i \left\{ -c_i(t)z_i(t) + \sum_{j=1}^{n} a_{ij}(t)[f_j(x_j(t-\gamma_{ij}(t))) \right. \\
&\quad - f_j(x_j^*(t-\gamma_{ij}(t)))] + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t) \left[\int_0^\infty k_{ij}(\theta)\left(g_j(x_j(t-\theta))\right.\right. \\
&\quad - g_j(x_j^*(t-\theta)))\Delta\theta \int_0^\infty k_{il}(\theta) g_l(x_l(t-\theta))\Delta\theta + \int_0^\infty k_{ij}(\theta) g_j(x_j^*(t-\theta))\Delta\theta \\
&\quad \left.\left. \times \int_0^\infty k_{il}(\theta)\left(g_l(x_l(t-\theta)) - g_l\left(x_l^*(t-\theta)\right)\right)\Delta\theta \right]\right\} \\
&\le e_\lambda(\sigma(t),\delta) \left\{ -(c_i^- - \lambda)|z_i(t)| + \sum_{j=1}^{n} a_{ij}^+ L_j |z_j(t-\gamma_{ij}(t))| \right. \\
&\quad + \quad \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^+ \left[N_l H_j \int_0^\infty k_{ij}(\theta)|z_j(x_j(t-\theta))|\Delta\theta \right. \\
&\quad \left.\left. + N_j H_l \int_0^\infty k_{il}(\theta)|z_l(x_l(t-\theta))|\Delta\theta \right]\right\} \\
&\le [1+(\mu(t))\lambda] \left\{ -(c_i^- - \lambda)G_i(t) + \sum_{j=1}^{n} a_{ij}^+ L_j e_\lambda(t, t-\gamma_{ij}(t)) G_j(t-\gamma_{ij}(t)) \right. \\
&\quad \left. + \quad 2\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^+ N_l H_j \int_0^\infty k_{ij}(\theta) e_\lambda(t, t-\theta) G_j(t-\theta)\Delta\theta \right\}, \qquad (7)
\end{aligned}
$$

where $i = 1, 2, \ldots, n$. Set

$$||\phi - \phi^*||_1 = \max_{1\le i\le n} \max_{s\in(-\infty,0]_{\mathbb{T}}} |\phi_i(s) - \phi_i^*(s)| > 0,$$

if it is not true, then $\phi(s) = \phi^*(s), \quad x(t) = x^*(t)$, and the conclusion of this lemma holds.

From (6), for every $\delta \in (-\infty, 0]_{\mathbb{T}}$ we can choose a constant $N = N(\delta) \ge 1$ such that

$$G_i(t) = e_\lambda(t,\delta)|z_i(t)| < N||\phi - \phi^*||_1, \quad t \in (-\infty, 0]_{\mathbb{T}}, \quad i = 1, 2, \ldots, n.$$

We claim that for every $\delta \in (-\infty, 0]_{\mathbb{T}}$,

$$G_i(t) = e_\lambda(t,\delta)|z_i(t)| < N||\phi - \phi^*||_1, \quad t \in (0, \infty)_{\mathbb{T}}, \quad i = 1, 2, \ldots, n.$$

If this is not true, there exists some $i \in \{1, 2, \ldots, n\}$ and the first time $t_1 \in (0, \infty)_{\mathbb{T}}$ such that

$$G_i(t_1) = N||\phi - \phi^*||_1, \quad G_i(t) < N||\phi - \phi^*||_1, \quad t \in (-\infty, t_1)_{\mathbb{T}},$$

$$G_j(t) < N||\phi - \phi^*||_1, \quad j \ne i, \quad t \in (-\infty, t_1]_{\mathbb{T}}, \quad j = 1, 2, \ldots, n.$$

By using Lemma 5, we have $e_\lambda(t_1, t_1 - k_0\omega) = e_\lambda(0, -k_0\omega)$. Noting the above equation and (7), we obtain

$$\begin{aligned}
0 &\le D^+\left(G_i^\Delta(t_1)\right)\\
&\le [1+\mu(t_1)\lambda]\Bigg\{-\left(c_i^- - \lambda\right)G_i(t_1) + \sum_{j=1}^n a_{ij}^+ L_j e_\lambda(t_1, t_1 - \gamma_{ij}(t_1))G_j(t_1 - \gamma_{ij}(t_1))\\
&\quad + \ 2\sum_{j=1}^n\sum_{l=1}^n b_{ijl}^+ N_l H_j \int_0^\infty k_{ij}(\theta)e_\lambda(t_1, t_1-\theta)G_j(t_1-\theta)\Delta\theta\Bigg\}\\
&< [1+\mu(t_1)\lambda]\Bigg\{-\left(c_i^- - \lambda\right) + \sum_{j=1}^n a_{ij}^+ L_j e_\lambda(t_1, t_1 - k_0\omega)\\
&\quad + \ 2\sum_{j=1}^n\sum_{l=1}^n b_{ijl}^+ N_l H_j \sum_{m=1}^\infty \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)e_\lambda(t_1, t_1 - m\omega)\Delta\theta\Bigg\} N||\phi-\phi^*||_1\\
&\le [1+\mu(t_1)\lambda]\Bigg\{-\left(c_i^- - \lambda\right) + \sum_{j=1}^n a_{ij}^+ L_j e_\lambda(0, -k_0\omega)\\
&\quad + 2\sum_{j=1}^n\sum_{l=1}^n b_{ijl}^+ N_l H_j \sum_{m=1}^\infty \int_{(m-1)\omega}^{m\omega} k_{ij}(\theta)e_\lambda(0, -m\omega)\Delta\theta\Bigg\} N||\phi-\phi^*||_1\\
&\le -[1+\mu(t_1)\lambda]A_i(\lambda)N||\phi-\phi^*||_1\\
&\le 0, \quad i = 1, 2, \ldots, n.
\end{aligned}$$

From the above inequality, we have: $0 \le D^+(G_i^\Delta(t_1)) < 0$, this is a contradiction. So $G_i(t) \le N||\phi - \phi^*||_1$, for $t \in (0, \infty)_{\mathbb{T}}$, $\delta \in (-\infty, 0]_{\mathbb{T}}$, $i = 1, 2, \ldots, n$, which implies $|z_i(t)| \le N||\phi-\phi^*||_1 e_{\ominus\lambda}(t, \delta)$. Thus, we obtain

$$|x_i(t) - x_i^*(t)| \le N||\phi - \phi^*||_1 e_{\ominus\lambda}(t, \delta), \quad t \in (0, \infty)_{\mathbb{T}}, \quad \delta \in (-\infty, 0]_{\mathbb{T}}, \quad i = 1, 2, \ldots, n.$$

The proof of Lemma 7 is complete.

Theorem 1 *Let* (H_1)–(H_4) *hold. Then (2) has an* ω*-anti-periodic solution, which is exponentially stable.*

Proof Let $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$ be a solution of (1) with initial conditions

$$u_i(s) = \phi_i^u(s), \quad |\phi_i^u(s)| < \frac{I + \sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ N_j N_l}{\eta}, \quad s \in (-\infty, 0]_{\mathbb{T}}, \quad i = 1, 2, \ldots, n.$$

By Lemma 6, the solution $u(t)$ is bounded and

$$|u_i(t)| < \frac{I + \sum_{j=1}^n \sum_{l=1}^n b_{ijl}^+ N_j N_l}{\eta}, \quad t \in (0, \infty)_{\mathbb{T}}, \quad i = 1, 2, \ldots, n.$$

From system (1), conditions $(H_1)-(H_4)$, and by using Lemma 3, for $i=1,2,\ldots,n,\quad r\in\mathbb{N}$, we have

$$\begin{aligned}
&[(-1)^{r+1}u_i(t+(r+1)\omega)]^{\Delta}=(-1)^{r+1}u_i^{\Delta}(t+(r+1)\omega)\\
&\quad=(-1)^{r+1}\left[-c_i(t+(r+1)\omega)u_i(t+(r+1)\omega)\right.\\
&\qquad+\sum_{j=1}^{n}a_{ij}(t+(r+1)\omega)f_j(u_j(t+(r+1)\omega-\gamma_{ij}(t+(r+1)\omega)))\\
&\qquad+\sum_{j=1}^{n}\sum_{l=1}^{n}b_{ijl}(t+(r+1)\omega)\int_0^{\infty}k_{ij}(\theta)g_j(u_j(t+(r+1)\omega-\theta))\Delta\theta\\
&\qquad\left.\times\int_0^{\infty}k_{il}(\theta)g_l(u_l(t+(r+1)\omega-\theta))\Delta\theta+I_i(t+(r+1)\omega)\right]\\
&\quad=(-1)^{r+1}\left[-c_i(t)u_i(t+(r+1)\omega)+\sum_{j=1}^{n}a_{ij}(t)f_j(u_j(t+(r+1)\omega-\gamma_{ij}(t)))\right.\\
&\qquad+\sum_{j=1}^{n}\sum_{l=1}^{n}(-1)^{r+1}b_{ijl}(t)\int_0^{\infty}k_{ij}(\theta)g_j(u_j(t+(r+1)\omega-\theta))\Delta\theta\\
&\qquad\left.\times\int_0^{\infty}k_{il}(\theta)g_l(u_l(t+(r+1)\omega-\theta))\Delta\theta+(-1)^{r+1}I_i(t)\right]\\
&\quad=-c_i(t)[(-1)^{r+1}u_i(t+(r+1)\omega)]\\
&\qquad+\sum_{j=1}^{n}a_{ij}(t)f_j((-1)^{r+1}u_j(t+(r+1)\omega)-\gamma_{ij}(t)))\\
&\qquad+\sum_{j=1}^{n}\sum_{l=1}^{n}b_{ijl}(t)\int_0^{\infty}k_{ij}(\theta)g_j((-1)^{r+1}u_j(t+(r+1)\omega-\theta))\Delta\theta\\
&\qquad\times\int_0^{\infty}k_{il}(\theta)g_l((-1)^{r+1}u_l(t+(r+1)\omega-\theta))\Delta\theta+I_i(t).
\end{aligned}\tag{8}$$

Thus, for any natural number r, $(-1)^{r+1}u_i(t+(r+1)\omega)$, $1\le i\le n$ are the solutions of (1). Then, by Lemma 7, for every $\delta\in(-\infty,0]_{\mathbb{T}}$ there exists a $N=N(\delta)\ge 1$, such that

$$\begin{aligned}
&|(-1)^{r+1}u_i(t+(r+1)\omega)-(-1)^{r}u_i(t+r\omega)|\\
&\quad\le N\max_{s\in(-\infty,0]}\max_{1\le i\le n}|u_i(s+\omega)-u_i(s)|e_{\ominus\lambda}(t+r\omega,\delta)\\
&\quad\le 2Ne_{\ominus\lambda}(t+r\omega,\delta)\frac{I+\sum_{j=1}^{n}\sum_{l=1}^{n}b_{ijl}^{+}N_jN_l}{\eta}\\
&\quad\le 2Ne_{\ominus\lambda}(t+r\omega,0)\frac{I+\sum_{j=1}^{n}\sum_{l=1}^{n}b_{ijl}^{+}N_jN_l}{\eta},
\end{aligned}\tag{9}$$

where $t + r\omega > 0, \quad i = 1, 2, \dots, n$. Thus, for any natural number p, we have

$$(-1)^{p+1}u_i(t + (p + 1)\omega) = u_i(t) + \sum_{r=0}^{p} \left[(-1)^{r+1}u_i(t + (r + 1)\omega) - (-1)^r u_i(t + r\omega)\right].$$

It follows that

$$\begin{aligned} |(-1)^{p+1}u_i(t + (p + 1)\omega)| &\le |u_i(t)| \\ &+ \sum_{r=0}^{p} |(-1)^{r+1}u_i(t + (r + 1)\omega) - (-1)^r u_i(t + r\omega)|. \end{aligned} \tag{10}$$

In view of (9), we have

$$\begin{aligned} &\left|(-1)^{r+1}u_i(t + (r + 1)\omega) - (-1)^r u_i(t + r\omega)\right| \\ &\le 2N\left(\frac{I + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l}{\eta}\right) e_{\ominus\lambda}(r\omega - \gamma, 0) \\ &= 2N\left(\frac{I + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l}{\eta}\right) e_{\ominus\lambda}(r\omega - \gamma, r\omega) e_{\ominus\lambda}(r\omega, 0) \\ &\le 2N\left(\frac{I + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}^{+} N_j N_l}{\eta}\right) e_{\ominus\lambda}(-\gamma, 0)(e_{\ominus\lambda}(\omega, 0))^r, \end{aligned} \tag{11}$$

where $t \in \mathbb{T}$. In view of $(e_{\ominus\lambda}(\omega, 0)) < 1$ and (10)–(11), we have $\{(-1)^p u_i(t+p\omega)\}$ uniformly converges to a rd-continuous function $x_i^*(t)$ on any compact subset of $\mathbb{T}$.

Now, we will show that $x^*(t)$ is an ω-anti-periodic solution of (1). First, $x_i^*(t)$ is ω-anti-periodic, since

$$x_i^*(t + \omega) = \lim_{p\to\infty}(-1)^p u_i(t + \omega + p\omega) = -\lim_{p\to\infty}(-1)^{p+1}u_i(t + (p + 1)\omega) = -x_i^*(t).$$

Next, we prove that $x^*(t)$ is a solution of (1). In fact, by using (1) and (8), we have $\{((-1)^{p+1}u_i(t+(p+1)\omega))^{\Delta}\}$ uniformly converges to a rd-continuous function $y_i^*(t), \quad i = 1, 2, \dots, n$ on any compact subset of $\mathbb{T}$. Since $y_i^*(t)$ is rd-continuous function, we have that every $y_i^*(t)$ has an antiderivative. So

$$\begin{aligned} \int_a^t y_i^*(t)\Delta t &= \int_a^t \lim_{p\to\infty} \left[(-1)^{p+1}u_i(t + (p + 1)\omega)\right]^{\Delta} \Delta t \\ &= \lim_{p\to\infty} \int_a^t \left[(-1)^{p+1}u_i(t + (p + 1)\omega)\right]^{\Delta} \Delta t \\ &= \lim_{p\to\infty} \left[(-1)^{p+1}u_i(t + (p + 1)\omega) - (-1)^{p+1}u_i(a + (p + 1)\omega)\right] \\ &= x_i^*(t) - x_i^*(a), \end{aligned} \tag{12}$$

where $a \in \mathbb{T}$ is a constant. From (12) we obtain $y_i^*(t) = (x_i^*(t))^\Delta$. Thus, by letting $p \to \infty$ in (8), we obtain

$$\begin{aligned}\left(x_i^*(t)\right)^\Delta &= -c_i(t)x_i^*(t) + \sum_{j=1}^n a_{ij}(t) f_j\left(x_j^*(t-\gamma_{ij}(t))\right)\\ &\quad + \sum_{j=1}^n\sum_{l=1}^n b_{ijl}(t)\int_0^\infty k_{ij}(\theta)g_j\left(x_j^*(t-\theta)\right)\Delta\theta\int_0^\infty k_{il}(\theta)g_l\left(x_l^*(t-\theta)\right)\Delta\theta\\ &\quad + I_i(t),\quad i=1,2,\ldots,n, \end{aligned}\tag{13}$$

that is, $x^*(t)$ is a solution of (1). Then by Lemma 7, we obtain that $x^*(t)$ is exponentially stable. The proof of Theorem 1 is complete.

An example

Consider the following neural networks

$$\begin{aligned}x_i^\Delta(t) &= -c_i(t)x_i(t) + \sum_{j=1}^2 a_{ij}(t) f_j(x_j(t-\gamma_{ij}(t)))\\ &\quad + \sum_{j=1}^2\sum_{l=1}^2 b_{ijl}(t)\int_0^\infty k_{ij}(\theta)g_j(x_j(t-\theta))\Delta\theta\int_0^\infty k_{il}(\theta)g_l(x_l(t-\theta))\Delta\theta\\ &\quad + I_i(t),\quad i=1,2,\quad t\in(0,\infty)_{\mathbb{T}}, \end{aligned}\tag{14}$$

where $\mathbb{T}$ is a 1-periodic time scale, and

$$f_1(x_1) = \sin\left(\frac{1}{2^{\frac{1}{2}}}x_1\right),\quad f_2(x_2) = \sin\left(\frac{1}{2^{\frac{3}{2}}}x_2\right),$$
$$g_1(x_1) = \left|\arctan\left(\frac{1}{2^{\frac{1}{2}}}x_1\right)\right|,\quad g_2(x_2) = \left|\arctan\left(\frac{1}{2^{\frac{3}{2}}}x_2\right)\right|,$$

$$\begin{aligned} a_{11}(t) &= 1+\cos(2\pi t),\quad a_{12}(t) = 2+\cos(2\pi t),\\ a_{21}(t) &= 2+\cos(2\pi t),\quad a_{22}(t) = 3+\cos(2\pi t), \end{aligned}$$

$$\begin{aligned} c_1(t) &= 25+5\sin(2\pi t),\quad c_2(t) = 35+16\sin(2\pi t),\\ I_1(t) &= 1+\sin(\pi t),\qquad I_2(t) = 1+\cos(\pi t), \end{aligned}$$

$$\begin{aligned} b_{111}(t) = b_{222}(t) &= \frac{1}{4}+\frac{1}{4}\sin(\pi t),\quad b_{112}(t) = b_{212}(t) = \frac{1}{3}+\frac{1}{3}\cos\pi t),\\ b_{121}(t) = b_{221}(t) &= \frac{1}{5}+\frac{1}{5}\cos(\pi t),\quad b_{122}(t) = b_{211}(t) = \frac{1}{6}+\frac{1}{6}\sin(\pi t), \end{aligned}$$

$$\begin{aligned} (k_{ij}(\theta))_{2\times 2} &= \begin{pmatrix} \frac{2}{1+2\mu(0)}e_{\ominus 2}(\theta,0) & \frac{3}{1+3\mu(0)}e_{\ominus 3}(\theta,0)\\ \frac{4}{1+4\mu(0)}e_{\ominus 4}(\theta,0) & \frac{5}{1+5\mu(0)}e_{\ominus 5}(\theta,0) \end{pmatrix},\\ (\gamma_{ij}(t))_{2\times 2} &= \begin{pmatrix} e^{\cos(2\pi t)} & e^{\sin^2(\pi t)}\\ \sin^2(\pi t) & \cos^2(\pi t) \end{pmatrix}. \end{aligned}$$

By calculating, we have $\omega = 1$, $c_1^- = 20$, $c_2^- = 19$, $a_{11}^+ = 2$, $a_{12}^+ = 3$, $a_{21}^+ = 3$, $a_{22}^+ = 4$, $b_{111}^+ = b_{222}^+ = \frac{1}{2}$, $b_{112}^+ = b_{212}^+ = \frac{2}{3}$, $b_{121}^+ = b_{221}^+ = \frac{2}{5}$, $b_{122}^+ = b_{211}^+ = \frac{1}{3}$, $L_1 = L_2 = H_1 = H_2 = 1$, $N_1 = N_2 = \frac{\pi}{2}$. It is not difficult to verify that (H_1)–(H_2) are satisfied.

Also by calculating, we have

$$(K_{ij}(\alpha))_{2\times 2} = \begin{pmatrix} -[e_{\alpha\ominus 2}(\omega,0) - e_\alpha(\omega,0)]\frac{1}{1-e_{\alpha\ominus 2}(\omega,0)} & -[e_{\alpha\ominus 3}(\omega,0)\frac{1}{1-e_{\alpha\ominus 3}(\omega,0)} \\ & -e_\alpha(\omega,0)] \\ -[e_{\alpha\ominus 4}(\omega,0) - e_\alpha(\omega,0)]\frac{1}{1-e_{\alpha\ominus 4}(\omega,0)} & -[e_{\alpha\ominus 5}(\omega,0) - e_\alpha(\omega,0)] \\ & \frac{1}{1-e_{\alpha\ominus 5}(\omega,0)} \end{pmatrix}$$

is right-dense continuous for $\alpha \in [0, 2)$ and $K_{ij}(0) = 1$, $i, j = 1, 2$, i.e., condition (H_3) holds. If we take $\eta = 3$, we can obtain

$$-c_1^- + \sum_{j=1}^{2} a_{1j}^+ L_j + 2\sum_{j=1}^{2}\sum_{l=1}^{2} b_{1jl}^+ H_j N_l = -20 + 5 + \frac{19}{10}\pi < -\eta < 0,$$
$$-c_2^- + \sum_{j=1}^{2} a_{2j}^+ L_j + 2\sum_{j=1}^{2}\sum_{l=1}^{2} b_{2jl}^+ H_j N_l = -19 + 7 + \frac{19}{10}\pi < -\eta < 0.$$

The condition (H_4) is satisfied. So, from Theorem 1, we know that system (14) has at least one 1-anti-periodic solution, and this solution is exponential stability.

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