Cognitive Computation (2018) 10:737–751
https://doi.org/10.1007/s12559-018-9552-2

A Projection-Based Outranking Method with Multi-Hesitant Fuzzy Linguistic Term Sets for Hotel Location Selection

Pu Ji [1] · Hong-Yu Zhang [1] · Jian-Qiang Wang [1]

Received: 2 November 2016 / Accepted: 23 March 2018 / Published online: 27 April 2018

Abstract

Keen competition drives hotel companies to enhance their position. One way to do this is to select a proper hotel location. However, hotel location selection is a complex problem. This study establishes a multi-criteria hotel location selection method. In this method, cognitive information is depicted by multi-hesitant fuzzy linguistic term sets (MHFLTSs). Moreover, the method considers the non-compensation of criteria. It introduces the elimination and choice translating reality (ELECTRE) method. Notably, the method utilizes projection to define concordance and discordance indices. A case study and comparative study are performed in this study. They exhibit the feasibility of the method. Results of the studies show that the method can solve such problems, and they reveal the method's advantages. One theoretical contribution lies in the characterization of cognitive information. MHFLTSs can handle vacillation of decision-makers caused by their complex cognition, and they express both conformity and divergence of opinions during cognitive processes. Our method has the advantages of the ELECTRE method. In addition, the ELECTRE method is improved by introducing the projection. The proposed method is promising in hotel location selection. Moreover, it is a potential option to address cognitive computation.

Keywords Multi-hesitant fuzzy linguistic term sets · Multi-criteria decision-making · Outranking method · Projection · Hotel location selection

Introduction

Hotel location selection is a decision-making process. It is a human activity based on cognitive information. Cognitive information refers to beliefs or thoughts associated with an attitude object [1]. The appropriate expression of cognitive information is essential in cognitive computation [2]. Much research has been conducted on the expression of cognitive information [3–6]. Fuzziness exists in cognitive information due to the limited cognition of human beings [7]. In view of this, fuzzy sets (FSs) [8] have been introduced to represent cognitive information [9, 10]. For example, Liu and Li [11] employed interval-valued intuitionistic FSs (IVIFSs) to denote cognitive information. Farhadinia [12] depicted cognitive information with interval-transformed hesitant FSs (ITHFSs), and Li and Wang [13] applied hesitant probabilistic FSs (HPFSs) to process it. Improvements in information technology may change the methodology of cognitive computation [14]. Studies on cognitive information will enrich the domain of cognitive computation.

Studies on hotel location selection have utilized FSs to process cognitive information. For instance, Chou et al. [15] presented a method using triangular fuzzy numbers (TFNs) to derive cognitive information. Wibowo [16] used IVIFSs to model cognitive information. However, TFNs and IVIFSs have two drawbacks. First, they cannot express the vacillation of decision-makers. Second, decision-makers prefer to express themselves qualitatively rather than quantitatively. That is, qualitative expressions [17] are better than quantitative notions to depict cognitive information. However, the above FSs are composed of quantitative elements. The current study overcomes these deficiencies. It employs multi-hesitant fuzzy linguistic term sets (MHFLTSs) [18] to characterize cognitive information.

An MHFLTS is composed of multiple repeatable linguistic variables. MHFLTS can be used to denote cognitive information of multiple decision-makers. As an example, a hotel company plans to build a new hotel. Three

✉ Jian-Qiang Wang
jqwang@csu.edu.cn

[1] School of Business, Central South University, Changsha 410083, China

managers are appointed to assess a certain location. They utilize linguistic variables to express their opinions. The linguistic variables cover the range within $\{h_0 =$ Extremely Poor, $h_1 =$ Very Poor, $h_2 =$ Poor, $h_3 =$ Slightly Poor, $h_4 =$ Fair, $h_5 =$ Slightly Good, $h_6 =$ Good, $h_7 =$ Very Good$\}$. The first manager may waver between "Slightly Good" and "Good"; the second may evaluate the location as "Slightly Poor," and the last may think the location is "Good" or "Very Good." This cognitive information can be characterized as one MHFLTS $\{h_3, h_5, h_6, h_6, h_7\}$. A hesitant fuzzy linguistic term set (HFLTS) [19] has the restriction that its elements must be consecutive. However, linguistic terms are non-consecutive in the above example; hence, an HFLTS cannot express the above cognitive information. This indicates that MHFLTSs can better depict cognitive information than HFLTSs. The elements of a hesitant fuzzy linguistic set (HFLS) [20] can be non-consecutive but non-repetitive. Cognitive information in the above example can be depicted as one HFLS $\{h_3, h_5, h_6, h_7\}$. Apparently, this HFLS cannot reflect consensus among managers' opinions, while $\{h_3, h_5, h_6, h_6, h_7\}$ can. This suggests that MHFLTS performs better than HFLS.

This study introduces a multi-criteria decision-making (MCDM) method in the computation of cognitive information. Hotel location selection is an MCDM process. Studies [15, 16, 21, 22] have indicated that multiple influential factors are involved in hotel location selection. Wibowo and Deng [21] recognized the determinants of hotel selection. These factors include geographical location, traffic conditions, hotel facilities, and operational convenience. These criteria are non-compensatory. For instance, terrible traffic conditions may cause a negative effect. The negative effect cannot be remedied by a good geographic position. A hotel company may refuse to select a location with a good geographic position but poor operational convenience. From these perspectives, the outranking method is introduced to establish the selection method. It considers the non-compensation of criteria.

In summary, the factors motivating this study are as follows.

1. In hotel location selection, hesitancy and fuzziness may exist in qualitative cognitive information. The MHFLTS is a perfect tool for denoting such cognitive information, and it can express decision-makers' conformity and divergence.
2. Non-complementary criteria may be involved in hotel location selection. This problem can be solved by the elimination and choice translating reality (ELECTRE) III method. ELECTRE III with MHFLTSs has not been studied.
3. Current ELECTRE methods measure two objects' difference with distance, but the difference between objects can be better reflected by projection than by distance. This is because the distance ignores the included angle between objects. Thus, projection is introduced to the ELECTRE III method in the proposed selection method.
4. Projection has not been studied under multi-hesitant fuzzy linguistic term environments. Moreover, the existing projection measurements have a shortcoming. The shortcoming will be discussed in the "Projection of MHFLTSs" section. A projection for MHFLTS is developed to overcome this.

This study aims to develop a new hotel location selection method. First, the correlation coefficient of MHFLTSs is defined. Based on this, the projection of MHFLTSs is proposed. Moreover, the projection-based difference is presented. Second, this study defines projection-based preference relations between MHFLTSs. Next, a hotel location selection method is established. It is a projection-based ELECTRE method. Subsequently, a case study is conducted to explain the application of the proposed method, and a comparative analysis is presented. Its results suggest that the proposed method outperforms previous methods. The main contributions of this study are summarized as follows.

1. The expression of cognitive information is different from that in studies [11–13]. This study proposes to characterize cognitive information with MHFLTSs. MHFLTSs can denote decision-makers' hesitancy, while IVIFSs [11] cannot. Moreover, ITHFSs [12] cannot reflect conformity of decision-makers' cognitive information, while MHFLTSs can. In addition, MHFLTSs are more practical than HPFSs [13] in hotel location selection. As mentioned above, cognitive information in hotel location selection may be qualitative. MHFLTSs can express qualitative information while HPFSs take quantitative values.
2. Different from studies [15, 16], this study introduces the ELECTRE III method. Studies [15, 16] utilized the method of technique for order preference by similarity to an ideal solution (TOPSIS). TOPSIS assumes criteria to be completely compensatory. However, non-compensatory criteria exist in hotel location selection. The ELECTRE III method in this study considers the non-compensation of criteria.
3. This study introduces projection to define preference relations. Preference relations in the traditional ELECTRE III method are based on distance. Projection considers not only distance but difference between objects' included angles.
4. The proposed method is applied to the problem of hotel location selection. Results indicate the validity of the method. The proposed method has the potential to solve problems in other fields.

The remainder of this paper is organized as follows. The "Literature Review" section includes reviews of studies on hotel location selection. Studies on ELECTRE III method and projection are also reviewed in this section. The definitions of MHFLTSs and normalized projection are presented in "Preliminaries." Some measurements of MHFLTSs are defined in the "Projection-Based Outranking Relations on MHFLTSs" section. They include the correlation coefficient, normalized projection, and projection-based difference measurements. Based on these, the preference relations between MHFLTSs are constructed. A fuzzy hotel location selection method is established in a designated section ("A Fuzzy Hotel Location Selection Method"). In the "Case Study" section, a representative scenario and comparative analysis are described in detail, and our concluding remarks are presented in "Conclusion."

Literature Review

Hotel location selection is a vital issue for hotel companies. The influential factors have been identified [23]. Four most important factors were identified by Wibowo [16]. They were geographical location, traffic conditions, hotel facilities, and operational convenience. The same criteria were adopted in other studies [15, 21], and the current study also utilizes them. We discuss why and how these four factors contribute to hotel location selection in "Case Study."

Some methods have previously been developed for hotel location selection. Khalili et al. [24] constructed a method using queuing theory. Some researchers have introduced MCDM methods [15, 16, 21]. For example, Wibowo and Deng [21] and Wibowo [16] constructed hotel location selection methods on TOPSIS. Chou et al. [15] presented a fuzzy method. It combined the analytic hierarchy process and TOPSIS.

Uncertainty may exist in cognitive information in hotel location selection [15, 24, 25]. Researchers have introduced FS theory [8] to characterize cognitive information. Jiang et al. [25] developed a method incorporating hierarchical fuzzy systems and fuzzy rule interpolation. Another fuzzy selection method [24] was based on fuzzy queuing theory. FS theory has also been combined with the MCDM method. Chou et al. [15] proposed a selection method using TOPSIS with TFNs. Wibowo [16] established a fuzzy selection method based on TOPSIS with IVIFSs. However, as we noted, the above FSs have certain shortcomings. MHFLTS [26] can overcome these shortcomings.

As an efficient MCDM method [27], the ELECTRE III method [28] can be applied to location selection. Its performance is immune to operations and measures, and it considers the non-compensation of criteria. Preference relations in the ELECTRE III method are determined by introducing three parameters. They are preference, indifference, and veto thresholds. The ELECTRE III method has been extended to fuzzy environments [29–31]. For example, Wu et al. [32] developed an intuitionistic fuzzy ELECTRE III method. They applied it to select offshore wind-power stations. Hashemi et al. [33] defined the outranking relations for IVIFSs. The ELECTRE III method has also been studied under hesitant fuzzy environments [34–36]. Furthermore, the ELECTRE III method has been applied to many fields [37–39], but ELECTRE III with MHFLTSs has not been studied.

Distance is utilized in the existing ELECTRE III methods to measure two objects' difference, and projection is also used to measure difference. Projection has an advantage over distance. It considers the included angle in addition to the distance between objects. Projection between real vectors was defined by Xu [40]. Subsequently, Yue and Jia [41] defined normalized projection. It overcame a shortcoming of the projection in study [40]. Projection has been extended into fuzzy environments. For instance, Zeng et al. [42] defined projection of intuitionistic FSs. Projection has also been extended to intuitionistic trapezoidal fuzzy environments [43], and some researchers have studied projection in neutrosophic environments [44]. However, projection of MHFLTSs has not been developed.

This study utilizes MHFLTSs to denote cognitive information. Then, a hotel location selection method is constructed based on ELECTRE III. This method utilizes projection to define outranking relations.

Preliminaries

We will now review two concepts involved in the proposed method: MHFLTS and normalized projection.

MHFLTS was initially presented by Wang et al. [26]. It will be utilized in the proposed method to characterize cognitive information. The definition of MHFLTS is as follows.

Definition 1 [26]. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set and H be a continuous linguistic term set. An MHFLTS M on X can be defined in terms of a function t that returns an ordered finite multi-subset of H. M is denoted as:

$$M = \{\langle x_i, t_M(x_i)\rangle\},$$

where $t_M(x_i) = \{m_\varphi(x_i) | m_\varphi(x_i) \in H, \varphi = 1, 2, \ldots, L\}$ represents a set including all possible degrees of $x_i (x_i \in X)$

to X, and L denotes the number of elements in $t_M(x_i)$. $t_M(x_i)$ represents a multi-hesitant fuzzy linguistic term element (MHFLTE). Notably, elements in $t_M(x_i)$ are not required to be consecutive or non-repeatable.

Normalized projection between vectors was initially defined by Yue and Jia [41]. Based on this, normalized projection of MHFLTSs will be defined in "Projection-Based Outranking Relations on MHFLTSs."

Definition 2 [41]. Let $\alpha=(\alpha_1,\alpha_2,\ldots,\alpha_n)$ and $\beta=(\beta_1,\beta_2,\ldots,\beta_n)$ be two real vectors. The normalized projection of α on β is defined as

$$NProj_\beta(\alpha)=\frac{RProj_\beta(\alpha)}{RProj_\beta(\alpha)+|1-RProj_\beta(\alpha)|},\qquad(1)$$

where $RProj_\beta(\alpha)$ is the relative projection of α on β defined by Xu and Liu [45], and

$$RProj_\beta(\alpha)=\frac{\alpha\cdot\beta}{|\beta|^2}.$$

Projection-Based Outranking Relations on MHFLTSs

This section defines the correlation coefficient of MHFLTSs. Based on it, the normalized projection of MHFLTSs is developed. We then present a projection-based difference of MHFLTSs, and define and discuss projection-based preference relations on MHFLTSs.

Correlation Coefficient of MHFLTSs

The normalized projection of MHFLTSs is defined on the basis of the correlation coefficient. In view of this, we first define the correlation coefficient of MHFLTSs.

Definition 3 Let $H=\{h_i|\,0\le_i\le_{2g},g\in N\}$ be a continuous linguistic term set. Let $M_1=\{\langle x_i,t_1(x_i)\rangle\}$ and $M_2=\{\langle x_i,t_2(x_i)\rangle\}$ be two MHFLTSs. The correlation between M_1 and M_2 is defined as

$$C(M_1,M_2)=\sum_{i=1}^{n}\left(\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(m_{\varphi_r^1(x_i)}\right)\cdot\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(m_{\varphi_r^2(x_i)}\right)\right),\qquad(2)$$

where L_{1i} and L_{2i} represent the lengths of $t_1(x_i)$ and $t_2(x_i)$, respectively; $f^*(\cdot)$ is the linguistic scale function introduced in Appendix 1; and $t_1(x_i)=\left\{m_{\varphi_r^1(x_i)}\middle|m_{\varphi_r^1(x_i)}\in H,r=1,2,\ldots,L_{1i}\right\}$ and $t_2(x_i)=\left\{m_{\varphi_r^2(x_i)}\middle|m_{\varphi_r^2(x_i)}\in H,\ r=1,2,\ldots,L_{2i}\right\}$. The modules of M_1 and M_2 are defined as

$$E(M_1)=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(m_{\varphi_r^1(x_i)}\right)\right)^2,\qquad(3)$$

$$E(M_2)=\sum_{i=1}^{n}\left(\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(m_{\varphi_r^2(x_i)}\right)\right)^2,\qquad(4)$$

and the correlation coefficient between M_1 and M_2 is defined as

$$\rho(M_1,M_2)=\frac{C(M_1,M_2)}{\sqrt{E(M_1)}\sqrt{E(M_2)}}=\frac{\sum_{i=1}^{n}\left(\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(m_{\varphi_r^1(x_i)}\right)\cdot\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(m_{\varphi_r^2(x_i)}\right)\right)}{\sqrt{\sum_{i=1}^{n}\left(\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(m_{\varphi_r^1(x_i)}\right)\right)^2}\sqrt{\sum_{i=1}^{n}\left(\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(m_{\varphi_r^2(x_i)}\right)\right)^2}}.\qquad(5)$$

Theorem 1 The correlation coefficient between two MHFLTSs M_1 and M_2 satisfies the following properties:

1. $0\le\rho(M_1,M_2)\le1$;
2. If $M_1=M_2$, then $\rho(M_1,M_2)=1$; and
3. $\rho(M_1,M_2)=\rho(M_2,M_1)$.

The proof of Theorem 1 is provided in Appendix 2.

Projection of MHFLTSs

We now develop the projection of MHFLTSs based on Eq. (1). The projection of MHFLTSs utilizes the correlation coefficient of MHFLTSs in Eq. (5).

Definition 4 Let $H=\{h_i|\,0\le_i\le_{2g},g\in N\}$ be a continuous linguistic term set. Let $M_1=\{\langle x_i,t_1(x_i)\rangle\}$ and $M_2=\{\langle x_i,t_2(x_i)\rangle\}$ be two MHFLTSs. The normalized projection of M_1 on M_2 is defined as

$$NProj_{M_2}(M_1)=\frac{C(M_1,M_2)}{C(M_1,M_2)+|E(M_2)-C(M_1,M_2)|}.\qquad(6)$$

This normalized projection has a shortcoming. The following example reveals the shortcoming.

Example 1 Let $H=\{h_0,h_1,h_2,h_3,h_4\}$ be a continuous linguistic term set. Let $M_1=\{\langle x_1,\{h_0,h_4\}\rangle,\langle x_2,\{h_0,h_4\}\rangle\}$ and $M_2=\{\langle x_1,\{h_0,h_4\}\rangle,\langle x_2,\{h_0,h_0\}\rangle\}$ be two MHFLTSs. The linguistic scale function f_1^* in Appendix 1 is utilized here. By Eq. (6), we obtain $NProj_{M_2}(M_1)=\frac{1/4}{1/4-|1/4-1/4|}=1$ and $Proj_{M_2}(M_2)=\frac{1/4}{1/4-|1/4-1/4|}=1$.Thus, $Proj_{M_2}(M_1)=Proj_{M_2}(M_2)$. Under such a circumstance, we fail to obtain that M_1 is less close to M_2 than M_2.

A novel normalized projection is defined by taking into account the module of M_1. The novel normalized projection overcomes the above drawback. Its definition is presented as follows.

Definition 5 Let $H = \{h_i | 0 \le i \le 2g, g \in N\}$ be a continuous linguistic term set. Let $M_1 = \{\langle x_i, t_1(x_i)\rangle\}$ and $M_2 = \{\langle x_i, t_2(x_i)\rangle\}$ be two MHFLTSs. The novel normalized projection of M_1 on M_2 is defined as

$$NProj^*_{M_2}(M_1) = \frac{C(M_1,M_2)}{C(M_1,M_2)+|E(M_2)-C(M_1,M_2)|}\cdot\frac{E(M_1)}{E(M_1)+|E(M_2)-E(M_1)|}. \quad (7)$$

The following example confirms that the novel normalized projection addresses the problem in Example 1.

Example 2 Let $H = \{h_0, h_1, h_2, h_3, h_4\}$ be a continuous linguistic term set. Let $M_1 = \{\langle x_1, \{h_0, h_4\}\rangle, \langle x_2, \{h_0, h_4\}\rangle\}$ and $M_2 = \{\langle x_1, \{h_0, h_4\}\rangle, \langle x_2, \{h_0, h_0\}\rangle\}$ be two MHFLTSs. The linguistic scale function f_1^* in Appendix 1 is utilized here. $NProj^*_{M_2}(M_1) = \frac{1/4}{1/4-|1/4-1/4|}\cdot\frac{1/2}{1/2-|1/4-1/2|} = 2/3$ and $NProj^*_{M_2}(M_2) = \frac{1/4}{1/4-|1/4-1/4|}\cdot\frac{1/4}{1/4-|1/4-1/4|} = 1$ by Eq. (7). Hence, $Proj_{M_2}(M_1) < Proj_{M_2}(M_2)$, and the novel normalized projection $NProj^*_{M_2}(M_1)$ overcomes the deficiency of $NProj_{M_2}(M_1)$ in Eq. (6).

Theorem 2 Let M_1, M_2, and M_3 be three MHFLTSs. The novel normalized projection of MHFLTSs satisfies the following properties:

(*N*1) $0 \le NProj^*_{M_2}(M_1) \le 1$;
(*N*2) When $M_1 = M_2$, then $NProj^*_{M_2}(M_1) = 1$; and
(*N*3) When $M_1 \subseteq M_2 \subseteq M_3$, then $NProj^*_{M_3}(M_1) \le NProj^*_{M_3}(M_2)$.

The proof of Theorem 2 can be found in Appendix 2.

Projection-Based Difference Between MHFLTSs

Ji et al. [46] defined a projection-based difference measurement. In this section, the projection-based difference is extended to multi-hesitant fuzzy linguistic term environments. We now define the projection-based difference between MHFLTSs.

Definition 6 Let $H = \{h_i | 0 \le i \le 2g, g \in N\}$ be a continuous linguistic term set. Let $M_1 = \{\langle x_i, t_1(x_i)\rangle\}$ and $M_2 = \{\langle x_i, t_2(x_i)\rangle\}$ be two MHFLTSs. The projection-based difference between M_1 and M_2 is

$$Diff(M_1, M_2) = NProj^*_{M^+}(M_1) - NProj^*_{M^+}(M_2), \quad (8)$$

where M^+ is the maximum MHFLTS, i.e., $M^+ = \{\langle x_i, (h_{2g})\rangle\}$.

Theorem 3 Let $H = \{h_i | 0 \le i \le 2g, g \in N\}$ be a continuous linguistic term set. Let $M_1 = \{\langle x_i, t_1(x_i)\rangle\}$ and $M_2 = \{\langle x_i, t_2(x_i)\rangle\}$ be two MHFLTSs. The projection-based difference between M_1 and M_2 by using Eq. (8) can be written as

$$Diff(M_1, M_2) = \frac{Proj_{M^+}(M_1)\cdot E(M_1) - Proj_{M^+}(M_2)\cdot E(M_2)}{\left(\sqrt{E(M^+)}\right)^3}. \quad (9)$$

Theorem 4 The projection-based difference between M_1 and M_2 satisfies the following properties:

(*D*1) $-1 \le Diff(M_1, M_2) \le 1$;
(*D*2) If $M_1 = M_2$, then $Diff(M_1, M_2) = 0$;
(*D*3) If $M_1 \subseteq M_2$, then $Diff(M_1, M_2) \le 0$;
(*D*4) If $M_2 \subseteq M_1$, then $Diff(M_1, M_2) \ge 0$;
(*D*5) $Diff(M_1, M_2) + Diff(M_2, M_1) = 0$;
(*D*6) If $Diff(M_1, M_2) = Diff(M_2, M_1)$, then $Diff(M_1, M_2) = Diff(M_2, M_1) = 0$; and
(*D*7) $Diff(M_1, M_2) + Diff(M_2, M_3) = Diff(M_1, M_3)$.

The proof of Theorem 4 is provided in Appendix 2.

Preference Relations on MHFLTSs Based on Projection-Based Difference

The ELECTRE III method involves three thresholds. The three thresholds include preference threshold p_j, indifference threshold q_j, and veto threshold v_j. They are utilized to construct preference relations and concordance and discordance indices. Preference relations and concordance/discordance are two significant notions in the ELECTRE III method. Preference relations in the proposed method are defined in this section; concordance and discordance indices are defined in "A Fuzzy Hotel Location Selection Method." Preference relations in the proposed method are developed using the projection-based difference in Eq. (9). The definitions of preference relations in the proposed method are as follows.

Definition 7 Let $H = \{h_i | 0 \le i \le 2g, g \in N\}$ be a continuous linguistic term set. Let M_1 and M_2 be two arbitrary MHFLTSs. The dominance relations between M_1 and M_2 can be divided into the following three categories:

1. When $Diff(M_1, M_2) \ge p_j$, then M_1 strongly dominates M_2, denoted by $M_1 \succ_S M_2$.
2. When $q_j < Diff(M_1, M_2) < p_j$, then M_1 weakly dominates M_2, denoted by $M_1 \succ_W M_2$.
3. When $|Diff(M_1, M_2)| \le q_j$, then A_1 is indifferent to A_2, denoted by $M_1 \sim_I M_2$.

Theorem 5 Let $H = \{h_i | 0 \le i \le 2g, g \in N\}$ be a continuous linguistic term set. Let M_1, M_2, and M_3 be three MHFLTSs. The dominance relations between MHFLTSs satisfy the following properties:

1. The strong dominance relation has

   ($SR1$) irreflexivity: for any MHFLTS M_1, $M_1 \succ_S M_1$ is false;
   ($SR2$) asymmetry: for any two MHFLTSs M_1 and M_2, $M_1 \succ_S M_2 \Rightarrow \neg(M_2 \succ_S M_1)$;
   ($SR3$) transitivity: for any three MHFLTSs M_1, $a_2 \succ a_3 \succ a_1$, and M_3, $M_1 \succ_S M_2$, $M_2 \succ_S M_3 \Rightarrow M_1 \succ_S M_3$.

2. The weak dominance relation has

   ($WR1$) irreflexivity: for any MHFLTS M_1, $M_1 \succ_W M_1$ is false;
   ($WR2$) asymmetry: for any two MHFLTSs M_1 and M_2, $M_1 \succ_W M_2 \Rightarrow \neg(M_2 \succ_W M_1)$;
   ($WR3$) non-transitivity: for any three MHFLTSs M_1, M_2, and M_3, $M_1 \succ_W M_2$, $M_2 \succ_W M_3 \not\Rightarrow M_1 \succ_W M_3$.

3. The indifference relation has

   ($IR1$) reflexivity: for any MHFLTS M_1, $M_1 \sim_I M_1$;
   ($IR2$) symmetry: for any two MHFLTSs M_1 and M_2, $M_1 \sim_I M_2 \Rightarrow M_2 \sim_I M_1$;
   ($IR3$) non-transitivity: for any three MHFLTSs M_1, M_2, and M_3, $M_1 \sim_I M_2$, $M_2 \sim_I M_3 \not\Rightarrow M_1 \sim_I M_3$.

Definition 8 Let $H = \{h_i | 0 \le i \le 2g, g \in N\}$ be a continuous linguistic term set. Let M_1 and M_2 be two arbitrary MHFLTSs. The opposition relations between M_1 and M_2 can be divided into the following three categories:

1. When $Diff(M_2, M_1) \ge v_j$, then M_1 strongly opposes M_2, denoted by $M_1 \succ_{SO} M_2$.
2. When $p_j < Diff(M_2, M_1) < v_j$, then M_1 weakly opposes M_2, denoted by $M_1 \succ_{WO} M_2$.
3. When $|Diff(M_2, M_1)| \le p_j$, then M_1 is indifferently opposed to M_2, denoted by $M_1 \sim_{IO} M_2$.

Theorem 6 Let $H = \{h_i | 0 \le i \le 2g, g \in N\}$ be a continuous linguistic term set. Let M_1, M_2, and M_3 be three MHFLTSs. The opposition relations between MHFLTSs satisfy the following properties:

1. The strong opposed relation has

   ($SO1$) irreflexivity: for any MHFLTS M_1, $M_1 \succ_{SO} M_1$ is false;
   ($SO2$) asymmetry: for any two MHFLTSs M_1 and M_2, $M_1 \succ_{SO} M_2 \Rightarrow \neg(M_2 \succ_{SO} M_1)$;
   ($SO3$) transitivity: for any three MHFLTSs M_1, M_2, and M_3, $M_1 \succ_{SO} M_2$, $M_2 \succ_{SO} M_3 \Rightarrow M_1 \succ_{SO} M_3$.

2. The weak dominance relation has

   ($WO1$) irreflexivity: for any MHFLTS M_1, $M_1 \succ_{WO} M_1$ is false;
   ($WO2$) asymmetry: for any two MHFLTSs M_1 and M_2, $M_1 \succ_{WO} M_2 \Rightarrow \neg(M_2 \succ_{WO} M_1)$;
   ($WO3$) non-transitivity: for any three MHFLTSs M_1, M_2, and M_3, $M_1 \succ_{WO} M_2$, $M_2 \succ_{WO} M_3 \not\Rightarrow M_1 \succ_{WO} M_3$.

3. The indifference relation has

   ($IO1$) reflexivity: for any MHFLTS M_1, $M_1 \sim_{IO} M_1$;
   ($IO2$) symmetry: for any two MHFLTSs M_1 and M_2, $M_1 \sim_{IO} M_2 \Rightarrow M_2 \sim_{IO} M_1$;
   ($IO3$) non-transitivity: for any three MHFLTSs M_1, M_2, and M_3, $M_1 \sim_{IO} M_2$, $M_2 \sim_{IO} M_3 \not\Rightarrow M_1 \sim_{IO} M_3$.

A Fuzzy Hotel Location Selection Method

We now construct a fuzzy hotel location selection method. It introduces the projection-based difference of MHFLTSs to the ELECTRE III method. Concordance and discordance indices are defined by the projection-based difference in Eq. (9).

Assume that m alternative locations $\{A_1, A_2, \cdots, A_m\}$ are evaluated against n criteria $\{C_1, C_2, \cdots, C_n\}$. Multiple decision-makers participate in the hotel location selection. They can evaluate locations with values in the linguistic term set $H = \{h_i | 0 \le i \le 2g, g \in N\}$. Their cognitive information about a certain location against one criterion is characterized by an MHFLTE. M_{ij} represents the MHFLTE of A_i $(i = 1, 2, \cdots, m)$ against C_j $(j = 1, 2, \cdots, n)$. The decision matrix in the form of MHFLTEs can be denoted as

$$M = \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1n} \\ M_{21} & M_{22} & \cdots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1} & M_{m2} & \cdots & M_{mn} \end{pmatrix}_{m\times n},$$

where $M_{ij} = \left\{ m_\varphi^{ij} \middle| m_\varphi^{ij} \in H \right\}$. In practice, the n criteria are usually of different importance; hence, their weights may be different. Here, the weight vector of criteria is given by the decision-makers as $w = (w_1, w_2, \cdots, w_n)^T$, where $w_j \geq 0$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$.

Concordance and Discordance Indices

Definition 9 Let criteria set $C = \{C_1, C_2, \cdots, C_n\}$ be of the maximizing type. Let A_1 and A_2 be two locations in alternative set A. Two thresholds against criterion C_j are involved in the construction of the concordance index: q_j and p_j $(0 \leq q_j < p_j)$. The concordance degree between A_1 and A_2 against C_j can be defined as follows:

1. When $Diff(M_{2j}, M_{1j}) \geq p_j$, then $con_j(A_1, A_2) = 0$;
2. When $Diff(M_{2j}, M_{1j}) \leq q_j$, then $con_j(A_1, A_2) = 1$; and
3. When $q_j < Diff(M_{2j}, M_{1j}) < p_j$, then $con_j(A_1, A_2) = \frac{p_j - Diff(M_{1j}, M_{2j})}{p_j - q_j}$.

Definition 10 Let criteria set $C = \{C_1, C_2, \cdots, C_n\}$ be of the maximizing type. Let A_1 and A_2 be two locations in alternative set A. Two thresholds against criterion C_j are involved in the construction of the discordance index: p_j and v_j $(0 < p_j < v_j)$. The discordance degree between A_1 and A_2 against C_j can be defined as follows:

1. When $Diff(M_{2j}, M_{1j}) \leq p_j$, then $dis_j(A_1, A_2) = 0$;
2. When $Diff(M_{2j}, M_{1j}) \geq v_j$, then $dis_j(A_1, A_2) = 1$; and
3. When $p_j < Diff(M_{2j}, M_{1j}) < v_j$, then $dis_j(A_1, A_2) = \frac{Diff(M_{2j}, M_{1j}) - p_j}{v_j - p_j}$.

Fuzzy Hotel Location Selection Method

A fuzzy hotel location selection method is introduced in this section. It is a projection-based ELECTRE method. The procedure is as follows:

Step 1: Normalize the decision matrix M.

The decision matrix must be normalized due to the existence of both cost and benefit criteria. If C_j is a cost criterion, M_{ij} must be normalized using a negation operator. The negation operator of MHFLTE is defined as.

$$M_{ij}^c = \underset{m_\varphi^{ij} \in M_{ij}}{\cup} \left\{ neg\left(m_\varphi^{ij} \right) \right\} = \underset{m_\varphi^{ij} \in M_{ij}}{\cup} \left\{ h_{2g - sub\left(m_\varphi^{ij} \right)} \right\},$$

where $sub(m_\varphi{}^{ij})$ represents the subscript of m_φ^{ij}. If C_j belongs to a benefit criterion, M_{ij} need not be normalized. For ease of description, we assume that the normalized decision matrix is $\tilde{M} = \left(\tilde{M}_{ij} \right)_{m\times n}$.

Step 2: Determine the concordance degree between each pair of locations against each criterion.

The concordance degree $con_j(A_i, A_k)$ between A_i and A_k against C_j can be obtained by Definition 9.

Step 3: Determine the discordance degree between each pair of locations against each criterion.

The discordance degree $dis_j(A_i, A_k)$ between A_i and A_k against C_j can be obtained by Definition 10.

Step 4: Determine the concordance index between each pair of locations.

The concordance index c_{ik} between A_i and A_k is

$$c_{ik} = \sum_{j=1}^{n} w_j con_j(A_i, A_k), \tag{10}$$

where w_j is the weight of C_j.

Step 5: Determine the reliability index between each pair of locations.

The reliability index r_{ik} denotes the reliability degree that A_i inferior to A_k. It is determined by the formula

$$r_{ik} = \begin{cases} c_{ik} & dis_j(A_i, A_k) \leq c_{ik}, \forall j \\ c_{ik} \prod_{j \in J_{ik}} \frac{1 - dis_j(A_i, A_k)}{1 - c_{ik}} & else, \end{cases} \tag{11}$$

where J_{ik} represents the set of subscripts of the criterion satisfying $dis_j(A_i, A_k) > c_{ik}$.

Step 6: Determine the net reliability index of each location.

The net reliability index of A_i can be obtained by the following formula:

$$net(r_i) = \sum_{\substack{k=1, \\ k \neq i}}^{m} r_{ik} - \sum_{\substack{k=1, \\ k \neq i}}^{m} r_{ki}. \tag{12}$$

Step 7: Rank the locations.

The locations can be ranked using the net reliability indices. A greater net reliability index indicates a better location.

Case Study

We now apply the proposed method to a hotel location selection problem.

The example is provided by Wibowo and Deng [21]. A hotel company intends to select a suitable site for a new hotel. After initial sifting, the four hotel locations A_1, A_2, A_3, and A_4 are selected for further consideration. As some studies [15, 16, 21] suggest, four criteria are involved: (1) geographical location (C_1), (2) traffic conditions (C_2), (3) hotel facilities (C_3), and (4) operational convenience (C_4). The descriptions of these four criteria are provided in Table 1. How and why these four criteria contribute to hotel location selection are discussed below.

1. From the perspective of the surrounding environment, security around the location matters in hotel location selection. An area in which robberies and fires occur occasionally is not very suitable for a hotel location. Moreover, Yang et al. [22] found that distance to the public service infrastructure influences hotel location selection. A location near to a theater or large park is better than one that is far from the public service infrastructure. The above factors relate to the geographical location.
2. Three kinds of accessibility are important factors in hotel location selection. They are accessibility to roads, subways, and tourism sites. This is revealed by the empirical results of Yang et al. [22]. Higher degrees of accessibility yield a more suitable hotel location. These three kinds of accessibility are reflected by criterion C_2 (traffic conditions). A higher value of C_2 indicates a more suitable location.
3. Diverse locations may have different local cultures. Amalgamation with the local culture will impact the development of a hotel [47]. Moreover, convenience of owning nearby land is important from the perspective of long-term development. In view of these, hotel facilities (C_3), which includes the above two factors, contribute to hotel location selection. A greater value of C_3 means a more suitable location.
4. Operational convenience refers to resources for supporting a hotel's business operations. These include human resources, quality of manpower, regulation restrictions, etc. Studies have supported the notion that human resources are critical to a hotel [48]. Hotel location selection must consider whether human resources are sufficient in the region. In addition, local legal rules greatly affect a hotel's development. Positive legal rules promote its development. Thus, local regulations must be considered in hotel location selection. Operational convenience (C_4), which comprises the above factors, contributes to hotel location selection. Better operational convenience indicates a more suitable location.

Table 1 Descriptions of the four criteria

Criterion	Description
Geographical location (C_1)	The competitive advantage of the hotel regarding location, including its proximity to public facilities, distance to competitors, security around the location, etc.
Traffic conditions (C_2)	The level of convenience of the hotel to various locations of interest, including the distance to representative scenic spots, railway stations or airports, etc.
Hotel facilities (C_3)	The capacity of satisfying guests' requirements, including the amalgamation with local culture, convenience of obtaining nearby land, etc.
Operational convenience (C_4)	The key resources to support the hotel's business operations, including sufficiency of human resources, quality of manpower, regulation restrictions, etc.

The four criteria are not of equal importance. Their weight vector is provided by integrating all managers' opinions. The weight vector of criteria is $w = (0.3, 0.4, 0.1, 0.2)^T$.

The four hotel locations are evaluated by three managers. Cognitive information provided by managers consists of linguistic values rather than quantitative evaluations. The reason is explained as follows. Humans have limited cognition. It is hard for managers to determine accurate values over a continuous range. In fact, the situation will dramatically improve if finite qualitative values (e.g., linguistic values) are involved. Linguistic values in this case study belong to linguistic term set $H = \{h_0 = \text{Extremely Poor},\ h_1 = \text{Very Poor},\ h_2 = \text{Poor},\ h_3 = \text{Slightly Poor},\ h_4 = \text{Fair},\ h_5 = \text{Slightly Good},\ h_6 = \text{Good},\ h_7 = \text{Very Good},\ h_8 = \text{Extremely Good}\}$.

The characterization of cognitive information is presented in the "Introduction" section. Each manager expresses his cognitive information on every location against each criterion. Linguistic values in H are utilized. Notably, each manager is allowed to use more than one linguistic value for a location against a criterion. This is because he may vacillate between two (or more) linguistic values. An MHFLTE is applied to characterize all managers' cognitive information on a location against a criterion. This MHFLTE comprises all linguistic values mentioned by managers. As an example, managers evaluated A_1 against C_1. Different opinions existed. The first manager abstained due to limited knowledge, the second manager vacillated between "Fair" and "Slightly Fair," and the last manager evaluated A_1 against C_1 as "Very Good." The above information can be denoted by an MHFLTE as $\{h_4, h_5, h_7\}$. Duplicate linguistic values cannot appear in a manager's evaluation on a location against a criterion, but they can exist in an MHFLTE. For instance, managers also evaluated A_1 against C_2. Both the first and second managers evaluated it as "Slightly Fair," while the third manager provided an evaluation of "Extremely Good." The above information can be denoted by an MHFLTE as $\{h_5, h_5, h_8\}$. Clearly, duplicate values do not exist in each manager's evaluation. In addition, h_5 repeats twice in the MHFLTE since h_5 appears in both the first and second managers' evaluations. Managers evaluated each location against each criterion. These evaluations were characterized by the MHFLTEs listed in Table 2. The MHFLTE for A_i against C_j is presented in the $(i+1)$-th row and $(j+1)$-th column of Table 2.

Steps of the Proposed Method

Step 1: Normalize the decision matrix M.

C_1, C_2, C_3, and C_4 are benefit criteria. Therefore, it was unnecessary to normalize decision matrix M, i.e., $\tilde{M} = M$.

Step 2: Determine the concordance degree between each pair of locations against each criterion.

Table 3 shows the concordance degree between each pair of locations under each criterion. They were determined using Definition 9.

Step 3: Determine the discordance degree between each pair of locations against each criterion.

Table 4 presents the discordance degree between each pair of locations against each criterion. They were obtained using Definition 10.

Step 4: Determine the concordance index between each pair of locations.

Table 2 Decision matrix M in the form of MHFLTEs

	C_1	C_2	C_3	C_4
A_1	$\{h_4, h_5, h_7\}$	$\{h_5, h_5, h_8\}$	$\{h_6, h_6, h_8\}$	$\{h_1, h_3, h_4\}$
A_2	$\{h_5, h_6\}$	$\{h_6, h_8\}$	$\{h_1, h_3, h_3\}$	$\{h_3, h_6\}$
A_3	$\{h_2, h_3\}$	$\{h_6\}$	$\{h_2, h_3, h_5\}$	$\{h_4, h_4\}$
A_4	$\{h_6, h_8\}$	$\{h_4, h_5\}$	$\{h_3, h_5, h_8\}$	$\{h_1, h_2, h_3\}$

The concordance index (Table 5) between each pair of locations was obtained using Eq. (10). In this case study, the values of q_j and p_j in Eq. (10) were set to 0.02 and 0.25, respectively, for all $j \in \{1, 2, \ldots, n\}$.

Step 5: Determine the reliability index between each pair of locations.

The reliability index (Table 6) between each pair of locations was determined by Eq. (11).

Step 6: Determine the net reliability index of each location.

The net reliability index (Table 7) of each location was obtained by Eq. (12).

Step 7: Rank the locations.

As shown in Table 7, $net(s_1) > net(s_2) > net(s_4) > net(s_3)$. Hence, the ranking of locations was determined as $A_1 \succ A_2 \succ A_4 \succ A_3$. The most suitable location was identified as A_1.

Comparative Analysis

We now compare the proposed method to several other methods to show its advantages. Three cases are included in the comparative analysis. The first is conducted under hesitant fuzzy linguistic term environments, the second is conducted under multi-hesitant fuzzy linguistic term environments, and the last one analyzes why MHFLTEs are used to characterize cognitive information. The details of these three cases are as follows:

Case 1. Comparative analysis under hesitant fuzzy linguistic term environments.

HFLTS is a special case of MHFLTS, i.e., the proposed MHFLTS method can tackle problems with HFLTSs. The following HFLTS methods are compared with the proposed method:

1. Method 1: Proposed by Rodriguez et al. [19], Method 1 determined ranking results according to non-dominance choice degrees.
2. Method 2: Established by Lee and Chen [49], Method 2 ranked alternatives according to aggregated weighted preference values. The aggregated weighted preference values were HFLTSs. Method 2 compared HFLTSs by a likelihood-based comparison method. A greater weighted preference value indicated a better alternative.
3. Method 3: The third method was an outranking method in [50]. Method 3 was constructed based on the ELECTRE I method. It ranked alternatives according to net dominance and disadvantage indices. A greater

Table 3 Degree of concordance between each pair of alternatives under each criterion

	C_1				C_2				C_3				C_4			
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
A_1		0.9643	1	0		0.2038	1	1		1	1	1		0.1944	0	1
A_2	1		1	0	1		1	1	0		1	0	1		1	1
A_3	0	0		0	1	0.2038		1	0	1		0	1	0.7982		1
A_4	1	1	1		0	0	0		0	1	1		1	0	0	

net dominance index and a smaller disadvantage index indicate a better alternative.
4. Method 4: Established by Tian et al. [51], Method 4 was based on the qualitative flexible (QUALIFLEX) multiple criteria method. Method 4 introduced likelihood to define the concordance/discordance index. The best permutation was identified according to concordance/discordance indices.

Methods 1–4 and the proposed method were applied to the problem from study [50]. This problem is an MCDM problem with HFLTSs. Three alternatives (a_1, a_2, and a_3) must be ranked. Table 8 presents each method's results. Comparison among these results will reveal the proposed method's advantages.

From Table 8, differences exist in results of these five methods. Two different best alternatives are contained in Table 8. a_2 is obtained as the best alternative by Method 1 and the proposed method. The other methods determine a_3 as the best alternative. In addition, two different worst alternatives are shown in Table 8. a_1 is identified as the worst alternative by Methods 1–4. The proposed method obtains a_3 as the worst alternative. Reasons for these differences are as follows:

1. First, we explain why different results are generated by Method 1 and the proposed method. Method 1 is established on the irrational assumption. It assumes that criteria are complementary. Conversely, the proposed method considers the non-compensation of criteria. Also, Method 1 transforms linguistic terms into interval values [50]. This may lead to the loss of cognitive information. Therefore, the result of Method 1 may be unreasonable.
2. Second, like Method 1, Method 2 may lead to information loss. The transformation of linguistic values does not exist in the proposed method. This mitigates information loss. Additionally, Method 2 supposes that criteria are complementary. Inversely, the proposed method considers the non-compensation of criteria. Hence, Method 2 and the proposed method may generate different results. What is more, the result of the proposed method is more rational than that of Method 2.
3. Third, different results are obtained by Methods 3–4 and the proposed method. All these methods are outranking methods. However, different concordance and discordance indices exist in these three methods. Method 3 derives indices using distance. Method 4 applies likelihood to obtain indices. The proposed method defines indices using projection. Projection considers the included angle between HFLTSs in addition to their distance. Distance and likelihood do not consider the included angle between HFLTSs. It is reasonable that the ranking order of the proposed method differs from those of Methods 3–4.

Case 2. Comparative analysis under multi-hesitant fuzzy linguistic term environments.

The proposed method is compared with existing MHFLTS methods in this case. Thus far, only the following two MHFLTS methods have been developed:

1. Method 5: Established by Wang et al. [26], Method 5 aggregates criteria values with 2-tuple linguistic

Table 4 Degree of discordance between each pair of alternatives under each criterion

	C_1				C_2				C_3				C_4			
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
A_1		0	0	0.3559		0	0	0		0	0	0		0	0	0
A_2	0		0	0.2148	0		0	0	1		0	0.5469	0		0	0
A_3	0.484	0.625		1	0	0		0	1	0		0.1042	0	0		0
A_4	0	0	0		0	0.9961	0		0	0	0		0	0.0195	0	

Table 5 Concordance index between each pair of alternatives

	A_1	A_2	A_3	A_4
A_1		0.5097	0.8966	0.7
A_2	0.9		0.9702	0.6
A_3	0.6	0.3412		0.6
A_4	0.4819	0.4	0.4611	

aggregation operators. It ranks alternatives according to aggregated values. A greater aggregated value indicates a better alternative.

2. Method 6: Proposed by Wang et al. [52], Method 6 is based on the TODIM (a Portuguese acronym for interactive and multi-criteria decision-making) method. It introduces likelihood to define partial dominance. Then, the global value of each location is obtained. Method 6 generates a ranking order according to global values. A greater global value means a better alternative.

Methods 5–6 are applied to solve the selection problem in our case study. Results of these two methods are compared with those of the proposed method. Table 9 lists results of Methods 5–6 and the proposed method. The comparison of these results reveals some benefits of the proposed method.

As shown in Table 9, the proposed method and Method 5 yield the same result. This result is different from that of Method 6. The same best location is obtained by the three methods. Different worst locations are generated by the three methods. Method 6 obtains A_4 as the worst location. A_3 is recognized as the worst location by the other two methods. Reasons for these differences are as follows:

1. First, differences exist between Method 5 and the proposed method. An aggregation operator is utilized in Method 5. This indicates that criteria are complementary. However, non-compensatory criteria may be involved in hotel location selection, and the proposed method considers this. Consequently, Method 5 and the proposed method may generate different results, and the proposed method outperforms Method 5.
2. Second, Method 6 is based on the TODIM method. It irrationally assumes the compensation of criteria, while the proposed method considers the non-compensation of criteria. In addition, partial dominance in Method 6 is determined using likelihood. The proposed method introduces projection. Likelihood of MHFLTSs only considers values of elements in MHFLTSs. It ignored the included angle between MHFLTSs. However, projection considers the included angle between MHFLTSs in addition to the values of their elements. Obviously, the result of Method 6 may differ from that of the proposed method. Moreover, the proposed method performs better than Method 6.

Table 6 Reliability index between each pair of alternatives

	A_1	A_2	A_3	A_4
A_1		0.5097	0.8966	0.7
A_2	0		0.9702	0.6
A_3	0	0.6791		0
A_4	0.4819	0.0118	0.4611	

Table 7 Net reliability index of each alternative

	A_1	A_2	A_3	A_4
Net reliability index	**1.6244**	0.3696	− 1.6489	− 0.3451

The greatest net reliability index is in boldface

Case 3. Analysis of why MHFLTEs are used to characterize cognitive information.

This case verifies why MHFLTEs are used to characterize cognitive information. The proposed method is applied to solve two problems. The only difference between these two problems is the decision matrix. The details of these two problems are as follows:

1. Problem 1: This is the selection problem in our case study. Cognitive information is characterized by MHFLTEs in Problem 1, i.e., the decision matrix (Table 2) is composed of MHFLTEs.
2. Problem 2: This is identical to Problem 1 except for the decision matrix. Cognitive information is characterized by hesitant fuzzy linguistic numbers (HFLNs), i.e., the decision matrix in Problem 2 comprises HFLNs. HFLNs require elements to be non-repetitive. Managers' evaluations are the same in Problems 1 and 2. The acquisition of the decision matrix in Problem 2 is as follows. An HFLN is used to depict evaluations for a location against a criterion. Notably, an HFLN requires elements to be non-repetitive. As an example, evaluations for A_1 against C_2 can be denoted by HFLN as $\{h_5, h_8\}$. h_5 appears once in this HFLN, even if two managers provide evaluations as "Slightly Fair" (i.e., h_5). Table 10 lists the decision matrix in Problem 2.

Table 8 Ranking orders of different methods

Method	Ranking order	Best alternative	Worst alternative
Method 1	$a_2 \succ a_3 \succ a_1$	a_2	a_1
Method 2	$a_3 \succ a_2 \succ a_1$	a_3	a_1
Method 3	$a_3 \succ a_2 \succ a_1$	a_3	a_1
Method 4	$a_3 \succ a_2 \succ a_1$	a_3	a_1
Proposed method	$a_2 \succ a_1 \succ a_3$	a_2	a_3

Table 9 Ranking results of different methods

Method	Ranking order	Best location	Worst location
Method 5	$A_1 \succ A_2 \succ A_4 \succ A_3$	A_1	A_3
Method 6	$A_1 \succ A_2 \succ A_3 \succ A_4$	A_1	A_4
Proposed method	$A_1 \succ A_2 \succ A_4 \succ A_3$	A_1	A_3

HFLTSs restrict elements to be consecutive. Apparently, linguistic values for a location against a criterion may be non-consecutive. That is to say, cognitive information cannot be characterized by HFLTSs. Hence, a problem with HFLTSs is not involved in this case. The proposed method is applied to solve Problems 1 and 2. Table 11 shows the results of these two problems.

Table 11 shows that the results of Problems 1 and 2 are different. The best location in the results of Problem 2 is A_2, while for Problem 1 it is A_1. Furthermore, the worst location in the results of Problem 2 is A_4, while for Problem 1 it is A_3. The differences exist for the following reasons. The decision matrix in Problem 2 comprises HFLNs. From its acquisition method, each possible linguistic value appears only once in an HFLN, while an MHFLTE in Problem 1 allows linguistic values to repeat. The utilization of HFLNs to depict cognitive information may lead to information loss. The utilization of MHFLTEs overcomes this defect. This case analyzes why an MHFLTS is used to express cognitive information.

Generally speaking, the proposed method can solve hotel location selection problems. In addition, the comparative analysis indicates some advantages of the proposed method:

1. This study proposes to characterize qualitative cognitive information with MHFLTEs. MHFLTEs can reflect conformity and divergence of cognitive information.
2. The proposed method utilizes the ELECTRE III method. It considers the non-compensation of criteria.
3. The proposed method introduces projection. Projection is applied to define preference relations between MHFLTSs. It considers not only the included angle between MHFLTSs but also their distance.

The proposed method can yield results consistent with practical hotel location selection.

Table 10 Decision matrix in Problem 2

	C_1	C_2	C_3	C_4
A_1	$\{h_4, h_5, h_7\}$	$\{h_5, h_8\}$	$\{h_6, h_8\}$	$\{h_1, h_3, h_4\}$
A_2	$\{h_5, h_6\}$	$\{h_6, h_8\}$	$\{h_1, h_3\}$	$\{h_3, h_6\}$
A_3	$\{h_2, h_3\}$	$\{h_6\}$	$\{h_2, h_3, h_5\}$	$\{h_4\}$
A_4	$\{h_6, h_8\}$	$\{h_4, h_5\}$	$\{h_3, h_5, h_8\}$	$\{h_1, h_2, h_3\}$

Table 11 Ranking results of different problems

Problem	Ranking order	Best location	Worst location
Problem 1	$A_1 \succ A_2 \succ A_4 \succ A_3$	A_1	A_3
Problem 2	$A_2 \succ A_1 \succ A_3 \succ A_4$	A_2	A_4

Conclusion

Hotel location selection is an MCDM problem with non-compensatory criteria. This paper has established a hotel location selection method. MHFLTSs have been proposed to characterize qualitative cognitive information. In addition, projection of MHFLTSs has been defined and improved. Based on this, we have proposed a projection-based ELECTRE III method. The preference relations are constructed based on projection-based differences.

The proposed method has been applied to a case study on hotel location selection. This case study explains the application of the proposed method in detail. In addition, a comparative study of three cases has been conducted. Results indicate some advantages of the proposed method.

Our theoretical contribution is threefold. (1) This study proposes to characterize cognitive information with MHFLTSs. This enriches studies on the expression of cognitive information. MHFLTSs can denote vacillation in qualitative cognitive information. Vacillation is caused by the limited cognition of human beings. In addition, MHFLTSs can reflect conformity and divergence of decision-makers' opinions. (2) This study defines and improves the projection of MHFLTSs. It also provides new definitions that enrich existing studies on MHFLTSs. (3) In the computation of cognitive information, this study combines projection with the ELECTRE III method. It is an improvement on existing hotel location selection methods. Results of a comparative analysis support the feasibility of the proposed method.

From an application perspective, the proposed method can be applied to solve hotel location selection problems. Results of case study indicate the feasibility of the method. Furthermore, the expression of cognitive information is vital to the cognitive system [14]. Our study provides a guide for the settlement of cognitive computation.

We provide three promising directions for future research. First, we will extend the proposed method to handle D-intuitionistic hesitant fuzzy sets [53], picture fuzzy linguistic sets [2] and ITHFSs [12], and we will further apply them to the area of cognitive computation. Second, the proposed method can be applied to various other fields, such as pattern recognition [6]. We will study the application of the proposed method in these fields. Third, decision-makers have bounded rationality in practical selection [54]. It will be interesting to consider this in our method.

Acknowledgements The authors are rather grateful to thank the editors and anonymous reviewers for their helpful comments and suggestions. This work was supported by the National Natural Science Foundation of China (nos. 71501192 and 71571193).

Compliance with Ethical Standards

All the authors have read and have abided by the statement of ethical standards for manuscripts. And we declare that:

(a) The material has not been published in whole or in part elsewhere;

(b) The paper is not currently being considered for publication elsewhere;

(c) All authors have been personally and actively involved in substantive work leading to the manuscript, and will hold themselves jointly and individually responsible for its content;

(d) Authors whose names appear on the submission have contributed sufficiently to the scientific work and therefore share collective responsibility and accountability for the results;

(e) All sources of funding of all the authors that may be relevant, including current funding of posts and funding for the research being reported;

(f) There is no conflict of interest.

Appendix 1. Linguistic scale function

Linguistic values cannot be operated directly. The linguistic scale function is an effective tool to handle linguistic values.

Definition A1

Assuming that linguistic term h_i exists in the linguistic term set $H=\{h_i|\,0\leq i\leq 2g,\, g\in N\}$, the linguistic scale function f of the mapping from h_i to real number θ_i can be denoted by

$f: h_i \rightarrow \theta_i,$

where $\theta_i \in [0, 1]$ for $i=0, 1, \ldots, 2g$.

The above linguistic scale function f can be expanded to $f^*: H \rightarrow R^+$, which satisfies $f^*(h_i)=\theta_i\ (\theta_i \in R^+)$. f^* represents a strictly monotonically increasing function. Here, one linguistic scale function is given. This is developed using the subscripts of linguistic terms, and its formula is $f_1^*: h_i \rightarrow \frac{i}{2g}$.

Appendix 2. Proof of Theorems

(a) **Proof of Theorem** 1

Let $M_1=\{\langle x_i, t_1(x_i)\rangle\}$ and $M_2=\{\langle x_i, t_2(x_i)\rangle\}$ be two MHFLTSs, where $t_1(x_i)=\left\{m_{\varphi_r^1(x_i)}\middle|m_{\varphi_r^1(x_i)}\in H, r=1,2,\ldots,L_{1i}\right\}$ and $t_2(x_i)=\left\{m_{\varphi_r^2(x_i)}\middle|m_{\varphi_r^2(x_i)}\in H, r=1,2,\ldots,L_{2i}\right\}$.

1. $f^*\left(m_{\varphi_r^1(x_i)}\right)\geq 0$ exists for $r\in(0,L_{1i}]$, $i\in(0,n]$, and $f^*\left(m_{\varphi_r^2(x_i)}\right)\geq 0$ exists for $r\in(0,L_{2i}]$, $i\in(0,n]$ by Definition A1. Hence, $C(M_1,M_2)=\sum_{i=1}^n\left(\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(m_{\varphi_r^1(x_i)}\right)\cdot\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(m_{\varphi_r^2(x_i)}\right)\right)\geq 0.$
Then, $\rho(M_1,M_2)\geq 0$ by Eq. (5). Moreover, the following inequality can be determined from the Cauchy–Schwarz inequality:

$$\sqrt{E(M_1)}\sqrt{E(M_2)}$$
$$=\sqrt{\sum_{i=1}^n\left(\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(m_{\varphi_r^1(x_i)}\right)\right)^2}\sqrt{\sum_{i=1}^n\left(\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(m_{\varphi_r^2(x_i)}\right)\right)^2}.$$
$$\geq\sum_{i=1}^n\left(\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(m_{\varphi_r^1(x_i)}\right)\cdot\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(m_{\varphi_r^2(x_i)}\right)\right)=C(M_1,M_2)$$

That is to say, $\rho(M_1,M_2)=\frac{C(M_1,M_2)}{\sqrt{E(M_1)}\sqrt{E(M_2)}}\leq 1$. Therefore, it is true that $0\leq\rho(M_1,M_2)\leq 1$.

2. If $M_1=M_2$, then $\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(m_{\varphi_r^1(x_i)}\right)=\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(m_{\varphi_r^2(x_i)}\right)$ exists for any $i\in[1,n]$. Therefore, $C(M_1,M_2)=\sqrt{E(M_1)}\sqrt{E(M_2)}$. That is, $\rho(M_1,M_2)=1$ holds.

3. $\rho(M_1,M_2)=\frac{C(M_1,M_2)}{\sqrt{E(M_1)}\sqrt{E(M_2)}}$ and $\rho(M_2,M_1)=\frac{C(M_2,M_1)}{\sqrt{E(M_1)}\sqrt{E(M_2)}}$ by Definition 3. $C(M_1,M_2)=\sum_{i=1}^n\left(\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(M_{\varphi_r^1(x_i)}\right)\cdot\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(M_{\varphi_r^2(x_i)}\right)\right)=\sum_{i=1}^n\left(\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(M_{\varphi_r^2(x_i)}\right)\cdot\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(M_{\varphi_r^1(x_i)}\right)\right)=C(M_1,M_2)$ by Eq. (2). Therefore, $\rho(M_1,M_2)=\rho(M_2,M_1)$.

Hence, Theorem 1 holds.

(b) **Proof of Theorem 2:**

Let $M_1=\{\langle x_i, t_1(x_i)\rangle\}$, $M_2=\{\langle x_i, t_2(x_i)\rangle\}$, and $M_3=\{\langle x_i, t_3(x_i)\rangle\}$ be three MHFLTSs, where $t_1(x_i)=\left\{m_{\varphi_r^1(x_i)}\middle|m_{\varphi_r^1(x_i)}\in H, r=1,2,\ldots,L_{1i}\right\}$, $t_2(x_i)=\left\{m_{\varphi_r^2(x_i)}\middle|m_{\varphi_r^2(x_i)}\in H, r=1,2,\ldots,L_{2i}\right\}$, and $t_3(x_i)=\left\{m_{\varphi_r^3(x_i)}\middle|m_{\varphi_r^3(x_i)}\in H,\ r=1,2,\ldots,\mathrm{L}_{3i}\right\}$.

(N1) $f^*\left(m_{\varphi_r^1(x_i)}\right)\geq 0$ exists for $r\in(0,L_{1i}]$, $i\in(0,n]$, and $f^*\left(m_{\varphi_r^2(x_i)}\right)\geq 0$ exists for $r\in(0,L_{2i}]$, $i\in(0,n]$ by Definition A1. Hence, $C(M_1,M_2)=\sum_{i=1}^n\left(\frac{1}{L_{1i}}\sum_{r=1}^{L_{1i}}f^*\left(m_{\varphi_r^1(x_i)}\right)\cdot\frac{1}{L_{2i}}\sum_{r=1}^{L_{2i}}f^*\left(m_{\varphi_r^2(x_i)}\right)\right)\geq 0.$

Then $NProj^{*}_{M_2}(M_1) = \frac{C(M_1,M_2)}{C(M_1,M_2)+|E(M_2)-C(M_1,M_2)|} \cdot \frac{E(M_1)}{E(M_1)+|E(M_2)-E(M_1)|} \geq 0$ by Eq. (7). Obviously, $C(M_1, M_2) \leq C(M_1, M_2) + |E(M_2) - C(M_1, M_2)|$ and $E(M_1) \leq E(M_1) + |E(M_2) - E(M_1)|$. That is, $NProj^{*}_{M_2}(M_1) \leq 1$. Thus, $0 \leq NProj^{*}_{M_2}(M_1) \leq 1$ holds.

($N2$) If $M_1 = M_2$, then $E(M_2) = C(M_1, M_2)$ and $E(M_1) = E(M_2)$. Therefore, it is true that $NProj^{*}_{M_2}(M_1) = \frac{C(M_1,M_2)}{C(M_1,M_2)} \cdot \frac{E(M_1)}{E(M_1)} = 1$.

($N3$) If $M_1 \subseteq M_2 \subseteq M_3$, then $L_{1i} \leq L_{2i} \leq L_{3i}$ and $m_{\varphi^1_{L_{1i}-r}(x_i)} \leq m_{\varphi^2_{L_{2i}-r}(x_i)} \leq m_{\varphi^3_{L_{3i}-r}(x_i)}$ exists for any $r \in [1, L_{1i}]$ and $i \in [1, n]$. Moreover, the linguistic scale function f^* is strictly monotonically increasing by Definition A1. Therefore, $\frac{1}{L_{1i}} \sum_{r=1}^{L_{1i}} f^*\left(m_{\varphi^1_r(x_i)}\right) \leq \frac{1}{L_{2i}} \sum_{r=1}^{L_{2i}} f^*\left(m_{\varphi^2_r(x_i)}\right) \leq \frac{1}{L_{3i}} \sum_{r=1}^{L_{3i}} f^*\left(m_{\varphi^2_r(x_i)}\right)$ holds for any $i \in [1, n]$. That is, $C(M_1, M_3) \leq C(M_2, M_3) \leq C(M_3, M_3) = E(M_3)$ and $E(M_1) \leq E(M_2) \leq E(M_3)$. Hence, $NProj^{*}_{M_2}(M_1) = \frac{C(M_1,M_3)}{C(M_1,M_3)+|E(M_3)-C(M_1,M_3)|} \cdot \frac{E(M_1)}{E(M_1)+|E(M_3)-E(M_1)|} = \frac{C(M_1,M_3) \cdot E(M_1)}{(E(M_2))^2}$, and $NProj^{*}_{M_3}(M_2) = \frac{C(M_2,M_3)}{C(M_2,M_3)+|E(M_3)-C(M_2,M_3)|} \cdot \frac{E(M_2)}{E(M_2)+|E(M_3)-E(M_2)|} = \frac{C(M_2,M_3) \cdot E(M_2)}{(E(M_3))^2}$ by Eq. (7). Thus, $NProj^{*}_{M_3}(M_1) \leq NProj^{*}_{M_3}(M_2)$ holds.

Therefore, Theorem 2 is true.

(c) **Proof of Theorem** 4:

($D1$) $0 \leq NProj^{*}_{M^+}(M_1) \leq 1$ and $0 \leq NProj^{*}_{M^+}(M_2) \leq 1$ by Theorem 2. Therefore, $Diff(M_1, M_2) = NProj^{*}_{M^+}(M_1) - NProj^{*}_{M^+}(M_2) \in [-1, 1]$.

($D2$) By Theorem 2, $NProj^{*}_{M^+}(M_1) = NProj^{*}_{M^+}(M_2)$ if $M_1 = M_2$. Hence, $Diff(M_1, M_2) = NProj^{*}_{M^+}(M_1) - NProj^{*}_{M^+}(M_2) = 0$.

($D3$) By Theorem 2, $NProj^{*}_{M^+}(M_1) \leq NProj^{*}_{M^+}(M_2)$ if $M_1 \subseteq M_2 \subseteq M^+$. Thus, $Diff(M_1, M_2) = NProj^{*}_{M^+}(M_1) - NProj^{*}_{M^+}(M_2) \leq 0$ holds.

($D4$) By Theorem 2, $NProj^{*}_{M^+}(M_1) \geq NProj^{*}_{M^+}(M_2)$ if $M_2 \subseteq M_1 \subseteq M^+$. Thus, we can obtain that $Diff(M_1, M_2) = NProj^{*}_{M^+}(M_1) - NProj^{*}_{M^+}(M_2) \geq 0$.

($D5$) $Diff(M_1, M_2) = NProj^{*}_{M^+}(M_1) - NProj^{*}_{M^+}(M_2)$ and $Diff(M_2, M_1) = NProj^{*}_{M^+}(M_2) - NProj^{*}_{M^+}(M_1)$ by Eq. (8). Therefore, $Diff(M_1, M_2) + Diff(M_2, M_1) = NProj^{*}_{M^+}(M_1) - NProj^{*}_{M^+}(M_2) + NProj^{*}_{M^+}(M_2) - NProj^{*}_{M^+}(M_1) = 0$.

($D6$) $Diff(M_1, M_2) + Diff(M_2, M_1) = 0$ by ($D5$). If $Diff(M_1, M_2) = Diff(M_2, M_1)$, then $Diff(M_1, M_2) = Diff(M_2, M_1) = 0$.

($D7$) $Diff(M_1, M_2) = NProj^{*}_{M^+}(M_1) - NProj^{*}_{M^+}(M_2)$ and $Diff(M_2, M_3) = NProj^{*}_{M^+}(M_2) - NProj^{*}_{M^+}(M_3)$ by Eq. (8). Therefore, $Diff(M_1, M_2) + Diff(M_2, M_3) = NProj^{*}_{M^+}(M_1) - NProj^{*}_{M^+}(M_2) + NProj^{*}_{M^+}(M_2) - NProj^{*}_{M^+}(M_3) = NProj_{M^+}{}^{*}(M_1) - NProj^{*}_{M^+}(M_3) = Diff(M_1, M_3)$. Hence, $Diff(M_1, M_2) + Diff(M_2, M_3) = Diff(M_1, M_3)$ holds.

Therefore, Theorem 4 holds.

References

1. Huskinson TLH, Haddock G. Individual differences in attitude structure: variance in the chronic reliance on affective and cognitive information. J Exp Soc Psychol. 2004;40(1):82–90.
2. Liu P, Zhang X. A novel picture fuzzy linguistic aggregation operator and its application to group decision-making. Cogn Comput. 2017; https://doi.org/10.1007/s12559-017-9523-z.
3. Zhao N, Xu Z, Liu F. Group decision making with dual hesitant fuzzy preference relations. Cogn Comput. 2016;8(6):1119–43.
4. Liu Y, Zhang L, Deng P, He Z. Common subspace learning via cross-domain extreme learning machine. Cogn Comput. 2017;9(4):555–63.
5. Thanh ND, Ali M, Le HS. A novel clustering algorithm in a neutrosophic recommender system for medical diagnosis. Cogn Comput. 2017;9(4):526–44.
6. Wootton AJ, Taylor SL, Day CR, Haycock PW. Optimizing echo state networks for static pattern recognition. Cogn Comput. 2017;9(3):391–9.
7. Zhang HY, Ji P, Wang JQ, Chen XH. A neutrosophic normal cloud and its application in decision-making. Cogn Comput. 2016;8(4): 649–69.
8. Zadeh LA. Fuzzy sets. Inf Control. 1965;8(3):338–53.
9. Peng HG, Zhang HY, Wang JQ. Cloud decision support model for selecting hotels on TripAdvisor.com with probabilistic linguistic information. Int J Hosp Manag. 2018;68:124–138.
10. Hu J, Yang Y, Zhang X, Chen X. Similarity and entropy measures for hesitant fuzzy sets. Int Trans Oper Res. 2018;25(3):857–86. https://doi.org/10.1111/itor.12477.
11. Liu P, Li H. Interval-valued intuitionistic fuzzy power Bonferroni aggregation operators and their application to group decision making. Cogn Comput. 2017;9(4):494–512.
12. Farhadinia B. A multiple criteria decision making model with entropy weight in an interval-transformed hesitant fuzzy environment. Cogn Comput. 2017;9(4):513–25.
13. Li J, Wang J. Multi-criteria outranking methods with hesitant probabilistic fuzzy sets. Cogn Comput. 2017;9(6):611–25.
14. Tao Z, Han B, Chen H. On intuitionistic fuzzy copula aggregation operators in multiple-attribute decision making. Cogn Comput. 2018; https://doi.org/10.1007/s12559-018-9545-1.
15. Chou TY, Hsu CL, Chen MC. A fuzzy multi-criteria decision model for international tourist hotels location selection. Int J Hosp Manag. 2008;27(2):293–301.
16. Wibowo S. Interval-valued intuitionistic fuzzy multicriteria group decision making approach for hotel selection. Int J Mach Learn Comput. 2013;3(1):65–9.
17. Yu SM, Wang J, Wang JQ, Li L. A multi-criteria decision-making model for hotel selection with linguistic distribution assessments. Appl Soft Comput. 2017; https://doi.org/10.1016/j.asoc.2017.08.009.
18. Wang J, J-q W, Z-p T, D-y Z. A multi-hesitant fuzzy linguistic multi-criteria decision-making approach for logistics outsourcing

with incomplete weight information. Int Trans Oper Res. 2018;25(3):831–56.
19. Rodriguez RM, MartíNez L, Herrera F. Hesitant fuzzy linguistic term sets for decision making. IEEE Trans Fuzzy Syst. 2012;20(1):109–19.
20. Zhang Z, Wu C. Hesitant fuzzy linguistic aggregation operators and their applications to multiple attribute group decision making. J Intell Fuzzy Syst. 2014;26(5):2185–202.
21. Wibowo S, Deng H. A fuzzy multi-criteria group decision making approach for hotel location evaluation and selection. Lect Notes Electr Eng. 2012;107:1599–608.
22. Yang Y, Wong KKF, Wang T. How do hotels choose their location? Evidence from hotels in Beijing. Int J Hosp Manag. 2012;31(3): 675–85.
23. Pan CM. Market structure and profitability in the international tourist hotel industry. Tourism Manage. 2005;26(6):845–50.
24. Khalili S, Zaremehrjerdi Y, Fallahnezhad MS, Mohammadzade H. Hotel location problem using Erlang queuing model under uncertainty. Int J Eng. 2014;27(12):1879–87.
25. Jiang Y, Jin S, Peng J. Hierarchical fuzzy rule interpolation and its application for hotels location selection. Int J Cogn Informatics Natural Intell. 2016;10(1):55–79.
26. Wang J, Wang JQ, Zhang HY, Chen XH. Multi-criteria group decision-making approach based on 2-tuple linguistic aggregation operators with multi-hesitant fuzzy linguistic information. Int J Fuzzy Syst. 2016;18(1):81–97.
27. Zhou H, Wang J, Zhang H. Stochastic multi-criteria decision-making approach based on SMAA-ELECTRE with extended gray numbers. Int Trans Oper Res. 2016; https://doi.org/10.1111/itor.12380.
28. Roy B. Electre III: un algorithme de classements fondé sur une représentation floue des préférences en présence de critères multiples. Cahiers Centre Études Rech Opér. 1978;20(1):3–24.
29. Vahdani B, Hadipour H. Extension of the ELECTRE method based on interval-valued fuzzy sets. Soft Comput. 2011;15(15):569–79.
30. Chen TY. An ELECTRE-based outranking method for multiple criteria group decision making using interval type-2 fuzzy sets. Inf Sci. 2014;263(3):1–21.
31. Leyva-López JC, Fernández-González E. A new method for group decision support based on ELECTRE III methodology. Eur J Oper Res. 2003;148(1):14–27.
32. Wu Y, Zhang J, Yuan J, Geng S, Zhang H. Study of decision framework of offshore wind power station site selection based on ELECTRE-III under intuitionistic fuzzy environment: a case of China. Energy Conv Manag. 2016;113:66–81.
33. Hashemi SS, Hajiagha SHR, Zavadskas EK, Mahdiraji HA. Multicriteria group decision making with ELECTRE III method based on interval-valued intuitionistic fuzzy information. Appl Math Model. 2016;40(2):1554–64.
34. Rashid T, Faizi S, Xu Z, Zafar S. ELECTRE-based outranking method for multi-criteria decision making using hesitant intuitionistic fuzzy linguistic term sets. Int J Fuzzy Syst. 2018;20(1):78–92.
35. Gitinavard H, Ghaderi H, Pishvaee MS. Green supplier evaluation in manufacturing systems: a novel interval-valued hesitant fuzzy group outranking approach. Soft Comput. 2017; https://doi.org/10.1007/s00500-017-2697-1.
36. Wang JQ, Peng JJ, Zhang HY, Chen XH. Outranking approach for multi-criteria decision-making problems with hesitant interval-valued fuzzy sets. Soft Comput. 2017; https://doi.org/10.1007/s00500-017-2791-4.
37. Chavira DAG, Lopez JCL, Noriega JJS, Valenzuela OA, Carrillo PAA. A credit ranking model for a parafinancial company based on the ELECTRE-III method and a multiobjective evolutionary algorithm. Appl Soft Comput. 2017;60:190–201.
38. Vezmelai AS, Lashgari Z, Keyghobadi A. Portfolio selection using ELECTRE III: evidence from Tehran stock exchange. Decis Sci Lett. 2015;4(2):227–36.
39. Montazer GA, Saremi HQ, Ramezani M. Design a new mixed expert decision aiding system using fuzzy ELECTRE III method for vendor selection. Expert Syst Appl. 2009;36(8):10837–47.
40. Xu Z. On method for uncertain multiple attribute decision making problems with uncertain multiplicative preference information on alternatives. Fuzzy Optim Decis Mak. 2005;4(2):131–9.
41. Yue Z, Jia Y. A direct projection-based group decision-making methodology with crisp values and interval data. Soft Comput. 2017;21(9):2395–405.
42. Zeng S, Bale Entis T, Chen J, Luo G. A projection method for multiple attribute group decision making with intuitionistic fuzzy information. Informatica. 2013;24(3):485–503.
43. Zhang X, Jin F, Liu P. A grey relational projection method for multi-attribute decision making based on intuitionistic trapezoidal fuzzy number. Appl Math Model. 2013;37(5):3467–77.
44. Ye J. Bidirectional projection method for multiple attribute group decision making with neutrosophic numbers. Neural Comput Appl. 2017;28(5):1021–9.
45. Xu GL, Liu F. An approach to group decision making based on interval multiplicative and fuzzy preference relations by using projection. Appl Math Model. 2013;37(6):3929–43.
46. Ji P, Zhang H, Wang J. A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection. Neural Comput Appl. 2018;29(1):221–34.
47. Adam I, Amuquandoh FE. Dimensions of hotel location in the Kumasi metropolis, Ghana. Tour Manag Perspect. 2013;8:1–8.
48. Youngthelin L, Boluk K. A case study of human resource practices in small hotels in Sweden. J Hum Resour Hosp Tour. 2012;11(4): 327–53.
49. Lee LW, Chen SM. Fuzzy decision making based on hesitant fuzzy linguistic term sets. 5th Asian Conference on Intelligent Information and Database Systems, Kuala Lumpur, Malaysia. Springer-Verlag Berlin, Heidelberg; 2013. p 21–30.
50. Wang JQ, Wang J, Chen QH, Zhang HY, Chen XH. An outranking approach for multi-criteria decision-making with hesitant fuzzy linguistic term sets. Inf Sci. 2014;280:224–36.
51. Tian ZP, Wang J, Wang JQ, Zhang HY. A likelihood-based qualitative flexible approach with hesitant fuzzy linguistic information. Cogn Comput. 2016;8(4):670–83.
52. Wang J, Wang JQ, Zhang HY. A likelihood-based TODIM approach based on multi-hesitant fuzzy linguistic information for evaluation in logistics outsourcing. Comput Ind Eng. 2016;99(C: 287–99.
53. Li X, Chen X. D-intuitionistic hesitant fuzzy sets and their application in multiple attribute decision making. Cogn Comput. 2018; https://doi.org/10.1007/s12559-018-9544-2.
54. Yu SM, Wang J, Wang JQ. An extended TODIM approach with intuitionistic linguistic numbers. Int Trans Oper Res. 2018;25(3):781–805.