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Output Feedback Consensus for High-order Stochastic Multi-agent Systems With Unknown Time-varying Delays

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Abstract: This paper addresses the dynamic output feedback leader-following consensus control for a class of high-order stochastic multi-agent systems characterized by unknown time-varying delays. The agents are modeled as a class of stochastic strict feedback nonlinear systems with unknown time-varying delays. Additionally, the states of the agents are unknown for control design. To address these challenges, observers are designed firstly to estimate the unknown states of each agent. Subsequently, a distributed observer-based output feedback consensus protocol, relying solely on the outputs of neighboring agents, is introduced. It is shown that the followers can effectively track the leader's output with a 1st-moment exponential rate. The effectiveness of the proposed control scheme is validated through simulation examples.

Keywords: Distributed control, output feedback control, stochastic multi-agent systems, unknown time-varying delays.

1. INTRODUCTION

The past decades have witnessed significant research interest in the distributed consensus control of multi-agent systems (MASs) owing to their practical applications ([1-3], and related references therein). In practice, numerous physical systems are susceptible to uncertainties and random disturbances, such as noise from unpredictable environmental conditions, resulting in stochastic nonlinearities in modelling of practical systems. As a result, it is of great importance to investigate the distributed consensus control of stochastic nonlinear MASs. Many related works have been proposed in literature. For example, for first- and second-order MASs, sufficient and necessary conditions are provided in [4] to ensure mean square consensus. In [5], the average consensus problem for first-order discrete-time MASs in uncertain communication network environments is investigated. For a category of high-order nonlinear stochastic multi-agent systems (SMASs), [6] proposes a set of distributed controllers designed to guarantee that the tracking error of each agent can be reduced to an arbitrarily small value. Additionally, [7] presents an observer-based distributed consensus protocol for a group of high-order nonlinear SMASs, which guarantees 1st moment exponential leader-following consensus of the systems. Despite the progress made in consensus control for SMASs, generalizing these findings to delayed nonlinear SMASs remains challenging.

In the presence of delays, system performance may deteriorate, potentially leading to instability. To date, research on time-delay consensus in multi-agent systems has primarily focused on deterministic systems. Insufficient attention has been given to the consensus of stochastic systems with time delays, with the majority of studies focusing on first- and second-order linear systems [8-14]. For instance, in cases where the dynamic models of continuous-time MASs are single integrators and the systems are subject to measurement noises and time delays, a distributed leader-following consensus control scheme is proposed in [10]. Another consensus algorithm is designed in [11] for first-order MASs with time delays and measurement noises, taking into account both directed fixed and switching topologies. The stochastic bounded consensus tracking problem is studied in [12] for second-order MASs corrupted by random noises and subject to general sampling delays caused by the signal sampling process. For heterogeneous MASs made up of first- and second-order agents with random time delays, a distributed consensus algorithm on the basis of the probability distribution of the time delays is constructed in [13]. In [14], sufficient conditions are provided based on the linear matrix inequalities for the existence of the expected dynamic output feedback consensus algorithm for SMASs with time delays. Although progress has been made regarding first-, second-order, and linear SMASs with time delays, as far as we know, there are no existing research

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achievements for higher-order nonlinear SMAs with time delays. Furthermore, when the states of systems are unknown, controlling time-delayed high-order SMASs with strict-feedback form becomes increasingly complicated.

In practice, acquiring all the states for each agent is not always feasible due to cost constraints or sensor failures. In such cases, where only output information is available, it is more realistic to investigate output feedback consensus than to investigate state feedback consensus. Significant advancements have been made in the study of output feedback consensus for MASs. For instance, in [15], for linear MASs with communication delays and switching networks, a distributed feedforward approach is employed to deal with the problem of cooperative output regulation. For MASs with external disturbances and Markovian jump parameters, the H-∞ consensus is studied in [16] by applying a static output feedback control method. In [17], an output-based control law is devised for a category of discrete-time heterogeneous linear MASs with actuator limitations on position and rate to, aiming to achieve semiglobal leader-following output consensus. In [18], an internal model-based control strategy is designed for a group of nonlinear MASs with arbitrary relative degree to realize cooperative semi-global output regulation. A distributed adaptive output feedback control strategy, based on the backstepping technique, is introduced in [19] to realize output consensus tracking for uncertain heterogeneous linear MASs with uncertainties. [20] presents a dynamic codec distributed output feedback protocol based on an extended state observer, aiming to achieve consensus for high-order nonlinear heterogeneous MASs.

As far as we know, addressing high-order SMASs in strict feedback form with unknown time-varying delays is rarely considered in literature. Only a few results could be obtained addressing similar problems. For instance, in [21], a distributed tracking consensus control scheme based on adaptive control and neural network is proposed for a category of high-order nonlinear SMASs with time delays. However, this approach solely guarantees that the tracking error converges to a small neighborhood around the origin. As far as we know, how to guarantee exponential convergence for the leader-following consensus under the problem formulation in this paper remains unknown.

Inspired by the aforementioned discussions, this paper studies the problem of 1st moment exponential leader-following consensus for a group of high-order stochastic nonlinear systems under a directed graph. In order to address this issue, in this paper, a hybrid control design and stability analysis paradigm is established, which includes the design of a state observer combined with dynamic gain, utilizing only the outputs of neighboring agents, and a set of Lypunov-Krasovskii functions. Through an appropriate state transformation, it is shown that the proposed protocol guarantees 1st moment exponential leader-following consensus for the system. By combining the designed state observer with dynamic gains, the problem is mainly transformed into determining the dynamics of the gains and stability analysis with a set of Lypunov-Krasovskii functions. The designed controllers ensure 1st moment exponential consensus of the strict-feedback SMASs with unknown time delays. The main contributions of this paper are as

1) This paper establishes a hybrid control design and stability analysis paradigm for output feedback consensus control of high-order strict-feedback SMASs with unknown time-varying delays, which includes designing a state observer combined with dynamic gains and utilizing a set of Lyapunov-Krasovskii functions. Compared with the methods proposed in [6,7], the controller designed in this paper can handle time-varying delays.
2) It is proved that the proposed protocol guarantees 1st moment exponential leader-following consensus through an appropriate state transformation. Furthermore, this paper addresses the exponential leader-following consensus in high-order strict-feedback SMASs with unknown time-varying delays, where previous approaches fell short of achieving exponential consensus. Compared with the method proposed in [21], which can only ensure the tracking error converges to a small neighborhood around the origin, the method proposed in this paper can ensure the 1st moment exponential leader-following consensus of the system.

The remaining sections are structured in the following: Section 2 demonstrates the preliminaries and problem formulation. The design of a universal output-feedback control law is detailed in Section 3. Section 4 presents simulation results to demonstrate the control scheme is effective. Lastly, conclusions are drawn in Section 5.

Notation: $\mathbb{R}$ denotes the set of reals, $\mathbb{R}^+$ denotes the set of positive reals, and $\mathbb{R}^n$ represents the real n-dimensional space. For a given real number h, $|h|$ denotes the absolute value of h. The N-dimensional identity matrix is represented by I_N. For a given vector or matrix Y, Y^T denotes its transpose, and $\|Y\|$ is the Euclidean norm of Y. Defining $\|A\| = (\Sigma_{k=1}^{n}\Sigma_{f=1}^{m}a_{kf}^2)^{1/2}$ for a matrix A. $E[b]$ denotes the expectation of b.

2. PRELIMINARY AND PROBLEM FORMULATION

We will introduce the graph and the problem formulation in this section.

2.1. Preliminary

If the controlled system consists of N agents, and then, an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is applied to indicate the information exchanging between agents, where

$\mathcal{A} = [a_{kf}] \in \mathbb{R}^{N\times N}$ is the corresponding connectivity matrix, $\mathcal{V} = \{1, 2, \cdots, N\}$ represents the node set of the agents, and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ indicates the edge set. The edge (k, f) in $\mathcal{G}$ means that agents k and f are able to communicate with each other to obtain information. When $(f,k) \in \mathcal{E}$, let $a_{kf} = 1$; otherwise $a_{kf} = 0$. The degree matrix of $\mathcal{G}$ can be represented by $\mathcal{D} = diag\{d_1, \cdots, d_N\}$, where $d_k = \sum_{f=1}^{n} a_{kf}$. The corresponding Laplacian matrix of $\mathcal{G}$ is denoted as $\bar{\mathcal{L}} = \mathcal{D} - \mathcal{A}$. When the agent k can receive information from the leader, note $b_k = 1$; otherwise $b_k = 0$. The corresponding connection weight matrix is represented by $\mathcal{B} = diag\{b_1, \cdots, b_N\}$. $\bar{\mathcal{G}}$ is an augmented graph containing the undirected graph $\mathcal{G}$ and a leader with edges between some followers and the leader.

2.2. Problem formulation

In this paper, we consider a SMAs consisting of $N+1$ agents in which the dynamics of the kth ($k = 0, 1, \cdots, N$) agent can be modeled as

$$\begin{aligned} d\chi_{k,f} &= \chi_{k,f+1}dt + h_f(\bar{\chi}_{k,f}, \bar{\chi}_{k,f}(t-\delta))dt \\ &\quad + g_f^T(\bar{\chi}_{k,f}, \bar{\chi}_{k,f}(t-\delta))dw, \\ &\quad f = 1, \cdots, n-1, \\ d\chi_{k,n} &= u_k dt + h_n(\bar{\chi}_{k,n}, \bar{\chi}_{k,n}(t-\delta))dt \\ &\quad + g_n^T(\bar{\chi}_{k,n}, \bar{\chi}_{k,n}(t-\delta))dw, \\ y_k &= \chi_{k,1}, \end{aligned} \tag{1}$$

where $\bar{\chi}_{k,f} = (\chi_{k,1}, \cdots, \chi_{k,f})^T \in \mathbb{R}^f$ and $\bar{\chi}_{k,n} = (\chi_{k,1}, \cdots, \chi_{k,n})^T \in \mathbb{R}^n$ represent the system state, $u_k \in \mathbb{R}$ represents the control input, and $y_k \in \mathbb{R}$ represents the measurable output of the system. The variable δ is an unknown time-varying delay of the states, and w is an r-dimensional independent standard Wiener process (or Brownian motion). When $t \in [-\delta^*, 0)$, for $k = 0, 1, \cdots, N$ and $f = 1, \cdots, n$, the function $\chi_{k,f}(t)$ is defined to be equal to its initial value $\chi_{k,f}(0)$, where δ^* is given in Assumption 2. For simplicity, $(t-\delta)$ in the equations will be represented as the subscript (δ). In the formulation, we refer to the agent with an index of 0 as the leader, and agents with indexes $1, \cdots, N$ as followers. For $f = 1, \cdots, n$, the uncertain nonlinear functions $h_f(\cdot) : \mathbb{R}^f \times \mathbb{R} \to \mathbb{R}$ and $g_f(\cdot) : \mathbb{R}^f \times \mathbb{R} \to \mathbb{R}^r$ are Borel measurable. Additionally, they satisfy Assumption 1. For example, $\bar{\chi}_{k,f}(t-\delta)$ will be replaced by $\bar{\chi}_{k,f}(\delta)$ later. To proceed, the following assumptions are presented.

Assumption 1: There are positive constants ρ_1 and ρ_2 holding the following inequalities for $k = 1, \cdots, N$ and $f = 1, \cdots, n$.

$$\begin{aligned} &|h_f(\bar{\chi}_{k,f}, \bar{\chi}_{k,f(\delta)}) - h_f(\bar{\chi}_{0,f}, \bar{\chi}_{0,f(\delta)})| \\ &\le \rho_1 \sum_{l=1}^{f} (|\chi_{k,l} - \chi_{0,l}| + |\chi_{k,l(\delta)} - \chi_{0,l(\delta)}|), \\ &\|g_f(\bar{\chi}_{k,f}, \bar{\chi}_{k,f(\delta)}) - g_f(\bar{\chi}_{0,f}, \bar{\chi}_{0,f(\delta)})\| \\ &\le \rho_2 \sum_{l=1}^{f} (|\chi_{k,l} - \chi_{0,l}| + |\chi_{k,l(\delta)} - \chi_{0,l(\delta)}|). \end{aligned} \tag{2}$$

Assumption 2: The unknown time-varying delay δ and its derivative $\dot{\delta}$ satisfy,

$$0 \le \delta \le \delta^*, \ \dot{\delta} \le \gamma < 1,$$

where δ^* and γ are positive scalars.

Assumption 3: The augmented graph $\bar{\mathcal{G}}$ contains a spanning tree whose root is node 0. u_0 is known to the followers.

Remark 1: For a deterministic system, one can observe a similar condition on nonlinear terms in Assumption 1 as presented in [22,23]. Additionally, [7] discusses a similar assumption for the diffusion and drift terms in stochastic systems without considering delay. Therefore, combined with the above analysis, it is fair to say that Assumption 1 is reasonable. The description of time-varying delay in Assumption 2 is common and can be found in [8,15,22]. The condition that the communication topology described in Assumption 3 should satisfy is a standard condition and is documented in [6,7,22]. In [6,7], u_0 is a function that depends solely on time, and in [22], u_0 is selected as $u_0 = 0$. Therefore, it can be considered as pre-known information for the followers.

Definition 1: For any twice continuously differentiable non-negative function $V(x,t)$ related to system (1), the differential operator $\mathcal{L}$ is defined as

$$\mathcal{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\Big\{h^T \frac{\partial V}{\partial t} h\Big\}, \tag{3}$$

where x, f, and h are the states and nonlinear functions, respectively. $Tr\{\cdot\}$ denotes the matrix trace.

Definition 2 [7]**:** If there are positive constants m_1 and m_2 such that $E[|\chi_{k,f} - \chi_{0,f}|] \le m_1 e^{-m_2 t}$, $t > 0$ for $k = 1, \cdots, N$ and $f = 1, \cdots, n$, then it can be said that the agents achieve 1st moment exponential leader-following consensus.

Lemma 1 [24]**:** If Assumption 3 is satisfied, then the symmetric matrix $\hat{\mathcal{L}} = \bar{\mathcal{L}} + \mathcal{B}$ associated with $\bar{\mathcal{G}}$ is positive definite.

Lemma 2 [24]**:** Let n-dimensional column vectors $\mathcal{B}_1 = (0, \cdots, 0, 1)^T$ and $\mathcal{B}_2 = (1, \cdots, 0, 0)^T$. $\tilde{D}$, $A \in \mathbb{R}^{n\times n}$ are matrices defined as $\tilde{D} = diag\{0, 1, \cdots, n-1\}$ and $A = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}$. Then, row vectors $K_\alpha = (\alpha_1, \cdots, \alpha_n) \in \mathbb{R}^{1\times n}$ and $K_\beta = (\beta_1, \cdots, \beta_n) \in \mathbb{R}^{1\times n}$ exist such that $M_1 = I_N \otimes A - (\bar{\mathcal{L}} + \mathcal{B}) \otimes (\mathcal{B}_1 K_\alpha)$ and $M_2 = I_N \otimes A - I_N \otimes (K_\beta^T \mathcal{B}_2^T)$ are Hurwitz matrices, where I_N is defined in Notation, $\bar{\mathcal{L}}$ and $\mathcal{B}$ are given in the graph subsection. In addition, there exist positive definite matrices $P_i \in \mathbb{R}^{Nn\times Nn}$ and a positive constant h that satisfy the following inequalities

$$M_i^T P_i + P_i M_i \le -I_{Nn},\ i=1,\ 2,$$
$$-hP_i \le P_i(I_N \otimes \tilde{D}) + (I_N \otimes \tilde{D})P_i \le hP_i.$$

Lemma 3 [25]**:** For $(\chi,\gamma)\in\mathbb{R}^2$, the following Young's inequality holds

$$\chi\gamma \le \frac{v^p}{p}|\chi|^p + \frac{1}{qv^q}|\gamma|^q,$$

where v is an arbitrary constant greater than 0, p, and q are constants greater than 1 and satisfy $(p-1)(q-1)=1$.

3. OUTPUT FEEDBACK CONTROL DESIGN

For the control system (1), where the full states are not available, a set of high-gain observers are designed for each agent ($k=0,\ 1,\ 2,\ \cdots,\ N$) to estimate the unknown states as follows:

$$\begin{aligned} dz_{k,f} &= \left(z_{k,f+1} + \beta_f L^f (y_k - z_{k,1})\right) dt, \\ &\quad f=1,\ \cdots,\ n-1, \\ dz_{k,n} &= (u_k + \beta_n L^n (y_k - z_{k,1}))\,dt, \end{aligned} \tag{4}$$

where β_f are the parameters designed in Lemma 2, L is a dynamic gain that satisfies $L(t)\ge 1$, which will be designed later.

Let

$$\begin{aligned} &\theta_{k,f} = \chi_{k,f} - \chi_{0,f},\ \eta_{k,f} = z_{k,f} - z_{0,f}, \\ &\theta_k = (\theta_{k,1},\theta_{k,2},\cdots,\theta_{k,n}), \\ &\eta_k = (\eta_{k,1},\eta_{k,2},\cdots,\eta_{k,n}), \\ &\theta = (\theta_1^T,\theta_2^T,\cdots,\theta_N^T)^T, \\ &\eta = (\eta_1^T,\eta_2^T,\cdots,\eta_N^T)^T, \end{aligned} \tag{5}$$

for $k=1,\ 2,\ \cdots,\ N$. Then, based on (1) and (4), we can obtain

$$\begin{aligned} d\theta_{k,f} &= \left(\theta_{k,f+1} + h_f(\bar{\chi}_{k,f},\bar{\chi}_{k,f(\delta)}) - h_f(\bar{\chi}_{0,f},\bar{\chi}_{0,f(\delta)})\right)dt \\ &\quad + \left(g_f^T(\bar{\chi}_{k,f},\bar{\chi}_{k,f(\delta)}) - g_f^T(\bar{\chi}_{0,f},\bar{\chi}_{0,f(\delta)})\right)dw, \\ &\quad f=1,\ 2,\ \cdots,\ n-1, \\ d\theta_{k,n} &= \left(u_k - u_0 + h_n(\bar{\chi}_{k,n},\bar{\chi}_{k,n(\delta)}) - h_n(\bar{\chi}_{0,n},\bar{\chi}_{0,n(\delta)})\right)dt \\ &\quad + \left(g_n^T(\bar{\chi}_{k,n},\bar{\chi}_{k,n(\delta)}) - g_n^T(\bar{\chi}_{0,n},\bar{\chi}_{0,n(\delta)})\right)dw, \end{aligned} \tag{6}$$

and

$$\begin{aligned} d\eta_{k,f} &= \left(\eta_{k,f+1} + \beta_f L^f(\theta_{k,1} - \eta_{k,1})\right)dt, \\ &\quad f=1,\ 2,\ \cdots,\ n-1, \\ d\eta_{k,n} &= \left(u_k - u_0 + \beta_n L^n(\theta_{k,1} - \eta_{k,1})\right)dt. \end{aligned} \tag{7}$$

As far as we can see, through appropriate state transformations, the output feedback consensus of system (1) can be solved by tackling the stabilization problem of systems (6) and (7).

For $k=1,\ 2,\ \cdots,\ N$, define

$$\tilde{e}_{k,f} = \theta_{k,f} - \eta_{k,f},\ f=1,\ 2,\ \cdots,\ n. \tag{8}$$

According to (6) and (7), the derivative of $\tilde{e}_{k,f}$ can be calculated as

$$\begin{aligned} d\tilde{e}_{k,f} &= \left(\tilde{e}_{k,f+1} - \beta_f L^f \tilde{e}_{k,1} + h_f(\bar{\chi}_{k,f},\bar{\chi}_{k,f(\delta)})\right. \\ &\quad \left. - h_f(\bar{\chi}_{0,f},\bar{\chi}_{0,f(\delta)})\right)dt + \left(g_f^T(\bar{\chi}_{k,f},\bar{\chi}_{k,f(\delta)})\right. \\ &\quad \left. - g_f^T(\bar{\chi}_{0,f},\bar{\chi}_{0,f(\delta)})\right)dw,\ f=1,\ 2,\ \cdots,\ n-1, \\ d\tilde{e}_{k,n} &= \left(h_n(\bar{\chi}_{k,n},\bar{\chi}_{k,n(\delta)}) - h_n(\bar{\chi}_{0,n},\bar{\chi}_{0,n(\delta)})\right. \\ &\quad \left. - \beta_n L^n \tilde{e}_{k,1}\right)dt + \left(g_n^T(\bar{\chi}_{k,n},\bar{\chi}_{k,n(\delta)})\right. \\ &\quad \left. - g_n^T(\bar{\chi}_{0,n},\bar{\chi}_{0,n(\delta)})\right)dw. \end{aligned}$$

For $k=1,\ 2,\ \cdots,\ N$, introduce the state transformations as

$$\hat{z}_{k,f} = \frac{\eta_{k,f}}{L^{f-1+h}},\ \hat{e}_{k,f} = \frac{\tilde{e}_{k,f}}{L^{f-1+h}},\ f=1,\ 2,\ \cdots,\ n, \tag{9}$$

with h defined in Lemma 2, and L being a dynamic gain to be designed, whose initial value is chosen as $L(0)=1$. Combining with (6) and (7), the converted systems can be obtained as

$$\begin{aligned} d\hat{z}_{k,f} &= \left(L\hat{z}_{k,f+1} - (f-1+h)\frac{dL/dt}{L}\hat{z}_{k,f}\right)dt \\ &\quad + \beta_f L\hat{e}_{k,1}dt,\ f=1,\ 2,\ \cdots,\ n-1, \\ d\hat{z}_{k,n} &= \left(L(u_k - u_0) - (n-1+h)\frac{dL/dt}{L}\hat{z}_{k,n}\right. \\ &\quad \left. + \beta_n L\hat{e}_{k,1}\right)dt, \end{aligned} \tag{10}$$

and

$$\begin{aligned} d\hat{e}_{k,f} &= \left(L\hat{e}_{k,f+1} - \beta_f L\hat{e}_{k,1} - (f-1+h)\frac{dL/dt}{L}\hat{e}_{k,f}\right. \\ &\quad \left. + \vartheta_{k,f}\right)dt + \varrho_{k,f}^T dw,\ f=1,\ 2,\ \cdots,\ n-1, \\ d\hat{e}_{k,n} &= \left(-L\beta_n\hat{e}_{k,1} - (n-1+h)\frac{dL/dt}{L}\hat{e}_{k,n} + \vartheta_{k,n}\right)dt \\ &\quad + \varrho_{k,n}^T dw, \end{aligned} \tag{11}$$

with $\vartheta_{k,f} = (h_f(\bar{\chi}_{k,f},\bar{\chi}_{k,f(\delta)}) - h_f(\bar{\chi}_{0,f},\bar{\chi}_{0,f(\delta)}))/L^{f-1+h}$ and $\varrho_{k,f} = (g_f(\bar{\chi}_{k,f},\bar{\chi}_{k,f(\delta)}) - g_f(\bar{\chi}_{0,f},\bar{\chi}_{0,f(\delta)}))/L^{f-1+h}$.

Proposition 1: If Assumptions 1-3 hold, then we can design a suitable dynamic gain $L(t)$ such that systems (10) and (11) can be stabilized by

$$u_k = -(\Gamma_k \otimes K_\alpha)\hat{z} + u_0, \tag{12}$$

where K_α is the vector given in Lemma 2, Γ_k is the kth row of $\hat{\mathcal{L}}$, and $\hat{z} = (\hat{z}_1^T, \hat{z}_2^T, \cdots, \hat{z}_N^T)^T$ with $\hat{z}_k = (\hat{z}_{k,1}, \cdots, \hat{z}_{k,n})^T$.

Proof: Design controllers u_k $(k = 1, 2, \cdots, N)$ for (10) as shown in (12). Then, by substituting (12) into (10), one can obtain that

$$\begin{aligned} d\hat{z}_{k,f} &= \left(L\hat{z}_{k,f+1} - (f-1+h)\frac{dL/dt}{L}\hat{z}_{k,f} + \beta_f L\hat{e}_{k,1}\right)dt, \\ &\quad f = 1, 2, \cdots, n-1, \\ d\hat{z}_{k,n} &= \Bigg(-L(\Gamma_k \otimes K_\alpha)\hat{z} - (n-1+h)\frac{dL/dt}{L}\hat{z}_{k,n} \\ &\quad + \beta_n L\hat{e}_{k,1}\Bigg)dt. \end{aligned} \tag{13}$$

Denote $\hat{e}_k = (\hat{e}_{k,1}^T, \cdots, \hat{e}_{k,n}^T)^T$. Then, (13) and (11) can be represented as the following compact forms

$$\begin{aligned} d\hat{z}_k = \Bigg(&LA\hat{z}_k - L(\Gamma_k \otimes (\mathcal{B}_1 K_\alpha))\hat{z} + LK_\beta^T \hat{e}_{k,1} \\ &- \frac{dL/dt}{L} D\hat{z}_k\Bigg)dt, \end{aligned} \tag{14}$$

and

$$d\hat{e}_k = \left(LG\hat{e}_k - \frac{dL/dt}{L}D\hat{z}_k + \Theta_k\right)dt + O_k^T dw, \tag{15}$$

respectively, where $D = diag(h, f-1+h, \cdots, n-1+h)$, $G = A - K_\beta^T \mathcal{B}_2^T$, $\Theta_k = \{\vartheta_{k,1}, \cdots, \vartheta_{k,n}\}^T$, and $O_k = \{\varrho_{k,1}, \cdots, \varrho_{k,n}\}$.

Let $\hat{e} = (\hat{e}_1^T, \hat{e}_2^T, \cdots, \hat{e}_N^T)^T$, then (14) and (15) can be expressed as

$$d\hat{z} = \left(LM_1\hat{z} - \frac{dL/dt}{L}(I_N \otimes D)\hat{z} + L(\Phi_e \otimes K_\beta^T)\right)dt, \tag{16}$$

and

$$d\hat{e} = \left(LM_2\hat{e} - \frac{dL/dt}{L}(I_N \otimes D)\hat{e} + \Theta\right)dt + O^T dw, \tag{17}$$

respectively, where $M_1 = I_N \otimes A - \hat{\mathcal{L}} \otimes (\mathcal{B}_1 K_\alpha)$ and $M_2 = (I_N \otimes G)$ are defined in Lemma 2, $\Phi_e = (\hat{e}_{1,1}, \hat{e}_{2,1}, \cdots, \hat{e}_{N,1})^T$, $\Theta = (\Theta_1^T, \Theta_2^T, \cdots, \Theta_N^T)^T$, and $O = (O_1, O_2, \cdots, O_N)$.

Next, we will select a suitable Lyapunov function V and design an appropriate dynamic gain $L(t)$.

Select $V_z = \hat{z}^T P_1 \hat{z}$ and $V_e = \hat{e}^T P_2 \hat{e}$, where the positive matrices P_1 and P_2 are presented in Lemma 2. And then, using $D = \tilde{D} + hI_n$ with $\tilde{D}$ provided by Lemma 2, we have

$$\begin{aligned} \mathcal{L}V_z = &L\hat{z}^T(M_1^T P_1 + P_1 M_1)\hat{z} + 2L\hat{z}^T P_1(\Phi_e \otimes K_\beta^T) \\ &- 2h\frac{dL/dt}{L}z^T P_1 z + (I_N \otimes \tilde{D})P_1)\hat{z} \\ &- \frac{dL/dt}{L}\hat{z}^T(P_1(I_N \otimes \tilde{D}), \end{aligned} \tag{18}$$

and

$$\begin{aligned} \mathcal{L}V_e = &L\hat{e}^T(M_2^T P_2 + P_2 M_2)\hat{e} + (I_N \otimes \tilde{D})P_2)\hat{e} \\ &- \frac{dL/dt}{L}\hat{e}^T(P_2(I_N \otimes \tilde{D}) + 2\hat{e}^T P_2 \Theta \\ &- 2h\frac{dL/dt}{L}e^T P_2 e + Tr\{OP_2 O^T\}. \end{aligned} \tag{19}$$

In the next step, we are going to estimate the right-hand side terms of (18) and (19). Based on Lemma 2, we know that

$$\begin{aligned} &L\hat{z}^T(M_1^T P_1 + P_1 M_1)\hat{z} \le -L\|\hat{z}\|^2, \\ &L\hat{e}^T(M_2^T P_2 + P_2 M_2)\hat{e} \le -L\|\hat{e}\|^2. \end{aligned}$$

Note

$$2L\hat{z}^T P_1(\Phi_e \otimes K_\beta^T) \le 2L\varsigma_1\|\hat{z}\| \cdot \|\hat{e}\|,$$

where $\varsigma_1 = \|P_1\|\sqrt{\sum_{f=1}^n \beta_f^2}$.

According to Lemma 3, the following inequality can be obtained

$$2\varsigma_1\|\hat{z}\| \cdot \|\hat{e}\| \le \frac{1}{2}\|\hat{z}\|^2 + 4\varsigma_1^2\|\hat{e}\|^2.$$

According to Assumption 1, (8) and (9), we have

$$\begin{aligned} &\left|h_f(\bar{\chi}_{k,f}, \bar{\chi}_{k,f(\delta)}) - h_f(\bar{\chi}_{0,f}, \bar{\chi}_{0,f(\delta)})\right|/L^{f-1+h} \\ &\le \rho_1/L^{f-1+h}\sum_{l=1}^f(|\chi_{k,l} - \chi_{0,l}| + |\chi_{k,l(\delta)} - \chi_{0,l(\delta)}|) \\ &\le \rho_1\sum_{l=1}^f \frac{1}{L^{f-l}}(|\hat{e}_{k,l}| + |\hat{z}_{k,l}| + |\hat{e}_{k,l(\delta)}| + |\hat{z}_{k,l(\delta)}|). \end{aligned}$$

Then, due to $L(t) \ge 1$ for $t \ge 0$, the following inequality can be obtained

$$\begin{aligned} &|h_f(\bar{\chi}_{k,f}, \bar{\chi}_{k,f(\delta)}) - h_f(\bar{\chi}_{0,f}, \bar{\chi}_{0,f(\delta)})|/L^{f-1+h} \\ &\le \rho_1\sum_{l=1}^f\left(|\hat{e}_{k,l}| + |\hat{z}_{k,l}| + |\hat{e}_{k,l(\delta)}| + |\hat{z}_{k,l(\delta)}|\right), \end{aligned}$$

which implies that

$$\|\Theta\| \le \rho_1 n\sqrt{nN}(\|\hat{e}\| + \|\hat{z}\| + \|\hat{e}_{(\delta)}\| + \|\hat{z}_{(\delta)}\|),$$

and

$$\begin{aligned} 2\hat{e}^T P_2\Theta \le 2\varsigma_2\big(&\|\hat{e}\|^2 + \|\hat{e}\|\|\hat{z}\| + \|\hat{e}\|\|\hat{e}_{(\delta)}\| \\ &+ \|\hat{e}\|\|\hat{z}_{(\delta)}\|\big), \end{aligned}$$

where $\varsigma_2 = n\sqrt{nN}\|P_2\|\rho_1$.

By applying Young's inequality, it can be got

$$2\varsigma_2\|\hat{e}\|\|\hat{z}\| \le \frac{\varsigma_2}{\sqrt{1+8\varsigma_1^2} - 1}\|\hat{z}\|^2$$

$$+\varsigma_2(\sqrt{1+8\varsigma_1^2}-1)\|\hat{e}\|^2,$$

and

$$2\|\hat{e}\|\|\hat{e}_{(\delta)}\| \le \|\hat{e}\|^2+\|\hat{e}_{(\delta)}\|^2,$$
$$2\|\hat{e}\|\|\hat{z}_{(\delta)}\| \le \|\hat{e}\|^2+\|\hat{z}_{(\delta)}\|^2.$$

Similarly,

$$|(g_f(\bar{\chi}_{k,f},\bar{\chi}_{k,f(\delta)})-g_f(\bar{\chi}_{0,f},\bar{\chi}_{0,f(\delta)}))|/L^{f-1+h} \le \rho_2\sum_{l=1}^{f}(|\hat{e}_{k,l}|+|\hat{z}_{k,l}|+|\hat{e}_{k,l(\delta)}|+|\hat{z}_{k,l(\delta)}|).$$

One can get

$$\|O\| \le \rho_2 n\sqrt{nN}(\|\hat{e}\|+\|\hat{z}\|+\|\hat{e}_{(\delta)}\|+\|\hat{z}_{(\delta)}\|),$$

and

$$Tr\{OP_2O^T\} \le \varsigma_3(\|\hat{e}\|^2+\|\hat{z}\|^2+\|\hat{e}_{(\delta)}\|^2+\|\hat{z}_{(\delta)}\|^2), \tag{20}$$

where $\varsigma_3=4n\sqrt{nN}\|P_2\|\rho_2$.

Moreover, according to $-hP_1 \le P_1(I_N\otimes\tilde{D})+(I_N\otimes\tilde{D})P_1 \le hP_1$ and $-hP_2 \le P_2(I_N\otimes\tilde{D})+(I_N\otimes\tilde{D})P_2 \le hP_2$ in Lemma 2, we have

$$\begin{aligned}&-\frac{dL/dt}{L}\hat{z}^T(P_1(I_N\otimes\tilde{D})+(I_N\otimes\tilde{D})P_1)\hat{z}\\ &\le h\frac{|dL/dt|}{L}\hat{z}^TP_1\hat{z},\\ &-\frac{dL/dt}{L}\hat{e}^T(P_2(I_N\otimes\tilde{D})+(I_N\otimes\tilde{D})P_2)\hat{e}\\ &\le h\frac{|dL/dt|}{L}\hat{e}^TP_2\hat{e}.\end{aligned} \tag{21}$$

Let $V=V_z+8\varsigma_1^2V_e+W$ with

$$W=8\frac{\varsigma_1^2(\varsigma_2+\varsigma_3)e^{\upsilon\delta^*}}{1-\gamma}\times\int_{t-\delta}^{t}e^{-\upsilon(t-s)}\Big(\hat{z}(s)^T\hat{z}(s)+\hat{e}(s)^T\hat{e}(s)\Big)ds.$$

According to (18)-(21), one can get

$$\begin{aligned}\mathcal{L}V \le\, & h\frac{(|dL/dt|-2dL/dt)}{L}\hat{z}^TP_1\hat{z}\\ &+\varsigma_2\left(\sqrt{1+8\varsigma_1^2}+1\right)\|\hat{z}\|^2-\frac{L}{2}\|\hat{z}\|^2\\ &+8\varsigma_1^2\varsigma_3\|\hat{z}\|^2+8\frac{\varsigma_1^2(\varsigma_2+\varsigma_3)e^{\upsilon\delta^*}}{1-\gamma}\|\hat{z}\|^2-\upsilon W\\ &+8\varsigma_1^2\Bigg(-\frac{L}{2}\|\hat{e}\|^2+h\frac{(|dL/dt|-2dL/dt)}{L}\hat{e}^TP_2\hat{e}\\ &+\varsigma_3\|\hat{e}\|^2+\varsigma_2\Big(3+\sqrt{1+8\varsigma_1^2}\Big)\|\hat{e}\|^2\\ &+\frac{(\varsigma_2+\varsigma_3)e^{\upsilon\delta^*}}{1-\gamma}\|\hat{e}\|^2\Bigg).\end{aligned} \tag{22}$$

Let

$$\begin{aligned}dL/dt=-\frac{L}{h}\Bigg(&\frac{(L-1)}{6\omega_1}-1-\frac{\varsigma_2}{\omega_2}\left(\sqrt{1+8\varsigma_1^2}+3\right)\\ &-\frac{\xi_1}{\omega_2}\left(\varsigma_3+\frac{(\varsigma_2+\varsigma_3)e^{\upsilon\delta^*}}{1-\gamma}\right)\Bigg),\end{aligned} \tag{23}$$

where $\omega_1=\max\{\lambda_{\max}(P_1),\lambda_{\max}(P_2)\}$, $\xi_1=\max\{8\varsigma_1^2,1\}$, and $\omega_2=\min\{\lambda_{\min}(P_1),\lambda_{\min}(P_2)\}$. From (23), we know that the upper bound of $L(t)$ is $L_{\max}=\Big(3\frac{\varsigma_2}{\omega_2}\big(\sqrt{1+8\varsigma_1^2}+3\big)+3\frac{\xi_1}{\omega_2}\Big(\varsigma_3+\frac{(\varsigma_2+\varsigma_3)e^{\upsilon\delta^*}}{1-\gamma}\Big)+3+\frac{1}{2\omega_1}\Big)2\omega_1$.

Substitute (23) into (22), one has

$$\mathcal{L}V \le -V_z-8\varsigma_1^2V_e-\upsilon W \le -\sigma V, \tag{24}$$

where $\sigma=\min\{\upsilon,1\}$.

From (24), we know that $E[V(t)] \le V(0)e^{-\sigma t}$. Therefore, the following inequalities are obtained.

$$\begin{aligned}E[\|\hat{z}(t)\|] &\le \sqrt{\frac{V(0)}{\lambda_{\min}(P_1)}}e^{-\frac{\sigma}{2}t},\\ E[\|\hat{e}(t)\|] &\le \sqrt{\frac{V(0)}{\lambda_{\min}(P_2)}}e^{-\frac{\sigma}{2}t}.\end{aligned} \tag{25}$$

Hence, the system composed of (10)-(12) is 1st moment exponential stable.

Theorem 1: If Assumptions 1-3 hold, then based on the constructed time-varying parameter $L(t)$, the 1st moment exponential leader-follower consensus of system (1) can be addressed through the following control protocol

$$u_k=-K_\alpha\Delta\left(\sum_{f=1}^{N}a_{k,f}(z_k-z_f)+b_k(z_k-z_0)\right)+u_0, \tag{26}$$

where $\Delta=diag\{L^{-h},L^{-1-h},\cdots,L^{-n+1-h}\}$, $z_k=(z_{k,1},\cdots,z_{k,n})^T$ are the states of high observers (4), and $K_\alpha\in R^{1\times n}$ is the vector given in Lemma 2.

Proof: From Proposition 1, it can be seen the system composed of (10)-(12) with the dynamic of $L(t)$ designed in (23) is 1st moment exponential stable.

According to (9) and $L(t)$ is bounded, we have

$$\begin{aligned}E[|\eta_{k,f}|] &= L^{f-1+h}E[|\hat{z}_{k,f}|] \le \kappa_{f1}e^{-\frac{\sigma}{2}t},\\ E[|\tilde{e}_{k,f}|] &= L^{f-1+h}E[|\hat{e}_{k,f}|] \le \kappa_{f2}e^{-\frac{\sigma}{2}t},\end{aligned} \tag{27}$$

where $\kappa_{f1}=L_{\max}^{f-1+h}\sqrt{\frac{V(0)}{\lambda_{\min}(P_1)}}$ and $\kappa_{f2}=L_{\max}^{f-1+h}\sqrt{\frac{V(0)}{\lambda_{\min}(P_2)}}$ for $f=1,\cdots,n$.

From (8) and (27), and the fact that $L(t)$ is bounded, it can be concluded that (6) and (7) are 1st moment exponential stable based on the following designed controller

$$u_k=-(\Gamma_k\otimes(K_\alpha\Delta))\eta+u_0,$$

where (9) and (12) are used.

According to (5), (8), and (27), one can get

$$E[|\chi_{k,f}-\chi_{0,f}|]=E[|\eta_{k,f}+\tilde{e}_{k,f}|] \\ \leq E[|\eta_{k,f}|]+E[|\tilde{e}_{k,f}|]\leq \kappa_f e^{-\frac{\sigma}{2}t}, \tag{28}$$

where $\kappa_f=\kappa_{f1}+\kappa_{f2}$. Therefore, we know that the 1st moment exponential leader-follower consensus of system (1) can be achieved under the control of protocol as (26) and (4). This completes the proof of Theorem 1. □

4. SIMULATION RESULTS AND ANALYSIS

In this section, the effectiveness of the proposed protocol is verified through simulation examples.

The dynamic model of a chemical reactor which contains delayed cyclic streams and random disturbances can be described as [22], is shown below

$$\dot{\chi}_{k,1}=-\frac{1}{\zeta_{k,1}}\chi_{k,1}-\iota_{k,1}\chi_{k,1}+\frac{1-\varepsilon_{k,2}}{\varpi_{k,1}}\chi_{k,2}+Q_1(\cdot), \\ \dot{\chi}_{k,2}=-\frac{1}{\zeta_{k,2}}\chi_{k,2}-\iota_{k,2}\chi_{k,2}+\frac{\varepsilon_{k,1}}{\varpi_{k,2}}\chi_{k,1(\delta)}+\frac{h_{k,2}}{\varpi_{k,2}}u_k \\ +Q_2(\cdot), \\ y_k=\chi_{k1}, \tag{29}$$

where $\chi_{k,1}$, $\chi_{k,2}$ represent the compositions; $\zeta_{k,1}$, $\zeta_{k,2}$ are the reactor residence times; $\iota_{k,1}$, $\iota_{k,2}$ stand for the reaction constants; $\varepsilon_{k,1}$ and $\varepsilon_{k,2}$ refer to the recycle flow rate; $\varpi_{k,1}$, $\varpi_{k,2}$ denote the reactor volumes; δ indicates the unknown time-varying delay; $Q_1(\cdot)$ and $Q_2(\cdot)$ are nonlinear functions that describe the system uncertainties and external disturbances. The corresponding simulation parameters are defined as: $\zeta_{k,1}=\zeta_{k,2}=10$; $\iota_{k,1}=0.02$; $\iota_{k,2}=0.05$, $\varepsilon_{k,1}=0.2$, $\varepsilon_{k,2}=0.2$, $\varpi_{k,1}=\varpi_{k,2}=h_{k,2}=0.8$. $Q_1(\cdot)=0.03\chi_{k,1}+c_1(\chi_{k,1}+\chi_{k,1(\delta)})\dot{w}$ and $Q_2(\cdot)=-0.25\chi_{k,2(\delta)}+c_2(\chi_{k,2}+\chi_{k,2(\delta)})\dot{w}$ are system uncertainties and external disturbances with stochastic disturbance w defined in system (1), and $\delta=0.6+0.2\sin(t)$, where c_1 and c_2 are constants. Substitute these parameters into (29), then it can be transformed into

$$d\chi_{k,1}=(\chi_{k,2}+\epsilon_1(\chi_{k,1}))dt+H_1(\chi_{k,1},\chi_{k,1(\delta)})dw, \\ d\chi_{k,2}=(u_k+\epsilon_2(\bar{\chi}_{k,2},\bar{\chi}_{k,2(\delta)}))+H_2(\bar{\chi}_{k,2},\bar{\chi}_{k,2(\delta)})dw, \\ y_k=\chi_{k1}, \tag{30}$$

where $\epsilon_1(\chi_{k,1})=-0.09\chi_{k,1}$, $\epsilon_2(\bar{\chi}_{k,2},\bar{\chi}_{k,2(\delta)})=-0.15\chi_{k,2}-0.25\chi_{k,1(\delta)}-0.25\chi_{k,2(\delta)}$, $H_1(\chi_{k,1}(t),\chi_{k,1(\delta)})=c_1(\chi_{k,1}+\chi_{k,1(\delta)})$, and $H_2(\bar{\chi}_{k,2}(t),\bar{\chi}_{k,2(\delta)})=c_2(\chi_{k,2}+\chi_{k,2(\delta)})$.

Furthermore, one has

$$|\epsilon_1|\leq 0.09|\chi_{k,1}|, \\ |\epsilon_2|\leq 0.25\sum_{f=1}^{2}\left(|\chi_{k,f}(t)|+|\chi_{k,f(\delta)}|\right),$$

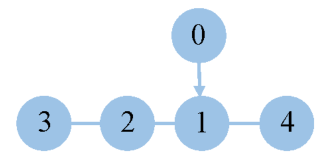

Fig. 1. Communication topology of multi-agent systems (30).

Table 1. Initial value of states and observer on $t\in[-\delta^*,0)$.

States	Initial value	Observer	Initial value
$\chi_{0,1}$	1.6	$z_{0,1}$	0.1
$\chi_{0,2}$	0.1	$z_{0,2}$	0.1
$\chi_{1,1}$	0.1	$z_{1,1}$	0.5
$\chi_{1,2}$	0.4	$z_{1,2}$	0.4
$\chi_{2,1}$	0.7	$z_{2,1}$	0.3
$\chi_{2,2}$	0.3	$z_{2,2}$	0.3
$\chi_{3,1}$	0.9	$z_{3,1}$	0.1
$\chi_{3,2}$	0.6	$z_{3,2}$	0.6
$\chi_{4,1}$	1.3	$z_{4,1}$	0.7
$\chi_{4,2}$	0.9	$z_{4,2}$	0.9

$$H_1\leq c_1(|\chi_{k,1}(t)|+|\chi_{k,1(\delta)}|), \\ H_2\leq c_2(|\chi_{k,2}(t)|+|\chi_{k,2(\delta)}|). \tag{31}$$

According to the formulas shown in (31), it is obvious that Assumption 1 is satisfied. It follows from (4) that the distributed dynamic observers of system (30) are constructed as

$$dz_{k,1}=(z_{k,2}+\beta_1 L(y_k-z_{k,1}))dt, \\ dz_{k,2}=\left(u_k+\beta_2 L^2(y_k-z_{k,1})\right)dt. \tag{32}$$

Fig. 1 represents the communication topology of the agents, where the index 0 represents the leader and the others represent the followers. From Fig. 1, the corresponding Laplacian matrix and connection weight matrix can be determined as follows:

$$\mathcal{L}=\begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix},\ \mathcal{B}=diag\{1,\ 0,\ 0,\ 0\}.$$

Let $\alpha_1=6$, $\alpha_2=6$, $\beta_1=0.6$, $\beta_2=0.8$, $c_1=c_2=0.1$. By directly computation, one has $\delta^*=0.8$, $\gamma=0.2$. The initial values of the system states and the designed observers are list in Table 1.

It can be seen that in [22], the nonlinear functions $Q_1(\cdot)$ and $Q_2(\cdot)$ are selected as $Q_1(\cdot)=0.03\chi_{k,1}$ and $Q_2(\cdot)=-0.25\chi_{k,2(\delta)}$, which do not consider randomness. Therefore, the control algorithm designed in [22] is not applicable to this paper. The simulation results are shown in Figs. 2-6, which further verify that the control algorithm

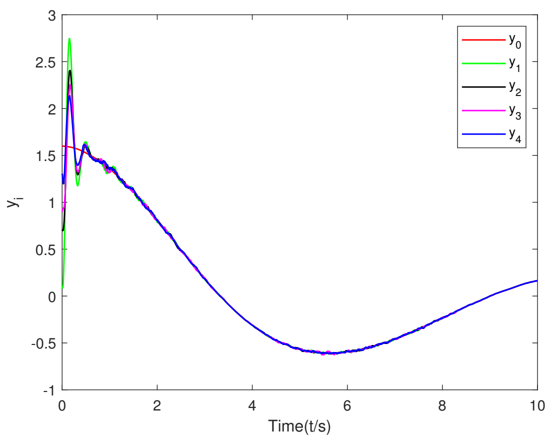

Fig. 2. Response curves of the output y_k of system (30).

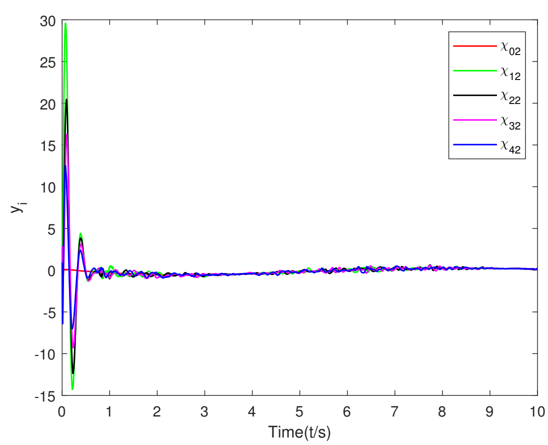

Fig. 3. Response curves of the state variables $\chi_{k,2}$ of system (30).

designed for stochastic system in this paper is effective. Figs. 2 and 3 show the signal response curves of the system (30). It can be observed from these that the outputs of the subsystems can track the desired trajectory under the constructed controllers. The control input curves are shown in Fig. 4. The observer states are shown in Figs. 5 and 6. Based on these, one can conclude that the validity of Theorem 1 is well-illustrated by the simulation example.

5. CONCLUSION

This paper achieves leader-following consensus tracking control for a class of stochastic multi-agent systems with unknown time delays. As existing methods suitable for high-order nonlinear deterministic multi-agent systems do not apply to the control systems addressed in this paper, a distributed controller combined with a dynamic-gained observer is designed. Additionally, it is shown that

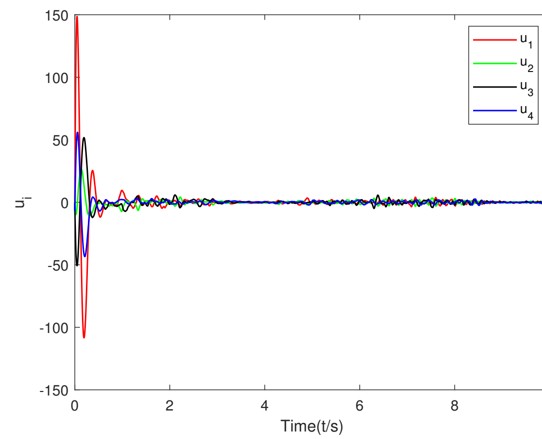

Fig. 4. Response curves of the control input u_k of system (30).

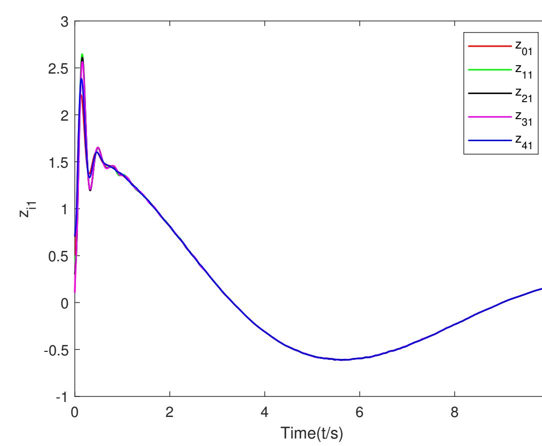

Fig. 5. Response curves of the output of the observer $z_{k,1}$ of system (30).

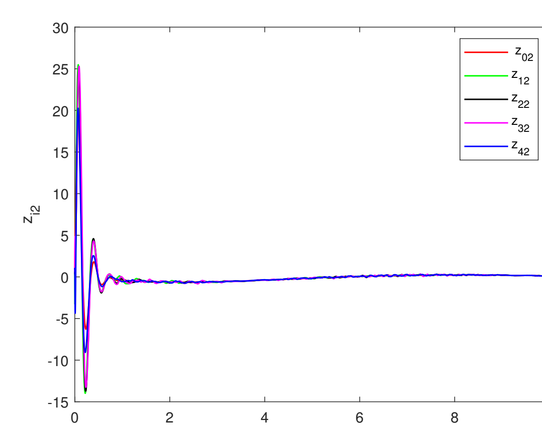

Fig. 6. Response curves of the output of the observer $z_{k,2}$ of system (30).

the proposed protocol guarantees 1st moment exponential leader-following consensus through state transformation. Finally, the simulation results demonstrate that each subsystem can track the leader with a 1st moment exponential rate, indicating the effectiveness of the designed control scheme.

CONFLICTS OF INTEREST STATEMENT

The authors declare that they have no conflict of interest.

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