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Stability Analysis of Asynchronous Impulsive Switched T-S Fuzzy Systems Based on the Admissible Edge-dependent Scheme

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Abstract: The main purpose of this paper is to study the stability of discrete-time impulsive switched T-S fuzzy systems with two kinds of asynchronous behaviors, including asynchronous behavior between impulse and switching, and asynchronous switching between controllers and subsystems. We divide the subsystems into stable and unstable subsystems, which respectively adopt slow switching and fast switching methods. Then, based on multiple Lyapunov functions, admissible edge-dependent average dwell time (AED-ADT) and admissible edge-dependent average impulsive interval (AED-AII) methods, sufficient conditions for global uniform exponential stability (GUES) of the closed-loop system are established, and the results are less conservative than that based on mode-dependent average dwell time (MDADT) and mode-dependent average impulsive interval (MDAII) methods. In addition, we provide the solvability conditions for the state feedback controller. Finally, several numerical examples are provided to verify the effectiveness of the results in this paper.

Keywords: Admissible edge-dependent scheme, asynchronous switching, impulsive switched systems, T-S fuzzy model.

1. INTRODUCTION

As a special kind of hybrid systems, switched systems are composed of several subsystems and switching rules that used to coordinate the switching between subsystems [1]. Switched systems are extensively applied in the research of complex systems in control field, such as transportation systems [2], network control systems [3], automotive steering systems [4] and robot control systems [5]. On the other hand, switched systems may be affected by some sudden changes during switching. Thus, uniting impulse with switched systems gives rise to impulsive switched systems [6]. Impulsive switched systems are popularly used in control, computer, communication and other fields. At present, many scholars are committed to this research and have achieved great fruits [7-9].

Stability analysis [10,11] is the primary issue in the exploration of impulsive switched systems. Literatures [12] and [13] both investigate the related stability problems of impulsive switched systems. However, these studies are conducted under the condition that the impulse and switching are synchronous, causing great limitations. In fact, due to the inevitable delay phenomenon in the system operation, impulse and switching often occur asynchronously. From another perspective, when the system switches, the switching of matching controller will lag behind the switching of subsystem mode, which is called asynchronous switching phenomenon. Asynchronous switching often leads to system performance degradation and even system instability. For this reason, it is of great significance to investigate asynchronous switching. For example, both [14] and [15] research other problems in view of asynchronous switching of switched systems. So far, the research on asynchronous switched systems is still limited, and further exploration is needed in the future.

How to constrain switching signals and impulsive signals is also a significant issue for analyzing impulsive switched systems. The commonly used methods for constraining switching signals mainly include average dwell time (ADT) [16] and mode-dependent average dwell time (MDADT) [17]. But both ADT switching and MDADT switching have a certain degree of conservation. Hence, a new concept named admissible edge-dependent average dwell time (AED-ADT) is proposed in [18], which makes switching parameters not only depend on the subsystems after switching, but also on the subsystems before switching. Relatively speaking, it has higher flexibility and less conservatism. Additionally, in consideration of the impact of impulse on system stability, average impulsive interval (AII) [19] and mode-dependent average impulsive interval (MDAII) [20] have been posed one after another, which are also conservative. Therefore, based on AED-ADT, the concept of admissible edge-dependent average impulsive interval (AED-AII) is constructed to

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improve this issue in [21].

In recent years, fuzzy control theory has become a hot topic in the field of control research, attracting more and more scholars' attention. The commonly used T-S fuzzy model [22] refers to describing nonlinear process as several locally linear input-output relations through several IF-THEN rules. When all subsystems are described by T-S fuzzy model, such systems are called switched T-S fuzzy systems, which have become a powerful tool for dealing with complex nonlinear systems [23,24]. Literatures [25,26] study the stability problem of continuous switched T-S fuzzy systems, while literatures [27,28] concern the stability analysis of discrete T-S fuzzy systems. But we find that these studies are all focused on switched systems. For all practical purposes, we can analyze a more complex situation where the presence of impulse will be considered in switched systems. Currently, there are few relevant papers on impulsive switched T-S fuzzy systems, which has aroused our interest. Affected by the above discussion, it is necessary to study the stability of impulsive switched T-S fuzzy systems based on AED-ADT and AED-AII methods.

The main contributions of this paper are as follows:

1) For nonlinear impulsive switched systems, we introduce the T-S fuzzy model. Compared with the complexity of dealing with nonlinear systems in [10], using T-S fuzzy model can transform complex nonlinear problem into easily solvable linear problem, simplifying the analysis process of the system.

2) Compared with the MDADT method in [17], we adopt AED-ADT and AED-AII methods to constrain switching and impulsive signals. The switching signals in this paper not only depend on the subsystem after switching, but also on the subsystem before switching. It follows that the results we obtain based on AED-ADT and AED-AII methods are less conservative and more flexible than those in [17].

3) This paper concerns the existence of asynchronous behavior. In [12], the switching and impulse are synchronous. In [28], the synchronous case of subsystem and controller is investigated. Compared with above results, this paper considers two asynchronous behaviors: impulsive instant and switching instant are asynchronous; subsystem mode and controller mode are asynchronous, making the results more general and less conservative.

4) We divide the subsystems into stable and unstable subsystems, which apply slow switching and fast switching methods, respectively. Different from [27], we not only consider the case where only includes stable subsystems or unstable subsystems, but also consider the case where there are both stable subsystems and unstable subsystems. Thus, we attain the sufficient conditions for less conservatism.

The rest of this paper is organized as follows: In Section 2, the expressions of the studied systems and some definitions are given. In Section 3, we discuss the sufficient conditions for GUES of impulsive switched T-S fuzzy systems. Section 4 gives several numerical examples. Finally, the paper is summarized in Section 5.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^m$ denote the n-dimensional and m-dimensional Euclidean spaces, respectively. The transpose of a matrix is typically expressed by 'T'. $\lambda_{max}(P)$ and $\lambda_{min}(P)$ represent the maximum and minimum eigenvalue of matrix P, respectively. The notation $\|\cdot\|$ indicates Euclidean vector norm. Denote $x(k_s^+) = \lim_{\Delta\to 0^+} x(k_s+\Delta)$ and $x(k_s) = x(k_s^-) = \lim_{\Delta\to 0^-} x(k_s+\Delta)$.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following discrete-time nonlinear impulsive switched system

$$\begin{cases} x(k+1) = f_{\sigma(k)}(x(k),u(k)),\ k\in[k_s,\ k_{s+1})/\Upsilon, \\ \Delta x(k) = g_{\sigma(k)}(x(k)),\ k\in\Upsilon, \\ x(k_0) = x_0, \end{cases} \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the system state with initial state $x(k_0) = x_0$, $u(k)\in\mathbb{R}^m$ is control input. $\Delta x(k_s) = x(k_s^+) - x(k_s^-) = x(k_s^+) - x(k_s)$. Switching signal $\sigma(k)$: $\mathcal{N} = \{1,\ 2,\ \cdots,\ N\}$ is a piecewise constant function, where N is the number of whole subsystems. Assume that $\mathcal{N} = \mathcal{N}_s \bigcup \mathcal{N}_u$. If $i\in\mathcal{N}_s = \{1,\ 2,\ \cdots,\ n\}$, then the ith subsystem is stable; if $i\in\mathcal{N}_u = \{n+1,\ n+2,\ \cdots,\ N\}$, then the ith subsystem is unstable. We suppose that $\Upsilon = \{k_{s,p},\ p = 1,\ 2,\ \cdots,\ m_s+h_s\}$ is the impulsive time sequence on $[k_s,\ k_{s+1})$, where m_s and h_s respectively denote the number of impulse on mismatching interval $[k_s,\ \Delta k_s)$ and matching $[\Delta k_s,\ k_{s+1})$. For any $i\in\mathcal{N}$, the ith subsystem is activated when $\sigma(k_s) = i$.

Each subsystem of system (1) can be described by T-S fuzzy model in the following form

Rule l for subsystem i:

IF $\theta_{il}(k)$ is M_{i1}^l and $\cdots$ and $\theta_{iv}(k)$ is M_{iv}^l, THEN

$$\begin{cases} x(k+1) = A_{il}x(k) + B_{il}u(k), \\ \Delta x(k) = C_{il}x(k), \end{cases}$$

where $M_{i1}^l\cdots M_{iv}^l$ is fuzzy set, $i\in\mathcal{N}$, $l\in R = \{1,\ 2,\ \cdots,\ r\}$, r is the number of the IF-THEN rules. $\theta_i(k) = [\theta_{i1}(k),\ \theta_{i2}(k),\ \cdots,\ \theta_{iv}(k)]^T$ is the premise variable vector. A_{il}, B_{il}, C_{il} are constant matrices with appropriate dimensions. So the fuzzy model of the ith subsystem is

$$\begin{cases} x(k+1) = \sum_{l=1}^r h_{il}(\theta_i(k))[A_{il}x(k) + B_{il}u(k)], \\ \Delta x(k) = \sum_{l=1}^r h_{il}(\theta_i(k))C_{il}x(k), \end{cases}$$

where $h_{il}(\theta_i(k))$ is normalized membership function with

$$h_{il}(\theta_i(k)) = \frac{\prod_{n=1}^v M_{in}^l(\theta_{in}(k))}{\sum_{l=1}^r \prod_{n=1}^v M_{in}^l(\theta_{in}(k))} \geq 0,$$

$$\sum_{l=1}^{r} h_{il}(\theta_i(k)) = 1,$$

where $M_{in}^l(\theta_{in}(k))$ represents the membership function of premise variable $\theta_{in}(k)$ with respect to fuzzy set M_{in}^l.

Owing to the existence of asynchronous switching, the running time interval $[k_s,\ k_{s+1})$ can be divided into three parts: $T_\uparrow(k_{s+1},k_s)$, $T_\downarrow(k_{s+1},k_s)$ and $k=\Delta k_s$. $T_\uparrow(k_{s+1},k_s)$, namely, $k\in[k_s,\ \Delta k_s)$, and $T_\downarrow(k_{s+1},k_s)$, namely, $k\in[\Delta k_s,\ k_{s+1})$, represent the running time when controller and subsystem mismatch or match, respectively. $k=\Delta k_s$ represents the matching instant of the system. The constant T_M here represents the maximum delay of asynchronous switching system. Then, the following controllers are constructed

$$u(k) = \begin{cases} K_{\sigma(k_{s-1})}x(k),\ k\in[k_s,\ \Delta k_s), \\ K_{\sigma(k_s)}x(k),\ k\in[\Delta k_s,\ k_{s+1}). \end{cases} \tag{2}$$

Let $\sigma(k_{s-1}) = j$, $\sigma(k_s) = i$. Similarly, the fuzzy model of the controller is given as

$$u(k) = \begin{cases} \sum_{m=1}^{r} h_{im}(\theta_i(k))K_{jm}x(k),\ k\in[k_s,\ \Delta k_s), \\ \sum_{m=1}^{r} h_{im}(\theta_i(k))K_{im}x(k),\ k\in[\Delta k_s,\ k_{s+1}). \end{cases}$$

As a result, we obtain the following discrete-time closed-loop impulsive switched T-S fuzzy system

$$\begin{cases} x(k+1) = H_{il}H_{im}(A_{il}+B_{il}K_{jm})x(k), \\ \quad k\in[k_s,\ \Delta k_s)/\Upsilon, \\ x(k+1) = H_{il}H_{im}(A_{il}+B_{il}K_{im})x(k), \\ \quad k\in[\Delta k_s,\ k_{s+1})/\Upsilon, \\ \Delta x(k) = H_{il}C_{il}x(k),\ k\in\Upsilon, \end{cases} \tag{3}$$

where $H_{il} = \sum_{l=1}^{r} h_{il}(\theta_i(k))$, $H_{im} = \sum_{m=1}^{r} h_{im}(\theta_i(k))$.

And we make clear that the relationship among switching instants, impulsive instants and matching instants satisfies $k_0 < k_1 < k_{1,1} < \cdots < k_{1,m_1} < \Delta k_1 < k_{1,m_1+1} < \cdots < k_{1,m_1+h_1} < k_2 < \cdots < k_s < k_{s,1} < \cdots < k_{s,m_s} < \Delta k_s < k_{s,m+1} < \cdots < k_{s,m_s+h_s} < k_{s+1}$. An example is given in Fig. 1.

Here are some definitions and lemmas involved in this paper.

Definition 1 [29]**:** Under a certain switching signal $\sigma(k)$, if there exist constants $\alpha>0$, $0<\varepsilon<1$, such that the solution of system (3) satisfies

$$\| x(k) \| \le \alpha\varepsilon^{k-k_0} \| x(k_0) \|,\ \forall k\ge k_0, \tag{4}$$

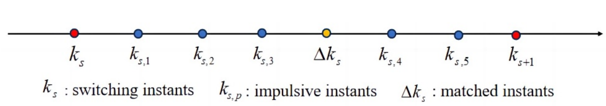

Fig. 1. The relationship of switching instants, impulsive instants, and matching instants.

then system (3) is globally uniformly exponentially stable (GUES) when control input $u(k)=0$.

Definition 2 [30]**:** For a switching signal $\sigma(k)$ and any $j,\ i\in\mathcal{N}$, let $N_{j,i}^{\sigma}(k_0,k)$ denote the switching number from j to i over the range $[k_0,\ k)$, and $T_{j,i}(k_0,k)$ express the entire duration of subsystem i in the range $[k_0,\ k)$, where j is the previous activated subsystem. We define $\tau_{j,i}^{\sigma}$ as a slow or fast admissible edge-dependent average dwell time (AED-ADT) of $\sigma(k)$, respectively, if there exist positive numbers $N_{j,i}^{\sigma,0}$ and $\tau_{j,i}^{\sigma}$ respectively satisfy

$$N_{j,i}^{\sigma}(k_0,k) \le N_{j,i}^{\sigma,0} + \frac{T_{j,i}(k_0,k)}{\tau_{j,i}^{\sigma}},\ \forall k\ge k_0\ge 0, \tag{5}$$

$$N_{j,i}^{\sigma}(k_0,k) \ge N_{j,i}^{\sigma,0} + \frac{T_{j,i}(k_0,k)}{\tau_{j,i}^{\sigma}},\ \forall k\ge k_0\ge 0, \tag{6}$$

where $N_{j,i}^{\sigma,0}$ is called as the admissible edge-dependent chatter bound.

Definition 3 [31]**:** Consider a hybrid system with impulsive and switching signals, for a switching signal $\sigma(k)$ and any $j,\ i\in\mathcal{N}$, let $N_{j,i}^{\delta}(k_0,k)$ denote the impulsive number from j to i over the range $[k_0,\ k)$, and $T_{j,i}(k_0,k)$ express the entire duration of subsystem j in the range $[k_0,\ k)$, where j is the previous activated subsystem. We define $\tau_{j,i}^{\delta}$ as a slow or fast admissible edge-dependent average impulsive interval (AED-AII), respectively, if there exist positive numbers $N_{j,i}^{\delta,0}$ and $\tau_{j,i}^{\delta}$ respectively satisfy

$$N_{j,i}^{\delta}(k_0,k) \le N_{j,i}^{\delta,0} + \frac{T_{j,i}(k_0,k)}{\tau_{j,i}^{\delta}},\ \forall k\ge k_0\ge 0, \tag{7}$$

$$N_{j,i}^{\delta}(k_0,k) \ge N_{j,i}^{\delta,0} + \frac{T_{j,i}(k_0,k)}{\tau_{j,i}^{\delta}},\ \forall k\ge k_0\ge 0, \tag{8}$$

where $N_{j,i}^{\delta,0}$ is called as the admissible edge-dependent elasticity number.

Lemma 1 [31]**:** As we all know, $P_i\in\mathbb{R}^{n\times n}$ as a symmetric positive definite matrix, the multiple Lyapunov functions $V_i(k) = x^T(k)P_ix(k)$ can be bounded as

$$\begin{aligned} \min_{i\in\mathcal{N}}(\lambda_{min}(P_i))\|x(k)\|^2 &\le V_i(k) \\ &\le \max_{i\in\mathcal{N}}(\lambda_{max}(P_i))\|x(k)\|^2. \end{aligned}$$

Lemma 2 [32]**:** For any given symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where S_{11} is $r\times r$ dimensional, the following three inequalities are equivalent.

1) $S<0$,
2) $S_{11}<0, S_{22}-S_{12}^TS_{11}^{-1}S_{12}<0$,
3) $S_{22}<0, S_{11}-S_{12}S_{22}^{-1}S_{12}^T<0$.

Remark 1: In this paper, slow switching and fast switching methods are introduced. When the subsystem is stable, slow switching method is applied, while when the subsystem is unstable, fast switching method is applied.

3. MAIN RESULTS

In this section, we mainly propose some sufficient conditions for GUES of system (3).

The following theorem gives the stability criteria for system (3) with both stable and unstable subsystems.

Theorem 1: Let $\lambda_{j,i} > 0$; $0 < \alpha_i < \beta_i < 1$, $\mu_{j,i} > 1$, $i \in \mathcal{N}_s$; $\beta_i > \alpha_i > 1$, $0 < u_{j,i} < 1$, $i \in \mathcal{N}_u$, if there exist symmetric definite matrices $P_i > 0$, for any $j, i \in \mathcal{N}$, such that

$$(I+C_{il})^T P_i (I+C_{il}) - \lambda_{j,i} P_i < 0, \tag{9}$$

$$(A_{il}+B_{il}K_{jm})^T P_i (A_{il}+B_{il}K_{jm}) - \beta_i P_i < 0, \tag{10}$$

$$(A_{il}+B_{il}K_{im})^T P_i (A_{il}+B_{il}K_{im}) - \alpha_i P_i < 0, \tag{11}$$

$$P_i - \mu_{j,i} P_j \leq 0, \tag{12}$$

and for any switching and impulsive signals satisfy

$$\ln\alpha_i + \frac{\ln\mu_{j,i} + T_M(\ln\beta_i - \ln\alpha_i)}{\tau^{\sigma}_{j,i}} + \frac{\ln\lambda_{j,i}}{\tau^{\delta}_{j,i}} < 0, \tag{13}$$

then, system (3) is GUES. And $T_M = \max\{T_{j,i\uparrow}(k_s, k_{s-1})\}$, $\forall s \geq 1$.

Proof: Construct the multiple Lyapunov functions

$$V_i(k) = x^T(k) P_i x(k).$$

At the impulsive instant $k_{s,p}$, we know that

$$\begin{aligned} V_i(k^+_{s,p}) &= [x(k_{s,p}) + \Delta x(k_{s,p})]^T P_i [x(k_{s,p}) + \Delta x(k_{s,p})] \\ &= [x(k_{s,p}) + H_{il}C_{il}x(k_{s,p})]^T P_i [x(k_{s,p}) \\ &\quad + H_{iq}C_{iq}x(k_{s,p})] \\ &\leq H_{il}x^T(k_{s,p})[(I+C_{il})^T P_i (I+C_{il}) \\ &\quad - \lambda_{j,i}P_i]x(k_{s,p}) + \lambda_{j,i}V_i(k_{s,p}). \end{aligned}$$

Based on (9), it gains

$$V_i(k^+_{s,p}) \leq \lambda_{j,i} V_i(k_{s,p}). \tag{14}$$

When $k \in [k_s, \Delta k_s)$, one has

$$\begin{aligned} V_i(k+1) &= x^T(k+1) P_i x(k+1) \\ &= x^T(k)[H_{il}H_{im}(A_{il}+B_{il}K_{jm})^T P_i H_{ig} H_{iu} \\ &\quad * (A_{ig}+B_{ig}K_{ju})]x(k) \\ &\leq H_{il}H_{im}x^T(k)[(A_{il}+B_{il}K_{jm})^T P_i \\ &\quad * (A_{il}+B_{il}K_{jm}) - \beta_i P_i]x(k) + \beta_i V_i(k). \end{aligned} \tag{15}$$

By (10), it can be deduced that

$$V_i(k+1) \leq \beta_i V_i(k). \tag{16}$$

When $k \in [\Delta k_s, k_{s+1})$, similar to (15), we summarize that

$$\begin{aligned} V_i(k+1) &\leq H_{il}H_{im}x^T(k)[(A_{il}+B_{il}K_{im})^T P_i \\ &\quad * (A_{il}+B_{il}K_{im}) - \alpha_i P_i]x(k) + \alpha_i V_i(k). \end{aligned}$$

According to (11), it indicates

$$V_i(k+1) \leq \alpha_i V_i(k). \tag{17}$$

Considering $\sigma(k_s) = i$, $\sigma(k_{s-1}) = j$, $i \neq j$, we obtain

$$\begin{aligned} & V_i(k_s) - \mu_{j,i} V_j(k_s) \\ &= x^T(k_s) P_i x(k_s) - \mu_{j,i} x^T(k_s) P_j x(k_s) \\ &= x^T(k_s)(P_i - \mu_{j,i} P_j) x(k_s). \end{aligned}$$

On the basis of (12), one can attain

$$V_i(k_s) \leq \mu_{j,i} V_j(k_s). \tag{18}$$

Then, by combining (14), (16), (17), (18), when $k \in [k_s, k_{s+1})$, we conclude that

$$\begin{aligned} V_{\sigma(k)}(k) &\leq \alpha^{k-k_{s,m_s+h_s}}_{\sigma(k_{s,m_s+h_s})} V_{\sigma(k_{s,m_s+h_s})}(k^+_{s,m_s+h_s}) \\ &\leq \lambda_{\delta(k_{s,m_s+h_s-1}),\delta(k_{s,m_s+h_s})} \alpha^{k-k_{s,m_s+h_s}}_{\sigma(k_{s,m_s+h_s})} \\ &\quad * V_{\sigma(k_{s,m_s+h_s})}(k_{s,m_s+h_s}) \\ &\leq \cdots \leq \pi_1 V_{\sigma(k_s)}(k_s), \end{aligned}$$

where $\pi_1 = \lambda_{\delta(k_{s,m_s+h_s-1}),\delta(k_{s,m_s+h_s})} \cdots \lambda_{\delta(k_{s,0}),\delta(k_{s,1})} \alpha^{k-k_{s,m_s+h_s}}_{\sigma(k_{s,m_s+h_s})} \cdots \alpha^{k_{s,m+1}-\Delta k_s}_{\sigma(\Delta k_s)} \beta^{\Delta k_s - k_{s,m_s}}_{\sigma(k_{s,m_s})} \cdots \beta^{k_{s,1}-k_s}_{\sigma(k_s)}$. Then

$$\begin{aligned} V_{\sigma(k)}(k) &\leq \pi_1 \mu_{\sigma(k_{s-1}),\sigma(k_s)} V_{\sigma(k_{s-1})}(k_s) \\ &\leq \cdots \leq N_1 N_2 N_3 N_4 V_{\sigma(k_0)}(k_0), \end{aligned}$$

where

$$N_1 = \prod_{l=1}^{s} \mu_{\sigma(k_{l-1}),\sigma(k_l)},$$

$$N_2 = \prod_{g=1}^{m_s+h_s} \lambda_{\delta(k_{s,g-1}),\delta(k_{s,g})} \prod_{l=1}^{s} \prod_{g=1}^{m_l+h_l} \lambda_{\delta(k_{l-1,g-1}),\delta(k_{l-1,g})},$$

$$\begin{aligned} N_3 &= \alpha^{k-k_{s,m_s+h_s}}_{\sigma(k_{s,m_s+h_s})} \alpha^{k_{s,m_s+1}-\Delta k_s}_{\sigma(\Delta k_s)} \prod_{g=m_s+1}^{m_s+h_s} \alpha^{k_{s,g}-k_{s,g-1}}_{\sigma(k_{s,g-1})} \\ &\quad \times \prod_{l=1}^{s} \prod_{g=m_l+1}^{m_l+h_l} \alpha^{k_{l-1,g}-k_{l-1,g-1}}_{\sigma(k_{l-1,g-1})} \\ &\quad \times \prod_{l=1}^{s} \alpha^{k_l - k_{l-1,m_l+h_l}}_{\sigma(k_{l-1,m_l+h_l})} \alpha^{k_{l-1,m_l+1}-\Delta k_{l-1}}_{\sigma(\Delta k_{l-1})}, \end{aligned}$$

$$\begin{aligned} N_4 &= \beta^{\Delta k_s - k_{s,m_s}}_{\sigma(k_{s,m_s})} \beta^{k_{s,1}-k_s}_{\sigma(k_s)} \prod_{g=2}^{m_s} \beta^{k_{s,g}-k_{s,g-1}}_{\sigma(k_{s,g-1})} \prod_{l=1}^{s} \prod_{g=2}^{m_l} \beta^{k_{l-1,g}-k_{l-1,g-1}}_{\sigma(k_{l-1,g-1})} \\ &\quad \times \prod_{l=1}^{s} \beta^{\Delta k_{l-1} - k_{l-1,m_l}}_{\sigma(k_{l-1,m_l})} \beta^{k_{l-1,1}-k_{l-1}}_{\sigma(k_{l-1})}. \end{aligned}$$

In the light of Definitions 2 and 3, it holds

$$N_1 = \exp\left\{ \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} N^{\sigma}_{j,i}(k_0, k) \ln\mu_{j,i} \right\}$$

$$*\exp\Big\{\sum_{i=n+1}^{N}\sum_{j=n+1,j\neq i}^{N}N_{j,i}^{\sigma}(k_0,k)\ln\mu_{j,i}\Big\},$$

$$N_4=\exp\Big\{\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}T_{j,i\uparrow}(k_0,k)\ln\beta_i\Big\}$$

$$*exp\Big\{\sum_{i=n+1}^{N}\sum_{j=n+1,j\neq i}^{N}T_{j,i\uparrow}(k_0,k)\ln\beta_i\Big\},$$

$$N_3=\exp\Big\{\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}T_{j,i\downarrow}(k_0,k)\ln\alpha_i\Big\}$$

$$*\exp\Big\{\sum_{i=n+1}^{N}\sum_{j=n+1,j\neq i}^{N}T_{j,i\downarrow}(k_0,k)\ln\alpha_i\Big\},$$

$$N_2=\exp\Big\{\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}N_{j,i}^{\delta}(k_0,k)\ln\lambda_{j,i}\Big\}$$

$$*\exp\Big\{\sum_{i=n+1}^{N}\sum_{j=n+1,j\neq i}^{N}N_{j,i}^{\delta}(k_0,k)\ln\lambda_{j,i}\Big\}.$$

Through above discussion, it can be launched that

$$\begin{aligned}V_{\sigma(k)}(k)\leq&\exp\Big\{\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}[N_{j,i}^{\delta}(k_0,k)\ln\lambda_{j,i}\\&+T_{j,i\uparrow}(k_0,k)\ln\beta_i+T_{j,i\downarrow}(k_0,k)\ln\alpha_i\\&+N_{j,i}^{\sigma}(k_0,k)\ln\mu_{j,i}]\Big\}\\&*\exp\Big\{\sum_{i=n+1}^{N}\sum_{j=n+1,j\neq i}^{N}[N_{j,i}^{\delta}(k_0,k)\ln\lambda_{j,i}\\&+T_{j,i\uparrow}(k_0,k)\ln\beta_i+T_{j,i\downarrow}(k_0,k)\ln\alpha_i\\&+N_{j,i}^{\sigma}(k_0,k)\ln\mu_{j,i}]\Big\}V_{\sigma(k_0)}(k_0).\end{aligned}$$

Owing to $T_{j,i}(k_0,k)=T_{j,i\downarrow}(k_0,k)+T_{j,i\uparrow}(k_0,k)$, it gives

$$\begin{aligned}V_{\sigma(k)}(k)\leq&\exp\Big\{\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}[N_{j,i}^{\delta}(k_0,k)\ln\lambda_{j,i}\\&+N_{j,i}^{\sigma}(k_0,k)(\ln\mu_{j,i}+T_M(\ln\beta_i-\ln\alpha_i))\\&+T_{j,i}(k_0,k)\ln\alpha_i]\Big\}\\&*\exp\Big\{\sum_{i=n+1}^{N}\sum_{j=n+1,j\neq i}^{N}[N_{j,i}^{\delta}(k_0,k)\ln\lambda_{j,i}\\&+N_{j,i}^{\sigma}(k_0,k)(\ln\mu_{j,i}+T_M(\ln\beta_i-\ln\alpha_i))\\&+T_{j,i}(k_0,k)\ln\alpha_i]\Big\}V_{\sigma(k_0)}(k_0).\end{aligned}$$

Combining Definitions 2 and 3, one can finally attain

$$\begin{aligned}V_{\sigma(k)}(k)\leq&g_0\exp\Big\{\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}g_{j,i}T_{j,i}(k_0,k)\Big\}\\&*\exp\Big\{\sum_{i=n+1}^{N}\sum_{j=n+1,j\neq i}^{N}g_{j,i}T_{j,i}(k_0,k)\Big\}\\&\times V_{\sigma(k_0)}(k_0),\end{aligned}\tag{19}$$

where $g_{j,i}=\ln\alpha_i+\frac{\ln\mu_{j,i}+T_M(\ln\beta_i-\ln\alpha_i)}{\tau_{j,i}^{\sigma}}+\frac{\ln\lambda_{j,i}}{\tau_{j,i}^{\delta}}$, $g_0=\exp\big\{\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}[N_{j,i}^{\sigma,0}(\ln\mu_{j,i}+T_M(\ln\beta_i-\ln\alpha_i))+N_{j,i}^{\delta,0}\ln\lambda_{j,i}]\big\}\exp\big\{\sum_{i=n+1}^{N}\sum_{j=n+1,j\neq i}^{N}[N_{j,i}^{\sigma,0}(\ln\mu_{j,i}+T_M(\ln\beta_i-\ln\alpha_i))+N_{j,i}^{\delta,0}\ln\lambda_{j,i}]\big\}$. Through Lemma 1, it can be summarized that

$$\|x(k)\|\leq\alpha e^{\beta(k-k_0)}\|x(k_0)\|,\tag{20}$$

where $\alpha=\sqrt{g_0\times\frac{\max_{i\in\mathcal{N}}(\lambda_{max}(P_i))}{\min_{i\in\mathcal{N}}(\lambda_{min}(P_i))}}$, $\beta=\max_{j,i\in\mathcal{N}}(\frac{1}{2}g_{j,i})$. United with condition (13), one has $\beta<0$. Then, on the basis of Definition 1, we deduce that system (3) is GUES. This proof is completed. □

Remark 2: In Theorem 1, β_i and α_i represent the growth or decay rate of Lyapunov function when subsystem and controller mismatch or match, respectively. And $\beta_i>\alpha_i>1$ indicates that the Lyapunov function is growing, namely, the subsystem is unstable, while $0<\alpha_i<\beta_i<1$ denotes that the Lyapunov function is decaying, namely, the subsystem is stable. Besides, $\mu_{j,i}$ and $\lambda_{j,i}$ represent switching parameters and impulsive parameters respectively, where $\mu_{j,i}$ and $\lambda_{j,i}$ are both admissible edge-dependent. $\mu_{j,i}<1$ and $\mu_{j,i}>1$ respectively indicate the switching is stable or unstable. And $\lambda_{j,i}>0$ means that the impulse may be stable or unstable. So we can choose appropriate values within the corresponding parameter range according to actual needs.

Next, consider the case where all subsystems are stable, and $i\in\mathcal{N}$ means $i\in\mathcal{N}_s$, that is, only a slow switching strategy needs to be designed.

Theorem 2: Given $\lambda_{j,i}>0$, $0<\alpha_i<\beta_i<1$, $\mu_{j,i}>1$, $i\in\mathcal{N}$, if there exist symmetric definite matrices $P_i>0$, for any j, $i\in\mathcal{N}$, such that the inequalities (9)-(12) hold, and for any slow switching signal and impulsive signal satisfy condition (13), system (3) is GUES.

Proof: The front proof is similar to (14)-(18) in Theorem 1. Next we combine Definitions 2 and 3 yields

$$N_1=\exp\{\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}N_{j,i}^{\sigma}(k_0,k)\ln\mu_{j,i}\},$$

$$N_4=\exp\{\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}T_{j,i\uparrow}(k_0,k)\ln\beta_i\},$$

$$N_3=\exp\{\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}T_{j,i\downarrow}(k_0,k)\ln\alpha_i\},$$

$$N_2=\exp\{\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}N_{j,i}^{\delta}(k_0,k)\ln\lambda_{j,i}\}.$$

Further, we will gain

$$V_{\sigma(k)}(k)\leq\exp\Big\{\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}[N_{j,i}^{\delta}(k_0,k)\ln\lambda_{j,i}$$

$$+T_{j,i\uparrow}(k_0,k)\ln\beta_i+T_{j,i\downarrow}(k_0,k)\ln\alpha_i$$
$$+N^{\sigma}_{j,i}(k_0,k)\ln\mu_{j,i}]\Big\}V_{\sigma(k_0)}(k_0).$$

The following proof is similar to Theorem 1, and we ultimately obtain

$$V_{\sigma(k)}(k)\leq\varepsilon_0\exp\Big\{\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\varepsilon_{j,i}T_{j,i}(k_0,k)\Big\}V_{\sigma(k_0)}(k_0),$$

where $\varepsilon_0=\exp\Big\{\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}[N^{\sigma,0}_{j,i}(\ln\mu_{j,i}+T_M(\ln\beta_i-\ln\alpha_i))+N^{\delta,0}_{j,i}\ln\lambda_{j,i}]\Big\}$, $\varepsilon_{j,i}=\ln\alpha_i+\frac{\ln\mu_{j,i}+T_M(\ln\beta_i-\ln\alpha_i)}{\tau^{\sigma}_{j,i}}+\frac{\ln\lambda_{j,i}}{\tau^{\delta}_{j,i}}$. Through Lemma 1, one eventually deduce that

$$\|x(k)\|\leq\alpha e^{\beta(k-k_0)}\|x(k_0)\|, \tag{21}$$

where $\alpha=\sqrt{\varepsilon_0\times\frac{\max_{i\in\mathcal{N}}(\lambda_{max}(P_i))}{\min_{i\in N}(\lambda_{min}(P_i))}}$, $\beta=\max_{j,i\in\mathcal{N}}(\frac{1}{2}\varepsilon_{j,i})$. Combining with condition (13), $\beta<0$ will be obtained. Then, according to Definition 1, system (3) is GUES. This completes the proof. □

Then, consider the other case where all subsystems are unstable. In this case, $i\in\mathcal{N}$ means $i\in\mathcal{N}_u$, so we only need to design fast switching method for system (3).

Theorem 3: Let $\lambda_{j,i}>0$, $\beta_i>\alpha_i>1$, $0<\mu_{j,i}<1$, $i\in\mathcal{N}$, if there exist symmetric definite matrices $P_i>0$, for any j, $i\in\mathcal{N}$, such that the inequalities (9)-(12) hold, and for any fast switching signal and impulsive signal satisfy condition (13), system (3) is GUES.

This proof, same as Theorem 2, also infers that system (3) is GUES. And we have omitted it here.

According to Theorems 1-3, an AED-ADT-based state feedback controller is designed in the following result for system (3).

Corollary 1: $\forall j$, $i\in\mathcal{N}$, if exist matrices S_{im}, $S_{jm}\in\mathbb{R}^{m\times n}$, positive definite symmetric matrix $X_i\in\mathbb{R}^{n\times n}$, the following inequalities holds

$$\begin{bmatrix}-X_i & A_{il}X_i+B_{il}S_{im}\\ * & -\alpha_iX_i\end{bmatrix}<0, \tag{22}$$
$$\begin{bmatrix}-X_i & A_{il}X_i+B_{il}S_{jm}\\ * & -\beta_iX_i\end{bmatrix}<0, \tag{23}$$
$$X_j\leq\mu_{j,i}X_i, \tag{24}$$

then system (3) is GUES. And the controller gain matrix is $K_{im}=S_{im}X_i^{-1}$.

Proof: Let $X_i=P_i^{-1}$, $S_{im}=K_{im}P_i^{-1}$, $S_{jm}=K_{jm}P_i^{-1}$, substituting them into (22) and (23), and it gives

$$\begin{bmatrix}-P_i^{-1} & A_{il}P_i^{-1}+B_{il}K_{im}P_i^{-1}\\ * & -\alpha_iP_i^{-1}\end{bmatrix}<0, \tag{25}$$
$$\begin{bmatrix}-P_i^{-1} & A_{il}P_i^{-1}+B_{il}K_{jm}P_i^{-1}\\ * & -\beta_iP_i^{-1}\end{bmatrix}<0. \tag{26}$$

Multiply diag$\{P_i,P_i\}$ on both sides of (25) and (26) at the same time, we will attain

$$\begin{bmatrix}-P_i & P_i(A_{il}+B_{il}K_{im})\\ * & -\alpha_iP_i\end{bmatrix}<0,$$
$$\begin{bmatrix}-P_i & P_i(A_{il}+B_{il}K_{jm})\\ * & -\beta_iP_i\end{bmatrix}<0.$$

Through Lemma 2, we deduce that $(A_{il}+B_{il}K_{im})^TP_i(A_{il}+B_{il}K_{im})-\alpha_iP_i<0$ and $(A_{il}+B_{il}K_{jm})^TP_i(A_{il}+B_{il}K_{jm})-\beta_iP_i<0$. And uniting other conditions in Theorem 1, it can be deduced that system (3) is GUES. The proof is completed. □

In order to highlight the advantages of AED-ADT and AED-AII methods than MDADT and MDAII methods, we present the following theorem based on MDADT and MDAII methods.

Theorem 4: Let $\lambda_i>0$; $0<\alpha_i<\beta_i<1$, $\mu_i>1$, $i\in\mathcal{N}_s$; $\beta_i>\alpha_i>1$, $0<u_i<1$, $i\in\mathcal{N}_u$, if there exist symmetric definite matrices $P_i>0$, for any j, $i\in\mathcal{N}$, $j\neq i$, such that

$$(I+C_{il})^TP_i(I+C_{il})-\lambda_iP_i<0, \tag{27}$$
$$(A_{il}+B_{il}K_{jm})^TP_i(A_{il}+B_{il}K_{jm})-\beta_iP_i<0, \tag{28}$$
$$(A_{il}+B_{il}K_{im})^TP_i(A_{il}+B_{il}K_{im})-\alpha_iP_i<0, \tag{29}$$
$$P_i-\mu_iP_j\leq0, \tag{30}$$

and for any switching and impulsive signals satisfy

$$\ln\alpha_i+\frac{\ln\mu_i+T_M(\ln\beta_i-\ln\alpha_i)}{\tau_i^{\sigma}}+\frac{\ln\lambda_i}{\tau_i^{\delta}}<0, \tag{31}$$

then system (3) is GUES. And $T_M=\max\{T_{i\uparrow}(k_s,k_{s-1})\}$, $\forall s\geq1$.

The proof of Theorem 4 will not be described here.

Remark 3: The main differences between Theorems 1 and 4 are as follows: 1) In Theorem 4, the switching parameters μ_i and impulsive parameters λ_i, regardless of which subsystem they are switched from, are only related to the switched subsystem. The contribution of other subsystems is ignored, which has some conservatism; In Theorem 1, the corresponding switching parameters $\mu_{j,i}$ and impulsive parameters $\lambda_{j,i}$ depend not only on the subsystem after switching, but also on the subsystem before switching. 2) In Theorem 4, τ_i^{σ} and τ_i^{δ} from any subsystem to subsystem i are constant, while $\tau^{\sigma}_{j,i}$ and $\tau^{\delta}_{j,i}$ in Theorem 1 may be different. Therefore, we conclude that AED-method is more adaptive and flexible than MD-method. And we will further validate this conclusion by comparing parameters and simulation results in Section 4.

4. NUMERICAL EXAMPLES

To show the main consequences, we provide several examples in this section. For convenience, the premise variable vectors and normalised membership functions of system (3) are given as $\theta_i(k) = x_1(k)$ and $h_{i1}(\theta_i(k)) = \sin^2(x_1(k))$, $h_{i2}(\theta_i(k)) = \cos^2(x_1(k))$, respectively.

Example 1: Consider system (3) including three subsystems. The parameters are shown as follows:

$$A_{11} = \begin{bmatrix} 1 & 0.3 \\ -0.49 & 0.1 \end{bmatrix},\ A_{12} = \begin{bmatrix} 1 & 0.3 \\ -0.42 & 0.1 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} -1 \\ -0.8 \end{bmatrix},\ B_{12} = \begin{bmatrix} -1.5 \\ -0.9 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix},\ C_{12} = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 1 & 0.3 \\ -0.85 & 0.85 \end{bmatrix},\ A_{22} = \begin{bmatrix} 1 & 0.3 \\ -0.74 & 0.85 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} -2.2 \\ -1.3 \end{bmatrix},\ B_{22} = \begin{bmatrix} -2.5 \\ -1.6 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} -0.5 & 0.1 \\ 1 & 0 \end{bmatrix},\ C_{22} = \begin{bmatrix} -1 & 0.1 \\ 1 & 0 \end{bmatrix},$$

$$A_{31} = \begin{bmatrix} 1 & 0.3 \\ -0.49 & 0.2 \end{bmatrix},\ A_{32} = \begin{bmatrix} 1 & 0.3 \\ -0.42 & 0.2 \end{bmatrix},$$

$$B_{31} = \begin{bmatrix} -10 \\ 9 \end{bmatrix},\ B_{32} = \begin{bmatrix} -8 \\ 4 \end{bmatrix},$$

$$C_{31} = \begin{bmatrix} 0.5 & 0.5 \\ 0 & 0.5 \end{bmatrix},\ C_{32} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix}.$$

It can be seen that the first subsystem and the third subsystem are stable, which should apply slow switching strategy. The second subsystem is unstable, which should design fast switching strategy. The switching path in this example is $1 \to 2 \to 3 \to 1 \to 2 \to 3 \cdots$.

For Theorem 1, we assume $T_M = 0.3$, $\alpha_1 = 0.5$, $\alpha_2 = 1.1$, $\alpha_3 = 0.3$, $\beta_1 = 0.6$, $\beta_2 = 1.2$, $\beta_3 = 0.8$. For Theorem 4, we assume $T_M = 0.3$, $\alpha_1 = 0.5$, $\alpha_2 = 1.01$, $\alpha_3 = 0.3$, $\beta_1 = 0.7$, $\beta_2 = 1.02$, $\beta_3 = 0.9$. The comparison results of the remaining relevant parameters for AED-method and MD-method are shown in Table 1. Besides, we choose $\tau^\sigma_{3,1} = 1.5$, $\tau^\sigma_{1,2} = 2$, $\tau^\sigma_{2,3} = 1.5$, $\tau^\delta_{3,1} = 2$, $\tau^\delta_{1,2} = 2.5$, $\tau^\delta_{2,3} = 2.4$. $\tau^\sigma_1 = 2$, $\tau^\sigma_2 = 2.5$, $\tau^\sigma_3 = 2$, $\tau^\delta_1 = 2.5$, $\tau^\delta_2 = 3$, $\tau^\delta_3 = 2.5$.

From the parameter comparison results in Table 1, it shows that the ADT and AII under MD-method are greater than those under AED-method, which implies that using AED-method can obtain a larger range of switching signals and impulsive signals, further relaxing the conditions of MD-method. Hence, we validate that AED-method is more flexible and less conservative than MD-method.

Given the initial state $x_0 = [0; -1]$, and the simulation results based on AED-method and MD-method are shown in Figs. 2-5, respectively. From the comparison between Figs. 3 and 5, it shows that the convergence time of the system state trajectory under AED-method is significantly shorter than that under MD-method, which once again indicates that AED-method is more flexible and effective than MD-method.

Table 1. The comparison between MD-method and AED-method in Example 1.

Method	MD-method	AED-method
Criteria	Theorem 4	Theorem 1
Parameters	$\mu_1 = 1.4$, $\mu_2 = 0.7$, $\mu_3 = 1.9$, $\lambda_1 = 2.7$, $\lambda_2 = 1.53$, $\lambda_3 = 4.5$	$\mu_{2,1} = 1.3 \mu_{3,1} = 1.2$, $\mu_{1,2} = 0.65 \mu_{3,2} = 0.6$, $\mu_{1,3} = 1.7 \mu_{2,3} = 3.6$, $\lambda_{2,1} = 2.4 \lambda_{3,1} = 2.5$, $\lambda_{1,2} = 1.3 \lambda_{3,2} = 1.5$, $\lambda_{1,3} = 3.5 \lambda_{2,3} = 3.6$
Switching signals	$\tau^{\sigma^*}_1 = 1.9$, $\tau^{\sigma^*}_2 = 2.55$, $\tau^{\sigma^*}_3 = 1.8$	$\tau^{\sigma^*}_{2,1} = 1.25 \tau^{\sigma^*}_{3,1} = 1.3$, $\tau^{\sigma^*}_{1,2} = 2.2 \tau^{\sigma^*}_{3,2} = 2.1$, $\tau^{\sigma^*}_{1,3} = 1.3 \tau^{\sigma^*}_{2,3} = 1.4$
Impulsive signals	$\tau^{\delta^*}_1 = 2.15$, $\tau^{\delta^*}_2 = 3.29$, $\tau^{\delta^*}_3 = 2.26$	$\tau^{\delta^*}_{2,1} = 1.79 \tau^{\delta^*}_{3,1} = 1.8$, $\tau^{\delta^*}_{1,2} = 2.94 \tau^{\delta^*}_{3,2} = 2.98$, $\tau^{\delta^*}_{1,3} = 2.2 \tau^{\delta^*}_{2,3} = 2.23$

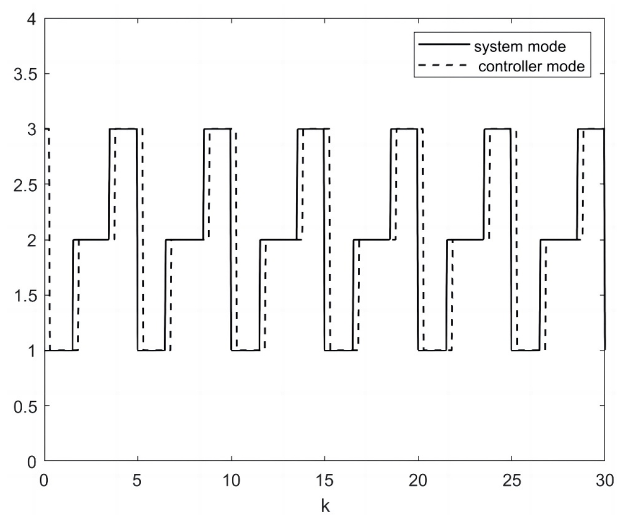

Fig. 2. Switching signals of AED-method for Example 1.

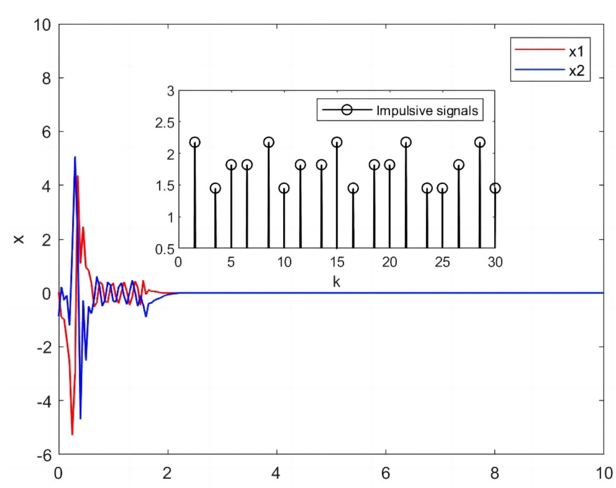

Fig. 3. Impulsive signals and state response of AED-method for Example 1.

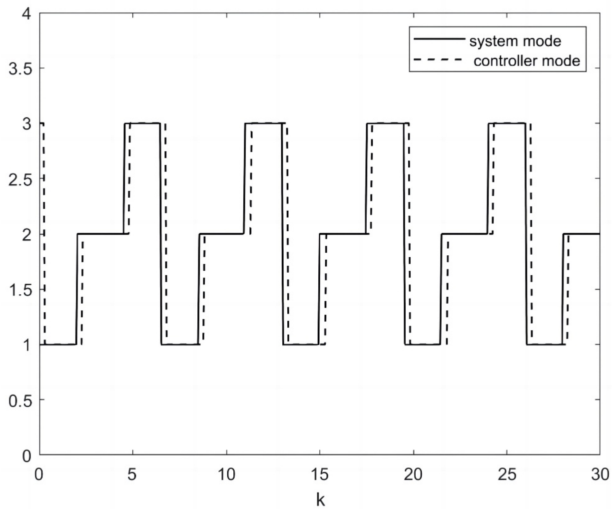

Fig. 4. Switching signals of MD-method for Example 1.

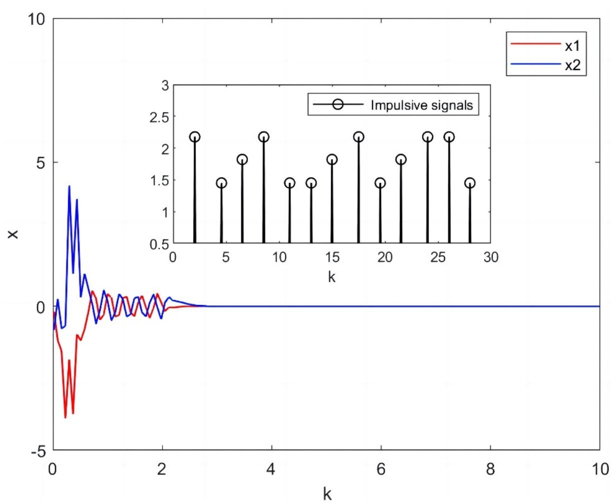

Fig. 5. Impulsive signals and state response of MD-method for Example 1.

Example 2: Consider system (3) including two stable subsystems. And

$$A_{11} = \begin{bmatrix} 0.9 & -0.5 \\ 0.5 & -0.3 \end{bmatrix},\ A_{12} = \begin{bmatrix} 0.9 & -0.5 \\ 0.6 & -0.32 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix},\ B_{12} = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 1.6 & 0.4 \\ 0.3 & 0.1 \end{bmatrix},\ C_{12} = \begin{bmatrix} 1.4 & -0.3 \\ 0.2 & -0.4 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0.9 & 0.32 \\ -1 & 0.45 \end{bmatrix},\ A_{22} = \begin{bmatrix} 0.85 & 0.32 \\ -1 & 0.44 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix},\ B_{22} = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} 1.2 & -0.5 \\ 0.3 & -0.3 \end{bmatrix},\ C_{22} = \begin{bmatrix} 1.3 & 0.2 \\ -0.3 & -0.5 \end{bmatrix}.$$

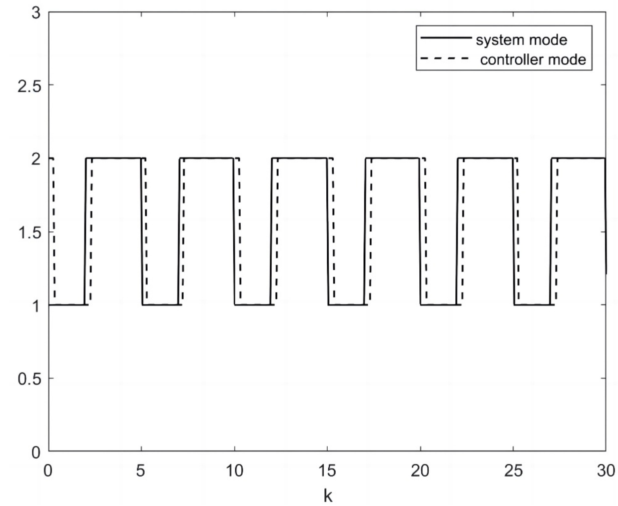

Fig. 6. Switching signals for Example 2.

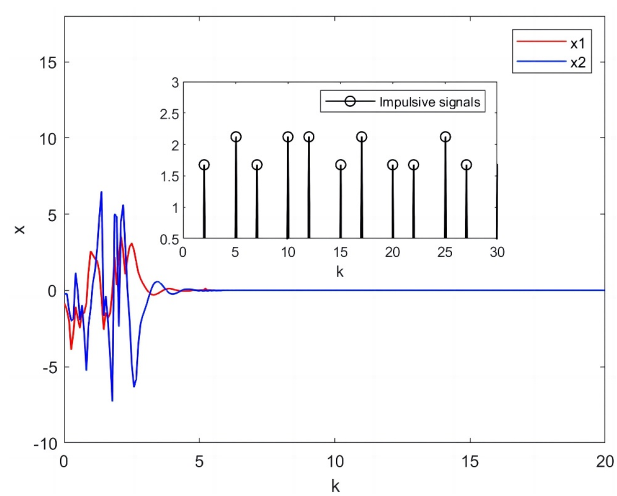

Fig. 7. Impulsive signals and state response for Example 2.

For these two stable subsystems, slow switching strategy is applied. Let $T_M = 0.3$, $\alpha_1 = 0.6$, $\alpha_2 = 0.6$, $\beta_1 = 0.7$, $\beta_2 = 0.8$, $\mu_{1,2} = 2.1$, $\mu_{2,1} = 2.2$, $\lambda_{2,1} = 1.15$, $\lambda_{1,2} = 1.9$, $\tau^{\sigma^*}_{2,1} = 1.8$, $\tau^{\sigma^*}_{1,2} = 2.6$. By Theorem 2, one will obtain $\tau^{\delta^*}_{2,1} = 2.97$, $\tau^{\delta^*}_{1,2} = 3.34$, and we select $\tau^{\sigma}_{2,1} = 2$, $\tau^{\sigma}_{1,2} = 3$, $\tau^{\delta}_{2,1} = 3$, $\tau^{\delta}_{1,2} = 3.5$. Then, the simulation results are shown in Figs. 6 and 7. It is evident that system (3) is GUES under AED-method.

Example 3: Consider system (3) including two unstable subsystems. And

$$A_{11} = \begin{bmatrix} 1.2 & 0.1 \\ -0.1 & 0.2 \end{bmatrix},\ A_{12} = \begin{bmatrix} 1.1 & 0.2 \\ -0.1 & 0.2 \end{bmatrix},$$

$$B_{11} = B_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} -1.71 & 0.62 \\ -0.93 & 0.46 \end{bmatrix},\ C_{12} = \begin{bmatrix} 2.41 & 0.82 \\ -0.93 & 0.46 \end{bmatrix},$$

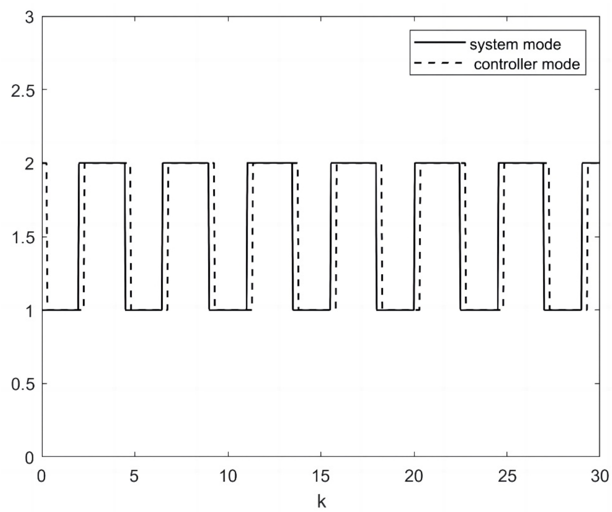

Fig. 8. Switching signals for Example 3.

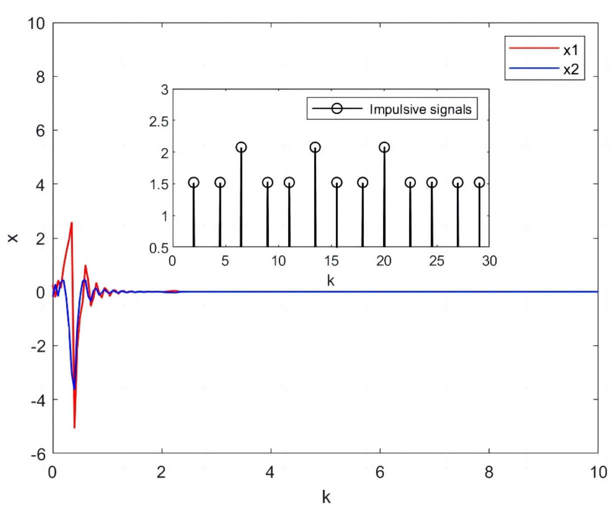

Fig. 9. Impulsive signals and state response for Example 3.

$$A_{21} = \begin{bmatrix} 0.4 & 0.2 \\ 0.5 & 1.1 \end{bmatrix}, A_{22} = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 1.2 \end{bmatrix},$$
$$B_{21} = B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C_{21} = \begin{bmatrix} 0.52 & -0.84 \\ 1.26 & 2.42 \end{bmatrix}, C_{22} = \begin{bmatrix} 0.32 & -0.61 \\ 0.91 & -1.8 \end{bmatrix}.$$

For these two unstable subsystems, we apply fast switching methods. Given $T_M = 0.3$, $\alpha_1 = 1.1$, $\alpha_2 = 1.3$, $\beta_1 = 1.5$, $\beta_2 = 1.8$, $\mu_{1,2} = 0.5$, $\mu_{2,1} = 0.7$, $\lambda_{2,1} = 1.06$, $\lambda_{1,2} = 0.86$, $\tau_{2,1}^{\sigma^*} = 1.8$, $\tau_{1,2}^{\sigma^*} = 2.6$. Based on Theorem 3, it gains $\tau_{2,1}^{\delta^*} = 2.33$, $\tau_{1,2}^{\delta^*} = 4.57$, and we choose $\tau_{2,1}^{\sigma} = 2$, $\tau_{1,2}^{\sigma} = 2.5$, $\tau_{2,1}^{\delta} = 2$, $\tau_{1,2}^{\delta} = 4.2$. The simulation results are shown in Figs. 8 and 9. It also infers that system (3) is GUES under AED-method.

5. CONCLUSION

This paper investigates the stability of discrete-time impulsive switched T-S fuzzy systems under two asynchronous behaviors. We divide the subsystems into stable subsystems and unstable subsystems, which adopt slow switching and fast switching strategies, respectively. Then, combining multiple Lyapunov functions, AED-ADT and AED-AII methods, three cases of the system including stable and unstable subsystems, only stable subsystems and only unstable subsystems are analyzed, and the criteria for GUES of the closed-loop system are established. Besides, the state feedback controller is designed. Finally, several examples are given to verify the effectiveness of the results in this paper.

CONFLICT OF INTEREST STATEMENT

The authors declare that they have no conflict of interest.

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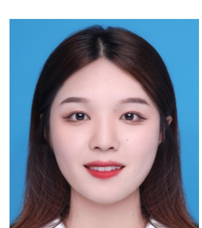

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