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Leader-following Adaptive Guaranteed-performance Consensus Control for Multi-agent Systems With Exogenous Disturbance

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Abstract: In this paper, the problem of leader-following distributed guaranteed-performance consensus for multi-agent systems (MASs) subject to exogenous disturbances is investigated. First, a disturbance observer is designed for each follower, which can be used to efficiently estimate the external disturbances. Next, an adaptive distributed state feedback consensus protocol with guaranteed performance constraints is proposed based on the above proposed observer. Most of the existing literature on guaranteed performance does not consider unknown disturbances. Unlike existing schemes, consensus control with fully distributed guaranteed-performance is accomplished using this protocol, which solves the consensus control problem of exogenous disturbances. The consensus criterion of adaptive guaranteed-performance is given by using Riccati inequality, and the adjustment method of consensus control gain is given by using linear matrix inequality for leader-following situation. At last, the derived analytical results are validated by presenting a simulation example.

Keywords: Adaptive consensus, disturbance observer, guaranteed-performance, multi-agent systems.

1. INTRODUCTION

Recent years, coordinated control technology is widely used in many fields, such as reconnaissance, surveillance and target striking, intelligent highways, environmental monitoring, and unknown environment exploration. The consensus question is a fundamental problem in the control of distributed collaborative MASs [1-4], which is mainly refers to a single agent in the systems in the network environment with the adjacent agent individual interaction measurable information (such as position, velocity, etc.). It generates a control protocol on the individual agent, making these agents consensus on the collaboration variables (data, parameter), and then performing the overall task spontaneously and in a coordinated manner. In other words, the convergence of the cooperative variables of all agents can be accomplished under the above protocol. Then, the key to solving the consensus problem translates into how to devise a reasonable control protocol. At present, many results have been achieved in consensus research on MASs [5-8].

In most of the literature, studies on consensus topics can be broadly separated into two kinds, namely leaderless consensus and leader-following consensus (or consensus tracking), by whether consensus is linked to the amount of leaders [9,10]. Aiming at leaderless consensus requires designing a distributed consensus protocol with a common function which lie with the original state of each agent and all information related to the agents in the network is tracked by this function [11-14]. As described in [15], motivated by some of the natural phenomena in life, such as swarms of bees, schools of fish, and flocks of geese, consensus seeking in MASs with a leader can be regarded as an energy conservation mechanism. Alternatively, the leader can be a true or a virtual agent, which can offer reference signals for all followers in leader-following MASs to follow. So far, considerable research work on leader-following consensus in MASs has been carried out. In [16], a distributed bipartite finite-time event-triggered protocol in a directed topology is devised to study the consensus tracking issue for stochastic nonlinear MASs. Along with this, adaptive control can be called another important advancement in the area of control, which has greatly enhanced the control performance of various questions [17-19]. In [20], a protocol for adaptive event-triggering is constructed. Using the approximation capability of neural networks, the consensus tracking error is shown to converge to the set of residuals at the ori-

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gin. The leader-following consensus question for MASs is studied under the trigger mechanism designed in [21] and an adaptive consensus strategy triggered by fully distributed edge events.

For MASs, consensus regulation performance is one of the significant factors while dealing with consensus questions. In the practical application of MASs, each agent might have only a restricted amount of energy available for performing certain tasks, such as perception, communicating, and moving. In [22], it is shown that the consensus error bound converges to a bounded region that relies on the total attack energy via the use of an extended diagonal matrix analysis. At the same time, consensus regulation is required to meet certain performance targets that are subject to practical constraints. For example, when scheduling multiple autonomous vehicle networks to perform a given inspection task, it is important to consider the distance performance to ensure that the utility is maximized. Therefore, designing a tradeoff between consensus adjustment performance and energy cost is a key challenge, of which can often be programmed as an optimal or second-best consensus issue. In [23], a performance function is built using the state error between the agents for leader-following nonlinear MASs, and a novel distributed nonlinear consensus protocol under state feedback is devised. The guaranteed performance consensus issue of nonlinear MASs with leaders is examined. In [24], for high-order MASs with Lipschitz nonlinearity and switching topology, a secondary function containing the consensus error among adjacent agents is presented, which can be used to tune the consensus performance. The issue of its non-fragile assured performance H_∞ leader-follower consensus is studied.

At the same time, due to the model uncertainty [25,26], parameter perturbation [27], external disturbance [28,29] and other factors, it is often necessary to design controllers with certain robustness in practical control systems. And the control framework based on disturbance observer can effectively eliminate the disturbance caused by the above factors. Therefore, it is more challenging and meaningful in theory and reality to deal with the disturbance rejection question in multi-agent control based on disturbance observer. In latest years, numerous researchers have studied the consensus of MASs with exogenous disturbances [30,31]. Wu and Liu [32] studied the distributed average tracking question for MASs with heterogeneous dynamics and external disturbances. In order to enable each agent to track the required signals accurately, they put forward controllers that do not require initialization. Ruan *et al.* [33] designed a distributed adaptive controller for each follower to settle the question of consensus tracking. Wu *et al.* [34] designed a controller that handles finite-time output regulation problems and rejects disturbances, and enables each agent to accurately track the reference signal.

Prompted by the discussions above, this essay investigates the leader-following consensus issue for the guaranteed performance of MASs under external disturbances. This work presents the following original contributions: (i) In contrast to the existing literature on guaranteed performance [35,36], exogenous disturbances are considered and an observer-based approach is explored to estimate them. A novel adaptive consensus protocol is presented for leader-follower MASs. (ii) Compared with [37-39], the consensus protocol put forward in this article achieves performance control in a fully distributed manner. Combined with the linear matrix inequality technique, a means of tuning the consensus control gain is presented, and the cost of guaranteed performance is defined. (iii) The disturbances are estimated by a disturbance observer, and this approach is helpful to reduce the impact of external disturbances on MASs, thus strengthening the robustness of the system. This is more meaningful and practical in both theory and application.

The rest of this paper is organized as follows: Section 2 presents some necessary preparatory knowledge and problem descriptions. Section 3 proposes a disturbance observer and a distributed consensus protocol to realize consensus disturbance suppression. Also, the consensus problem of leader-following adaptive guaranteed-performance is investigated, and the consensus criteria is presented. Section 4 uses an example to demonstrate the validity of the above mathematical analysis. Finally, Section 5 gives concluding remarks, and the next direction of research.

2. PRELIMINARIES AND PROBLEM DESCRIPTION

2.1. Notation

Some symbolic meanings about this paper are given as follows:

$R^{p\times q}$	$p\times q$ real matrice
$diag\{.\}$	Diagonal matrix
$\mathbf{1}$	A column vector with each term being 1
I_N	A matrix of identity of dimension N
A^T	Transpose of A
$A\otimes B$	The Kronecker product of matrices A and B
$\lambda_{\max}(N)$	The maximum eigenvalue of matrix N
$\lambda_{\min}(N)$	The minimum eigenvalue of matrix N

2.2. Basic graph theory

Assume a graph $G=\{\mathcal{J},\mathcal{R}\}$, in which $\mathcal{J}=\{j_1,\ j_2,\ ...,\ j_N\}$ stands for is the set of nodes. The edge set is represented by $\mathcal{R}=\{(j_i,j_k): j_i,\ j_k\in\mathcal{J},\ i\neq k\}$. If the edge $e_{ik}=(j_i,j_k)$ (or $e_{ki}=(j_k,j_i)$) between i and k, then agent i can receive messages from agent k. The neighbor set $\mathcal{N}_i=\{j_k\in\mathcal{J}\mid (j_k,j_i)\in\mathcal{R}\}$. The graph G is undirected if and only if $e_{ik} \in \mathcal{R}$ implies $e_{ki} \in \mathcal{R}$, for any j_i, $j_k \in \mathcal{J}$. The adjacency matrix $\mathcal{M} = [d_{ik}]_{N\times N} \in R^{N\times N}$, which is defined as $d_{ik} = 1$ if $(j_k, j_i) \in \mathcal{R}$, otherwise $d_{ik} = 0$. For the graph G, define the Laplacian matrix as $\mathcal{L} = [l_{ik}] \in R^{N\times N}$ with $l_{ii} = \sum_{k=0,k\neq i}^{N} d_{ik}$ and $l_{ik} = -d_{ik}(i \neq k)$. $\mathcal{L}$ can be denoted as $\mathcal{L} = \begin{bmatrix} 0 & \mathbf{0} \\ L_{fl} & L_{ff} \end{bmatrix}$, where $L_{fl} \in R^N$ denotes the interaction between leader and followers, and $L_{ff} \in R^{N\times N}$ represents interactions among followers and is symmetric.

2.3. Problem description

The MASs of $N+1$ agents. Suppose there are N followers marked 1, 2, ..., N, and a leader marked 0, and that the leader cannot get any information from the followers. Its dynamics are as follows:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + B\left(u_i(t) + d_i(t)\right), \\ y_i(t) = Dx_i(t), \end{cases} \tag{1}$$

in which $i \in \{1, 2, ..., N\}$, $A \in R^{s\times s}$, $B \in R^{s\times v}$ and $D \in R^{l\times s}$ are constant matrices. $x_i(t) \in R^s$, $y_i(t) \in R^l$ and $u_i(t) \in R^v$ are the state, the output and control input, respectively. $d_i(t) \in R^v$ is the external disturbance.

The expression for the leader is represented by

$$\begin{cases} \dot{x}_0(t) = Ax_0(t), \\ y_0(t) = Dx_0(t), \end{cases} \tag{2}$$

where $x_0(t) \in R^s$ and $y_0(t) \in R^l$ are the state and the output. Assumed the disturbance $d_i(t)$, $i = 1, 2, ..., N$ is produced by the next external system

$$\begin{cases} \dot{w}_i(t) = Sw_i(t), \\ d_i(t) = Fw_i(t), \end{cases} \tag{3}$$

in which $w_i(t) \in R^q$ is state of the linear external system, $S \in R^{q\times q}$ and $F \in R^{v\times q}$ are coefficient matrices.

Assumption 1: A topology graph is assumed to be connected, and the edges between followers are undirected.

Assumption 2: Matrix pair (A,B) is stable.

Remark 1: Under Assumption 2, there necessarily exists a symmetric positive definite matrix P that guarantees that (4) holds. The matrix P can be resolved by the LMI or the algebraic Riccati equation.

Definition 1: If for any bounded initial state $x_i(0)$ ($i =$ 1, 2, ..., N), MASs (1) and (2) are considered to achieve leader-following adaptive guaranteed-performance consensus via protocol (7), with gain matrices H_u and H_a and $\lim_{t\to+\infty}(x_i(t) - x_0(t)) = 0$, ($i = 1, 2, ..., N$) and $J \leq J^*$, where J^* is called the guaranteed performance cost.

Lemma 1: Under the condition that Assumption 2 holds, the solution to the algebraic Riccati inequality, for any $\gamma > 0$, has a unique symmetric matrix $P > 0$ as follows:

$$PA + A^T P - \gamma PBB^T P + 3\mathrm{E} < 0. \tag{4}$$

Lemma 2 (Schur complement) [40]**:** The following inequalities are equivalent assuming that Q, W, Y are matrices with appropriate dimensions

1) $\begin{pmatrix} Q & W \\ W^T & Y \end{pmatrix} < 0$;
2) $Q < 0$ and $Y - W^T Q^{-1} W < 0$;
3) $Y < 0$ and $Q - WY^{-1}W^T < 0$.

3. GUARANTEED PERFORMANCE CONSENSUS PROTOCOL DESIGN AND ANALYSIS UNDER EXTERNAL DISTURBANCE

3.1. Consensus problem for multi-agent systems with disturbance observer

Design the following disturbance observer

$$\begin{cases} \dot{v}_i(t) = (S + TBF)(v_i(t) - Tx_i(t)) + T(Ax_i(t) \\ \qquad\qquad + Bu_i(t)), \\ \hat{w}_i(t) = v_i(t) - Tx_i(t), \\ \hat{d}_i(t) = F\hat{w}_i(t), \end{cases} \tag{5}$$

where $v_i(t) \in R^q$ and $T \in R^{q\times s}$ are is the observer's internal state variable and the gain matrix of observer, separately. $\hat{w}_i(t) \in R^q$ and $\hat{d}_i(t) \in R^v$ are the estimates of $w_i(t)$ and $d_i(t)$. Denoting the estimation error $e_i(t) = w_i(t) - \hat{w}_i(t)$. Subsequently, from (3) and (5), the estimated error system is expressed as

$$\dot{e}_i(t) = (S + TBF)e_i(t). \tag{6}$$

To achieve adaptive guaranteed-performance tracking in the case of external disturbances, the following distributed protocol is proposed

$$\begin{cases} u_i(t) = l_{i0}a_{i0}(t)H_u(x_0(t) - x_i(t)) - \hat{d}_i(t) \\ \qquad\qquad + H_u \sum\limits_{y\in N_i, y\neq 0} a_{iy}(x_y(t) - x_i(t)), \\ \dot{a}_{i0}(t) = (x_0(t) - x_i(t))^T H_a(x_0(t) - x_i(t)), \\ J = J_{fl} + J_{ff}, \end{cases} \tag{7}$$

in which $H_u \in R^{v\times s}$ and $H_a \in R^{s\times s}$ are gain matrices with ${H_a}^T = H_a \geq 0$, $l_{00} = 0$. $l_{i0} = 1$ means that agent i is able to receive messages from the leader, otherwise $l_{i0} \equiv 0$. $l_{i0}a_{i0}(t)$ with $a_{i0}(0) = 1$ denotes the intensity of the adaptive regulation of the reaction between the leader and the

ith follower. Assume that γ_{i0} is the upper limit of $a_{i0}(t)$, and $E^T = E > 0$,

$$\begin{cases} J_{fl} = \sum_{i=1}^{N} \int_0^{+\infty} l_{i0}(x_0(t) - x_i(t))^T E(x_0(t) - x_i(t))dt, \\ J_{ff} = \frac{1}{N} \sum_{i=1}^{N} \sum_{y=1}^{N} \int_0^{+\infty} (x_y(t) - x_i(t))^T E(x_y(t) - x_i(t))dt. \end{cases} \tag{8}$$

The performance function J_{fl} is linked to the state error of the relative state motion between the leader and followers, and J_{ff} is associated with the state error of the relative state motion between adjacent followers. Therefore, it can be used to ensure regulation performance of the consensus control.

Theorem 1: Let's assume that Assumptions 1 and 2 were valid. For systems (1) and (2), the leader-following consensus question under external disturbances can be settled using the protocol (7), and the error of the disturbance observer converges to 0. If the gain matrix $H_u = B^T P$, $H_a = PBB^T P$ and the following conditions are satisfied

i) $S + TBF$ is Hurwitz;
ii) $P > 0$ is satisfied by algebraic Riccati inequality (4);
iii) λ_1 is the minimum eigenvalue of $L_{ff} + \Lambda_{a(0),fl}$.

Proof: Make $x(t) = [x_0^T(t), x_1{}^T(t), ..., x_N^T(t)]^T$, define a new variable error $\vartheta_i(t) = x_i(t) - x_0(t)$ $(i = 1, 2, ..., N)$ and $\vartheta(t) = [\vartheta_1^T(t), \vartheta_2^T(t), ..., \vartheta_N^T(t)]^T$, from (1), (2) and (7), we have

$$\begin{aligned} \dot{\vartheta}(t) = &(I_N \otimes A - (L_{ff} + \Lambda_{a(t),fl}) \otimes BH_u)\vartheta(t) \\ &+ (I_N \otimes BF)e(t), \end{aligned} \tag{9}$$

where $\Lambda_{a(t),fl} = diag\{l_{10}a_{10}(t),\ l_{20}a_{20}(t),\ ...,\ l_{N0}a_{N0}(t)\}$ expresses the action from leader to followers. If $lim_{t\to+\infty}\vartheta(t) = 0$, it then indicates that MASs (1) and (2) can achieve leader-following consensus in protocol (7).

Design the following Lyapunov function

$$\begin{aligned} V(t) = &\vartheta^T(t)(I_N \otimes P)\vartheta(t) + \sum_{i=1}^{N} l_{i0}(a_{i0}(t) - a_{i0}(0))^2 \\ &+ \gamma \sum_{i=1}^{N} (\gamma_{i0} - a_{i0}(t)) + \alpha e^T(t)e(t), \end{aligned} \tag{10}$$

with $\alpha > 0$ is a large enough constant and $\gamma > 0$ is any given translation factor. Since $P = P^T > 0$, and $\gamma_{i0} \geq a_{i0}(t)$ $(i = 1, 2, ..., N)$, it is possible to conclude that $V(t) \geq 0$.

The derivative of time for $V(t)$ is

$$\begin{aligned} \dot{V}(t) = &2\vartheta^T(t)(I_N \otimes PBF)e(t) \\ &+ \vartheta^T(t)(I_N \otimes (PA + A^T P) \\ &- 2(L_{ff} + \Lambda_{a(t),fl}) \otimes H_a)\vartheta(t) \\ &+ 2\alpha e^T(t)(I_N \otimes (S + TBF))e(t) \\ &+ \sum_{i=1}^{N} 2l_{i0}(a_{i0}(t) - a_{i0}(0))\dot{a}_{i0}(t) - \gamma \sum_{i=1}^{N} \dot{a}_{i0}(t). \end{aligned} \tag{11}$$

According to (7), we have

$$\begin{aligned} &\sum_{i=1}^{N} l_{i0}(a_{i0}(t) - a_{i0}(0))\dot{a}_{i0}(t) \\ &= \vartheta^T(t)((\Lambda_{a(t),fl} - \Lambda_{a(0),fl}) \otimes PBB^T P)\vartheta(t), \end{aligned} \tag{12}$$

$$\sum_{i=1}^{N} \dot{a}_{i0}(t) = \vartheta^T(t)(I_N \otimes PBB^T P)\vartheta(t). \tag{13}$$

Let $\widehat{\vartheta}(t) = (U^T \otimes I_s)\vartheta(t)$ and $\widehat{e}(t) = (U^T \otimes I_q)e(t)$, where $U^T(L_{ff} + \Lambda_{a(0),fl})U = diag\{\lambda_1, \lambda_2, ..., \lambda_N\}$. From (11) to (13), one has

$$\begin{aligned} &\dot{V}(t) \\ &\leq 2\widehat{\vartheta}^T(t)(I_N \otimes PBF)\widehat{e}(t) \\ &\quad + 2\alpha\widehat{e}^T(t)(I_N \otimes (S + TBF))\widehat{e}(t) \\ &\quad + \widehat{\vartheta}^T(t)(I_N \otimes (PA + A^T P - (2\lambda_1 + \gamma)PBB^T P))\widehat{\vartheta}(t) \\ &= \delta^T(t) \begin{pmatrix} \Phi & I_N \otimes PBF \\ I_N \otimes (PBF)^T & \alpha\Pi \end{pmatrix} \delta(t), \end{aligned} \tag{14}$$

where $\delta(t) = (\widehat{\vartheta}^T(t), \widehat{e}^T(t))^T$, $\widehat{e}^T(t))$, $\Phi = I_N \otimes (PA + A^T P - (2\lambda_1 + \gamma))PBB^T P$, $\Pi = I_N \otimes ((S + TBF) + (S + TBF)^T)$. Due to $L_{ff} + \Lambda_{a(0),fl} > 0$, one gets $\lambda_i > 0$, $(i = 1, 2, ..., N)$, then $PA + A^T P - (2\lambda_1 + \gamma))PBB^T P < PA + A^T P - \gamma PBB^T P + 3\mathrm{E} < 0$, we can know $\Phi < 0$. If $S + TBF$ is Hurwitz, we obtain $\Pi < 0$. Choosing a suffciently large α, PBF is a constant matrix. By Schur Complement Lemma 2, it is able to learn about $\dot{V}(t) < 0$. So $\widehat{\vartheta}(t) \to 0$ and $\widehat{e}(t) \to 0$ as $t \to \infty$, then $\vartheta(t) \to 0$ and $e(t) \to 0$ as $t \to \infty$, which means Theorem 1 holds. □

3.2. Adaptive guaranteed-performance consensus design for leader-following

The leader-following adaptive guaranteed-performance consensus is discussed next. A gain factor γ is introduced to adjust the control gain. Based on Ricatti's inequality and matrix inequality, the sufficient conditions for guaranteed-performance consensus are proposed.

Theorem 2: For any $\gamma > 0$, MASs (1), (2) are leader-following adaptively guaranteed-performance consensualizable by protocol (7) if condition i), ii), iii) in Theorem 1 is satisfied and the cost of guaranteed-performance meets the following conditions

$$\begin{aligned} J^* = &x^T(0)\left(\begin{bmatrix} N & -1_N^T \\ -1_N & I_N \end{bmatrix} \otimes P\right)x(0) \\ &+ \gamma \int_0^{+\infty} x^T(t)\left(\begin{bmatrix} N & -1_N^T \\ -1_N & I_N \end{bmatrix} \otimes H_a\right)x(t)dt. \end{aligned}$$

Proof: Let $h \geq 0$, from (8) it can get

$$J_{fl}^h = \sum_{i=1}^{N}\int_0^h l_{i0}(x_0(t)-x_i(t))^T E(x_0(t)-x_i(t))dt$$
$$= \int_0^h \vartheta^T(t)(\Lambda_{a(0),fl}\otimes E)\vartheta(t)dt, \quad (15)$$
$$J_{ff}^h = \frac{1}{N}\sum_{i=1}^{N}\sum_{y=1}^{N}\int_0^h (x_y(t)-x_i(t))^T E(x_y(t)-x_i(t))dt$$
$$= \int_0^h \vartheta^T(t)(2L_N\otimes E)\vartheta(t)dt. \quad (16)$$

By using (7), we have

$$J^h = \int_0^h \vartheta^T(t)((2L_N+\Lambda_{a(0),fl})\otimes E)\vartheta(t)dt. \quad (17)$$

Since all eigenvalues of the Laplacian matrix L_N with weight N of the topological graph are equal to 1. The eigenvalues of $2L_N+\Lambda_{a(0),fl}$ are all greater than 0 and less than 3. Thus by (16) and (17), from this it can be seen that

$$J^h \leq \int_0^h \vartheta^T(t)(3I_N\otimes E)\vartheta(t)dt. \quad (18)$$

Furthermore, one can prove that

$$\int_0^h \dot{V}(t)dt + V(0) - V(h) = 0. \quad (19)$$

Similar to (14), by using (18), (19) and after a series of mathematical operations, and thus get

$$J^h \leq \int_0^h \delta^T(t)\begin{pmatrix}\Theta & I_N\otimes PBF \\ I_N\otimes(PBF)^T & \alpha\Pi\end{pmatrix}\delta(t)dt$$
$$-V(h)+V(0), \quad (20)$$

where $\delta(t) = (\widehat{\vartheta}^T(t), \widehat{e}^T(t))^T$, $\Theta = I_N\otimes(PA+A^TP-(2\lambda_{\min}+\gamma)PBB^TP+3E)$ and $\Pi = I_N\otimes((S+TBF)+(S+TBF)^T)$. If $PA+A^TP-\gamma PBB^TP+3E<0$, it is obtained $\Theta<0$. According to $S+TBF$ is Hurwitz, then $\Pi<0$.

Due to $\lim_{t\to+\infty}(a_{i0}(t)-\gamma)=0$, $(i=1, 2, ..., N)$, it can get

$$\lim_{h\to+\infty}\sum_{i=1}^{N}(\gamma - a_{i0}(h)) = 0. \quad (21)$$

Hence, it can be easily derived from (20) and (21) that

$$\lim_{h\to+\infty} J^h \leq \lim_{h\to+\infty} V(0)$$
$$= \vartheta^T(0)(I_N\otimes P)\vartheta(0) + \alpha e^T(0)e(0)$$
$$+\gamma\sum_{i=1}^{N}(\gamma_{i0}-a_{i0}(0)). \quad (22)$$

By $U^TU = UU^T = I_N$, it can be concluded that $\begin{bmatrix}-\mathbf{1}_N^T U \\ U\end{bmatrix}\begin{bmatrix}-U^T\mathbf{1}_N & U^T\end{bmatrix} = \begin{bmatrix}N & -\mathbf{1}_N^T \\ -\mathbf{1}_N & I_N\end{bmatrix}$.

From this we can get

$$\vartheta^T(0)(I_N\otimes P)\vartheta(0)$$
$$= x^T(0)\left(\begin{bmatrix}N & -1_N^T \\ -1_N & I_N\end{bmatrix}\otimes P\right)x(0), \quad (23)$$
$$\sum_{i=1}^{N}(\gamma_{i0}-a_{i0}(0)) = \sum_{i=1}^{N}\int_0^{+\infty}\dot{a}_{i0}(t)dt$$
$$= \sum_{i=1}^{N}\int_0^{+\infty}(x_0(t)-x_i(t))^T H_a(x_0(t)-x_i(t))dt$$
$$= \int_0^{+\infty} x^T(t)\left(\begin{bmatrix}N & -1_N^T \\ -1_N & I_N\end{bmatrix}\otimes H_a\right)x(t)dt. \quad (24)$$

From (22) to (24), Theorem 2 can be obtained. To make $PA+A^TP-PBB^TP+3E=0$ have the only positive definite solution, it must be guaranteed that (A,B) is stable. As a consequence, (A,B) being stable is a sufficient condition to ensure that the leader-following consensus on adaptive guaranteed-performance can be fulfilled. In addition, γ can be taken as the right translation of the non-zero eigenvalues of $2(L_{ff}+\Lambda_{a(0),fl})$. The next corollary gives a way to adjust the consensus gain using a gain factor $\sigma>0$, which is $P\leq\sigma I$.

Corollary 1: For any given $\sigma>0$, when $\lambda_{\max}(BB^T)\leq 1$, $\gamma>0$ and $\widehat{P}=\widehat{P}^T\geq\sigma^{-1}I$ exists, the performance of the MASs (1) and (2) can be guaranteed by consensus through an adaptive protocol (7), where

$$\Psi = \begin{pmatrix}A\widehat{P}+\widehat{P}A^T-\gamma BB^T & 3\widehat{P}E \\ * & -3E\end{pmatrix} < 0,$$

and $H_u = B^T\widehat{P}^{-1}$, $H_a = \widehat{P}^{-1}BB^T\widehat{P}^{-1}$, we know that the guaranteed-performance cost satisfies this condition

$$J^* = \sum_{i=1}^{N}\left(\sigma\|\vartheta_i(0)\|^2 + \gamma\sigma^2\int_0^{+\infty}\left\|B^T\vartheta_i(t)\right\|^2 dt\right).$$

According to the characteristics of leader-following structure, in protocol (7), the strength of interactions between leaders and followers is adaptively time-varying, but the intensity of interactions between followers is fixed. It is clear that fixing the interaction strength between followers simplifies the analysis of the whole system. In this case, the performance cost J^* is solely linked to the state error between the leader and the followers, and the state error between the followers has no effect on it.

4. SIMULATIONS

In order to verify the correctness of the above mathematical derivation, a numerical example is used in this chapter to support it. Consider a topological graph consisting of a leader (noted as 0) and six followers (noted as

1 to 6). The communication topology between all agents expressed in Fig. 1. The system is represented by (1) and (2). The following system matrices are selected:

$$A=\begin{bmatrix}-1 & 1.1 & -1\\ 0 & 0.2 & 2\\ -1 & -1 & -1.9\end{bmatrix},\ B=\begin{bmatrix}-0.09\\ 0\\ 1\end{bmatrix},\ F=\begin{bmatrix}0\\ 0\\ -1\end{bmatrix}^T,$$

$$S=\begin{bmatrix}-0.8 & -1 & 1.4\\ 7.5 & -0.9 & -0.1\\ -6 & -1.2 & -1\end{bmatrix},\ D=\begin{bmatrix}1 & 0 & -1\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix},$$

$$T=\begin{bmatrix}1.1 & 1 & 0\\ -1 & 0 & 0.8\\ 1 & 0.2 & -0.6\end{bmatrix},\ E=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},$$

in which $x_i=[x_{i1},\ x_{i2},\ x_{i3}]^T$ with $i=0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6$. From this we can conclude that the matrix $S+TBF$ is Hurwitz, and thus condition 1 of Theorem 1 is satisfied. The solution of the algebraic Riccati inequality (4) can be presented as the following

$$P=\begin{bmatrix}0.4073 & -0.1063 & 0.0359\\ -0.1063 & 0.1769 & -0.1690\\ 0.0359 & -0.1690 & 0.3174\end{bmatrix}.$$

Thus, one can obtain from Theorem 1 that

$$H_u=\begin{bmatrix}1.0459 & 7.1697 & 6.8487\end{bmatrix},$$

$$H_a=\begin{bmatrix}1.0938 & 7.4986 & 7.1628\\ 7.4986 & 51.4042 & 49.1027\\ 7.1628 & 49.1027 & 46.9042\end{bmatrix}.$$

The coupling weight a_{i0} are displayed in Fig. 2. From this, we can obtain the fact that coupling weight a_{i0} reaches to some finite values. The three plots in Fig. 3 shows the state trajectory of the leader-following MASs. It can be noted that the state trajectories of all followers are converged to the leader's state trajectory. The three plots in Fig. 4 depicts that the disturbance observer error converges asymptotically to 0, indicating that the disturbance is observable. Fig. 5 shows that the performance function J has converged to a limited numerical value, which is $J<J^*$. This implies that guaranteed-performance consensus of the MASs is achieved under adaptive protocol (7) by using the gain matrices H_u and H_a without considering the global information. In addition, since the key results in this article are done without the global information of the topology graph, the calculation complexity does not increase with the increase of the number of agents.

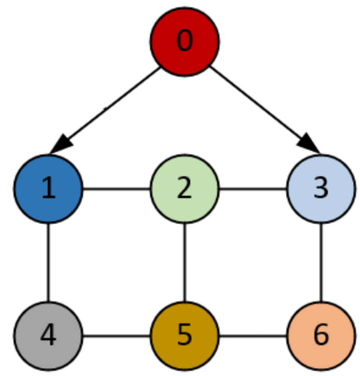

Fig. 1. Communication topology of agents.

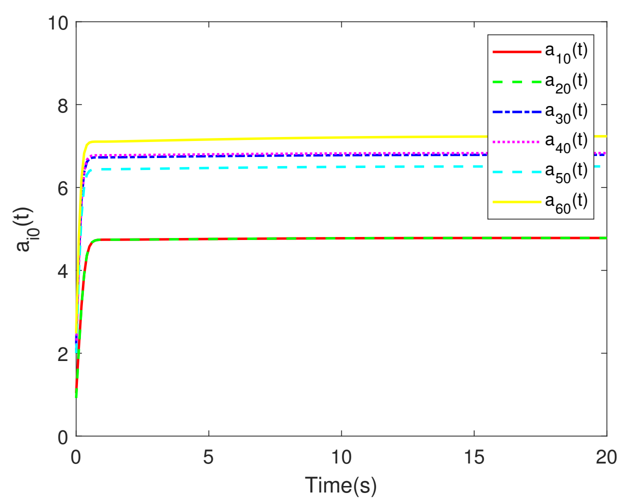

Fig. 2. Coupling weights a_{i0} in (7).

5. CONCLUSIONS

For MASs with external disturbances, this paper presentes a new adaptive consensus control scheme to implement completely distributed guaranteed-performance consensus control, in which the gain matrix of the distributed protocol is designed by algebraic Riccati inequalities. An unambiguous expression for the cost of guaranteed performance is also given. In addition, by choosing different translation factor, it can be used to adjust the control gain. The effect of non-zero eigenvalues of the Laplacian matrix with adaptively adjusted weights is eliminated by shifting to the right instead of scaling. We currently consider only the use of symmetry of Laplacian matrices in an undirected case to deal with the energy consumption issue of system with external disturbances. The next study orientation is to consider the energy consumption question of nonlinear MASs with a directed topology.

CONFLICTS OF INTERESTS

There is no conflict of interest in this article.

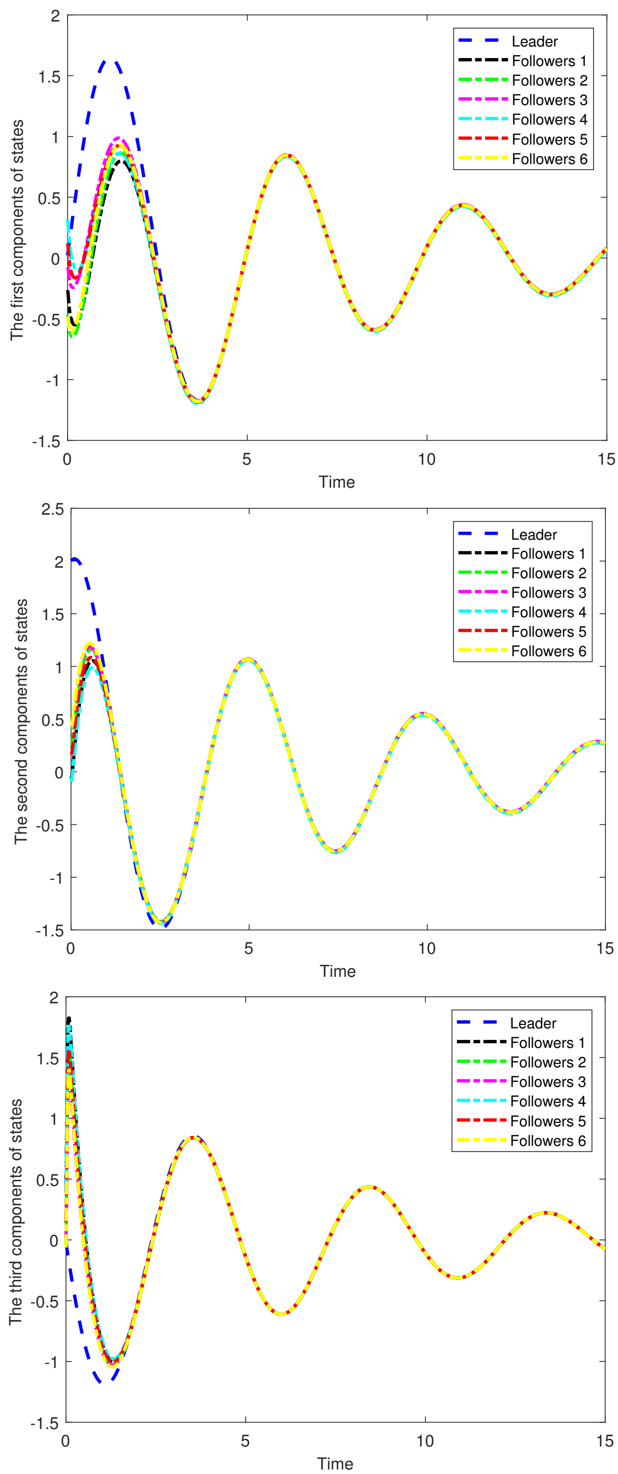

Fig. 3. State trajectory.

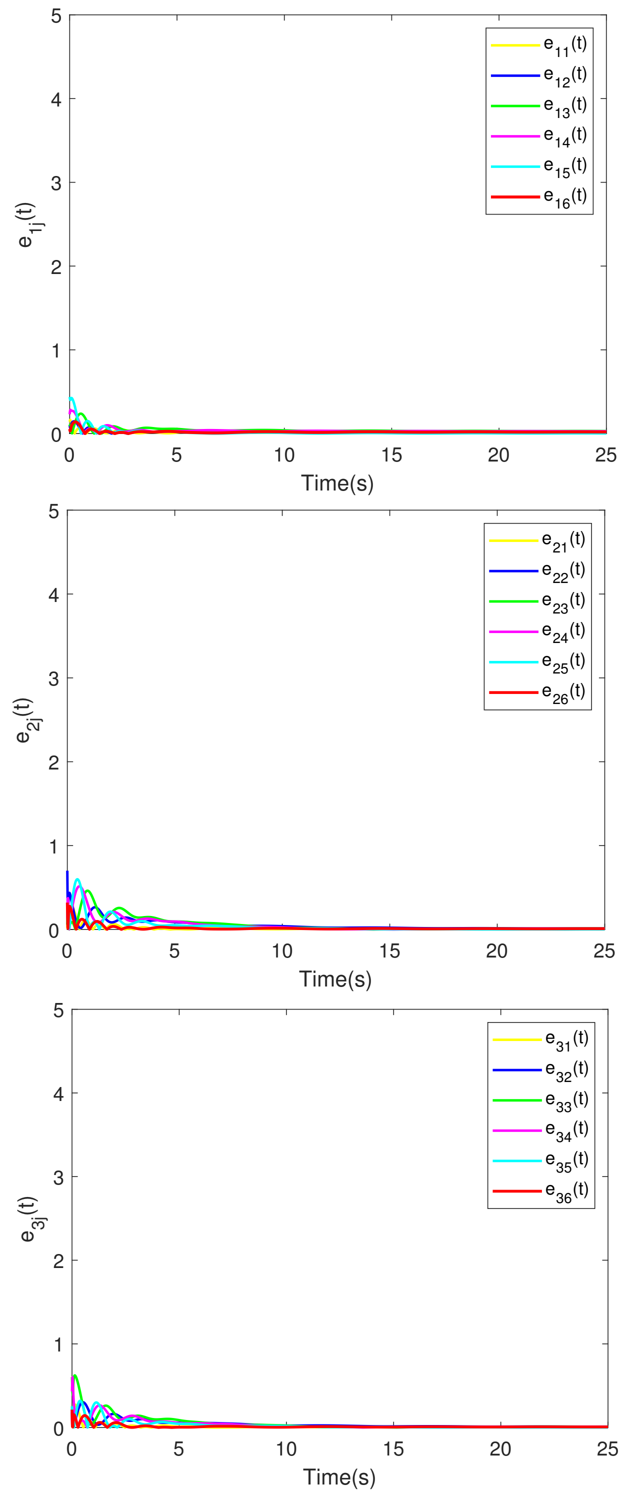

Fig. 4. Observation error.

REFERENCES

[1] Y.-W. Wang, Z.-H. Zeng, X.-K. Liu, and Z.-W. Liu, "Input-to-state stability of switched linear systems with unstabilizable modes under DoS attacks," *Automatica*, vol. 146, 110607, 2022.

[2] X. Zheng, H. Li, C. K. Ahn, and D. Yao, "NN-based fixed-time attitude tracking control for multiple unmanned aerial vehicles with nonlinear faults," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 59, no. 2, pp. 1738-1748, 2023.

[3] X.-K. Liu, Y.-W. Wang, P. Lin, and P. Wang, "Distributed

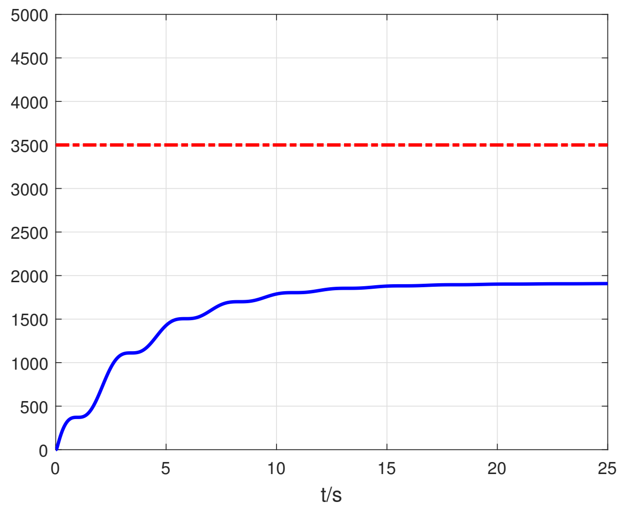

Fig. 5. Performance function.

supervisory secondary control for a DC microgrid," *IEEE Transactions on Energy Conversion*, vol. 35, no. 4, pp. 1736-1746, 2020.

[4] X. Du, X. Zhan, J. Wu, and H. Yan, "Performance analysis of MIMO information time-delay system under bandwidth, cyber-attack, and Gaussian white noise," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 53, no. 4, pp. 2329-2338, 2023.

[5] X. Zhang, J. Wu, X. Zhan, T. Han, and H. Yan, "Observer-based adaptive time-varying formation-containment tracking for multiagent system With bounded unknown input," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 53, no. 3, pp. 1479-1491, 2023.

[6] Z. Liu, X. Zhan, T. Han, and H. Yan, "Distributed adaptive finite-time bipartite containment control of linear multi-agent systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 11, pp. 4354-4358, 2022.

[7] Z.-W. Liu, X. Yu, Z.-H. Guan, B. Hu, and C. Li, "Pulse-modulated intermittent control in consensus of multiagent systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 5, pp. 783-793, 2017.

[8] Z.-W. Liu, G. Wen, X. Yu, Z.-H. Guan, and T. Huang, "Delayed impulsive control for consensus of multiagent systems with switching communication graphs," *IEEE Transactions on Cybernetics*, vol. 50, no. 7, pp. 3045-3055, 2020.

[9] G. Lin, H. Li, C. K. Ahn, and D. Yao, "Event-based finite-time neural control for human-in-the-loop UAV attitude systems," *IEEE Transactions on Neural Networks and Learning Systems*, pp. 1-11, 2022, DOI: 10.1109/TNNLS.2022.3166531

[10] X. Wang, H. Wu, and J. Cao, "Global leader-following consensus in finite time for fractional-order multi-agent systems with discontinuous inherent dynamics subject to nonlinear growth," *Nonlinear Analysis: Hybrid Systems*, vol. 37, 100888, 2020.

[11] F. Sun, X. Liao, and J. Kurths, "Mean-square consensus for heterogeneous multi-agent systems with probabilistic time delay," *Information Sciences*, vol. 543, pp. 112-124, 2021.

[12] J. Fu, G. Wen, T. Huang, and Z. Duan, "Consensus of multi-agent systems with heterogeneous input saturation levels," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 66, no. 6, pp. 1053-1057, 2019.

[13] X. Liu, K. Zhang, and W.-C. Xie, "Consensus seeking in multi-agent systems via hybrid protocols with impulse delays," *Nonlinear Analysis: Hybrid Systems*, vol. 25, pp. 90-98, 2017.

[14] X. Jiang, G. Xia, and Z. Feng, "Non-fragile consensus control for singular multi-agent systems with Lipschitz nonlinear dynamics," *Neurocomputing*, vol. 351, pp. 123-133, 2019.

[15] K. Li and X. Mu, "Necessary and sufficient conditions for leader-following consensus of multi-agent systems with random switching topologies," *Nonlinear Analysis: Hybrid Systems*, vol. 37, 100905, 2020.

[16] H. Liang, L. Chen, Y. Pan, and H.-K. Lam, "Fuzzy-based robust precision consensus tracking for uncertain networked systems with cooperative-antagonistic interactions," *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 4, pp. 1362-1376, 2023.

[17] Y. Wu, Q. Liang, Y. Zhao, J. Hu, and L. Xiang, "Adaptive bipartite consensus control of general linear multi-agent systems using noisy measurements," *European Journal of Control*, vol. 59, pp. 123-128, 2021.

[18] H. Zhang, Y. Liu, and Y. Wang, "Observer-based finite-time adaptive fuzzy control for nontriangular nonlinear systems with full-state constraints," *IEEE Transactions on Cybernetics*, vol. 51, no. 3, pp. 1110-1120, 2021.

[19] H. Zhang, Y. Liu, J. Dai, and Y. Wang, "Command filter based adaptive fuzzy finite-time control for a class of uncertain nonlinear systems with hysteresis," *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 9, pp. 2553-2564, 2021.

[20] L. Cao, Y. Pan, H. Liang, and T. Huang, "Observer-based dynamic event-triggered control for multiagent systems with time-varying delay," *IEEE Transactions on Cybernetics*, vol. 53, no. 5, pp. 3376-3387, 2023.

[21] J. Zhang, H. Zhang, S. Sun, and Z. Gao, "Leader-follower consensus control for linear multi-agent systems by fully distributed edge-event-triggered adaptive strategies," *Information Sciences*, vol. 555, pp. 314-338, 2021.

[22] Z.-W. Liu, Y.-L. Shi, H. Yan, B.-X. Han, and Z.-H. Guan, "Secure consensus of multiagent systems via impulsive control subject to deception attacks," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 70, no. 1, pp. 166-170, 2023.

[23] Z. Gao, H. Zhang, J. Duan, and Y. Cai, "Guaranteed-performance consensus for descriptor nonlinear multi-agent systems based on distributed nonlinear consensus protocol," *Neurocomputing*, vol. 383, pp. 359-367, 2020.

[24] X. Jiang, G. Xia, Z. Feng, and Z. Jiang, "Non-fragile guaranteed-performance H_∞ leader-following consensus of Lipschitz nonlinear multi-agent systems with switching topologies," *Nonlinear Analysis: Hybrid Systems*, vol. 38, 100913, 2020.

[25] Z. Zhang, S. Zhang, H. Li, and W. Yan, "Cooperative robust optimal control of uncertain multi-agent systems," *Journal of the Franklin Institute*, vol. 357, no. 14, pp. 9467-9483, 2020.

[26] J Zhang, D.-W. Ding, Y. Ren, and X. Sun, "Distributed robust group output synchronization control for heterogeneous uncertain linear multi-agent systems," *ISA Transactions*, vol. 134, pp. 108-121, 2023.

[27] N. Li, X. Wu, J. Feng, Y. Xu, and J. Lü, "Fixed-time synchronization of coupled neural networks with discontinuous activation and mismatched parameters," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 32, no. 6, pp. 2470-2482, 2021.

[28] Y.-Y. Qian, L. Liu, and G. Feng, "Distributed event-triggered adaptive control for consensus of linear multi-agent systems with external disturbances," *IEEE Transactions on Cybernetics*, vol. 50, no. 5, pp. 2197-2208, 2020.

[29] A. Mei, G. Wen, Z. Peng, A. Rahmani, and T. Huang, "Vortex formation control for linear multiagent systems with unknown leader input and external disturbances," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 70, no. 2, pp. 636-640, 2023.

[30] H. Su, Y. Ye, Y. Qiu, Y. Cao and M. Z. Q. Chen, "Semi-global output consensus for discrete-time switching networked systems subject to input saturation and external disturbances," *IEEE Transactions on Cybernetics*, vol. 49, no. 11, pp. 3934-3945, 2019.

[31] W. Jia and J. Wang, "Partial-nodes-based distributed fault detection and isolation for second-order multiagent systems with exogenous disturbances," *IEEE Transactions on Cybernetics*, vol. 52, no. 4, pp. 2518-2530, 2022.

[32] Y. Wu and L. Liu, "Distributed average tracking for linear heterogeneous multi-agent systems with external disturbances," *IEEE Transactions on Network Science and Engineering*, vol. 8, no. 4, pp. 3491-3500, 2021.

[33] X. Ruan, J. Feng, C. Xu, and J. Wang, "Observer-based dynamic event-triggered strategies for leader-following consensus of multi-agent systems with disturbances," *IEEE Transactions on Network Science and Engineering*, vol. 7, no. 4, pp. 3148-3158, 2020.

[34] Y. Wu, J. Hu, L. Xiang, Q. Liang, and K. Shi, "Finite-time output regulation of linear heterogeneous multi-agent systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 3, pp. 1248-1252, 2022.

[35] X. Jiang, G. Xia, and Z. Feng, "Non-fragile consensus control for singular multi-agent systems with Lipschitz nonlinear dynamics," *Neurocomputing*, vol. 351, pp. 123-133, 2019.

[36] Z. Gao, H. Zhang, J. Duan, and Y. Cai, "Guaranteed-performance consensus for descriptor nonlinear multi-agent systems based on distributed nonlinear consensus protocol," *Neurocomputing*, vol. 383, pp. 359-367, 2020.

[37] J. Xu, G. Zhang, J. Zeng, J. Xi, and B. Du, "Robust guaranteed cost consensus for high-order discrete-time multi-agent systems with parameter uncertainties and time-varying delays," *IET Control Theory & Applications*, vol. 11, no. 5, pp. 647-667, 2017.

[38] J. Xi, Z. Fan, H. Liu, and T. Zheng, "Guaranteed-cost consensus for multiagent networks with Lipschitz nonlinear dynamics and switching topologies," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 7, pp. 2841-2852, 2018.

[39] Y. Zhao and W. Zhang, "Guaranteed cost consensus protocol design for linear multi-agent systems with sampled-data information: An input delay approach," *ISA Transactions*, vol. 67, pp. 87-97, 2017.

[40] Y. Li, Y. Wu, and S. He, "Network-based leader-following formation control of second-order autonomous unmanned systems," *Journal of the Franklin Institute*, vol. 358, no. 1, pp. 757-775, 2021.

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