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Stabilization of an Uncertain Maglev Train System Using Finite Time Adaptive Back-stepping Controller

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Abstract: In this paper, we have presented an adaptive controller for a class of nonlinear systems. The proposed control law includes some terms to eliminate nonlinear parts. Also, an adaptive term is considered dealing with the uncertainties of the system. This controller is designed in several steps by establishing the finite time stability condition. Finite time stability of the each step is proved using Lyapunov theorem. Also, the relation of the convergence time depending on the initial conditions is presented. Numerical simulations are presented in this paper for a Maglev system to evaluate the analysis and effectiveness of the controller. Robustness of the control schemes in the presence of uncertainty is also investigated.

Keywords: Adaptive control, finite time convergence, nonlinear system, uncertainty.

1. INTRODUCTION

In many applications, the time of convergence is very important. Finite time stable systems might enjoy not only faster convergence but also better robustness and disturbance rejection properties [1]. Finite-time control owns the advantages of precision and rapidity and making the system state reach the desired point/manifold in finite time [2].

The problem of finite-time stabilization for nonlinear systems is studied in the literatures such as attitude tracking control for rigid spacecraft [3] and Finite/fixed-time control of unmanned aerial vehicles [4]. Based on the research done in [4], the sliding mode method can guarantee the finite time stability of uncertain nonlinear systems. But due to chattering, this method requires smoothing, and in this case, finite time stability will not be achieved. Also in the sliding mode control, to ensure finite time convergence of the system states, the control law will involve a non-smooth sign function that cause chattering of control commands. Due to chattering, the application of the sliding mode control theory usually is not possible [5]. Therefore, require modifications for its application. Usually an approximation of control law is used for the purpose of removing the chattering. However, this method brings a finite steady state error and lead to tracking within a guaranteed precision rather than perfect tracking [6]. The other way to eliminate chattering is to use second order sliding mode (SOSM) control. For the SOSMs, commonly the finite time convergence analysis has been done without the use of the Lyapunov function approach in uncertain nonlinear systems. Some approaches depend on the state derivatives on, so-called, homogeneity principle [7], which also does not allow to estimate the reaching time. Some studies are based on disturbance observer [8,9]. In [8], observer-based sliding mode control for fractional order singular fuzzy systems is studied. An observer is designed to reconstruct the unmeasured states and the fractional order sliding mode control is constructed to ensure the reachability of the sliding surface. In [9], a new nonsingular fast terminal sliding mode back-stepping control is presented for uncertain nonlinear systems subjected to unknown mismatched disturbance based on an adaptive super-twisting sliding mode nonlinear disturbance observer. The main drawbacks in these methods are the absence of a formal closed-loop system stability proof and existence of sign function.

Recently, adaptive nonlinear controllers have been proposed, the interest being the adaptation of uncertainty effects. Then, a reduced gain induces lower chattering [10,11]. Plestan *et al.* [11] propose new methodologies for the design of adaptive sliding mode control. The goal is to obtain a robust sliding mode adaptive-gain control law with respect to uncertainties without the knowledge of their bound. These approaches guarantee finite time stability but have non-smooth control signal. The gain-adaptation laws in this article do not overestimate uncertainties magnitude and this is the main difference between

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this method and conventional adaptive sliding mode.

Sliding mode control with adaptive gains for an electro pneumatic actuator and a micromanipulator system are designed in [12,13], respectively. The main drawback of these approaches is the chattering which is not completely eliminated. In [13], based on the boundary layer technique, the sign function is replaced by saturation function that affects accuracy and robustness.

The other problem in the sliding mode control is the asymptotic stability of the system states due to introducing conventional linear sliding variable. In the conventional SMC theory, the most commonly used sliding surface is based on linear combination of the system states. Next the SM controller is designed, which drive the system to reach and remain on this sliding surface in finite reaching time. Then in the sliding phase, the convergence of state variables can be made faster by utilizing a larger valued coefficient in the linear sliding surface, but the system states cannot converge to the equilibrium point in finite time. Instead of using a linear sliding variable, terminal SM control with a nonlinear sliding surface is presented, which grantee the finite time stability of the state variables [14-16]. Although terminal sliding mode methods guarantee finite time convergence in the sliding phase, these methods have two basic problems. First, the terminal sliding variable can be defined in normal form for systems. For example, in second order systems where the second state variable is the derivative of the first state variable. The second problem is that chattering will still occur in the terminal sliding mode methods and these methods require smoothing.

Magnetic levitation (Maglev) technique has been widely applied into many engineering systems such as frictionless bearings and high speed magnetic levitation trains, because of its contactless, low noise and low friction characteristics. These kinds of systems are nonlinear and usually open-loop unstable [17,18]. In recent years, a lot of works have been reported in the literature for controlling magnetic levitation systems. For traditional methods, the controllers have been designed based on linearized models about nominal operating points [19-22], and thus the performance deteriorates rapidly with increasing deviations from these points. So this type of linearization cannot be considered in the situation that the gap is a large range of variation. Therefore it may lead to the performance degradation or instability of the Maglev system. Hence, a new solution should be proposed. Moreover the control performances are degraded due to the existence of uncertainties and neglected dynamics [23,24]. Based on linearization, a robust disturbance observer based controller [20], a fault tolerant controller [19], a discrete-time mixed LQR/H_∞ controller [21] and a finite frequency H_∞ controller [22] for a Maglev system are designed. Some references are directly based on nonlinear techniques such as feedback linearization [27,28] and the nonlinear super twisting [29]. Because of the uncertainties and unknown parameters of the nonlinear Maglev system, the adaptive sliding mode controller is proposed in [23]. The main drawback of this approach is the chattering which is not completely eliminated.

The main contributions of this article have been listed as the following items:

1) An adaptive nonlinear controller is proposed for achieving finite time convergence of the nonlinear and uncertain systems. In order to stabilization of state variables, virtual controllers are introduced based on the terminal sliding mode theory in a back stepping procedure.
2) An adaptive term is designed in order to estimation of these uncertainties and is used in the feedback control law.
3) The finite time stability of the proposed algorithm in each step is proved by using Lyapunov method.
4) The performance of this algorithm in stabilization of a magnetic levitation system is addressed. Unlike the work done in this field, in this article the finite time convergence of the position of the suspended object to a desired point in the presence of model and parametric uncertainties is considered. Also, another goal considered in this article is to produce a smooth control signal.

The rest of the paper is organized as follows: Section 2 contains the introducing the proposed control scheme. Section 3 deals with the mathematical modeling of the uncertain magnetic levitation system. In Section 4, the proposed controller is designed for the magnetic levitation system. Section 5 presents and discusses the simulation results of the proposed control schemes. Conclusions are reported in Section 6.

2. ADAPTIVE FINITE-TIME CONTROLLER FOR UNCERTAIN NONLINEAR SYSTEMS

In this section, the proposed adaptive finite-time controller for a class of uncertain nonlinear systems is presented. Consider a single-input uncertain nonlinear system in the form

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \quad \vdots \\ \dot{x}_{n-1} = x_n, \\ \dot{x}_n = f(x) + g(x)u + d(t), \\ y = x_1, \end{cases} \tag{1}$$

where $x_i \in X \subset \mathbb{R}^n$ is a state variable, $u \in \mathbb{R}$ is a control function, $f(x)$, $g(x) \in \mathbb{R}$ are nonlinear functions. Assume that the functions $f(x)$ and $g(x)$, $g(x)$ are certain and the uncertain function $d(t)$ can be presented as

$$\begin{aligned} &|d(t)| \le L_d, \\ &\left|\dot{d}(t)\right| \le L_{\dot{d}}. \end{aligned} \tag{2}$$

The problem is to design a smooth control u that drives the output $y = x_1$ to x_{1d} in a finite time in the presence of the uncertainty $d(t)$.

To design the controller, we start with the first equation of relation (1)

$$\dot{x}_1 = x_2. \tag{3}$$

To finite time tracking of the first state variable, the desired value of the second state variable is selected as follows:

$$\begin{cases} \phi_1 = x_{1d}, \\ \phi_2 = -k_1 (x_1 - \phi_1)^{\gamma_1} + \dot{\phi}_1, \end{cases} \quad 0 < \gamma_1 < 1, \tag{4}$$

and by introducing error

$$e_1 = x_1 - \phi_1, \tag{5}$$

the resulting error dynamics can be written

$$\dot{e}_1 = -k_1 (e_1)^{\gamma_1}. \tag{6}$$

Integrating from relation (6) implies

$$t_1 = \frac{1}{k_1(1-\gamma_1)} {e_1}^{1-\gamma_1}, \tag{7}$$

where t_1 is the convergence time of e_1. Therefore the finite time stability of e_1 is proved and the convergence of $e_1 = x_1 - x_{1d}$ will be achieved after t_1 seconds.

Then for the x_2 to x_{n-1} state variables, we have in a similar way

$$\begin{cases} \phi_3 = -k_2 (x_2 - \phi_2)^{\gamma_2} + \dot{\phi}_2, \\ \quad \vdots \\ \phi_n = -k_{n-1} (x_{n-1} - \phi_{n-1})^{\gamma_{n-1}} + \dot{\phi}_{n-1}, \end{cases} \tag{8}$$

which guarantees finite time tracking of x_2 to x_{n-1} variables.

And finally in the n-th dynamics of the system (1) we use

$$\begin{cases} u = \dfrac{1}{g(x)} \left(-f(x) - k_n (x_n - \phi_n) + \dot{\phi}_n - A_1 + A_2 \right), \\ \dot{A}_1 = k_{n+1} (x_n - \phi_n), \\ \dot{A}_2 = -k_{n+1} (x_n - \phi_n) |A_2| - k_{n+2} \dfrac{A_2}{|A_2|}. \end{cases} \tag{9}$$

And we have

$$\begin{aligned} &\dot{x}_n = -k_n (x_n - \phi_n) + \dot{\phi}_n - A_1 + A_2 + d(t), \\ &\dot{A}_1 = k_{n+1} (x_n - \phi_n), \\ &\dot{A}_2 = -k_{n+1} (x_n - \phi_n) |A_2| - k_{n+2} \frac{A_2}{|A_2|}. \end{aligned} \tag{10}$$

By introducing the system errors in the final step as

$$\begin{cases} e_n = x_n - \phi_n, \\ e_{n+1} = d(t) - A_1, \\ e_{n+2} = A_2, \end{cases} \tag{11}$$

the resulting error dynamics can be written

$$\begin{cases} \dot{e}_n = -k_n e_n + e_{n+1} + e_{n+2}, \\ \dot{e}_{n+1} = \dot{d}(t) - k_{n+1} e_n, \\ \dot{e}_{n+2} = -k_{n+1} e_n |e_{n+2}| - k_{n+2} sgn(e_{n+2}). \end{cases} \tag{12}$$

Consider the following Lyapunov function candidate

$$V = \frac{k_{n+1}}{2} {e_n}^2 + \frac{1}{2} {e_{n+1}}^2 + |e_{n+2}|, \tag{13}$$

which is positive definite with $k_{n+1} > 0$. By taking the time derivative of V, we obtain

$$\begin{aligned} \dot{V} &= k_{n+1} e_n (-k_n e_n + e_{n+1} + e_{n+2}) \\ &\quad + e_{n+1} \left(\dot{d}(t) - k_{n+1} e_n \right) \\ &\quad + \frac{e_{n+2}}{|e_{n+2}|} \left(-k_{n+1} e_n |e_{n+2}| - k_{n+2} \frac{e_{n+2}}{|e_{n+2}|} \right) \\ &= -k_n k_{n+1} {e_n}^2 + e_{n+1} \dot{d}(t) - k_{n+2}. \end{aligned} \tag{14}$$

By choosing $k_n k_{n+1} > 0$ and $k_{n+2} = (L_d + |A_1|) L_{\dot{d}} + \eta$, where η is a positive constant, yields

$$\begin{aligned} \dot{V} &= -k_n k_{n+1} {e_n}^2 + e_{n+1} \dot{d}(t) \\ &\quad - (L_d + |A_1|) L_{\dot{d}} - \eta \le -\eta. \end{aligned} \tag{15}$$

The condition (15) that implies [24,25]

$$0 \le V(x(t)) = V(x(0)) - \eta t, \tag{16}$$

defines

$$t_r \le \frac{V(e_n(0), e_{n+1}(0), e_{n+2}(0))}{\eta}. \tag{17}$$

Therefore condition (15) guarantees the finite time convergence of e_n, e_{n+1} and e_{n+2}. Hence, these errors converge in t_n and we have $x_n = \phi_n$ and $A_1 = d(t)$.

Also in the other steps we have a dynamics of the system as follows:

$$\dot{x}_i = -k_i (x_i - \phi_i)^{\gamma_i} + \dot{\phi}_i, \ (i = 1, 2, \ldots, n-1), \tag{18}$$

and by introducing errors

$$e_i = x_i - \phi_i, \ (i = 1, 2, \ldots, n-1), \tag{19}$$

the resulting error dynamics can be written

$$\dot{e}_i = -k_i (e_i)^{\gamma_i}, \ (i = 1, 2, ..., n-1). \quad (20)$$

Relation (20) implies

$$t_i = \frac{1}{k_i(1-\gamma_i)} e_i^{1-\gamma_i}, \ (i = 1, 2, ..., n-1), \quad (21)$$

where t_i is the convergence time of e_i. Finite time stability of e_i, $(i = 1, 2, ..., n-1)$ are proved and the convergence of $e_1 = x_1 - x_{1d}$ will be achieved after $t_f = t_1 + ... + t_r$.

Therefore, the control input is proposed to finite time control of uncertain and nonlinear system (1) as follows:

$$\begin{cases} \phi_1 = x_{1d}, \\ \phi_2 = -k_1 (x_1 - \phi_1)^{\gamma_1} + \dot{\phi}_1, \\ \phi_3 = -k_2 (x_2 - \phi_2)^{\gamma_2} + \dot{\phi}_2, \\ \quad \vdots \\ \phi_n = -k_{n-1} (x_{n-1} - \phi_{n-1})^{\gamma_{n-1}} + \dot{\phi}_{n-1}, \\ u = \dfrac{1}{g(x)} \left(-f(x) - k_n (x_n - \phi_n) + \dot{\phi}_n - A_1 + A_2\right), \\ \dot{A}_1 = k_{n+1} (x_n - \phi_n), \\ \dot{A}_2 = -k_{n+1} (x_n - \phi_n) |A_2| - k_{n+2} \dfrac{A_2}{|A_2|}, \end{cases} \quad (22)$$

where $0 < \gamma_i < 1$ and ϕ_i $(i = 2, 3, ..., n)$ are virtual controllers. This controller stabilizes $x_1 - x_{1d}$ in a finite time. Also in this algorithm the first adaptive term A_1 converges to $d(t)$ and the second adaptive term A_2 converges to zero in a finite time.

3. MODEL OF MAGNETIC LEVITATION SYSTEM

In this section, the dynamics model of the magnetic levitation system is presented. Only the vertical motion is considered. The schematic of this Maglev system is shown in Fig. 1. The parameters and variables are introduced in Table 1.

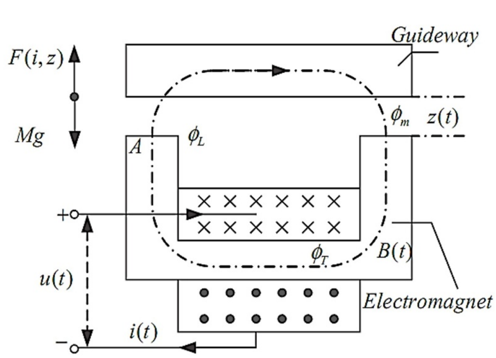

Fig. 1. Schematic diagram of magnetic levitation system.

Table 1. Parameters and variables of the Maglev system.

Variable	Significations
ϕ_m	Gap flux
ϕ_T	Main flux
ϕ_L	Leaking flux
$u(t)$	Voltage of the magnetic coil
R	Resistance of the magnetic coil
$B(t)$	Gap flux density
μ_0	Magnetic permeability of atmosphere
F	Electromagnetic force
A	Magnetic pole area
$z(t)$	Gap among electromagnet and Track
N	Number of magnetic coil
M	Total mass
$i(t)$	Current of magnetic coil

The dynamic model of this Maglev system can be written as [26].

$$\begin{aligned} B(t) &= \frac{\mu_0 N i(t)}{2z(t)}, \\ F &= \frac{B(t)^2 A}{\mu_0}, \\ M\ddot{z}(t) &= Mg - F, \\ u(t) &= \frac{2Rz(t)}{\mu_0 N} B(t) + NA\dot{B}(t). \end{aligned} \quad (23)$$

Let the states, the output and the control input are chosen such that

$$\begin{aligned} x &= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T = \begin{bmatrix} z & \dot{z} & B \end{bmatrix}^T, \\ y &= z = x_1, \\ u &= u(t). \end{aligned} \quad (24)$$

Thus, the nonlinear state-space model of the magnetic levitation system can be written as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = g - \dfrac{A}{\mu_0 M} {x_3}^2, \\ \dot{x}_3 = -\dfrac{2R}{\mu_0 A N^2} x_1 x_3 + \dfrac{1}{NA} u, \\ y = x_1. \end{cases} \quad (25)$$

4. PROPOSED CONTROLLER FOR MAGLEV SYSTEM

In this section the design procedure of the proposed finite-time adaptive nonlinear controller for the magnetic levitation system is discussed. Let x_{1d} is the desired values of x_1. The objective of the control schemes is to drive the state x_1 to their desired constant value. For this purpose first consider the following nonlinear change of coordinates

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = g - \dfrac{A}{\mu_0 M} x_3^2, \\ \dot{x}_3 = -\dfrac{2R}{\mu_0 A N^2} x_1 x_3 + \dfrac{1}{NA} u, \\ y = x_1. \end{cases} \tag{26}$$

Remark 1: If z_1 is driven to zero, then x_1 converge to x_{1d}. The dynamic model of the magnetic levitation system in the new coordinates system can be written as

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= z_3, \\ \dot{z}_3 &= f(x) + g(x)u + d(t), \end{aligned} \tag{27}$$

where

$$\begin{aligned} f(x) &= \frac{4R}{{\mu_0}^2 M N^2} x_1 x_3^2, \\ g(x) &= -\frac{2}{\mu_0 M N} x_3, \end{aligned} \tag{28}$$

and $d(t)$ is considered as the total uncertainties including parameter variations, un-modeled dynamics and unknown external disturbances.

Based on the proposed algorithm, the finite time adaptive controller for the Maglev system (27) is designed as

$$\begin{cases} \phi_1 = z_{1d}, \\ \phi_2 = -k_1 (z_1 - \phi_1)^{\gamma_1} + \dot{\phi}_1, \\ \phi_3 = -k_2 (z_2 - \phi_2)^{\gamma_2} + \dot{\phi}_2, \\ u = \dfrac{1}{g(x)} \left(-f(x) - k_3 (z_3 - \phi_3) + \dot{\phi}_3 - A_1 + A_2 \right), \\ \dot{A}_1 = k_4 (z_3 - \phi_3), \\ \dot{A}_2 = -k_4 (z_3 - \phi_3) |A_2| - k_5 \dfrac{A_2}{|A_2|}, \end{cases} \tag{29}$$

where $0 < \gamma_1, \gamma_2 < 1$, and $z_{1d} = 0$. This controller guarantees the stabilization of $x_1 - x_{1d}$ and $A_1 - d(t)$ in a finite time. To prove the stability, in the first step by controller (29) and by choosing $k_3 k_4 > 0$ and $k_5 = (L_d + |A_1|) L_d + \eta$, the convergence of $z_3 - \phi_3$, $A_1 - d(t)$ and A_2 in t_3 seconds are achieved. Then in the next steps we have a dynamics of the system as

$$\dot{z}_i = -k_i (z_i - \phi_i)^{\gamma_i} + \dot{\phi}_i, \; i = 1, \, 2, \tag{30}$$

and by introducing errors

$$e_i = z_i - \phi_i, \; i = 1, \, 2, \tag{31}$$

the resulting error dynamics can be written

$$\dot{e}_i = -k_i (e_i)^{\gamma_i}, \; i = 1, \, 2. \tag{32}$$

Relation (16) implies

$$t_i = \frac{1}{k_i (1 - \gamma_i)} {e_i}^{1-\gamma_i}, \; i = 1, \, 2, \tag{33}$$

where t_i is the convergence time of e_i. Therefore the finite time stability of e_i, $i = 1, 2$ are proved and the convergence of $e_1 = z_1 - z_{1d}$ will be achieved after $t = t_1 + t_2 + t_3$ seconds. Note that it is assumed that the nonlinear terms in (22) and (29) are known. Of course, if an error occurs in eliminating these sentences, it will be added to the uncertainty $d(t)$ and will be eliminated with the controller adaptive term.

5. SIMULATION RESULTS

Simulations are performed in this section and the proposed controller is compared with the classical and the approximated sliding mode controllers. The curve of uncertainty $d(t)$ is shown in Fig. 2. The parameters of the magnetic levitation system and the values of the controller's gains are as listed in Table 2.

The curves of the first adaptive term in the proposed algorithm $A_1$and the uncertainty $d(t)$ are plotted in Fig. 2. As shown in this figure, the uncertainty is tracked by the first adaptive term in the proposed controller with high precision in a finite time. Also, Fig. 3 shows that the second adaptive term of the proposed algorithm is converged.

The control input signal that is generated by the controllers, is plotted in Fig. 4. As shown in this figure, the chattering is occurred in the classical sliding mode controller and this control law is not applicable. The control signals in the approximated sliding mode controller and the proposed controller are smooth but the maximum magnitude of the proposed control signal is lower than approximated SMC. In both standard and approximate methods, it has been tried to select the gains in such a way that the best result is obtained. If the gains are chosen smaller in

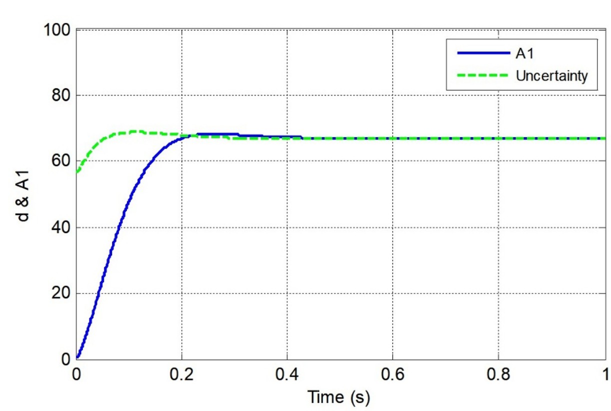

Fig. 2. Uncertainty $d(t)$ and the first adaptive term of the proposed controller.

Table 2. Values of the parameters and the controller gains.

Parameter or gain		Value
Maglev system [26]	R	0.5
	μ_0	$4\pi \times 10^{-7}$
	A	0.0235
	N	324
	M	635
	z_d	0.01
Finite time adaptive nonlinear controller	k_1	5
	k_2	30
	k_3	40
	k_4	500
	k_5	100
	γ_1	0.9
	γ_2	0.95
Classical and approximated sliding mode controllers	k	200
	ε	1
	λ_1	30
	λ_2	2000

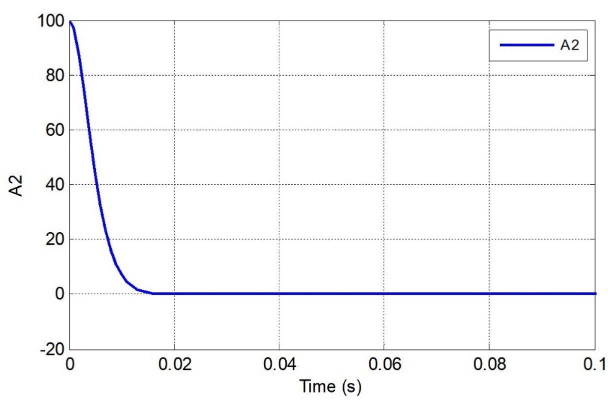

Fig. 3. The second adaptive term of the proposed controller.

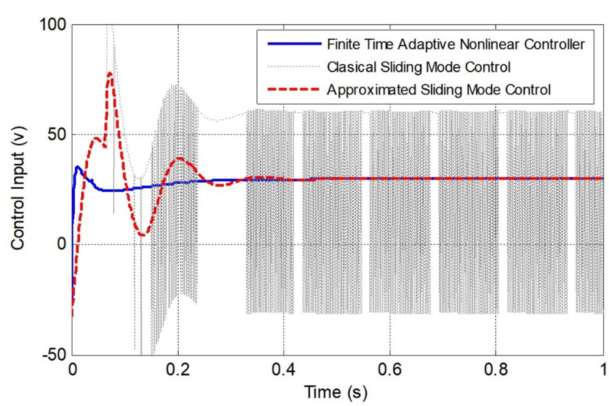

Fig. 4. Control input signals that are generated by the proposed finite time adaptive nonlinear, the classical and the approximated sliding mode controllers.

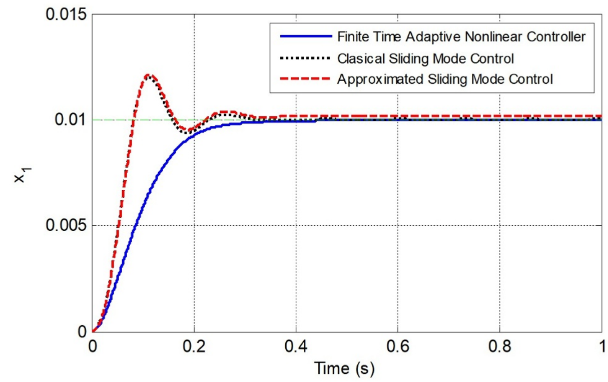

Fig. 5. The position of the mass by applying the proposed finite time adaptive nonlinear, the classical and the approximated sliding mode controllers.

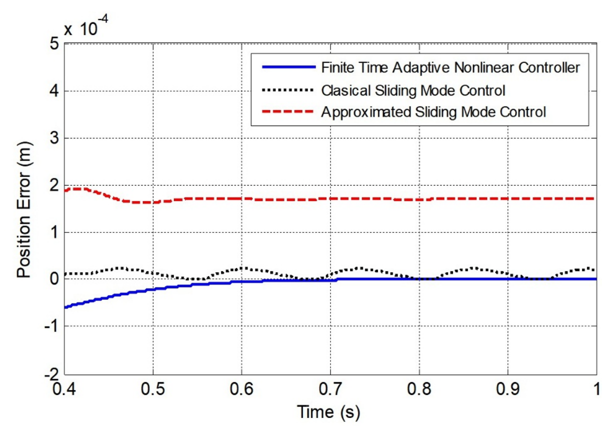

Fig. 6. The position error by applying the proposed finite time adaptive nonlinear, the classical and the approximated sliding mode controllers.

these two methods, the convergence time will increase and the accuracy will decrease in the approximated method.

Position of the mass in Fig. 5, error position in Fig. 6, mass velocity in Fig. 7 and the flux of the gap in Fig. 8 are plotted. These figures show that by applying the proposed controller the mass position is controlled in the desired position with higher precision in comparison with the approximated sliding mode control. Also, this controller is able to stabilize the mass velocity and gap flux with lower overshot in comparison with the sliding mode controllers.

In Fig. 6, the position tracking error is plotted and it can be seen that the accuracy of the suspended object stabilization in the proposed method is more than the sliding mode method, while a smaller control input signal is issued in the proposed method. Although the proposed method is not particularly superior compared to the standard sliding mode method in terms of accuracy, but in terms of the control signal, the proposed method is completely smooth and the standard sliding mode method is inapplicable due to chattering.

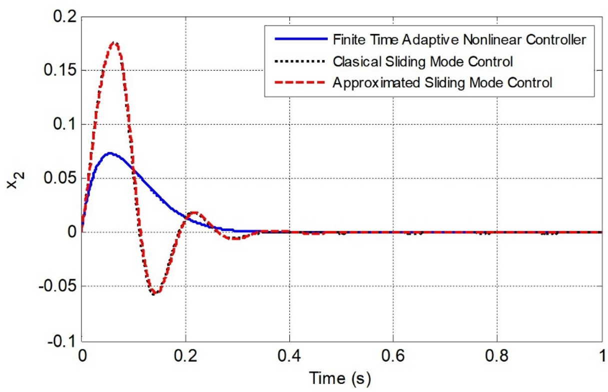

Fig. 7. The velocity of the mass by applying the proposed finite time adaptive nonlinear, the classical and the approximated sliding mode controllers.

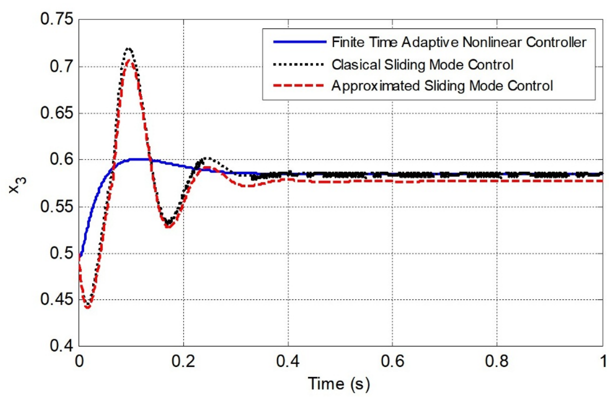

Fig. 8. The gap flux density by applying the proposed finite time adaptive nonlinear, the classical and the approximated sliding mode controllers.

Table 3. Some data to compare control methods.

Controller	Maximum magnetude of control signal	Error range	Contol signal
Finite time adaptive nonlinear controller	38	0	Smooth
Classical sliding mode controllers	100	$< 2\times 10^{-4}$	Non smooth
Approximated sliding mode controllers	73	$< 2\times 10^{-3}$	Smooth

Finally some data to compare control methods are presented in Table 3. The gains are chosen in such a way that the convergence time is almost the same in all three methods, but in terms of the maximum size and smoothness of the control signal, the proposed method performs better.

6. CONCLUTION

In this paper, a finite time adaptive controller for a class of uncertain nonlinear systems is proposed. In this algorithm, the virtual controllers are introduced based on finite time control idea in a back-stepping procedure. Also, an adaptive nonlinear control law is designed that is able to estimate uncertainty in a finite time. The stability of the proposed algorithm is proved by using Lyapunov method in each step. This control scheme is used to control a magnetic levitation system. Simulation results show that the first and second adaptive terms of the proposed controller are converged to the uncertainty and zero, respectively and this controller is able to control the position of mass in a finite time. Also, this controller generates smooth control signals and has higher precision in comparison with classical and approximated order sliding mode controllers.

CONFLICT OF INTEREST

The authors declare that there is no competing financial interest or personal relationship that could have appeared to influence the work reported in this paper.

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