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Iterative Algorithm for Feedback Nonlinear Systems by Using the Maximum Likelihood Principle

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Abstract: This paper aims to find a maximum likelihood least squares-based iterative algorithm to solve the identification issues of closed-loop input nonlinear equation-error systems. By adopting the key term separation technique, the parameters of the forward channel are identified separately from the parameters of the feedback channel to address the cross-product terms. The hierarchical identification principle is introduced to decompose the original system into two subsystems for reduced computational complexity. The iterative estimation theory and the maximum likelihood principle are applied to design a new least-squares algorithm with high estimation accuracy by taking full use of all the measured input-output data at each iterative computation. Compared with the recursive least-squares (RELS) method. The simulation results verify theoretical findings, and the proposed algorithm can generate more accurate parameter estimates than the RELS algorithm.

Keywords: Feedback nonlinear system, iterative identification theory, key term separation technique, least-squares, maximum likelihood.

1. INTRODUCTION

Parameter estimation is very important for system analysis and synthesis [1-4]. Some identification methods are developed for open-loop systems [5-8]. However, in industrial processes, system identification is best carried out in closed-loop conditions for safety and system stability [9]. Because the feedback channel causes noise coupling to the input signal under closed-loop, so it is desired to separate the forward channel and the feedback channel for identification. In addition, different inputs may lead to the same output results because of the feedback channel. So the closed-loop identification is more difficult than the open-loop identification. In this regard, many closed-loop identification methods have been developed such as the subspace identification method [10,11], the recursive identification methods [12], and the iterative identification methods [13-15].

Most industrial processes are nonlinear systems with complex structures in nature [16,17]. Hammerstein model has become a widely used block-oriented model owning to its computational efficiency and easy identification [18]. Many methods have been investigated, such as the blind identification methods [19], the recursive identification methods and the iterative identification methods [20]. For the fractional order Hammerstein nonlinear systems, a multiple innovation Levenberg-Marquardt algorithm hybrid identification method was proposed to estimate the linear block parameters and fractional order, and experiments were provided to verify the effectiveness of the proposed method [21].

Based on the decomposition-coordination principle, the hierarchical identification principle can be used to solve the identification problems of large-scale system identification with high dimensions and many variables [22]. The key is to decompose the system into several fictitious subsystems and identify the parameters of their corresponding parts respectively [23]. The hierarchical identification method includes model decomposition, subsystem identification and coordination of the related items [18]. During identification, the unknown parameters and the related terms are replaced with their previous estimates until satisfactory parameter estimates are obtained [24].

Taking the output of the nonlinear block as the key term can solve the bilinear parameter identification problem [18]. Then the model output can be expressed as a linear combination containing all unknown parameters without redundant parameters [25,26]. Therefore, the computational cost is reduced, and the unknown key terms in the information vector can be replaced with the estimates of

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the corresponding auxiliary model [27]. Because of good statistical properties, the maximum likelihood methods are extensively used [28,29].

This paper aims to explore a new least-squares optimal estimation algorithm for the concerned feedback nonlinear systems. The basic idea is to transform the nonlinear system into two subsystems by using the hierarchical identification principle, to apply the key term separation technique for solving the identification difficulty of the associated terms, to identify the subsystem parameters interactively. The main contributions are as follows:

1) The proposed method identifies the parameters of the forward channel and the parameters of the feedback channel interactively.
2) The proposed method has high estimation accuracy and reduced computational burden.

This paper is structured as follows: Section 2 describes the closed-loop nonlinear equation-error systems, gives the sub-identification models after applying the hierarchical identification principle, and forms the identification problem. Section 3 derives a maximum likelihood least squares-based iterative algorithm. Section 4 gives numerical examples to demonstrate the effectiveness of the proposed methods. Finally, the conclusion is offered in Section 5.

2. PROBLEM STATEMENT

Consider the following closed-loop input nonlinear equation-error system

$$\begin{aligned}
&R(q)s(t) = B(q)\bar{u}(t) + G(q)v(t),\\
&\bar{u}(t) = f(u(t))\\
&\quad = c_1 f_1(u(t)) + c_2 f_2(u(t)) + \cdots + c_\gamma f_\gamma(u(t))\\
&\quad = \boldsymbol{f}(u(t))\boldsymbol{c},
\end{aligned}$$

with the system output $s(t)$, the reference input $r(t)$, the control input $u(t) = r(t) - s(t)$, the output of the nonlinear block $\bar{u}(t)$, the unknown coefficients c_i $(i = 1, 2, \cdots, \gamma)$, the known basis functions f_i $(i = 1, 2, \cdots, \gamma)$, the noise $v(t)$, and measurable polynomials $R(q)$, $B(q)$ and $G(q)$, defined as

$$\begin{aligned}
R(q) &:= 1 + r_1 q^{-1} + r_2 q^{-2} + \cdots + r_{n_r} q^{-n_r},\\
B(q) &:= b_0 + b_1 q^{-1} + b_2 q^{-2} + \cdots + b_{n_b} q^{-n_b},\\
G(q) &:= 1 + g_1 q^{-1} + g_2 q^{-2} + \cdots + g_{n_g} q^{-n_g}.
\end{aligned}$$

Assume that the degrees n_r, n_b and n_g are known, and $s(t) = 0$, $r(t) = 0$, $v(t) = 0$ for $t \leqslant 0$. Let $n = n_r + n_b + n_g$. Introduce the parameter vectors $\boldsymbol{r}$, $\boldsymbol{b}$, $\boldsymbol{g}$, ϑ, $\boldsymbol{c}$ and $\boldsymbol{\theta}$

$$\begin{aligned}
\boldsymbol{r} &:= [r_1,\ r_2,\ \cdots,\ r_{n_r}]^{\mathrm{T}} \in \mathbb{R}^{n_r},\\
\boldsymbol{b} &:= [b_1,\ b_2,\ \cdots,\ b_{n_b}]^{\mathrm{T}} \in \mathbb{R}^{n_b},\\
\boldsymbol{g} &:= [g_1,\ g_2,\ \cdots,\ g_{n_g}]^{\mathrm{T}} \in \mathbb{R}^{n_g},\\
\vartheta &:= [\boldsymbol{r}^{\mathrm{T}},\ \boldsymbol{b}^{\mathrm{T}},\ \boldsymbol{g}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n},\\
\boldsymbol{c} &:= [c_1,\ c_2,\ \cdots,\ c_\gamma]^{\mathrm{T}} \in \mathbb{R}^{\gamma},\\
\boldsymbol{\theta} &:= [\boldsymbol{r}^{\mathrm{T}},\ \boldsymbol{b}^{\mathrm{T}},\ \boldsymbol{g}^{\mathrm{T}},\ \boldsymbol{c}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n+\gamma},
\end{aligned}$$

with the information vectors $\varphi(t)$ and $\boldsymbol{f}(t)$

$$\begin{aligned}
\varphi(t) &:= [-s(t-1),\ \cdots,\ -s(t-n_r),\ \bar{u}(t-1),\ \cdots,\\
&\qquad \bar{u}(t-n_b),\ v(t-1),\ \cdots,\ v(t-n_g)]^{\mathrm{T}},\\
\boldsymbol{f}(t) &:= [f_1(u(t)),\ f_2(u(t)),\ \cdots,\ f_\gamma(u(t))]^{\mathrm{T}} \in \mathbb{R}^{\gamma}.
\end{aligned}$$

From the above derivation, we have

$$\begin{aligned}
s(t) &= -\sum_{l=1}^{n_r} r_l q^{-l} s(t) + \sum_{l=0}^{n_b} b_l q^{-l} \bar{u}(t)\\
&\quad + \sum_{l=1}^{n_g} g_l q^{-l} v(t) + v(t),
\end{aligned} \tag{1}$$

or

$$\begin{aligned}
&v(t)\\
&= G^{-1}(q)\left[\sum_{l=0}^{n_r} r_l q^{-l} s(t) - \sum_{l=0}^{n_b}\sum_{i=1}^{\gamma} c_i b_l q^{-l} f_i(u(t))\right].
\end{aligned} \tag{2}$$

Clearly, there contain the products of the parameters b_l of the linear block and c_i of the nonlinear block. To identify the model, we assume that $b_0 = 1$ and choose $\bar{u}(t)$ as a separated key term. Then (1) becomes

$$\begin{aligned}
s(t) &= -\sum_{l=1}^{n_r} r_l q^{-l} s(t) + \sum_{l=1}^{n_b} b_l q^{-l} \bar{u}(t) + \sum_{i=1}^{\gamma} c_i f_i(u(t))\\
&\quad + \sum_{l=1}^{n_g} g_l q^{-l} v(t) + v(t).
\end{aligned}$$

The measured output $s(t)$ in (1) can be represented as

$$s(t) = \varphi^{\mathrm{T}}(t)\vartheta + \boldsymbol{f}^{\mathrm{T}}(t)\boldsymbol{c} + v(t). \tag{3}$$

Here, by using the hierarchical identification principle [30-33], it gives rise to the following two fictitious sub-identification models:

$$S1: \quad s_1(t) := s(t) - \boldsymbol{f}^{\mathrm{T}}(t)\boldsymbol{c} = \varphi^{\mathrm{T}}(t)\vartheta + v(t), \tag{4}$$

$$S2: \quad s_2(t) := s(t) - \varphi^{\mathrm{T}}(t)\vartheta = \boldsymbol{f}^{\mathrm{T}}(t)\boldsymbol{c} + v(t). \tag{5}$$

The parameter vector ϑ of the linear block and the parameter vector $\boldsymbol{c}$ of the nonlinear block are in the above subsystem models respectively. They are the interrelated items. The proposed parameter estimation algorithms in this paper are based on these two identification models in (4) and (5). Many identification methods are derived based on the identification models of the systems [34-37] and these methods can be used to estimate the parameters of other linear systems and nonlinear systems [38-41] and can be applied to other fields [42-46] such as chemical process control systems.

Remark 1: The key term separation technique is used to separate the parameters of the linear block from the parameters of the nonlinear block for reducing computational cost.

Remark 2: The hierarchical identification principle is adopted to decompose the original system into two sub-identification models for reducing computational complexity.

3. THE MAXIMUM LIKELIHOOD LEAST SQUARES-BASED ITERATIVE ALGORITHM

This section derives a maximum likelihood least squares-based iterative (ML-LSI) algorithm to obtain the estimates of ϑ and $\boldsymbol{c}$ in (3). The hierarchical identification principle is utilized to estimate the associated items of ϑ and $\boldsymbol{c}$ interactively.

Observing (4) and according to the maximum likelihood principle, the maximum likelihood estimation of ϑ can be obtained by minimizing the following cost function

$$J_1(\vartheta)\Big|_{\hat{\vartheta}_{\mathrm{ML}}} = \frac{1}{2}\sum_{k=1}^{t} v^2(k)\Big|_{\hat{\vartheta}_{\mathrm{ML}}}.$$

Let $k = 1, 2, 3, \cdots$ be an iterative variable, and $\hat{\vartheta}_k$ be the iterative estimate of ϑ. Thus the estimates of $\hat{R}_k(t,q)$, $\hat{B}_k(t,q)$ and $\hat{G}_k(t,q)$ can be recast as

$$\begin{aligned}
\hat{R}_k(t,q) &= 1+\hat{r}_{1,k}(t)q^{-1}+\hat{r}_{2,k}(t)q^{-2}+\cdots\\
&\quad +\hat{r}_{n_r,k}(t)q^{-n_r},\\
\hat{B}_k(t,q) &= 1+\hat{b}_{1,k}(t)q^{-1}+\hat{b}_{2,k}(t)q^{-2}+\cdots\\
&\quad +\hat{b}_{n_b,k}(t)q^{-n_b},\\
\hat{G}_k(t,q) &= 1+\hat{g}_{1,k}(t)q^{-1}+\hat{g}_{2,k}(t)q^{-2}+\cdots\\
&\quad +\hat{g}_{n_g,k}(t)q^{-n_g},\\
\hat{\bar{u}}_k(t) &= \hat{c}_{1,k}(t)f_1(u(t))+\hat{c}_{2,k}(t)f_2(u(t))+\cdots\\
&\quad +\hat{c}_{\gamma,k}(t)f_\gamma(u(t)).
\end{aligned}$$

Let $\hat{\vartheta}_k(t)$, $\hat{\boldsymbol{c}}_k(t)$ and $\hat{\varphi}_k(t)$ be the estimates of ϑ, $\boldsymbol{c}(t)$ and $\varphi(t)$ at iteration k, respectively, which can be represented as

$$\begin{aligned}
\hat{\vartheta}_k(t) &:= [\hat{\boldsymbol{r}}_k^{\mathrm{T}}(t),\hat{\boldsymbol{b}}_k^{\mathrm{T}}(t),\hat{\boldsymbol{g}}_k^{\mathrm{T}}(t)]^{\mathrm{T}}\\
&= [\hat{r}_{1,k}(t),\cdots,\hat{r}_{n_r,k}(t),\hat{b}_{1,k}(t),\cdots,\hat{b}_{n_b,k}(t),\\
&\quad \hat{g}_{1,k}(t),\cdots,\hat{g}_{n_g,k}(t)]^{\mathrm{T}} \in \mathbb{R}^{n_r+n_b+n_g},\\
\hat{\boldsymbol{c}}_k(t) &:= [\hat{c}_{1,k}(t),\hat{c}_{2,k}(t),\cdots,\hat{c}_{\gamma,k}(t)]^{\mathrm{T}} \in \mathbb{R}^{\gamma},\\
\hat{\varphi}_k(t) &:= [-s(t-1),\cdots,-s(t-n_r),\hat{\bar{u}}_k(t-1),\cdots,\\
&\quad \hat{\bar{u}}_k(t-n_b),\hat{v}_k(t-1),\cdots,\hat{v}_k(t-n_g)]^{\mathrm{T}}.
\end{aligned}$$

Observing (3), $\hat{v}_k(t)$ can be computed by replacing ϑ, $\boldsymbol{c}(t)$ and $\varphi(t)$ with their corresponding estimates $\hat{\vartheta}_k(t)$, $\hat{\boldsymbol{c}}_k(t)$ and $\hat{\varphi}_k(t)$, that is

$$\hat{v}_k(t) = s(t)-\hat{\varphi}_k^{\mathrm{T}}(t)\hat{\vartheta}_k(t)-\boldsymbol{f}^{\mathrm{T}}(t)\hat{\boldsymbol{c}}_k(t).$$

The partial derivatives of $v(t)$ in (2) relative to r_l, b_l and g_l at the point $\hat{\vartheta}_{k-1}(t)$ can be computed by

$$\begin{aligned}
\frac{\partial v(t)}{\partial r_l}\Big|_{\hat{\vartheta}_{k-1}(t)} &= [\hat{G}_{k-1}(t,q)]^{-1}q^{-l}s(t) =: q^{-l}\hat{s}_{\mathrm{f},k}(t),\\
\frac{\partial v(t)}{\partial b_l}\Big|_{\hat{\vartheta}_{k-1}(t)} &= -[\hat{G}_{k-1}(t,q)]^{-1}q^{-l}\hat{\bar{u}}(t) =: -q^{-l}\hat{\bar{u}}_{\mathrm{f},k}(t),\\
\frac{\partial v(t)}{\partial g_l}\Big|_{\hat{\vartheta}_{k-1}(t)} &= -[\hat{G}_{k-1}(t,q)]^{-1}q^{-l}\hat{v}(t) =: -q^{-l}\hat{v}_{\mathrm{f},k}(t).
\end{aligned}$$

Define the filtered information vector

$$\varphi_{\mathrm{f}}(t) := -\frac{\partial v(t)}{\partial \vartheta}\Big|_{\hat{\vartheta}_{k-1}(t)} = -\begin{bmatrix}\frac{\partial v(t)}{\partial \boldsymbol{r}_l}\\ \frac{\partial v(t)}{\partial \boldsymbol{b}_l}\\ \frac{\partial v(t)}{\partial \boldsymbol{g}_l}\end{bmatrix}_{\hat{\vartheta}_{k-1}(t)}.$$

Then $\hat{\varphi}_{\mathrm{f},k}(t)$ can be presented as

$$\begin{aligned}
\hat{\varphi}_{\mathrm{f},k}(t) = [&-\hat{s}_{\mathrm{f},k}(t-1),\ \cdots,\ -\hat{s}_{\mathrm{f},k}(t-n_r),\\
&\hat{\bar{u}}_{\mathrm{f},k}(t-1),\ \cdots,\ \hat{\bar{u}}_{\mathrm{f},k}(t-n_b),\\
&\hat{v}_{\mathrm{f},k}(t-1),\ \hat{v}_{\mathrm{f},k}(t-2),\ \cdots,\ \hat{v}_{\mathrm{f},k}(t-n_g)]^{\mathrm{T}}.
\end{aligned}$$

Collect N input and output data, and define

$$\begin{aligned}
\boldsymbol{S}(N) &:= [s(N),s(N-1),\cdots,s(1)]^{\mathrm{T}} \in \mathbb{R}^{N},\\
\boldsymbol{V}(N) &:= [V(N),V(N-1),\cdots,V(1)]^{\mathrm{T}} \in \mathbb{R}^{N},\\
\boldsymbol{\Phi}(N) &:= [\varphi(N),\varphi(N-1),\cdots,\varphi(1)]^{\mathrm{T}} \in \mathbb{R}^{N\times n},\\
\boldsymbol{F}(N) &:= [\boldsymbol{f}(N),\boldsymbol{f}(N-1),\cdots,\boldsymbol{f}(1)]^{\mathrm{T}} \in \mathbb{R}^{N\times \gamma},\\
\boldsymbol{\Phi}_{\mathrm{f}}(N) &:= [\varphi_{\mathrm{f}}(N),\varphi_{\mathrm{f}}(N-1),\cdots,\varphi_{\mathrm{f}}(1)]^{\mathrm{T}} \in \mathbb{R}^{N\times n}.
\end{aligned}$$

Equation (3) becomes

$$\boldsymbol{S}(N) = \boldsymbol{\Phi}(N)\vartheta+\boldsymbol{F}(N)\boldsymbol{c}+\boldsymbol{V}(N).$$

For submodel S1, the cost function can be restated as

$$J_2(\vartheta)\Big|_{\hat{\vartheta}_{\mathrm{ML}}} = \frac{1}{2}\|\boldsymbol{V}(N)\|^2_{\hat{\vartheta}_{\mathrm{ML}}}.$$

Minimizing the above cost function $J_2(\vartheta)$ and letting its gradient with respect to ϑ at $\vartheta = \vartheta_{\mathrm{ML}}$ be zero yield

$$\frac{\partial J_2(\vartheta)}{\partial \vartheta}\Big|_{\hat{\vartheta}_{\mathrm{ML}}} = \frac{\partial \boldsymbol{V}(N)}{\partial \vartheta}\boldsymbol{V}(N)\Big|_{\hat{\vartheta}_{\mathrm{ML}}} = 0.$$

Summing up the above, we have

$$\begin{aligned}
\frac{\partial J_2(\vartheta)}{\partial \vartheta}\Big|_{\hat{\vartheta}_{\mathrm{ML}}} &= \boldsymbol{\Phi}_{\mathrm{f}}^{\mathrm{T}}(N)[\boldsymbol{S}(N)-\boldsymbol{\Phi}(N)\vartheta-\boldsymbol{F}(N)\boldsymbol{c}]_{\hat{\vartheta}_{\mathrm{ML}}}\\
&= 0.
\end{aligned}$$

That is

$$\boldsymbol{\Phi}_{\mathrm{f}}^{\mathrm{T}}(N)\boldsymbol{\Phi}(N)\hat{\vartheta}_{\mathrm{ML}} = \boldsymbol{\Phi}_{\mathrm{f}}^{\mathrm{T}}(N)[\boldsymbol{S}(N)-\boldsymbol{F}(N)\boldsymbol{c}].$$

Provided that $\boldsymbol{\Phi}_{\mathrm{f}}(N)$ and $\boldsymbol{\Phi}(N)$ are persistently exciting. Using the maximum likelihood least-squares estimate of submodel S1 to update the parameter estimates yields

$$\hat{\vartheta}(t) = [\boldsymbol{\Phi}_{\mathrm{f}}^{\mathrm{T}}(N)\boldsymbol{\Phi}(N)]^{-1}\boldsymbol{\Phi}_{\mathrm{f}}^{\mathrm{T}}(N)[\boldsymbol{S}(N) - \boldsymbol{F}(N)\boldsymbol{c}]. \quad (6)$$

Define the estimates of $\boldsymbol{V}(N)$, $\boldsymbol{\Phi}(N)$ and $\boldsymbol{\Phi}_{\mathrm{f}}(L)$ at iteration k as

$$\hat{\boldsymbol{V}}_k(N) := [\hat{v}_k(N), \hat{v}_k(N-1), \cdots, \hat{v}_k(1)]^{\mathrm{T}} \in \mathbb{R}^N,$$
$$\hat{\boldsymbol{\Phi}}_k(N) := [\hat{\varphi}_k(N), \hat{\varphi}_k(N-1), \cdots, \hat{\varphi}_k(1)]^{\mathrm{T}} \in \mathbb{R}^{N\times n},$$
$$\hat{\boldsymbol{\Phi}}_{\mathrm{f},k}(N) := [\hat{\varphi}_{\mathrm{f},k}(N), \hat{\varphi}_{\mathrm{f},k}(N-1), \cdots, \hat{\varphi}_{\mathrm{f},k}(1)]^{\mathrm{T}}.$$

Thus, according to the hierarchical identification principle, (6) can be restated as

$$\hat{\vartheta}_k(t) = [\hat{\mathbf{\Phi}}_{\mathrm{f},k}^{\mathrm{T}}(N)\hat{\mathbf{\Phi}}_k(N)]^{-1}\hat{\mathbf{\Phi}}_{\mathrm{f},k}^{\mathrm{T}}(N) \times [\boldsymbol{S}(N) - \boldsymbol{F}(N)\hat{\boldsymbol{c}}_{k-1}(t)].$$

Observing (5), applying the least-squares iterative method for the cost function

$$J(\boldsymbol{c}) := \sum_{t=1}^{N} \|s_2(t) - \boldsymbol{f}^{\mathrm{T}}(t)\boldsymbol{c}\|^2,$$

the iterative estimate of $\hat{\boldsymbol{c}}_k(t)$ can be acquired. To sum up, we can obtain a maximum likelihood least squares-based iterative (ML-LSI) algorithm. The proposed estimation methods in this paper can combine some identification algorithms [47-50] to explore parameter estimation issues of linear and nonlinear stochastic systems [51-56] and can be applied to other areas [57-63] such as engineering application systems.

Remark 3: The input and output signals of the closed-loop systems are usually assumed to satisfy the following conditions. Firstly, $r(t)$ is a stationary stochastic process. Secondly, the noise $v(t)$ is statistically uncorrelated with $r(t)$.

Remark 4: Owning to sufficient use of all the measured data $\{r(t), s(t) : t = 0, 1, 2, \cdots, N\}$, the improved parameter estimation accuracy can be obtained.

Remark 5: $\hat{\vartheta}_k(t)$ and $\hat{\boldsymbol{c}}_k(t)$ are acquired by using the input and output data with a window of length N. Since the data window changes with time t, the proposed algorithm can be used to track time-varying parameters, so as to realize online identification.

4. EXAMPLES

Example 1: Consider the following closed-loop input nonlinear system

$$R(q)s(t) = B(q)\bar{u}(t) + G(q)v(t),$$
$$u(t) = r(t) - s(t),$$
$$R(q) = 1 + r_1q^{-1} + r_2q^{-2}$$

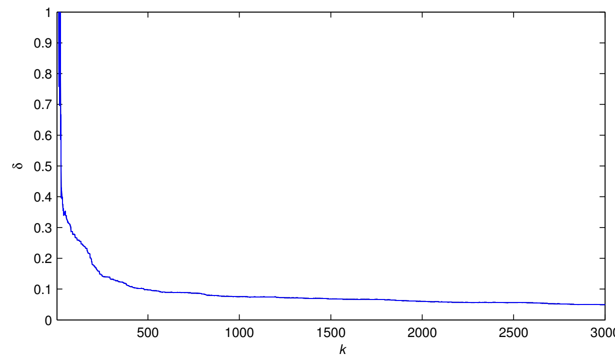

Fig. 1. The RELS parameter estimation errors δ versus t under $\sigma^2 = 0.10^2$.

$$= 1 + 1.72q^{-1} + 1.02q^{-2},$$
$$B(q) = b_0 + b_1q^{-1} + b_2q^{-2}$$
$$= 1 + 1.05q^{-1} - 0.61q^{-2},$$
$$G(q) = 1 + g_1q^{-1} = 1 - 0.30q^{-1},$$
$$\bar{u}(t) = f(u(t)) = 0.15u(t) + 0.35u^2(t).$$

The parameter vectors ϑ, $\boldsymbol{c}$ and θ can be presented below

$$\vartheta = [r_1,\ r_2,\ b_1,\ b_2,\ g_1]^{\mathrm{T}}$$
$$= [1.72,\ 1.02,\ 1.05,\ -0.61,\ -0.30]^{\mathrm{T}},$$
$$\boldsymbol{c} = [c_1,\ c_2]^{\mathrm{T}} = [0.15,\ 0.35]^{\mathrm{T}},$$
$$\theta == [1.72,\ 1.02,\ 1.05, -0.61, -0.30,\ 0.15,\ 0.35]^{\mathrm{T}}.$$

In simulation, the input $r(t)$ is taken as an independent persistent excitation signal sequence with zero mean and unit variance, and $v(t)$ is taken as an uncorrelated white noise sequence with zero mean and variance σ^2.

Applying the RELS algorithm with the data length $t = 3000$ to estimate θ, the parameter estimates and their errors $\delta := \|\hat{\theta}(t) - \theta\|/\|\theta\|$ under $\sigma^2 = 0.10^2$ are shown in Table 1, and their estimation errors versus t are shown in Fig. 1.

Taking $\sigma_1^2 = 0.10^2$, $\sigma_2^2 = 0.20^2$ and $\sigma_3^2 = 0.30^2$, respectively, applying the proposed ML-LSI algorithm with the data length $t = N = 1000$ to estimate the parameters of this example system, the parameter estimates and their errors $\delta := \|\hat{\theta}_k(t) - \theta\|/\|\theta\|$ are shown in Table 2, and their estimation errors versus k are shown in Figs. 2-4.

Example 2: Consider another system

$$R(q)s(t) = B(q)\bar{u}(t) + G(q)v(t),$$
$$u(t) = r(t) - s(t),$$
$$R(q) = 1 + r_1q^{-1} + r_2q^{-2}$$
$$= 1 + 0.96q^{-1} + 0.90q^{-2},$$
$$B(q) = b_0 + b_1q^{-1} = 1 + 0.86q^{-1},$$
$$G(q) = 1 + g_1q^{-1} + g_2q^{-2}$$
$$= 1 - 0.31q^{-1} + 0.33q^{-2},$$

Table 1. The RELS estimates and errors of θ under $\sigma^2 = 0.10^2$.

t	r_1	r_2	b_1	b_2	g_1	c_1	c_2	δ (%)
100	1.71238	1.01449	0.71890	-0.09243	-0.08292	0.21172	0.31856	27.43055
200	1.71564	1.01792	0.83031	-0.27219	-0.16998	0.18998	0.30463	17.90317
500	1.71997	1.02000	1.01270	-0.46143	-0.12642	0.16497	0.33052	9.74386
1000	1.72000	1.02000	1.05412	-0.53429	-0.13728	0.15496	0.33317	7.54947
2000	1.72000	1.02000	1.08378	-0.57953	-0.16385	0.15108	0.34013	6.02234
3000	1.72000	1.02000	1.08058	-0.59322	-0.18764	0.15150	0.34178	4.93664
True values	1.72000	1.02000	1.05000	-0.61000	-0.30000	0.15000	0.35000	

Table 2. The ML-LSI estimates and errors of θ with different noise variances.

σ^2	k	r_1	r_2	b_1	b_2	g_1	c_1	c_2	δ (%)
0.10^2	1	1.72000	1.02000	1.33521	-0.38455	-0.14426	-0.01249	0.00124	23.09478
	2	1.72000	1.02000	-2.04282	5.61713	0.38733	0.09261	0.24627	292.47083
	5	1.72000	1.02000	1.01175	-0.59790	-0.28584	0.11108	0.24515	5.00824
	10	1.72000	1.02000	1.05354	-0.56979	-0.31194	0.12458	0.24410	4.88692
	15	1.72000	1.02000	1.04016	-0.57442	-0.30871	0.11448	0.24557	4.88229
	20	1.72000	1.02000	1.04685	-0.57017	-0.31060	0.12249	0.24474	4.87107
0.20^2	1	1.72000	1.02000	1.33867	-0.37695	-0.24568	-0.01323	0.00131	22.49366
	2	1.72000	1.02000	-2.17622	5.80846	0.34703	0.09707	0.25838	301.91903
	5	1.72000	1.02000	0.91837	-0.55236	-0.27511	0.11780	0.25691	7.36572
	10	1.72000	1.02000	0.94802	-0.53172	-0.29374	0.12610	0.25607	6.74362
	15	1.72000	1.02000	0.94462	-0.53386	-0.29267	0.12235	0.25645	6.80992
	20	1.72000	1.02000	0.94707	-0.53325	-0.29317	0.12403	0.25616	6.75039
0.30^2	1	1.72000	1.02000	1.34213	-0.36935	-0.27949	-0.01397	0.00138	22.62073
	2	1.72000	1.02000	-2.18076	5.63087	0.32187	0.10154	0.27053	295.30948
	5	1.72000	1.02000	0.83152	-0.51289	-0.26542	0.12430	0.26877	10.72130
	10	1.72000	1.02000	0.85295	-0.49657	-0.27936	0.12975	0.26814	10.18647
	15	1.72000	1.02000	0.85216	-0.49759	-0.27896	0.12830	0.26821	10.19924
	20	1.72000	1.02000	0.85327	-0.49761	-0.27912	0.12863	0.26807	10.16102
True values		1.72000	1.02000	1.05000	-0.61000	-0.30000	0.15000	0.35000	

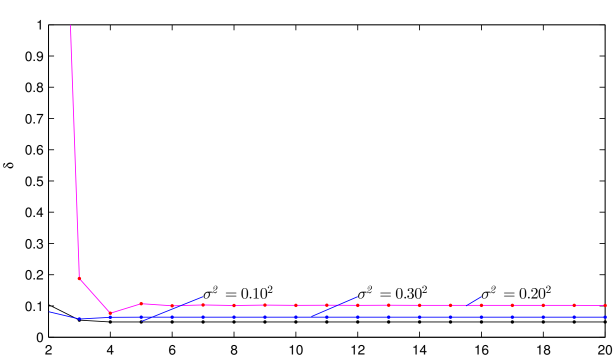

Fig. 2. The parameter estimation errors δ versus k with different noise variances.

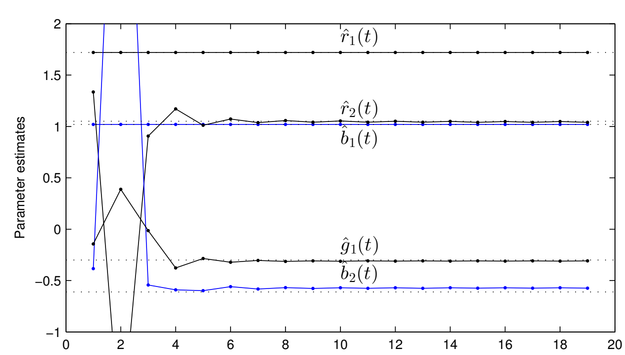

Fig. 3. The ML-LSI estimate $\hat{\vartheta}$ versus k.

$$\bar{u}(t) = f(u(t)) = 0.10\sin(u(t)) + 0.23\cos(u(t)^2).$$

The simulation conditions are similar to those of Example 1. Applying the proposed algorithm to estimate this example system, the simulation results are shown in Figs. 5 and 6.

From the simulation results of Tables 1-2 and Figs. 1-6, we can draw the following conclusions.

1) Table 2 illustrate that the parameter estimation errors through the ML-LSI algorithm are smaller under a lower noise level.

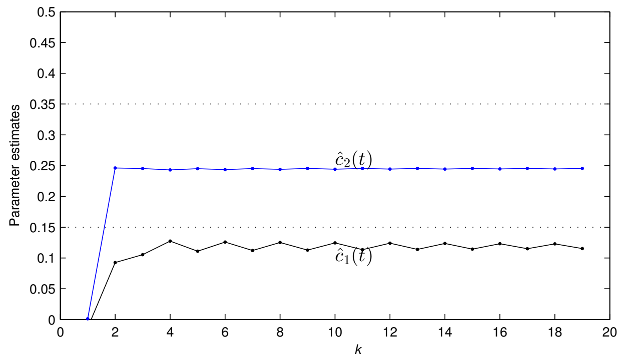

Fig. 4. The ML-LSI estimate $\hat{\boldsymbol{c}}$ versus k.

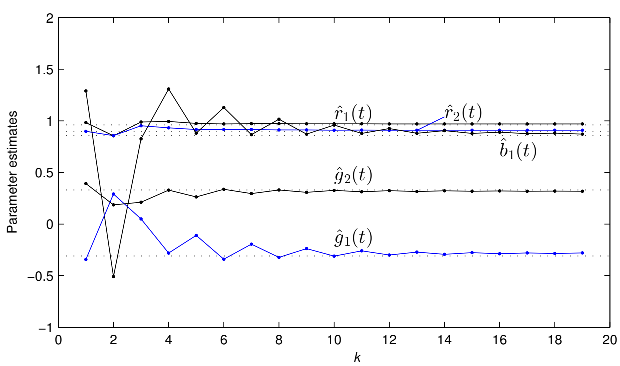

Fig. 5. The ML-LSI estimate $\hat{\vartheta}$ versus k of Example 2.

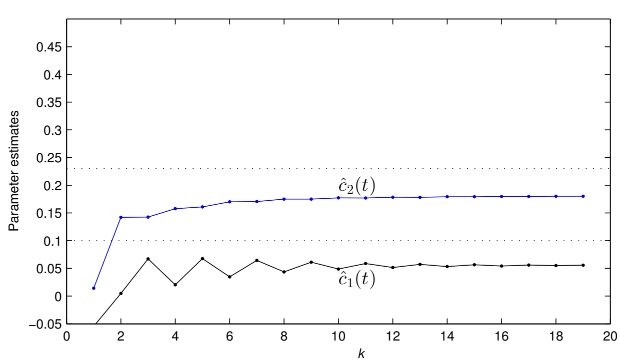

Fig. 6. The ML-LSI estimate $\hat{\boldsymbol{c}}$ versus k of Example 2.

2) Fig. 2 witnesses that the ML-LSI algorithm can produce more accurate estimates after four iterations under a lower noise level.
3) Figs. 3-4 and Figs. 5-6 declare that the ML-LSI parameter estimates are very close to their true values with k increasing.
4) Figs. 2-4 and Figs. 5-6 demonstrate that the parameter estimation errors of the ML-LSI method decrease with the iteration k increases.
5) Tables 1-2 and Figs. 1-2 show that the parameter estimates of the ML-LSI algorithm is a little better than those of the RELS algorithm under $\sigma^2 = 0.10^2$.

5. CONCLUSIONS

This paper presents an ML-LSI algorithm for identifying the closed-loop input nonlinear equation-error system. By applying the key term separation technique, the identification difficulty of the parameter interaction between the linear and nonlinear blocks is solved. By means of the hierarchical identification principle, the original system is transformed into two sub-identification models for reduced computational complexity. The iterative identification theory is utilized to generate highly accurate parameter estimation by updating the estimate with a fixed data batch under a finite length. The simulation results verify theoretical findings. In the future, we will study the identification problems of time delay nonlinear system. The proposed algorithms in this paper can joint other identification methods [64-69] to study new parameter estimation approaches of linear and nonlinear systems [70-73] and can be applied to other literatures [74-79] such as information processing and engineering application systems.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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