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Global Output Feedback Stabilization for Switched Nonlinear Systems Under Arbitrary Switching

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Abstract: Global stabilization of a class of switched nonlinear systems with unknown arbitrarily switched output function is achieved via output feedback. A couple of improved high-gain observer and output feedback control law are proposed by using dual high-gain scaling technique. In the presence of non-differentiable output function, a common dynamic output feedback controller is established, where a generalized Lyapunov inequality is introduced with explicit calculation. By constructinga single Lyapunov function for all subsystems, it shows the closed-loop switched systems are globally asymptotically stable under arbitrary switching. Ultimately, two examples are given to illustrate the effectiveness of our control scheme.

Keywords: Common Lyapunov function, output feedback stabilization, switched nonlinear systems, unknown output function.

1. INTRODUCTION

In the past decades, switched nonlinear systems have been studied deeply due to their wide applications in physical factories in the real world, such as mobile robots, chemical processes, automotive engineering and many others [1-4]. In particular, it has been shown in [1] that the existence of a common Lyapunov function is a sufficient and necessary condition for the stability of a system with arbitrary switching. For this reason, a lot of control design methods have been proposed based on constructing a common Lyapunov function. For example, for a class of switched nonlinear systems in triangular-structure form, a common Lyapunov function that for closed-loop system stability under arbitrary switching was constructed via backstepping technique in [5,6]. Similar results were extended to switched nonlinear feedforward systems with the help of integrator forwarding technique [7]. Also, triangular-structure systems under arbitrary switching were considered in [8-12], especially, [11] and [12] addressed the adaptive fuzzy control problem and presented observer-based output feedback schemes for uncertain nonlinear systems with unknown nonlinearities. In addition, average dwell-time method was extensively used by introducing multiple Lyapunov functions, see, e.g., [13-18]. Some other interesting results have also been reported in [19-25] and the references therein.

On the other hand, output-feedback control problem for switched nonlinear systems has been extensively studied since only part of states, as system output, can be measured in many practical systems. As far as the author knows, most of the existing results are about the situation where the output function is completely known (with an ideal case $y = x_1$), see, e.g., [7,8,14,16,17,23] and so on. However, in practical applications, output-constrained nonlinear control systems are often encountered [26-30]. In the presence of uncertain output function, only a few results have been reported recently in [31-35]. To be specific, [31] concentrated on a class of switched nonlinear time-delay systems under arbitrary switching and [32,33] proposed respectively average dwell time control methods for switched nonlinear systems with/without time-delay; in [34] and [35], the output feedback control for switched nonlinear systems was addressed with their output functions unknown but differentiable. It is worth pointing out that the results [31-35] have made the assumption that either the output function is differentiable or the output parameter belongs to a small allowable interval.

In this paper, global output feedback asymptotic stabilization for switched nonlinear systems with arbitrary switching is achieved, where a new pair of high-gain observer and output feedback controller are constructed. Less restrictive output parameter is allowed, and our main contributions lie in

1) Different from [7,8,14,23], an output feedback control scheme is proposed under the condition that the

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system output function is uncertain. Compared with [31-35], the constraint on output function is less conservative, that is, the output function in this paper is allowed to be non-differentiable and vary in a wider range. Specifically, one removes the restrictive requirement that the output parameter only varies in a small admissible interval as assumed in [31].

2) An extension of Lyapunov inequality with an unknown time-varying parameter is introduced with the solution being calculated explicitly. With this inequality, the output feedback stabilization problem for a class of switched nonlinear systems with non-differentiable switched output function is successfully handled by combining with the dual-gain domination technique [26,28].

3) Compared with the average-dwell-time based results [32,33], this paper presents a common Lyapunov function method which can guarantee the resulting closed-loop system is globally asymptotically stable under arbitrary switching. In particular, a method to construct a common Lyapunov function is presented when there is any switching in the unknown output function.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Problem formulation

Consider a class of switched nonlinear systems with unknown switched output function

$$\begin{cases} \dot{x}_1 = x_2 + f_{1\sigma(t)}(t,x), \\ \dot{x}_2 = x_3 + f_{2\sigma(t)}(t,x), \\ \quad \vdots \\ \dot{x}_n = u + f_{n\sigma(t)}(t,x), \\ y = \vartheta_{\sigma(t)}(t)x_1, \end{cases} \tag{1}$$

where $x = [x_1,\ x_2,\ \ldots,\ x_n]^T \in \mathcal{R}^n$, $u \in \mathcal{R}$ and $y \in \mathcal{R}$ denote the state vectors, system input and measurable output, respectively; $\sigma : [0, +\infty) \to \mathcal{P} = \{1, 2, \ldots, m\}$ is the piecewise constant switching signal, and $\sigma(t) = p\ (p \in \mathcal{P})$ implies that the subsystem p has been activated. $\vartheta_{\sigma(t)} > 0$ is a bounded switched function with ϑ_p being the measurement sensitivity of the subsystem p which maybe non-differentiable. Assume that the system states don't jump at any switching instants.

The control objective of this paper is to globally asymptotically stabilize system (1) under arbitrary switching via designing a dynamic output feedback controller. To this end, the following assumptions are required.

Assumption 1: For arbitrary switching $\sigma(t)$, there exist two known positive constants $\underline{\vartheta}$ and $\overline{\vartheta}$ such that

$$0 < \underline{\vartheta} \le \vartheta_{\sigma(t)} \le \overline{\vartheta}.$$

Assumption 2: There exist a set of known constants $c_p > 0$, $p \in \mathcal{P}$ such that

$$|f_{ip}(t,x)| \le c_p(|x_1| + |x_2| + \cdots + |x_i|),\quad p \in \mathcal{P},\ i = 1,\ 2,\ \cdots,\ n. \tag{2}$$

Remark 1: A lot of work has been done in [31-35] to address the influence of unknown measurement sensitivity. Compared with the existing work, Assumption 1 is less conservative by noting the non-differentiable switching parameter. Assumption 2 is a switching version of the commonly used linear growth condition [5-7].

Remark 2: The model (1) with Assumptions 1 and 2 contains the following switched nonlinear system

$$\begin{aligned} &\dot{x}_i = \delta_i x_{i+1} + f_i(\cdot),\ i = 1,\ \ldots,\ n-1, \\ &\dot{x}_n = \delta_n u + f_n(\cdot), \\ &y = \vartheta_{\sigma(t)}(t)x_1, \end{aligned}$$

with unknown control coefficients δ_i's satisfying

$$0 < \delta_0 \le \delta_i \le \delta_\infty,\ i = 1,\ \ldots,\ n,$$

for two positive constants δ_0 and δ_∞. In fact, such system can be changed into system (1) satisfying Assumptions 1 and 2 by introducing a coordinate change $\chi_i = \frac{x_i}{\delta_i \cdots \delta_n}$, $i = 1,\ \ldots,\ n$.

2.2. Preliminaries

Definition 1 (Common Lyapunov function) [1]**:** For a switched system, if there exists a positive definite continuously differentiable function $V : \mathcal{R}^n \to \mathcal{R}$ and a positive definite continuous function $W : \mathcal{R}^n \to \mathcal{R}$ such that $\frac{\partial V}{\partial x}\dot{x} \le -W(x)$, then we will say that V is a common Lyapunov function.

In the following, an extension of Lyapunov inequality is introduced which plays a central role in this paper.

Lemma 1: For a constant $\alpha > 0$, there exist a set of positive constants ℓ_i, $i = 1, 2, \ldots, n$ and a constant matrix $P = P^T > 0$ to satisfy

$$A_\vartheta^T P + P A_\vartheta \le -\alpha I, \tag{3}$$

where $I \in \mathcal{R}^{n\times n}$ is an identity matrix, A_ϑ is defined as

$$A_\vartheta = \begin{bmatrix} -\vartheta(t)\ell_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\vartheta(t)\ell_{n-1} & 0 & \cdots & 1 \\ -\vartheta(t)\ell_n & 0 & \cdots & 0 \end{bmatrix} \in \mathcal{R}^{n\times n},$$

with an unknown parameter $\vartheta(t)$ being bounded below by $\underline{\vartheta} > 0$ and ℓ_i's being design freedoms.

Proof: See Appendix A. □

3. MAIN RESULTS

3.1. Design of high-gain observer-controller

With the help of Lemma 1, we construct an observer-based controller as

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + L_1\ell_1(y-\hat{x}_1), \\ \dot{\hat{x}}_2 = \hat{x}_3 + L_1^2\ell_2(y-\hat{x}_1), \\ \quad\vdots \\ \dot{\hat{x}}_n = u + L_1^n\ell_n(y-\hat{x}_1), \\ u = -\sum_{i=1}^{n}(L_2L_1)^{n+1-i}k_i\hat{x}_i, \end{cases} \tag{4}$$

where $L_1 \geq 1$ and $L_2 \geq 1$ are scaling gains determined in (17); the control gains k_i's are the coefficients of the Hurwitz polynomial $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$; the observer gains ℓ_i's are explicitly calculated by Lemma 1 with the following algorithm

Step 1: Initialize $\delta > 0$, $g_1 > 0$.

Step 2: Determine parameters ρ, a_i, g_i, $\bar{a}$ by

$$\begin{cases} \rho = 0.5\delta^2, \\ a_i = \sum_{j=i}^{n}\delta\rho^{(n-j)},\ i = 2,\ 3,\ \cdots,\ n, \\ g_i = a_i g_{i-1},\ i = 2,\ 3,\ \cdots,\ n, \\ \bar{a} = \begin{cases} 0.75,\ n = 2, \\ \max\{a_3,\ \cdots,\ a_n\},\ n \geq 3. \end{cases} \end{cases} \tag{5}$$

Step 3: Check whether the parameters ρ, $\bar{a}$, g_1 meet the following constraints

$$\begin{cases} \rho \leq 1, \\ \bar{a} \leq \min\left\{\frac{0.75}{n-1},\ \frac{0.25}{(n-i)\rho+i-1}\ \middle|\ i = 2,\ \ldots,\ n-1\right\}, \\ g_1 \geq \frac{1}{2\underline{\vartheta}}\left[0.75\delta + \frac{a_2^2}{\bar{a}}(n-1)\delta\rho + 2\rho a_2\right]. \end{cases} \tag{6}$$

Step 4: If it is true in Step 3, the parameters $\bar{\ell}C^T = \mathcal{T}GC^T\mathcal{T}^{-1}$ with

$$\begin{cases} C = [1,\ 0,\ \cdots,\ 0]^T, \\ \mathcal{T} = diag\{1,\ \rho,\ \rho^{1+2},\ \cdots,\ \rho^{1+2+\cdots+(n-1)}\}, \\ \bar{\ell} = [\ell_1,\ \cdots,\ \ell_n]^T, \\ G = [g_1,\ \cdots, g_n]^T. \end{cases} \tag{7}$$

Remark 3: In essence, the control scheme (4) is a switching system by noting the switched output function $y = \vartheta_{\sigma(t)}(t)x_1$. However, we do not directly use any switching information in the controller design and its implementation. As a result, our controller is suitable for arbitrary switching and a common Lyapunov function can be found as shown in Subsection 3.2.

3.2. Stability analysis

Based on the above output feedback controller, we can draw a conclusion as follows:

Theorem 1: Under Assumptions 1 and 2, the closed-loop system comprised of (1) and (4) with arbitrary switching is globally asymptotically stable.

Proof: First, introduce a dual-scaling change of the form

$$\varepsilon_i = \frac{x_i - \hat{x}_i}{L_1^{i-1}} := \frac{e_i}{L_1^{i-1}},\ z_i = \frac{x_i}{L_1^{i-1}}, \\ \hat{z}_i = \frac{\hat{x}_i}{(L_2L_1)^{i-1}},\ i = 1,\ \ldots,\ n. \tag{8}$$

From (4), (5) and $y - \hat{x}_1 = \vartheta_\sigma e_1 + (\vartheta_\sigma - 1)\hat{x}_1$, we get

$$\dot{\varepsilon} = L_1A_\vartheta\varepsilon + L_1\bar{\ell}(1-\vartheta_\sigma)\hat{z}_1 + \mathcal{F}, \\ \dot{\hat{z}} = L_2L_1B\hat{z} + \vartheta_\sigma L_1\mathcal{M}\varepsilon_1 + (\vartheta_\sigma - 1)L_1\mathcal{M}\hat{z}_1, \tag{9}$$

where $\varepsilon = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n]^T$, $\hat{z} = [\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_n]^T$, $\bar{\ell} = [\ell_1, \ell_2, \ldots, \ell_n]^T$, $\mathcal{F} = [f_{1\sigma}, \frac{f_{2\sigma}}{L_1}, \ldots, \frac{f_{n\sigma}}{L_1^{n-1}}]^T$, $\mathcal{M} = [\ell_1, \frac{\ell_2}{L_2}, \ldots, \frac{\ell_n}{L_2^{n-1}}]^T$, A_ϑ is given in Lemma 1, and matrix B is defined by

$$B = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -k_1 & -k_2 & \cdots & -k_n \end{bmatrix} \in \mathcal{R}^{n\times n}.$$

Note that B is a Hurwitz matrix. Thus, there exists a matrix $Q = Q^T > 0$ to satisfy (see [36])

$$B^TQ + QB \leq -\beta I, \tag{10}$$

for a constant $\beta > 0$. Then, by choosing a common Lyapunov function candidate

$$V = \varepsilon^TP\varepsilon + \hat{z}^TQ\hat{z}, \tag{11}$$

with $P > 0$ satisfying (3), we take its derivative along system (9) and arrive at

$$\dot{V} \leq -\alpha L_1\|\varepsilon\|^2 - \beta L_2L_1\|\hat{z}\|^2 \\ + 2\varepsilon^TPL_1\bar{\ell}(1-\vartheta_\sigma)\hat{z}_1 + 2\varepsilon^TP\mathcal{F} \\ + 2\vartheta_\sigma L_1\hat{z}^TQ\mathcal{M}\varepsilon_1 + 2(\vartheta_\sigma - 1)L_1\hat{z}^TQ\mathcal{M}\hat{z}_1. \tag{12}$$

In what follows, we will estimate the redundant terms in (9). First, from Assumption 2, one has

$$\|\mathcal{F}\| \leq \sum_{i=1}^{n}\left|\frac{f_{i\sigma}}{L_1^{i-1}}\right|$$

$$\leq \sum_{i=1}^{n}\sum_{j=1}^{i}\bar{c}\left(\frac{|e_j+\hat{x}_j|}{L_1^{i-1}}\right)$$
$$\leq \sum_{i=1}^{n}\bar{c}(n+1-i)(|\varepsilon_i|+L_2^{i-1}|\hat{z}_i|), \tag{13}$$

where $\bar{c}=\max\{c_1, c_2, \ldots, c_m\}$. Further, it follows from the complete square formula that

$$2\varepsilon^T PL_1\bar{\ell}(1-\vartheta_\sigma)\hat{z}_1+2\varepsilon^T P\mathcal{F}$$
$$\leq 0.5\alpha L_1\|\varepsilon\|^2+\bar{c}_0L_1\hat{z}_1^2+\bar{c}_1\|\varepsilon\|^2+\bar{c}_2\|\hat{z}\|^2, \tag{14}$$

where $\bar{c}_i$'s are known positive constants.

Since $L_2\geq 1$, we arrive at

$$\|\mathcal{M}\|\leq\|\bar{\ell}\|,$$

which together with Assumption 1 leads to

$$2\vartheta_\sigma L_1\hat{z}^T Q\mathcal{M}\varepsilon_1+2(\vartheta_\sigma-1)L_1\hat{z}^T Q\mathcal{M}\hat{z}_1$$
$$\leq 2\vartheta_\sigma L_1\|\hat{z}\|\|Q\|\|\mathcal{M}\||\varepsilon_1|$$
$$+2|\vartheta_\sigma-1|L_1\|\hat{z}\|\|Q\|\|\mathcal{M}\||\hat{z}_1|$$
$$\leq m_1L_1\varepsilon_1^2+m_2L_1\|\hat{z}\|^2, \tag{15}$$

where $m_1>0$ and $m_2>0$ are suitable constants independent of L_1 and L_2.

Plugging (14) and (15) into (12) yields

$$\dot{V}\leq -[(0.5\alpha-m_1)L_1-\bar{c}_1]\,\|\varepsilon\|^2$$
$$-[(\beta L_2-\bar{c}_0-m_2)L_1-\bar{c}_2]\,\|\hat{z}\|^2. \tag{16}$$

In Lemma 1, we pick a sufficiently large constant α to satisfy $0.5\alpha>m_1$. Then, the design constants L_1 and L_2 can always be found such that

$$\begin{cases}(0.5\alpha-m_1)L_1-\bar{c}_1\geq\gamma_1,\\ (\beta L_2-\bar{c}_0-m_2)L_1-\bar{c}_2\geq\gamma_2,\end{cases} \tag{17}$$

for two suitable constants $\gamma_1>0$ and $\gamma_2>0$.

As a consequence, it gets from (16) and (17) that

$$\dot{V}\leq-\gamma_1\|\varepsilon\|^2-\gamma_2\|\hat{z}\|^2. \tag{18}$$

According to Lyapunov stability theory for switched systems [1], it is easy to deduce the asymptotic stability of the closed-loop system under arbitrary switching.

This completes the proof of Theorem 1. □

4. EXAMPLES AND SIMULATIONS

In this section, two examples are given to illustrate the effectiveness of the control scheme in Theorem 1.

Example 1: Consider a switched nonlinear system

$$\dot{x}_1=x_2+f_{1\sigma}(t,x_1),$$
$$\dot{x}_2=x_3+f_{2\sigma}(t,x_1,x_2),$$
$$\dot{x}_3=u+f_{3\sigma}(t,x_1,x_2,x_3),$$
$$y=\vartheta_\sigma(t)x_1, \tag{19}$$

where $\sigma_i(t)\in\{1,2\}$, $i=1,\ 2$, $f_{11}=-\sin(x_1)$, $f_{12}=\frac{3x_1}{1+x_1^2}$, $f_{21}=2\cos(t)\ln(1+x_2^2)$, $f_{22}=\frac{3x_1+5x_2}{1+x_1^2+x_2^2}$, $f_{31}=\sin(x_3)$, $f_{32}=\frac{x_1+x_2+x_3}{1+x_1^2+x_2^2+x_3^2}$, $\vartheta_1(t)=1+|\sin(2t)|$, and $\vartheta_2(t)=1+|\cos(5t)|$. Obviously, Assumption 2 is fulfilled with $c_1=2$, $c_2=5$, i.e.,

$$|f_{11}|=|-\sin(x_1)|\leq|x_1|,$$
$$|f_{21}|=|2\cos(t)\ln(1+x_2^2)|\leq 2(|x_1|+|x_2|),$$
$$|f_{31}|=|\sin(x_3)|\leq|x_3|,$$
$$|f_{12}|=|\frac{3x_1}{1+x_1^2}|\leq 3|x_1|,$$
$$|f_{22}|=|\frac{3x_1+5x_2}{1+x_1^2+x_2^2}|\leq 5(|x_1|+|x_2|),$$
$$|f_{32}|=|\frac{x_1+x_2+x_3}{1+x_1^2+x_2^2+x_3^2}|\leq(|x_1|+|x_2|+|x_3|).$$

By Theorem 1, we can design an observer-controller as

$$\dot{\hat{x}}_1=\hat{x}_2+L_1\ell_1(y-\hat{x}_1),$$
$$\dot{\hat{x}}_2=\hat{x}_3+L_1^2\ell_2(y-\hat{x}_1),$$
$$\dot{\hat{x}}_3=u+L_1^3\ell_3(y-\hat{x}_1), \tag{20}$$
$$u=-[(L_2L_1)^3k_1\hat{x}_1+(L_2L_1)^2k_2\hat{x}_2+L_2L_1k_3\hat{x}_3],$$

where the design parameters are $k_1=0.2$, $k_2=0.6$, $k_3=1$, $\ell_1=1$, $\ell_2=0.7$, $\ell_3=0.3$, $L_1=10$, and $L_2=16$.

Notably, the control scheme (20) is applicable to any switching signal by Theorem 1. Without loss of generality, we choose the switching signals $\sigma_i(t)$ as given in Figs. 1 and 2. Letting the initial condition $x_1(0)=4$, $x_2(0)=-1$, $x_3(0)=-5$, $\hat{x}_1(0)=0$, $\hat{x}_2(0)=0$, $\hat{x}_3(0)=0$, and performing the simulation, we get Figs. 3 and 4, which shows the

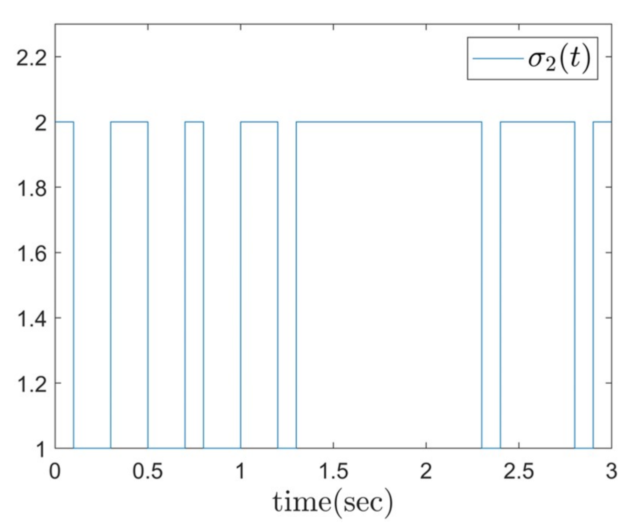

Fig. 1. The switching signal σ_1.

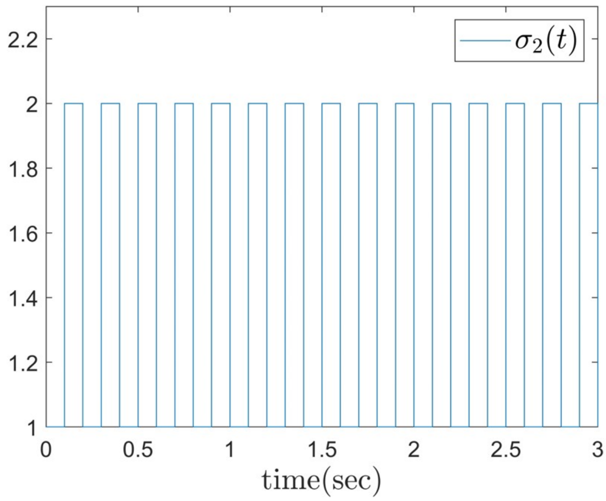

Fig. 2. The switching signal σ_2.

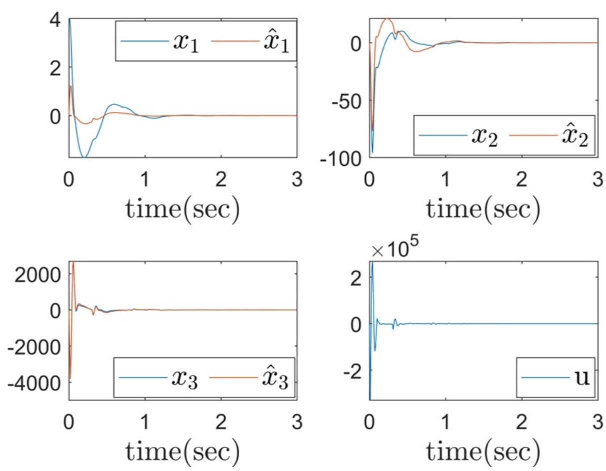

Fig. 3. The closed-loop response under switching signal σ_1.

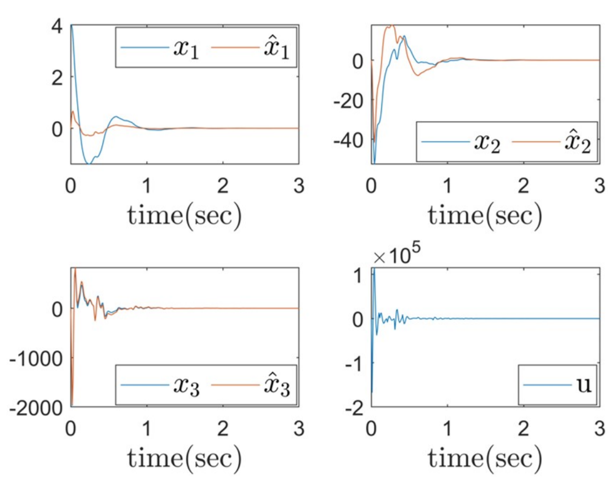

Fig. 4. The closed-loop response under switching signal σ_2.

resulting closed-loop system (19)-(20) is asymptotically stable.

Example 2: A practical example, continuous stirred tank reactor with the model borrowed from [38]

$$\begin{aligned}\dot{C}_A &= \frac{q_{\sigma(t)}}{V}(C_{Af\sigma(t)} - C_A) - a_0 \exp(-\frac{E}{RT})C_A,\\ \dot{T} &= \frac{q_{\sigma(t)}}{V}(T_{f\sigma(t)} - T) - a_1 \exp(-\frac{E}{RT})C_A\\ &\quad + a_2(T_c - T),\end{aligned} \tag{21}$$

where the physical meaning of the system parameters are the same to [38]. Under a coordinate transformation and smooth feedback in [38], the new system is

$$\begin{aligned}\dot{z}_1 &= g_1 z_2 + h_{\sigma(t)}(z_1),\\ \dot{z}_2 &= g_2 u,\\ y &= \vartheta(t) z_1,\end{aligned} \tag{22}$$

where $\sigma(t) \in \{1, 2\}$, g_i, $i = 1, 2$ are unknown bounded control coefficients, bounded by $g_i \in [0.9, 1.5]$, and $h_1(z_1) = 0.5 g_1 g_2 x_1$, $h_2(z_1) = 2 g_1 g_2 x_1$.

Defining the states $x_1 = \frac{1}{g_1 g_2} z_1$, $x_2 = \frac{1}{g_2} z_2$, $f_{\sigma(t)}(x_1) = \frac{1}{g_1 g_2} h_{\sigma(t)}(z_1)$, then system (22) can be written as the follow form.

$$\begin{aligned}\dot{x}_1 &= x_2 + f_{\sigma(t)}(x_1),\\ \dot{x}_2 &= u,\\ y &= g_1 g_2 \vartheta(t) x_1,\end{aligned} \tag{23}$$

where $f_1 = 0.5x_1$, $f_2 = 2x_1$. It is clear that Assumption 2 is fulfilled with $c_1 = 0.5$, $c_2 = 2$. Then, the observer and controller can be constructed as

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + L_1 \ell_1 (y - \hat{x}_1),\\ \dot{\hat{x}}_2 &= u + L_1^2 \ell_2 (y - \hat{x}_1),\\ u &= -[(L_2 L_1)^2 k_1 \hat{x}_1 + L_2 L_1 k_2 \hat{x}_2],\end{aligned} \tag{24}$$

with the design parameters being chosen as $k_1 = 0.75$, $k_2 = 0.35$, $\ell_1 = 1$, $\ell_2 = 0.75$, $L_1 = 3.5$, and $L_2 = 8$.

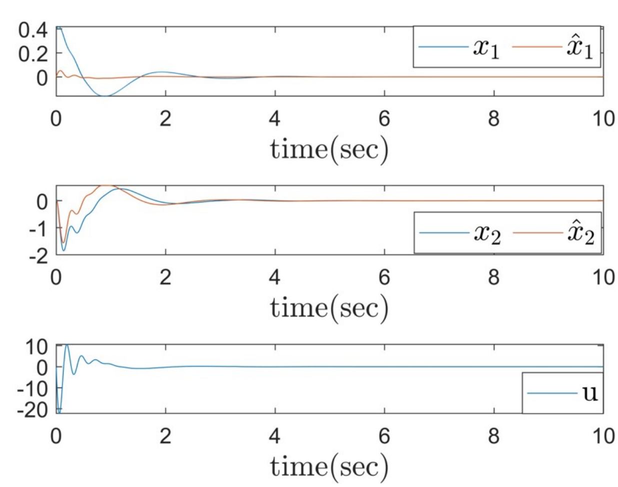

Fig. 5. The closed-loop response.

To perform the simulation, we let the initial condition $x_1(0) = 1.5$, $x_2(0) = 6.5$, $\hat{x}_1(0) = 2$, $\hat{x}_2(0) = -10$, $\vartheta(t) = 1+|0.42\sin(t)|$, and the switching time between the switched system on and off is 0.001 s. The simulation results are shown in Fig. 5.

5. CONCLUSION

In this paper, an output feedback control method is proposed to achieve global stabilization of a class of switched nonlinear systems subject to non-differentiable output parameters. By constructing a common Lyapunov function in the spirit of dual domination idea, it shows that the proposed control scheme is applicable to arbitrary switching signal and unknown switching output coefficient. In particular, we introduce a generalized Lyapunov inequality and give an algorithm of its solution in Section 3.

Moreover, it seems that the linear growth condition is restrictive, and how to further relax the assumption is the direction of our future efforts. Also, the combination of switched nonlinear systems and constraint control of nonlinear systems should be better studied in the future.

APPENDIX A: PROOF OF LEMMA 1

First, define a set of positive constants by $(n \geq 2)$

$$\rho = 0.5\delta^2,\ a_i = \sum_{j=i}^{n} \delta\rho^{n-j},\ g_i = a_i g_{i-1},\quad i = 2,\ \ldots,\ n, \tag{A.1}$$

with the constants $\delta > 0$, $g_1 > 0$ and

$$\bar{a} = \begin{cases} 0.75, & \text{when} n = 2, \\ \max\{a_3,\ \ldots,\ a_n\}, & \text{when} n \geq 3, \end{cases} \tag{A.2}$$

satisfying

$$\begin{cases} \rho \leq 1, \\ \bar{a} \leq \min\left\{ \dfrac{0.75}{n-1},\ \dfrac{0.25}{(n-i)\rho + i - 1} \right| \\ \qquad\qquad i = 2,\ \ldots,\ n-1 \Big\}, \\ g_1 \geq 0.5\underline{\vartheta}^{-1}\left[0.75\delta + \dfrac{a_2^2}{\bar{a}}(n-1)\delta\rho + 2\rho a_2\right]. \end{cases} \tag{A.3}$$

Next, let $C = [1,\ 0,\ \ldots,\ 0]^T$, $G = [g_1,\ \ldots,\ g_n]^T$, $\bar{\ell} = [\ell_1,\ \ldots,\ \ell_n]^T$ such that

$$\bar{\ell}C^T = \mathcal{T}GC^T\mathcal{T}^{-1},\quad \mathcal{T} = diag\{1,\ \rho,\ \rho^{1+2},\ \ldots,\ \rho^{1+2+\cdots+n-1}\}. \tag{A.4}$$

As done in [28], we consider a system $\dot{\omega} = \mathcal{T}^{-1}A_{\vartheta}\mathcal{T}\omega = \mathcal{A}\omega$ with $\omega = [\omega_1,\ \ldots,\ \omega_n]^T$, and define a coordinate change

$$\Omega_1 = \omega_1,\ \Omega_i = \omega_i - a_i\omega_{i-1},\ i = 2,\ \ldots,\ n. \tag{A.5}$$

By referring to the proof in [28,37], we immediately get Lemma 1. □

CONFLICT OF INTEREST

The authors declare that there is no competing financial interest or personal relationship that could have appeared to influence the work reported in this paper.

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